Game-Theoretic Analysis of Trade-In Services in Closed-Loop Supply Chains[1]

Yuting Quan[2], Jiangtao Hong[3], Jingpu Song[4], Mingming Leng[5]

January, April, June, and July 2021

Accepted July 2021

To appear in *Transportation Research Part E*

[1]The authors are grateful to the Editor (Professor Jason Choi) and three anonymous reviewers for their insightful comments that helped improve this paper.

[2]School of Business, Shanghai University of International Business and Economics, Shanghai, 201620, China. (Email: kallirainer@163.com)

[3]School of Business, Shanghai University of International Business and Economics, Shanghai, 201620, China. (Email: hongjiangtao@suibe.edu.cn)

[4]School of Business, Shanghai University of International Business and Economics, Shanghai, 201620, China. (Email: songjingpu@suibe.edu.cn)

[5]**Corresponding author**. Faculty of Business, Lingnan University, Hong Kong. (Email: mmleng@ln.edu.hk; Tel: +852 2616-8104; Fax: +852 2892-2442)

Abstract

We develop a two-period model to investigate the trade-in service in a closed-loop supply chain consisting of a manufacturer and a retailer. In the supply chain, we consider two options for the trade-in service. The first one is Scenario M, in which the manufacturer collects the used products from trade-in customers by herself; and the second one is Scenario R, in which the manufacturer outsources the trade-in service to the retailer and obtains the used products from the retailer at a buy-back price. Accordingly, the firm in charge of the service needs to determine a rebate rate. We show that both firms prefer to operate the trade-in service by themselves in most cases, and we derive the condition for each firm to prefer a scenario and also the condition under which the supply chain is better off from a scenario. Compared to Scenario M, trade-in customers can enjoy lower prices in two periods in Scenario R, although the second-period sales price for new customers is higher. We also perform a numerical study to investigate the impacts of major parameters on the optimal prices, rebate rate, profits, and the conditions for each firm's and the supply chain's scenario preferences. The difference between trade-in prices in the two scenarios is larger when trade-in customers are more sensitive to the price, whereas the difference becomes smaller when the manufacturer obtains a higher net gain from handling returned products.

Key words: Trade-in service, closed-loop supply chain, rebate, game theory.

1 Introduction

The trade-in service has been widely provided in business practices in which customers are encouraged to return their used old products for new ones at a discounted price and firms can not only realize higher demand but also achieve a cost saving (Zhang and Zhang 2018 and Tang, Ma, and Dai 2020). In the markets of durable products such as automobiles, electronics, and household electrical appliances, the trade-in transactions considerably increase (Ray, Boyaci, and Aras 2005, Li, Fong, and Xu 2011, Agrawal, Ferguson, and Souza 2016, and Zhang and Zhang 2018). In the automobile industry, around 57% of new cars are sold through trade-ins (Zhu, Chen, and Dasgupta 2008, and Tang et al. 2020), and it is reported that Volkswagen reduces its production cost by 70% via the recovery of engines and parts from used cars (Hu, Ma, and Sheu 2019). Analogously, in some highly saturated markets such as those for electric heaters and refrigerators, 70-80% of annual sales in the United States are replacement purchases (Ray et al. 2005 and Tang et al. 2020). More trade-in examples can be found in the industries for the products ranging from cell phones and fashion apparel (Choi, Chow, Lee, and Shen 2018) to gold jewelry and musical instruments (Srivastava and Chakravarti 2011). In the business-to-business (B2B) market, many original equipment manufacturers (OEMs), such as IT equipment manufacturers and medical equipment suppliers, also use trade-in rebates to entice existing customers to purchase new ones and serve/maintain customers (Ackere and Reyniers 1995 and Agrawal et al. 2016).

The COVID-19 pandemic has significantly impacted not only our human life but also the markets, industries, economy, and society in the world. Many problems regarding manufacturing, supply chain, and resource shortage have appeared and attracted great attention. Medical supplies like oxygen concentrators and masks with various specifications are extensively short. In the post COVID-19 pandemic era, it may be no longer realistic to continuously consider globalization and economic growth as drivers of sustainable development (Naidoo and Fisher 2020). The adoption of circular economy has been touted to be a viable solution, as Ibn-Mohammed et al. (2021) argued. In the medical and healthcare industry, the trade-in service also serves as a solution for medical equipment manufacturers who make products like ventilators, oxygen concentrators, and computed tomography (CT) machines to serve or maintain customers.

In the trade-in service, the customers who desire to return their used products should evaluate the price they need to pay, which is related to the sales price of a new product and the rebate they receive (Genc and De Giovanni 2018). In practice, manufacturers (e.g., Canon and IBM) operate the trade-in service themselves or outsource the service to retailers (e.g., Ford), centralized organizations (e.g., Apple), and third-party platforms (e.g., Amazon, JD, and Suning). The Canon Marketing (Malaysia) Sdn. Bhd. offers a trade-in promotion that allows customers to exchange their used Canon devices for trade-in rebates. IBM runs a trade-in program, in which customers who return specified IBM or non-IBM machines can get a trade-in credit for buying a new IBM machine. Ford's retailer in China offers a trade-in program, in which customers can trade in their on-hand old Ford cars or other brands of vehicles to the Ford dealers for a new vehicle. Amazon, as a third-party platform, offers a trade-in program that encourages customers to trade in their used products for an Amazon gift card to exchange thousands of products such as video games, books, electronics, etc. (Tang et al. 2020). The largest two third-party retail platforms of household electrical appliances in China, JD.COM and Suning, have been actively promoting trade-in projects for household electrical appliances.

The trade-in service can both promote repurchases and play a reverse channel role in a closed-loop supply chain (CLSC). Therefore, the trade-in service is actually at the intersection point of marketing, customer behavior, and supply chain (or operations) management, because of its inherent properties. The CLSC literature have discussed such trade-in service but mainly from the perspectives of reverse channels and remanufacturing (e.g., Genc and De Giovanni 2017, Huang et al. 2019, and Tang et al. 2020), whereas many other papers analyzed trade-in services with customer choice behavior (e.g., Rao et al. 2009 , Li et al. 2011, Xiao 2017, and Xiao and Zhou 2019). Different from existing publications, in this paper we consider both direct sales and trade-in product flows and accordingly develop a two-period CLSC game with a manufacturer and a retailer. Because the manufacturer may operate the trade-in service by herself (called, Scenario M) or may outsource the service to the retailer (called, Scenario R), we examine the two scenarios in which the customers can trade in their products bought in period 1 for new ones in period 2. In Scenarios M and R, the manufacturer and the retailer make the rebate rate decision in addition to the wholesale and retail prices, respectively. Naturally, in each period, the manufacturer and the retailer make their decisions as a “leader” and a “follower,” respectively. Accordingly, we develop a leader-follower game and find the corresponding Stackelberg equilibrium in each scenario.

In this paper, dependent demands in two periods (Sibdari and Pyke 2010) are involved into our model to describe market fundamentals in different periods in the presence of the trade-in service. The following three novelties can distinguish our model from previous two-period CLSC models. Firstly, in our model, we consider both the product returns and direct sales under the trade-in service. When customers trade in their old products for new ones, they not only return their old products for rebates but also use the rebates to buy new products. Most of previous CLSC models do not consider the impact of trade-in service on direct sales but mainly focus on return policies, rebate mechanisms (e.g., Genc and De Giovanni 2018) and environmental benefits (e.g., Mondal and Giri 2020) to the supply chain. The literature that discuss trade-in services from the marketing and customer behavior perspectives (e.g. Agrawal et al. 2016, Zhang and Zhang 2018, and Tang et al. 2020) mainly investigate its repurchase promotion instead of the CLSC structure. Differently, our analysis focuses on the CLSC optimal trade-in strategies and supply chain structure, considering the impact of trade-in service on direct sales.

Secondly, we develop a dependent demand function for the two-period problem in which only those products sold in period 1 can be returned via trade-in service. The return of an “old” product under the trade-in scheme always results in the sale of a new product in period 2. Therefore, the first-period realized demand is the demand base for the trade-in service in period 2, and the trade-in quantity should be included in the second-period demand function. Note that those customers who buy in period 1 do not purchase a new product as a new customer but decide on whether to trade in their used products or not in period 2.

Thirdly, the trade-in rebate rate in our model is a decision variable as in practice, which differs from majority of extant CLSC models that assumed an exogeneous collection rate (e.g., Chuang et al. 2014), a fixed rebate (e.g., Miao et al. 2017 and Cao et al. 2020), or an exogeneous rebate rate (e.g., Genc and De Giovanni 2018). One can learn from Genc and De Giovanni (2018) that there are two rebate policies in practice: the first is one with a fixed rebate (i.e., a direct rebate) and the second is one with a variable rebate (i.e., a price-based rebate with a rebate rate). They compared the two policies with some examples. In addition to those examples by Genc and De Giovanni (2018), we find that Bevmarks (a mattress manufacturer), and Pantone (a color tool seller) use the first rebate strategy, whereas some firms choose the second strategy. For instance, the Navionic offers the price-based trade-in rebates for its products with the rebate rates of up to 50%; see `https://www.navionics.com/fin/lp/rebate`. Samsung provides new smart octopus customers in Hong Kong with up to 50% spending rebate, see `https://www.samsung.com/hk_en/samsungpay/promotion/`.

The remainder of this paper is organized as follows. In Section 2, we review the relevant literature to show the originality of our research issues. In Section 3, we describe two scenarios for our study and develop the profit functions for the manufacturer and the retailer in our trade-in CLSC model. In Section 4 we analyze the leader-follower game model, derive the Stackelberg equilibrium, and expose managerial implications. Section 5 provides a numerical study and sensitivity analysis. This paper ends with a summary of major findings in Section 6. The proofs of all propositions and corollaries are presented in online Appendix A.

2 Literature Review

This paper is mainly relevant to three streams of literature: (a) the models concerning trade-in issues from the economics, marketing, and operations management (OM) perspectives; (b) the models regarding the remanufacturing and reverse logistics; (c) the models about the CLSC mechanism, especially in a two-period game setting. Our research activities are expected to bridge the gap among these distinct streams by investigating the trade-in service in a two-period CLSC that considers the dependent demand in two periods and treat the trade-in rebate as a decision variable.

The stream of literature on remanufacturing and closed-loop supply chain rapidly grows. Guide and Van Wassenhove (2009) provided a comprehensive reviews of the literature. Shekarian (2020) also conducted a literature review about the factors influencing CLSC models. Early CLSC papers mainly focused on the models regarding remanufacturing process. Majumder and Groenevelt (2001) developed a two-period game model of remanufacturing in the presence of competition (i.e., an OEM and a local manufacturer). Savaskan et al. (2004) developed a Stackelberg game model to study a manufacturer's choice of reverse channel structures for collecting used products from customers. Ferrer and Swaminathan (2006) discussed the remanufacturing in two-period and multiperiod scenarios for the monopoly and duopoly cases. Atasu et al. (2008) provided an alternative and complementary approach that can address demand-related issues and showed that remanufacturing can become an effective marketing strategy. The process of product collection and reverse channel structures were further discussed by the following researchers. Heese et al. (2005) constructed an one-period, buy-back model for durable products in a duopoly market with two manufacturers to examine the consequences of used product returns on firms, industry, and customers. Atasu et al. (2013) further extended Savaskan et al. (2004) by studying the manufacturer's choice of reverse channel structures under two alternative collection cost structures that exhibit economies and diseconomies of scale. Atasu and Souza (2013) studied the impact of product recovery on a monopoly firm's product quality choice, considering heterogeneous customers and two different recovery rate types (i.e., exogenous recovery rate and endogenous one as a decision variable). De Giovanni and Zaccour (2014) investigated a two-period CLSC game for three scenarios (i.e., a manufacturer exclusively manages the collection process; the manufacturer outsources the collection process to the retailer; the manufacturer outsources it to a third-service provider). De Giovanni et al. (2016) modeled a dynamic CLSC model in which both the manufacturer and the retailer invest in a product recovery program to increase the return rate in an infinite time horizon. Esenduran et al. (2017) developed a stylized one-period CLSC game between an OEM and a retailer with a take-back regulation.

Trade-in mechanism, which has been widely adopted in various industries, is related to a reverse channel structure from the CLSC perspective. It is actually regarded as a green/closed-loop supply chain practice and also a sales promotion strategy that can enhance trade-in customers' repeat purchases. Moreover, the relevant literature mainly address the trade-in issues from the perspective of marketing, and customer choice behavior rather than green supply chain operations.

From the market perspective, Ray et al. (2005) studied the trade-in strategies for durable and reproducible products, separating the trade-in customers from new customers. Three price schemes (i.e., uniform pricing with no trade-in rebate, age-independent and age-dependent price differentiations) were discussed and the optimal pricing decisions were obtained. Chen and Hsu (2015) developed analytic models involving a deterioration rate and a recovery cost to address the problem of when and how a durable product manufacturer should offer a trade-in rebate to collect and recover used products for a better price discrimination and a weaker competition from third-party remanufacturers. Agrawal et al. (2016) modeled a trade-in program of an IT equipment OEM to discuss whether the OEM should compete with a third-party remanufacturer by providing a trade-in service, making reproducible products, or both. Zhu et al. (2016) built a two-period, duopoly competition model to investigate whether adopting the trade-in service could raise the competitive advantage for a firm in a duopoly situation. Chen and Hsu (2017) studied market segmentation effect of trade-ins and certified pre-owned options for a durable product firm. Cao et al. (2019) examined trade-in strategies for a B2C platform with a dual-format (i.e., gift card or cash coupon) retailing model. Huang et al. (2019) investigated two different recycling approaches for trade-in rebates (i.e., fixed timing recycling and continuous period recycling) launched by a firm who sells both new and remanufactured items to the market. Feng et al. (2020) explored the optimal price and quality with the coexistence of a secondary market and a trade-in program.

From the perspective of customer heterogeneity and choice behavior, Bruce et al. (2006) considered a durable-product manufacturer who provides customers with a refund enabling their willing to replace used products with new ones. Rao et al. (2009) and Li et al. (2011) involves customer heterogeneity into their models and investigate the trade-in program in durable product markets and B2B markets, respectively. Yin and Tang (2014) modeled a two-period, trade-in service with up-front fees to examine the program effectiveness and investigated the optimal purchasing behavior of rational customers. Xiao (2017) investigated two models for "exchange-old-for-new" program (i.e., Scenarios M and R) from the customer behavior perspective. Mahmoudzadeh (2020) provided an experimental evidence that alternative frames (i.e., trade-in or upgrade) are not equivalent and the framing effect induces customers' reference points for their current version. Xiao and Zhou (2019) developed a dynamic pricing model with T selling horizons, using a customer choice behavior model for a hybrid trade-in program in which a firm offers two options(i.e., trade-in-for-upgrade and trade-in-for-cash). Xiao et al. (2020) studied dynamic decisions on the selling price of a new product, a trade-in price, and a semi-dynamic trade-in program with a given sale price, using a vertical product differentiation choice model from the perspective of customer choice behavior. Ma et al. (2020) modeled a manufacturer's trade-in service for remanufactured products (rather than new products), considering customers' double reference effects and a government's incentives.

From the perspective of green supply chain operations, Genc and De Giovanni (2017) treated the trade-in as the reverse channel of CLSC in a two-period game in which the return rate is dependent on both price and quality. Miao et al. (2017) discussed three types of optimal collection strategies (no collection, partial collection, and full collection) in three scenarios (centralized, retailer, and manufacturer collection) in the presence of environmental performance. Genc and De Giovanni (2018) studied two different return and rebate mechanisms (i.e., a fixed rebate and a variable rebate) in a two-period CLSC game under Markovian equilibrium and open-loop strategies.

Most relevant publications are concerned with the discussions on trade-in service from the marketing or customer behavior perspective; and, in some publications, the demand functions were derived from the consumer utility. Nevertheless, Genc and De Giovanni (2017) and Genc and De Giovanni (2018) characterized the market demand by using a linear demand function. Genc and De Giovanni (2018) also provided an empirical evidence for the validity of their linear return function. However, they neglected the forward sales promotion effect of trade-in mechanism. Differently, we develop a dependent demand function that exhibits (1) the forward sales promotion effect of trade-ins including the return function in Genc and De Giovanni (2018) and (2) the separation between trade-in demands and new sales as by Feng et al. (2019) and Tang et al. (2020). Our model characterizes the demands in both direct sales and trade-in channels, with the price-based rebate rate as a decision variable.

3 Scenario Description and Profit Functions

We investigate the trade-in service in a CLSC including a manufacturer and a retailer who serves customers in a market. Under such service, customers can trade in their used products for new ones with a price-based rebate. To properly characterize the trade-in services in our model, we consider a two-period problem in which there are two cohorts of potential customers, i.e., one in period 1 and the other in period 2. For simplicity, we hereafter call them the first and second cohorts. The first cohort of customers purchase products in period 1 and can trade in their used products in period 2, in which the second cohort of customers buy new products directly. In each period, customers are sensitive to the retail price. Moreover, in period 2, the rebate rate influences the trade-in quantity.

In practice, the manufacturer may operate the trade-in service by herself or may outsource such service to the retailer. Accordingly, we consider two scenarios. The first scenario is called "Scenario M" in which the manufacturer collects used products by herself at a trade-in rebate rate. The second scenario is "Scenario R" in which the manufacturer outsources the trade-in service to the retailer, and the latter is then responsible for the collection of used products at a trade-in rebate rate. In Scenario M, the manufacturer determines her wholesale price w_1 in period 1, and decides on a wholesale price w_2 and a rebate rate τ_m in period 2. In period i ($i = 1, 2$), the retailer purchases the product at wholesale price w_i and then sells the product to customers at a retail price p_i. We plot Figure 1 to indicate the sequence of events in Scenario M. In Scenario R, for period i, the manufacturer determines her wholesale price w_i only, and the retailer make a decision on his retail price p_i. In addition, the retailer needs to decide on a rebate rate τ_r in period 2. For Scenario R, see Figure 1.

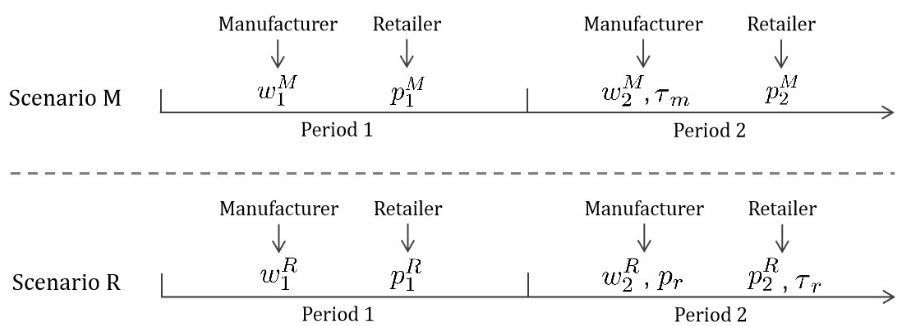

Figure 1: Decision-makings sequences in two scenarios

In period i ($i = 1, 2$), the purchase quantity of new customers in cohort i is dependent on retail price p_i, i.e.,

$$q_i = \alpha_i - \beta p_i, \tag{1}$$

where α_i is the market potential in period i, and β denotes the price sensitivity of new customers. Naturally, the value of β is significantly smaller than the value of α_i, i.e., $\beta < \alpha_i$, for $i = 1, 2$. As the first cohort of customers may trade in their used products (purchased in period 1) for new ones in period 2, we compute the trade-in quantity as

$$\hat{q}_2 = q_1 - \gamma(p_2 - \tau_j p_1), \tag{2}$$

where γ is the price sensitivity of trade-in customers, and τ_j ($j = m, r$) denotes the rebate rate determined by the manufacturer in Scenario M or by the retailer in Scenario R. Similar to β, the value γ is significantly smaller than the value of α_i ($i = 1, 2$). The justification for (2) is as follows: if a customer who buys in period 1 trades in his or her used product for a new one in period 2, then the customer receives the rebate $\tau_j p_1$ but pays the purchase cost p_2. Thus, each trade-in customer's net payment is $p_2 - \tau_j p_1$. As the net payment increases, the first cohort of customers are more reluctant to take the trade-in service. Thus, the term $\gamma(p_2 - \tau_j p_1)$ describes the unwillingness of the customers in the first cohort to return their used products according to the retail price differential over the two periods, similar to Genc and De Giovanni (2018). Since the total purchase quantity in period 1 is q_1, we can construct the trade-in quantity model as in (2). According to the above, the total demands in periods 1 and 2 are q_1 and $Q_2 \equiv q_2 + \hat{q}_2$, respectively.

Next, we build each firm's profit function for different scenarios. In Scenario M, the manufacturer and the retailer obtain their profits $q_1(w_1 - c_1)$—where c_1 denotes the manufacturer's unit acquisition cost—and $q_1(p_1 - w_1)$, respectively, in period 1. In period 2, the manufacturer is responsible for the trade-in service, under which the customers who choose the service (i) directly return their used products to, and also receive a rebate from, the manufacturer and (ii) spend the rebate (and possibly, additional financial resources) to buy new products from the retailer. As a result, the manufacturer enjoys (1) a profit that is generated by new customers' purchases and old customers' repurchases via trade-in service and (2) a profit resulting from the disposal of used products. Differently, the retailer receives a profit generated by the sales specified in (1) only. The manufacturer's and the retailer's profits from the "new" purchases in period 2 are $\delta q_2(w_2 - c_2)$ and $\delta q_2(p_2 - w_2)$, where c_2 is the manufacturer's unit acquisition cost in period 2 and δ means the discount rate for present (period 1) values of the profits in period 2. Moreover, the value of c_i $(i = 1, 2)$ is significantly smaller than α_i.

When the manufacturer receives a returned product, she enjoys a salvage value $s \geq 0$ but incurs a handling cost $g \geq 0$, which is associated with the reverse logistics activities. Following Genc and De Giovanni (2018), we compute the manufacturer's net gain from processing a used product as $\Delta_m \equiv s - g > 0$. When a customer returns a used product (that is bought in period 1) for a new product in period 2, the manufacturer needs to pay the p_1-based rebate $\tau_m p_1$ and also obtains the profit from selling the new product as $w_2 - c_2$. Therefore, the present value of the manufacturer's profit generated by her trade-in service in period 2 is $\delta\hat{q}_2(w_2 - c_2 + \Delta_m - \tau_m p_1)$, where $\hat{q}_2 = q_1 - \gamma(p_2 - \tau_m p_1)$. The manufacture's total profit in period 2 is computed as

$$\delta q_2(w_2 - c_2) + \delta\hat{q}_2(w_2 - c_2 + \Delta_m - \tau_m p_1) = \delta[Q_2^M(w_2 - c_2) + \hat{q}_2(\Delta_m - \tau_m p_1),$$

where $Q_2^M \equiv q_2 + \hat{q}_2$ is the total demand in period 2 and is calculated as

$$Q_2^M = \alpha_2 - \beta p_2 + q_1 - \gamma(p_2 - \tau_m p_1). \tag{3}$$

According to the above, the manufacturer's two-period total profit is

$$\Pi_m^M = q_1(w_1 - c_1) + \delta[Q_2^M(w_2 - c_2) + \hat{q}_2(\Delta_m - \tau_m p_1)]. \tag{4}$$

We also calculate the retailer's two-period total profit as

$$\Pi_r^M = q_1(p_1 - w_1) + \delta Q_2^M(p_2 - w_2). \tag{5}$$

The manufacturer's and the retailer's optimization problems are $\max_{w_1,w_2,\tau_m} \Pi_m^M$ and $\max_{p_1,p_2} \Pi_r^M$, respectively.

In Scenario R, the manufacturer outsources the trade-in service to the retailer. As a result, the manufacturer only needs to decide on her wholesale prices in periods 1 and 2, and the retailer should determine his retail prices in the two periods as well as a rebate rate τ_r in period 2. The total demand in this period is

$$Q_2^R \equiv q_2 + \hat{q}_2 = \alpha_2 - \beta p_2 + q_1 - \gamma(p_2 - \tau_r p_1). \tag{6}$$

When a customer returns the used product to the retailer for a new one in period 2, the retailer incurs the handling cost g and pays the rebate $\tau_r p_1$. Since the product is made by the manufacturer rather than the retailer, a common practice is that the manufacturer buys the returned products back from the retailer at the buyback price p_r, which is the manufacturer's decision variable in Scenario R. That is, for each returned product, the retailer's net gain is $p_r - g$. In addition, similar to our analysis for Scenario M, as the retailer sells a new product for each returned one in period 2, he gains the sale profit $p_2 - w_2$ when receiving a returned product. Therefore, the present value of the retailer's profit generated by his trade-in service in period 2 is $\delta\hat{q}_2(p_2 - w_2 + p_r - g - \tau_r p_1)$, where $\hat{q}_2 = q_1 - \gamma(p_2 - \tau_r p_1)$. The retailer's total profit in this period is

$$\delta q_2(p_2 - w_2) + \delta\hat{q}_2(p_2 - w_2 + p_r - g - \tau_r p_1) = \delta[Q_2^R(p_2 - w_2) + \hat{q}_2(p_r - g - \tau_r p_1)],$$

and his total profit in two periods is

$$\Pi_r^R = q_1(p_1 - w_1) + \delta[Q_2^R(p_2 - w_2) + \hat{q}_2(p_r - g - \tau_r p_1)]. \tag{7}$$

In Scenario R, for each returned product, the manufacturer obtains a salvage value s but pays a buyback cost p_r to the retailer. Therefore, the manufacturer's profit for each returned product is $s - p_r = s - g + g - p_r = \Delta_m - p_r + g$. As the manufacturer can obtain $w_2 - c_2$ from selling a new product for the return of each used product, the present value of her total profit in period 2 is

$$\delta q_2(w_2 - c_2) + \delta\hat{q}_2(w_2 - c_2 + \Delta_m - p_r + g) = \delta[Q_2^R(w_2 - c_2) + \hat{q}_2(\Delta_m - p_r + g)].$$

We then compute the manufacturer's total profit in two periods as

$$\Pi_m^R = q_1(w_1 - c_1) + \delta[Q_2^R(w_2 - c_2) + \hat{q}_2(\Delta_m - p_r + g)]. \tag{8}$$

The manufacturer's and the retailer's optimization problems in Scenario R are $\max_{w_1,w_2,p_r} \Pi_m^R$ and $\max_{p_1,p_2,\tau_r} \Pi_r^R$, respectively.

In the above, we construct the manufacturer's and the retailer's profits for both Scenario M and Scenario R. In each scenario, the manufacturer plays the role of the "leader" and announces her decisions to the retailer, who then makes his decisions as the "follower." Thus, we can view each scenario as a sequential-move game, for which the firms' decisions are characterized in Stackelberg equilibrium. To find the solution, we should use the backward approach. Specifically, in Scenario M, for period 2, given the manufacturer's decisions on the wholesale price and rebate rate, the retailer makes his optimal retail price as his best response. Using the best response, the manufacturer determines her optimal wholesale pricing and rebate rate decisions. Similarly, for period 1, the manufacturer and the retailer sequentially make their optimal pricing decisions. The decision sequence in Scenario R is similar to but differs from that in Scenario M because the manufacturer needs to determine a buy-back price p_r in addition to wholesale price w_2, and the rebate rate decision in period 2 is made by the retailer in Scenario R.

4 Stackelberg Equilibrium and Management Implications

In this section, we solve the sequential-move game to find Stackelberg Equilibrium for each scenario. We also analyze the impact of some important parameters on the supply chain to obtain managerial implications.

Proposition 1 *For Scenario M, the two-period decisions in Stackelberg equilibrium can be uniquely obtained. In period 1, the manufacturer's wholesale price and the retailer's retail price are*

$$\begin{cases} w_1^{*M} = \dfrac{\Sigma^M}{32\beta(\beta+\gamma)^2(8\gamma-\delta\beta)}, \\ p_1^{*M} = \dfrac{[8(\beta+\gamma)-\delta\beta]\alpha_1 - \delta\beta[\alpha_2-(\beta+\gamma)c_2+\gamma\Delta_m] + 8\beta(\beta+\gamma)w_1^{*M}}{\beta[16(\beta+\gamma)-\delta\beta]}, \end{cases}$$

where

$$\begin{aligned} \Sigma^M \equiv\ & 8[16\gamma(\beta+\gamma)^2 - 4\delta\beta^3 - 7\delta\gamma\beta^2 - 3\delta\gamma^2\beta]\alpha_1 + \delta[48\gamma\beta(\beta+\gamma) - 4\delta\beta^2(\beta+\gamma) - \delta\gamma\beta^2]\alpha_2 \\ & +8[16\gamma\beta(\beta+\gamma)^2 - \delta\gamma\beta^2(\beta+\gamma)]c_1 + \delta[16\gamma\beta(\beta+\gamma)^2 + \delta\beta^2(4\beta^2+5\gamma\beta+\gamma^2)]c_2 \\ & -\delta[16\gamma\beta(4\beta^2+5\gamma\beta+\gamma^2) + \delta\gamma^2\beta^2]\Delta_m. \end{aligned} \tag{9}$$

In period 2, the wholesale and retail prices are

$$w_2^{*M} = \frac{\alpha_2+\beta c_2}{2\beta} \text{ and } p_2^{*M} = \frac{(3\beta+2\gamma)\alpha_2 + (\beta+\gamma)\beta c_2 + \gamma\beta\Delta_m + \beta q_1^M}{4\beta(\beta+\gamma)};$$

and the manufacturer's rebate rate is

$$\tau_m^* = \frac{2\gamma\alpha_2 + 2\gamma\beta\Delta_m - \beta\alpha_1 + \beta^2 w_1^M}{2\gamma(\alpha_1+\beta w_1^M)}.$$

Noting from Section 3 that the values of c_i, β, Δ_m and γ are all significantly smaller than α_i, we find that the value of Σ^M in (9) is positive, i.e., $\Sigma^M > 0$, which ensures that all decisions in Stackelberg equilibrium are positive. In addition, we recall from Section 3 that β and γ denote "new" customers' sensitivity to the retail price and "old" customers' sensitivity to the price that they need to pay for the new products obtained by trading in their used products in period 2. If there is no any promotion mechanism for the trade-in service, then the value of β should be naturally equal to the value of γ, i.e., $\beta = \gamma$. Otherwise, if the manufacturer or the retailer provides some incentive policies (e.g., gifts or bonus points) for motivating the customers who buy products in period 1 to trade in their used products, then the value of γ is lower than the value of β, i.e., $\gamma < \beta$.

Proposition 2 *In Scenario R, the pricing and rebate rate decisions in Stackelberg equilibrium can be uniquely determined as follows: in period 1, the wholesale and retail prices are*

$$\begin{cases} w_1^{*R} = \dfrac{8(16\gamma - 3\delta\beta)\alpha_1 + \beta[8(16\gamma - \delta\beta)c_1 + (16\gamma + \delta\beta)(c_2 - \Delta_m)]}{32\beta(8\gamma - \delta\beta)}, \\ p_1^{*R} = \dfrac{4(6\gamma - \delta\beta)\alpha_1 + 8\beta\gamma c_1 + 3\delta\beta\gamma c_2 - 3\delta\beta\gamma\Delta_m}{4\beta(8\gamma - \delta\beta)}. \end{cases}$$

In period 2, the wholesale and retail prices are

$$w_2^{*R} = \frac{\alpha_2 + \beta c_2}{2\beta} \text{ and } p_2^{*R} = \frac{3\alpha_2 + \beta c_2}{4\beta} \ ;$$

and the rebate rate and buy-back price are

$$\begin{cases} \tau_r^* = \dfrac{12[(8\gamma - \delta\beta)\alpha_2 - 2\beta\alpha_1] + \beta(32\gamma - 13\delta\beta)\Delta_m + 3\beta^2(8c_1 + 3\delta c_2)}{4\gamma[4(6\gamma - \delta\beta)\alpha_1 + \beta(8c_1 + 3\delta c_2) - 3\delta\beta\Delta_m]}, \\ p_r^* = \dfrac{4(8\gamma - \delta\beta)\alpha_2 - 8\beta\alpha_1 + \beta(32\gamma - 7\delta\beta)\Delta_m + \beta^2(8c_1 + 3\delta c_2) + 8\beta(8\gamma - \delta\beta)g}{8\beta(8\gamma - \delta\beta)}. \end{cases}$$

Similarly, the values of c_i, β, Δ_m, γ, and g are significantly smaller than α_i. Therefore, all decisions in Stackelberg equilibrium in Proposition 2 are positive.

Proposition 3 *The first-period wholesale and retail prices in each scenario (i.e.,* w_1^{*M}*,* p_1^{*M}*,* w_1^{*R}*, and* p_1^{*R}*) decrease with the manufacturer's net gain from processing a used product (i.e.,* Δ_m*). In period 2, only* p_2^{*M} *in Scenario M increases with* Δ_m*, and other pricing decisions are independent of* Δ_m*.*

The findings in Proposition 3 differ from those in existing publications (including, e.g. Savaskan et al. 2004 and Genc and De Giovanni 2018). They find that the first-period prices increase with the manufacturer's net gain from processing used products, whereas the second-period prices decrease in the net gain when the manufacturer collects and remanufactures used products. The differences are attributed to the followings facts: firstly, the rebate rate in our paper is a decision variable rather than an exogenous one as in those publications. Secondly, we build the price- and rebate-dependent demand functions in two periods, while in previous two-period CLSC game models, e.g., Genc and De Giovanni (2018), the demand in a period only depends on the player's decision variables in that period, and the intercept in the return function is also an independent parameter, which is not related to how many products have been sold that can be returned for the trade-in service. As in practice, only those products sold in period 1 can be collected for trade-in, viz., the first-period demand affects the number of returned products in period 2. Accordingly, we use the first-period demand as the intercept of the trade-in function rather than an exogenous independent parameter, as shown in (2).

In both Scenarios, when the manufacturer obtains a higher net gain from processing a used product, the wholesale and retail prices (i.e., w_1^{*M} and p_1^{*M}) in period 1 should decrease. This occurs mainly because the manufacturer's higher gain generated by her trade-in service helps offset the costs in the supply chain, thus inducing both firms to decrease their prices for attracting more customers to buy.

In Scenario M, an increase in net gain Δ_m results in a higher retail price but has no impact on the wholesale price in period 2. The manufacturer jointly determines wholesale price w_2 and trade-in rebate rate τ_m in this period. Although Δ_m has no impact on w_2^{*M}, its higher value may increase the optimal trade-in rebate rate τ_m^*, as indicated by Proposition 4. In Scenario R, Δ_m has no impact on the second-period optimal price decisions (i.e., w_2^{*R}, p_2^{*R}) because both the manufacturer and the retailer make two decisions in this period, and the impact of Δ_m on their decisions are dependent on the buy-back price p_r^* (as in Proposition 7) and trade-in rebate rate τ_r^* (as in Proposition 4), respectively.

For our subsequent analysis, we set conditions $\delta = 1$ and $c_i = 0$, which are mild, because of the following facts: in the operations management area, many publications do not consider the discount rate for present value, which means $\delta = 1$ (e.g., Genc and De Giovanni 2018, Dey et al. 2019, and Tang et al. 2020). Moreover, we can normalize the values of c_i to be zero, as done by Genc and De Giovanni (2018). Under such conditions, the first-period wholesale prices in Scenarios M and R can be given as

$$\begin{aligned} w_1^{*M} &= \frac{8(\beta+\gamma)(16\gamma^2-13\gamma\beta+4\beta^2)\alpha_1+(48\gamma^2+43\gamma\beta-4\beta^2)\beta\alpha_2-(16\gamma^2+81\gamma\beta+64\beta^2)\gamma\beta\Delta_m}{32(8\gamma-\beta)(\beta+\gamma)^2}, \\ w_1^{*R} &= \frac{8(16\gamma-3\beta)\alpha_1-(16\gamma+\beta)\Delta_m}{32(8\gamma-\beta)}. \end{aligned}$$

Proposition 4 *When the manufacturer achieves a higher net gain from handling returned products (i.e., the value of Δ_m increases), the trade-in service provider should determine a greater rebate rate (i.e., τ_m^* and τ_r^*) in each scenario.*

In Scenario M, the manufacturer benefits more from the return of used product when the value of Δ_m increases. As a consequence, she has an incentive to determine a higher trade-in rebate rate and transfers a part of her net gain from processing used products to the first cohort of customers, encouraging these customers to trade in used products, in order to increase the second-period demand and her total profit in the two periods. Although the second-period retail price increases, the demand Q_2^{*M} increases because of the higher rebate rate, viz., the manufacturer's higher rebate rate mitigates the impact of the higher retail price on the demand. This implies that the retailer can attain a portion of the manufacturer's net gain from processing used products by increasing the second-period retail price. It is concluded that the manufacturer's profit increases as a result of a higher value of Δ_m, and the retailer enjoys a higher profit because of his higher retail price and an increasing demand in period 2.

In Scenario R, the manufacturer would like to pay a higher buy-back price (as in Proposition 7) for collected products when she enjoys a higher Δ_m, and to keep the second-period wholesale price the same (as in Proposition 3). Therefore, the retailer has an incentive to offer a higher rebate rate and make the retail price in this period unchanged to encourage first-cohort customers to trade in their used products and increase the second-period demand via trade-in service. Consequently, he enjoys a higher total profit from the greater demand, with the forward markup unchanged.

Proposition 5 *The first-cohort customers' lower price sensitivity in the trade-in service (i.e., a smaller value of γ) can reduce the wholesale and retail prices in period 1 in each scenario. However, a higher price sensitivity raises the retail price in period 2 in Scenario M (i.e., p_2^{*M}), and it does not impact w_2^{*M} and the second-period prices in Scenario R (i.e., w_2^{*R}, p_2^{*R}).*

The price sensitivity of trade-in customers (i.e., γ) from the first cohort has an important impact on pricing decisions, especially first-period ones, in each scenario. The first-period wholesale and retail prices both increase with γ in two scenarios, whereas the second-period retail price in each scenario and the wholesale price in Scenario R decrease with γ. It indicates that if first-cohort customers who buy products in period 1 have a lower price sensitivity when deciding whether or not to trade in their old products for new ones, then the manufacturer is likely to set a lower first-period wholesale price in each period, and the retailer responds by also reducing his price in that period. Because the first-period realized demand is the base of the second-period trade-in customers. Otherwise, the firms tend to benefit from a high first-period markup instead of a large trade-in demand base when trade-in customers' price sensitivity is high. Note that the first-period retail price decrement in each scenario is less than the wholesale price decrement, which means the retailer enjoys a higher profit margin from direct sales when the value of γ decreases. In period 2, the manufacturer adjusts her rebate rate τ_m^* (as in Proposition 6) rather than her wholesale pricing decision in Scenario M, whereas the retailer increases his retail price as γ is reduced. In Scenario R, the impacts of γ on decisions are dependent on the buy-back price p_r^* (as in Proposition 7) and trade-in rebate rate τ_r^* (as in Proposition 6).

Proposition 6 *The optimal trade-in rebate rate in each scenario (i.e., τ_m^* and τ_r^*) increases, when the first cohort of customers have a higher price sensitivity to the trade-in service, and τ_r^* is more sensitive to γ.*

The price sensitivity of trade-in customers (i.e., γ) influences not only the pricing decisions but also the rebate rate decision in each scenario. Both the manufacturer and the retailer tend to offer a lower rebate rate when the value of γ is reduced in two scenarios, and the optimal rebate rate τ_r^* in Scenario R changes more sharply in γ than τ_m^* set by the manufacturer in Scenario M (as indicated by Table 1), because the manufacturer influences the retailer by using the buy-back price (as in Proposition 7). If the first cohort of customers are more sensitive to the price when they consider whether or not to trade in their used products, the firms should set a higher rebate rate (i.e., reduce the price that trade-in customers need to pay) to improve their profitability. Observing the impact of γ on pricing decisions, we find that γ is one of the most important factors for firms to consider the trade-in service, as it has important impacts on pricing decisions and profits. The manufacturer's profit in Scenario R increases sharply as the value of γ decreases, and a lower value of γ makes Scenario R more preferable to the two firms, see the following section.

Proposition 7 *The manufacturer has to pay a higher buy-back price in Scenario R when one of the following two cases happens: first, she enjoys a higher net gain from handling returned product (i.e., the value of Δ_m is larger) when $\gamma > 7\beta/32$; second, she serves trade-in customers with a higher price sensitivity (i.e., the value of γ is larger).*

The manufacturer pays the retailer a buy-back price p_r for each collected product in Scenario R, and such decision is affected by the value of returned products and trade-in customers' price sensitivity. We can learn from Propositions 3 and 5 that the manufacturer determines w_2 and p_r simultaneously in period 2 in this scenario, and both Δ_m and γ do not impact her forward wholesale price decision but influence the backward buy-back price. When the manufacturer enjoys a higher Δ_m, she has an incentive to pay a higher buy-back price to encourage the retailer to offer a greater rebate rate, in order to benefit more from a higher demand via trade-in service. We note that $\gamma > 7\beta/32$ is a very mild restriction, because it is abnormal if and only if the two price sensitivities are very different. In addition, the manufacturer pays a higher p_r^* and "enforces" the retailer to offer a higher τ_r^* (as in Proposition 6) when γ is higher.

Proposition 8 *We compare the prices in each period in two scenarios and draw the following findings:*

1. *In period 1, $p_1^{*M} > p_1^{*R}$; and when $\gamma > \beta/8$ and $\alpha_2 > \kappa_1(\alpha_1) \equiv [8(\beta+\gamma)\beta\alpha_1 + (48\gamma^2 + 46\gamma\beta - \beta^2)\beta\Delta_m]/[48\gamma^2 + 43\gamma\beta - 4\beta^2]$, $w_1^{*M} > w_1^{*R}$.*
2. *In period 2, $w_2^{*M} = w_2^{*R}$, and when $\gamma > \beta/8$ and $\alpha_2 > \kappa_2(\alpha_1) \equiv [8(1+\gamma)\beta\alpha_1 + (32\gamma + 31\beta)\gamma\beta\Delta_m]/[32\gamma^2 + 28\gamma\beta - 3\beta^2]$, $p_2^{*M} < p_2^{*R}$.*
3. *When $\gamma > \beta/8$ and $\alpha_2 > \kappa_2(\alpha_1)$, the retailer enjoys a higher forward markup in each period in Scenario R.*
4. *When $\gamma > \beta/8$ and $\alpha_2 > \kappa_3(\alpha_1) \equiv [8\beta(\beta+\gamma)\alpha_1 + (32\gamma^2 + 28\beta\gamma - 3\beta^2)\beta\Delta_m]/[32\gamma^2 + 25\gamma\beta - 6\beta^2]$, trade-in customers can enjoy lower prices in Scenario R .*

Using previous arguments, we can similarly find that $\gamma > \beta/8$ is not a strong condition. The first point in Proposition 8 exposes that the optimal first-period prices in Scenario R are smaller than those in Scenario M, when the market potential of the second cohort of customers is not much smaller than that of the first cohort. The manufacturer sets a lower wholesale price when she outsources the trade-in service as an incentive for the retailer to lower his price and stimulate the first-period demand, which is also the demand base for the trade in service in period 2. The third point in Proposition 8 indicates that, although the two firms set lower first-period prices in Scenario R, the retailer enjoys a higher forward markup in this scenario (i.e., $p_1^{*R} - w_1^{*R} > p_1^{*M} - w_1^{*M}$). It is interesting to note from the second point in Proposition 8 that the manufacturer sets the same second-period wholesale price in two scenarios, when she simultaneously make her decisions. In Scenario R, the retailer sets a higher second-period price when he is the trade-in service provider and gains a higher markup. As a result, the retailer enjoys a higher forward markup in each period in Scenario R.

The fourth point in Proposition 8 shows that customers enjoy a lower first-period retail price when the trade-in service is provided by the retailer. Moreover, if customers buy products in period 1, then, when they trade in used products for new ones in period 2 in Scenario R, they can enjoy a lower price because (1) trade-in customers actually pay $p_2 - \tau p_1$ in each scenario and (2) $p_2^{*M} - \tau_m^* p_1^{*M} - (p_2^{*R} - \tau_r^* p_1^{*R}) > 0$ when $\alpha_2 > \kappa_2(\alpha_1)$.

Proposition 9 *We compare the demands in each period in two scenarios and find the following results: In period 1, $q_1^{*M} < q_1^{*R}$. In period 2, $Q_2^{*M} < Q_2^{*R}$; and $\hat{q}_2^{*M} < \hat{q}_2^{*R}$ when $\alpha_2 > \kappa_4(\alpha_1) \equiv [8\beta(\beta+\gamma)\alpha_1 + (32\gamma^2 + 32\gamma\beta + \beta^2)\beta\Delta_m]/[32\gamma^2 + 28\gamma\beta - 3\beta^2]$.*

Proposition 9 indicates that $q_1^{*M} < q_1^{*R}$, which results from the result $p_1^{*M} > p_1^{*R}$ as shown in Proposition8. The demand in period 2 is dependent on both the retail prices and trade-in rebate rate (i.e., τ_m^* or τ_r^*). Although the second-period retail price in Scenario R is higher than that in Scenario M, the trade-in rebate rate τ_r^* is also higher than τ_m^* (as exposed by Table 2). This is attributed to the fact that the retailer has an incentive to offer a higher rebate to boost the demand via trade-in service, when he enjoys a higher forward markup and backward profit from the buy-back price paid by the manufacturer.

Proposition 10 *There exist three α_1- and α_2-dependent functions $u(\alpha_1, \alpha_2)$, $v(\alpha_1, \alpha_2)$, and $f(\alpha_1, \alpha_2)$ that can be used to determine the conditions under which the manufacturer, the retailer, and the supply chain prefer a scenario to the other, respectively. We specify the conditions below.*

1. *If $u(\alpha_1, \alpha_2) \geq 0$, then $\Pi_m^{*M} \geq \Pi_m^{*R}$, i.e., the manufacturer prefers Scenario M to R. Otherwise, if $u(\alpha_1, \alpha_2) < 0$, then $\Pi_m^{*M} < \Pi_m^{*R}$, i.e., the manufacturer prefers to choose Scenario R.*
2. *If $v(\alpha_1, \alpha_2) \geq 0$, then $\Pi_r^{*M} \leq \Pi_r^{*R}$, i.e., the retailer prefers Scenario R to M. Otherwise, if $v(\alpha_1, \alpha_2) < 0$, then $\Pi_r^{*M} > \Pi_r^{*R}$, and the retailer prefers Scenario M.*
3. *If $f(\alpha_1, \alpha_2) \geq 0$, then $\Pi^{*M} \geq \Pi^{*R}$, i.e., the supply chain benefits more from Scenario M, and the two firms should prefers Scenario M as a whole. Otherwise, if $f(\alpha_1, \alpha_2) < 0$, then $\Pi^{*M} < \Pi^{*R}$, the supply chain is better off from Scenario R.*

Xiao (2017) found that a firm can obtain a higher profit by acting as a "free-rider" than by operating the trade-in service, treating each firm's pricing decision as an exogenous factor in the other firm's profit function. Different from Xiao's result (2017), Proposition 10 reveals that no scenario is dominant, and whether to operate or to outsource the trade-in service is dependent on the market potential of each customer cohort. Specifically, the supply chain-wide profit is higher in Scenario M when $f(\alpha_1, \alpha_2) > 0$; otherwise, the supply chain is better off in Scenario R. The manufacturer enjoys a higher profit in Scenario M when $u(\alpha_1, \alpha_2) > 0$, and the retailer obtains a higher profit in Scenario R when $v(\alpha_1, \alpha_2) > 0$. The three curves ($u(\alpha_1, \alpha_2) = 0$, $v(\alpha_1, \alpha_2) = 0$, and $f(\alpha_1, \alpha_2) = 0$) divide the (α_1, α_2)-plane into four zones, as depicted by Figure 2. In different zones, the optimal trade-in strategies for the supply chain and the firms differ. We learn from Figure 2 that, (1) $\Pi^{*M} > \Pi^{*R}$ in zone A, but $\Pi^{*M} < \Pi^{*R}$ in $B \cup C \cup D$; (2) $\Pi_m^{*M} > \Pi_m^{*R}$ in $A \cup B$, but $\Pi_m^{*M} < \Pi_m^{*R}$ in $C \cup D$; and (3) $\Pi_r^{*M} < \Pi_r^{*R}$ in $A \cup B \cup C$, but $\Pi_r^{*M} > \Pi_r^{*R}$ in D.

The above results expose the following implications. First, if $(\alpha_1, \alpha_2) \in A$, then the manufacturer should provide the trade-in service herself and reach an agreement with the retailer using a transfer payment. Secondly, if $(\alpha_1, \alpha_2) \in B$, then the manufacturer should outsource the trade-in service and charge the retailer a transfer payment. Thirdly, if $(\alpha_1, \alpha_2) \in C$, then the manufacturer and the retailer achieve higher profits from the retailer's trade-in service, thus reaching a unanimous agreement. Fourthly, if $(\alpha_1, \alpha_2) \in D$, then the manufacturer should outsource the trade-in service and reach an agreement with the retailer using a transfer payment. It thus follows that there is a unanimous agreement that the manufacturer outsources the trade-in service to the retailer when $f(\alpha_1, \alpha_2) < 0$, $u(\alpha_1, \alpha_2) < 0$ and $v(\alpha_1, \alpha_2) > 0$.

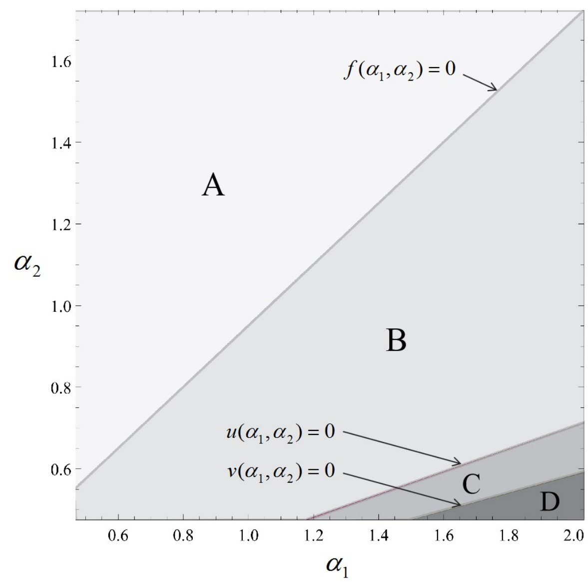

Figure 2: Three curves $f = 0$, $u = 0$, and $v = 0$ on the (α_1, α_2)-plane.

5 Numerical Study with Managerial Discussions

We numerically compare solutions in Scenarios M and R for different cases each corresponding to a set of parameter values. The numerical analysis starts from a baseline set, in which $\alpha_1 = \alpha_2 = 1$, $\beta = 0.5$, $\gamma = 0.5$, $c_1 = c_2 = 0.01$, $\delta = 1$, $\Delta_m = 0.3$, and $g = 0$. The baseline dataset is based on some recent studies on CLSC and our model setting. To obtain managerial implications regarding the impact of parameters on optimal solutions and supply chain performance, we vary the parameter values and compute corresponding results as shown in Table 1. We also compare the computational results between two scenarios, as given in Table 2. In each of the two tables, the first row presents the results obtained by using the baseline parameter values.

5.1 Numerical Analyses

According to Tables 1 and 2, we can draw the following insights. First, when the market potential for the first cohort of customers (i.e., α_1) increases, both the manufacturer and the retailer achieve higher profits in Scenario M and R. The market potential in period 1 has a positive impact on both the first-period demands (i.e., q_1^{*M} and q_1^{*R}) and second-period demands (i.e., Q_2^{*M} and Q_2^{*R}), mainly because the trade-in service is available to customers. A greater market potential induces the manufacturer and the retailer to raise their prices in period 1, and the retailer increases his second-period retail price with α_1 in Scenario M. The manufacturer charges an identical wholesale price in period 2 for each scenario.

The first-cohort market potential, by and large, has a positive impact on the supply chain-wide profit as well as firms' individual profits. In Scenario M, the impact of α_1 on the retailer's profit is related to the higher profit margins (from direct sales) and demands in two periods. The manufacturer in this scenario enjoys a higher direct-sales profit in period 2 due to a higher first-period market

	The Computational Results in Scenario M														
Baseline results	$\Pi_m^{*M} = 0.627$			$\Pi_r^{*M} = 0.238$			$w_1^{*M} = 1.040$			$p_1^{*M} = 1.431$			$q_1^{*M} = 0.285$		
$\alpha_1(1.1;1.2;1.3)$	0.686	0.752	0.823	0.271	0.308	0.347	1.129	1.218	1.307	1.574	1.716	1.859	0.313	0.342	0.370
$\alpha_2(0.8;0.9;1.1)$	0.507	0.563	0.698	0.216	0.227	0.251	1.001	1.020	1.059	1.423	1.427	1.434	0.288	0.286	0.283
$\beta(.6;.7;.8)$	0.522	0.449	0.397	0.214	0.197	0.184	0.846	0.703	0.593	1.175	0.991	0.851	0.295	0.307	0.319
$\gamma(.25;.3;.4)$	0.630	0.624	0.623	0.299	0.277	0.252	0.928	0.976	1.021	1.348	1.379	1.413	0.326	0.310	0.293
$\delta(.9;.95;.98)$	0.588	0.607	0.619	0.226	0.232	0.236	1.037	1.039	1.039	1.439	1.435	1.432	0.281	0.283	0.284
$\Delta_m(.4;.5;.6)$	0.633	0.642	0.653	0.252	0.267	0.282	1.022	1.004	0.986	1.418	1.406	1.393	0.291	0.297	0.303
$c_1(.02;.05;.1)$	0.624	0.615	0.602	0.237	0.232	0.224	1.045	1.062	1.089	1.434	1.442	1.456	0.283	0.279	0.272
$c_2(.02;.05;.1)$	0.623	0.611	0.592	0.236	0.230	0.221	1.040	1.043	1.047	1.432	1.435	1.440	0.284	0.283	0.280
Baseline results	$w_2^{*M} = 1.005$			$p_2^{*M} = 1.361$			$\tau_m^* = 0.605$			$Q_2^{*M} = 0.356$					
$\alpha_1(1.1;1.2;1.3)$	1.005	1.005	1.005	1.368	1.375	1.383	0.532	0.471	0.419	0.363	0.370	0.378			
$\alpha_2(0.8;0.9;1.1)$	0.805	0.905	1.105	1.112	1.237	1.486	0.465	0.535	0.674	0.307	0.332	0.381			
$\beta(.6;.7;.8)$	0.838	0.719	0.630	1.164	1.020	0.910	0.586	0.563	0.536	0.359	0.361	0.364			
$\gamma(.25;.3;.4)$	1.005	1.005	1.005	1.470	1.440	1.395	0.369	0.459	0.554	0.348	0.348	0.351		—	
$\delta(.9;.95;.98)$	1.005	1.005	1.005	1.361	1.361	1.361	0.604	0.605	0.606	0.355	0.356	0.356			
$\Delta_m(.4;.5;.6)$	1.005	1.005	1.005	1.375	1.389	1.389	0.641	0.678	0.715	0.370	0.384	0.398			
$c_1(.02;.05;.1)$	1.005	1.005	1.005	1.361	1.360	1.360	0.506	0.604	0.603	0.356	0.355	0.353			
$c_2(.02;.05;.1)$	1.010	1.025	1.050	1.364	1.371	1.371	0.605	0.605	0.604	0.354	0.346	0.332			
	The Computational Results in Scenario R														
Baseline results	$\Pi_m^{*R} = 0.537$			$\Pi_r^{*R} = 0.325$			$w_1^{*R} = 0.912$			$p_1^{*R} = 1.400$			$q_1^{*R} = 0.300$		
$\alpha_1(1.1;1.2;1.3)$	0.599	0.668	0.742	0.359	0.396	0.436	1.005	1.098	1.190	1.543	1.686	1.829	0.328	0.357	0.386
$\alpha_2(0.8;0.9;1.1)$	0.458	0.495	0.583	0.270	0.296	0.357	0.912	0.912	0.912	1.400	1.400	1.400	0.300	0.300	0.300
$\beta(.6;.7;.8)$	0.465	0.415	0.378	0.276	0.242	0.216	0.742	0.620	0.528	1.147	0.966	0.829	0.312	0.324	0.337
$\gamma(.25;.3;.4)$	0.609	0.583	0.553	0.347	0.338	0.329	0.812	0.849	0.890	1.300	1.337	1.378	0.350	0.331	0.311
$\delta(.9;.95;.98)$	0.507	0.522	0.531	0.304	0.315	0.321	0.923	0.917	0.914	1.412	1.406	1.403	0.294	0.297	0.299
$\Delta_m(.4;.5;.6)$	0.552	0.570	0.589	0.334	0.343	0.351	0.904	0.897	0.889	1.390	1.379	1.368	0.305	0.311	0.316
$c_1(.02;.05;.1)$	0.534	0.525	0.511	0.324	0.319	0.311	0.917	0.933	0.960	1.403	1.412	1.426	0.298	0.294	0.287
$c_2(.02;.05;.1)$	0.533	0.523	0.505	0.323	0.316	0.304	0.913	0.915	0.919	1.401	1.405	1.410	0.299	0.298	0.295
Baseline results	$w_2^{*R} = 1.005$			$p_r^* = 0.850$			$p_2^{*R} = 1.503$			$\tau_r^* = 0.804$			$Q_2^{*R} = 0.360$		
$\alpha_1(1.1;1.2;1.3)$	1.005	1.005	1.005	0.822	0.793	0.764	1.503	1.503	1.503	0.701	0.617	0.545	0.367	0.374	0.381
$\alpha_2(0.8;0.9;1.1)$	0.805	0.905	1.105	0.650	0.750	0.950	1.203	1.353	1.653	0.589	0.696	0.911	0.310	0.335	0.385
$\beta(.6;.7;.8)$	0.838	0.719	0.630	0.672	0.540	0.438	1.253	1.074	0.940	0.747	0.684	0.612	0.363	0.365	0.369
$\gamma(.25;.3;.4)$	1.005	1.005	1.005	0.450	0.598	0.761	1.503	1.503	1.503	0.404	0.558	0.720	0.354	0.353	0.355
$\delta(.9;.95;.98)$	1.005	1.005	1.005	0.856	0.853	0.851	1.503	1.503	1.503	0.803	0.803	0.803	0.359	0.359	0.360
$\Delta_m(.4;.5;.6)$	1.005	1.005	1.005	0.895	0.939	0.984	1.503	1.503	1.503	0.822	0.841	0.860	0.374	0.388	0.401
$c_1(.02;.05;.1)$	1.005	1.005	1.005	0.852	0.856	0.863	1.503	1.503	1.503	0.803	0.803	0.803	0.360	0.359	0.357
$c_2(.02;.05;.1)$	1.01	1.025	1.05	0.851	0.852	0.855	1.505	1.513	1.525	0.804	0.804	0.803	0.357	0.349	0.336

Table 1: The computational results in Scenarios M and R.

Baseline results	$\Pi^{*M} - \Pi^{*R} = 0.0025$			$\Pi_m^{*M} - \Pi_m^{*R} = 0.090$			$\Pi_r^{*M} - \Pi_r^{*R} = -0.087$			$\tau_m^* - \tau_r^* = -0.199$			$w_1^{*M} - w_1^{*R} = 0.128$		
$\alpha_1(1.1;1.2;1.3)$	-0.0011	-0.0046	-0.0079	0.087	0.084	0.081	-0.088	-0.089	-0.089	-0.170	-0.146	-0.126	0.124	0.120	0.117
$\alpha_2(0.8;0.9;1.1)$	-0.0050	-0.0019	0.0083	0.048	0.068	0.114	-0.053	-0.070	-0.106	-0.124	-0.161	-0.237	0.089	0.108	0.147
$\beta(.6;.7;.8)$	-0.0059	-0.0102	-0.0123	0.057	0.036	0.019	-0.062	-0.045	-0.031	-0.162	-0.121	-0.076	0.104	0.083	0.065
$\gamma(.25;.3;.4)$	-0.0279	-0.0203	-0.0075	0.020	0.041	0.069	-0.048	-0.061	-0.077	-0.035	-0.100	-0.166	0.116	0.127	0.132
$\delta(.9;.95;.98)$	0.0030	0.0028	0.0026	0.081	0.086	0.088	-0.078	-0.083	-0.086	-0.199	-0.199	-0.199	0.115	0.121	0.125
$\Delta_m(.4;.5;.6)$	-0.0005	-0.0030	-0.0052	0.081	0.072	0.063	-0.082	-0.075	-0.069	-0.181	-0.163	-0.144	0.117	0.107	0.096
$c_1(.02;.05;.1)$	0.0027	0.0033	0.0042	0.090	0.090	0.091	-0.087	-0.087	-0.087	-0.199	-0.199	-0.200	0.128	0.128	0.129
$c_2(.02;.05;.1)$	0.0026	0.0028	0.0032	0.090	0.089	0.087	-0.087	-0.086	-0.084	-0.199	-0.199	-0.199	0.128	0.128	0.128
Baseline results	$p_1^{*M} - p_1^{*R} = 0.030$			$q_1^{*M} - q_1^{*R} = -0.015$			$w_2^{*M} - w_2^{*R} = 0$			$p_2^{*M} - p_2^{*R} = -0.141$			$Q_2^{*M} - Q_2^{*R} = -0.0038$		
$\alpha_1(1.1;1.2;1.3)$	0.030	0.030	0.030	-0.015	-0.015	-0.015	0	0	0	-0.134	-0.127	-0.120	-0.0038	-0.0038	-0.0038
$\alpha_2(0.8;0.9;1.1)$	0.023	0.027	0.034	-0.012	-0.013	-0.017	0	0	0	-0.090	-0.116	-0.167	-0.0029	-0.0033	-0.0042
$\beta(.6;.7;.8)$	0.027	0.025	0.023	-0.016	-0.017	-0.018	0	0	0	-0.088	-0.054	-0.030	-0.0041	-0.0044	-0.0046
$\gamma(.25;.3;.4)$	0.047	0.042	0.035	-0.024	-0.021	-0.017	0	0	0	-0.033	-0.062	-0.107	-0.0059	-0.0052	-0.0044
$\delta(.9;.95;.98)$	0.027	0.029	0.030	-0.013	-0.014	-0.015	0	0	0	-0.142	-0.142	-0.142	-0.0034	-0.0036	-0.0037
$\Delta_m(.4;.5;.6)$	0.029	0.027	0.025	-0.014	-0.013	-0.012	0	0	0	-0.127	-0.113	-0.099	-0.0036	-0.0033	-0.0031
$c_1(.02;.05;.1)$	0.030	0.030	0.030	-0.015	-0.015	-0.015	0	0	0	-0.142	-0.142	-0.145	-0.0038	-0.0038	-0.0038
$c_2(.02;.05;.1)$	0.030	0.030	0.030	-0.015	-0.015	-0.015	0	0	0	-0.141	-0.142	-0.143	-0.0038	-0.0038	-0.0038

Table 2: The computational comparison between two scenarios.

potential under the trade-in mechanism. In Scenario R, the retailer, the one who collects used products, shares the trade-in profit related to α_1 in period 2.

Similarly, the second-cohort market potential (i.e., α_2) has an impact on pricing decisions. However, such impacts in two scenarios differ, as shown in Table 1. In Scenario M, the wholesale and retail prices in two periods are positively affected by α_2. However, in Scenario R, the prices and the total demand in period 2 increase with α_2 sharply, while those in period 1 are independent of α_2. This indicates that in Scenario R, as the retailer is close to the customers, the two firms do not need to predict the second-cohort market potential when making their first-period decisions. In general, although, in two scenarios, the value of α_2 impacts the prices in a different manner, the firms still enjoy higher profits when the market potential of the second-cohort customers in period 2 increases.

The optimal rebate rate τ_j^* $(j = m, r)$ in each scenario decreases in the first-period market potential α_1 and increases with α_2. A larger value of α_1 leads to a larger demand q_1 in period 1 as well as a larger number of trade-in base from first cohort in period 2. That is, the intercept in the second-period return function (i.e., $\hat{q}_2$) increases with α_1, and the firm who operates the trade-in service intends to decrease the rebate rate to increase the price paid by trade-in customers. In contrast, the second-period retail price increases sharply with α_2, therefore, the optimal rebate rate τ_j^* $(j = m, r)$ for trade-in customers should also be increased when the price sensitivity of the first cohort (i.e., γ) remains unchanged. Moreover, we find that, for any values of α_1 and α_2, the rebate rate set by the manufacturer in Scenario M is always higher than that set by the retailer in Scenario R, as shown in Table 2.

The price sensitivity of the first cohort of customers who decide on whether to trade in their used products in period 2 (i.e., γ) plays an important role in the trade-in service, influencing the pricing and rebate rate decisions as well as two firms' profits. As shown in Table 1, the first-period wholesale and retail prices increase with γ in two scenarios, whereas the second-period retail prices in two scenarios and the wholesale price in Scenario R decrease with γ. As a higher value of γ implies that lower first-period prices is less attractive to a large number of trade-in customers, the firms prefer a greater markup in period 1. The trade-in rebate rate increases sharply with γ in each scenario, because the service provider should offer a more attractive trade-in price when those customers are more sensitive to it. In addition, any increase in the rebate rate set by the retailer in Scenario R is greater than that set by the manufacturer in Scenario M, because the manufacturer pays a higher buy-back price when the value of γ increases.

The manufacturer's increasing net gain $\Delta_m \equiv s - g$ has a positive impact on firms' profits in each scenario. In Scenario M, the rebate rate increases greatly with Δ_m, when the manufacturer provides the trade-in service and enjoys the total reverse profit. The manufacturer transfers a part of her net gain to customers for a higher trade-in rate and a higher demand in period 2. Therefore, the demand Q_2^M increases when the second-period retail price increases. Consequently, the retailer enjoys a higher profit as a result of a higher forward markup and a higher demand in period 2, and the manufacturer's profit increases because of her higher net gain Δ_m. In Scenario R, the retailer also has an incentive to offer a higher τ_r^*, because the manufacturer pays a higher buy-back price when the value of Δ_m is higher.

Except for most of the above, the impacts of δ, β, and c_i on pricing decisions and profits are consistent with the findings in extant publications including, e.g. Genc and De Giovanni (2018).

5.2 Sensitivity Analysis

Table 1 also presents the sensitivity analysis for Scenario M and R. Table 2 aims to indicate the differences between two scenarios and to identify the most suitable trade-in strategies for the firms in the supply chain. We observe that the wholesale prices for Scenario R in two periods are significantly smaller than those in Scenario M, whereas the retail prices in Scenario R are lower than those in Scenario M with a small difference. Consequently, the demand in each period in Scenario R is higher.

We find that γ, β, α_1, α_2 and Δ_m are the major parameters significantly impacting the optimal prices, rebate rate and supply chain-wide profit in each scenario as well as the two firms' preferences. The first-cohort market potential α_1 affects the supply chain-wide profit through the direct sales in period 2 and the return of used products under the trade-in service. A higher value of α_1 results in higher total profits in two scenario; and, the profit increase in Scenario R is higher, that is, $\Pi^{*M} - \Pi^{*R}$ decreases in α_1 according to Table 2. We find from Figure 3—which shows $\Pi_m^{*M} - \Pi_m^{*R}$, $\Pi_r^{*M} - \Pi_r^{*R}$, and $\Pi^{*M} - \Pi^{*R}$ in the (α_1, α_2)-space—that $\Pi^{*M} - \Pi^{*R}$ does not monotonically change with α_2. The intersection lines of the curved surfaces and the gray ones are $u(\alpha_1, \alpha_2) = 0$, $v(\alpha_1, \alpha_2) = 0$, and $f(\alpha_1, \alpha_2) = 0$, as illustrated in (α_1, α_2)-plane in Figure 2. Note that $\Pi_m^{*M} - \Pi_m^{*R} > 0$ when $u(\alpha_1, \alpha_2) > 0$; $\Pi_r^{*M} - \Pi_r^{*R} < 0$ when $v(\alpha_1, \alpha_2) > 0$; and $\Pi^{*M} - \Pi^{*R} > 0$ when $f(\alpha_1, \alpha_2) > 0$.

As shown in Tables 1 and 2, the two firms' profits decrease when price sensitivities γ and β increase. The manufacturer's profit and the supply chain-wide profit in Scenario M are more sensitive to β (less sensitive to γ) than those in Scenario R, while $\Pi_m^{*M} - \Pi_m^{*R}$ and $\Pi^{*M} - \Pi^{*R}$ decrease in β but increase in γ, and $\Pi_r^{*M} - \Pi_r^{*R}$ increases in β but decreases in γ. The firms benefit more from a higher value of Δ_m. The manufacturer's profit and the supply chain-wide profit in Scenario M are more sensitive to Δ_m than those in Scenario R, whereas $\Pi_m^{*M} - \Pi_m^{*R}$ and $\Pi^{*M} - \Pi^{*R}$ decrease in Δ_m, but $\Pi_r^{*M} - \Pi_r^{*R}$ increases in Δ_m.

The price that trade-in customers actually pay in each scenario also significantly depends on these major parameters, since it is related to retail prices in two periods and the rebate rate as such customers actually pay $p_2 - \tau_j p_1$ $(j = m, r)$ in each scenario. As exposed by the fourth point in Proposition 8, trade-in customers enjoy a lower price when the retailer provides trade-in service. We find that the

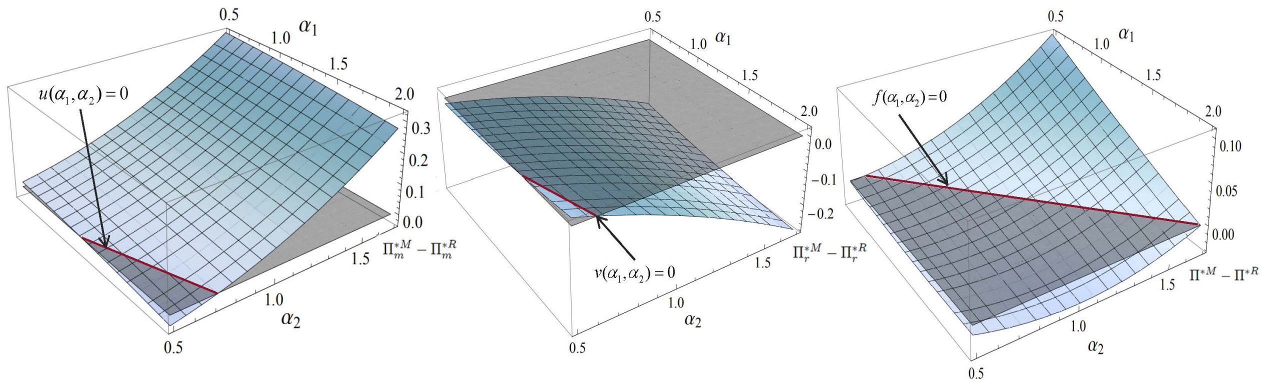

Figure 3: $\Pi_m^{*M} - \Pi_m^{*R}$, $\Pi_r^{*M} - \Pi_r^{*R}$, and $\Pi^{*M} - \Pi^{*R}$ in the (α_1, α_2)-space.

difference between the trade-in prices in two scenarios is larger when the value of γ is higher, as shown by Figure 4. The customers pay for a lower price, when the value of Δ_m is higher, because the trade-in service provider offers a larger rebate rate. The price gap between two scenarios is smaller when the value of Δ_m is greater, because the trade-in price in Scenario M is more sensitive to Δ_m, according to Figure 4.

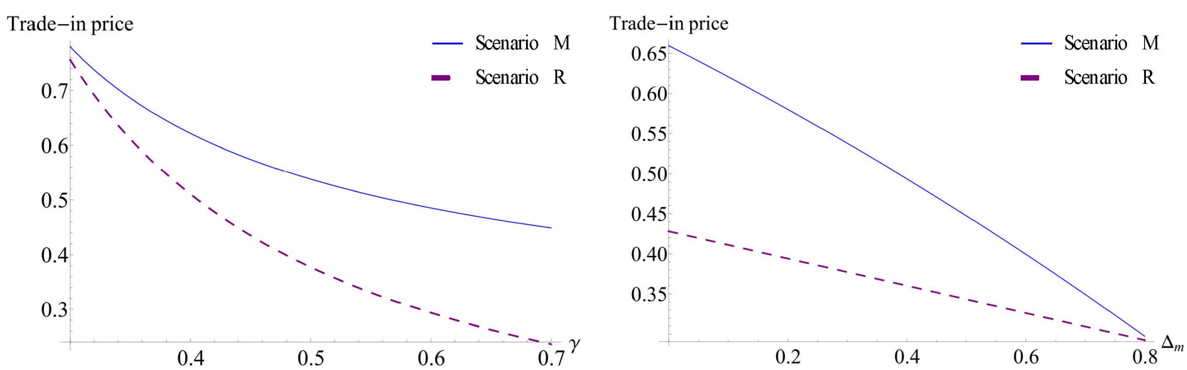

Figure 4: Trade-in prices in two scenarios

6 Summary and Concluding Remarks

The recent COVID-19 has illuminated the environmental folly of extract-produce-use-dump economic model of material and energy flows, and the adoption of circular economy has been touted to be a viable solution (Ibn-Mohammed et al., 2020). The trade-in service is a typical representative of the circular economic model, which can promote environmental friendly repurchase and save the manufacturing costs and resource inputs.

In this paper, we develop and analyze a game-theoretic model to investigate the trade-in service in a two-period CLSC. We develop a dependent demand across periods based on (1) the return function by Genc and De Giovanni (2018) and (2) the characteristics of trade-in activities. We also involve a price-based trade-in rebate rate into our model as a decision variable other than a fixed or exogenous term as in extant publications. Our model can help managers decide whether to outsource the trade-in service or not under different market conditions, and assist them in the optimal decisions on prices and rebate rate in different scenarios.

We make methodological and conceptual contributions to the CLSC literature by introducing a dependent demand function and the rebate rate as a decision variable as well as considering both direct sales and the sales achieved under the trade-in service. For a decentralized supply chain consisting of a manufacturer and a retailer, we investigate two different scenarios and obtain some important findings. Firstly, we find that both two firms can benefit more when they provide the trade-in service by themselves in most cases. Xiao (2017) found that a firm can obtain a higher profit from being a "free-rider" rather than operating the trade-in service, as the author developed a trade-in model by analyzing the customer choice behavior and treating each firm's pricing decision as an exogenous factor in the other firm's profit function. Differently, under the trade-in mechanism in a two-period CLSC game, firms can enjoy higher profits when they provide trade-in service by themselves in most cases, and there is no dominant scenario in the supply chain. We derive the conditions for two firms to individually or jointly choose the optimal trade-in strategy under different market conditions. Secondly, we obtain the conditions under which both the manufacturer and the retailer unanimously prefer that the manufacturer outsources the trade-in service to the retailer, and the conditions for the optimal trade-in strategies in other cases. Thirdly, we find that customers enjoy a lower price under the trade-in service (and also a lower first-period price) when trade-in service is provided by the retailer. The demand in each period is larger in Scenario R, although the second-period retail price is higher. In our numerical study and sensitivity analysis, we numerically investigate the impact of major parameters on the optimal prices, rebate rate, profits, and the conditions for firms and CLSC to choose their optimal trade-in strategies. The difference between the trade-in prices in two scenarios is larger when trade-in customers are more sensitive to the price, but it is smaller when the manufacturer enjoys a higher net gain from handling returned products.

Acknowledgement 1 *The second author (Jiangtao Hong) was sponsored by the National Social Science Fund of China (#17FGL010) and the Postgraduate Innovation Project of Shanghai University of International Business and Economics. The fourth author (Mingming Leng) was supported by the General Research Fund (GRF) of the Hong Kong Research Grants Council under Research Project No. LU13500020.* ■

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Online Supplements

"Game-Theoretic Analysis of Trade-In Services in Closed-Loop Supply Chains"

Yuting Quan, Jiangtao Hong, Jingpu Song, Mingming Leng

Appendix A Proofs

Proof of Proposition 1. The two firms optimize their objective functions in two periods, in each of which there are two decision-making stages. In Scenario M, the two firms' optimization problems are

$$\begin{aligned}\max_{w_1,w_2,\tau_m} \Pi_m^M &= q_1(w_1-c_1)+\delta[Q_2^M(w_2-c_2)+\hat{q}_2(\Delta_m-\tau_m p_1)];\\ \max_{p_1,p_2} \Pi_r^M &= q_1(p_1-w_1)+\delta Q_2^M(p_2-w_2).\end{aligned}$$

Given the manufacturer's decisions, the retailer optimizes his second-period profit to find the optimal retail price as

$$p_2=\frac{\alpha_2+q_1+\gamma\tau_m p_1+(\beta+\gamma)w_2}{2(\beta+\gamma)}.$$

Then, using the above, the manufacturer optimizes its second-period profit by choosing the wholesale price w_2 and trade-in rebate rate τ_m. The Hessian matrix is computed a

$$H=\begin{pmatrix}\partial^2\pi_{m_2}^M/\partial w_2^2 & \partial^2\pi_{m_2}^M/(\partial w_2\partial\tau_m)\\ \partial^2\pi_{m_2}^M/(\partial\tau_m\partial w_2) & \partial^2\pi_{m_2}^M/\partial\tau_m^2\end{pmatrix}=\begin{pmatrix}-(\beta+\gamma) & \gamma p_1\\ \gamma p_1 & -\gamma(2\beta+\gamma)p_1^2/(\beta+\gamma)\end{pmatrix},$$

which is negative definite, that is, the profit function is strictly concave in w_2 and τ_m. Hence, the unique solutions can be found as

$$w_2=\frac{\alpha_2+\beta c_2}{2\beta}>0 \text{ and } \tau_m=\frac{\gamma\alpha_2+\gamma\beta\Delta_m-\beta q_1}{2\gamma\beta p_1}>0, \text{ where } \gamma>\frac{\beta q_1}{\alpha_2+\beta\Delta_m}.$$

Substituting the above into p_2 and Q_2^M gives

$$\begin{aligned}p_2 &= \frac{(3\beta+2\gamma)\alpha_2+\beta q_1+(\beta+\gamma)\beta c_2+\gamma\beta\Delta_m}{4\beta(\beta+\gamma)},\\ Q_2^M &= q_2+\hat{q}_2=\frac{1}{4}[\alpha_2+q_1-(\beta+\gamma)c_2+\gamma\Delta_m].\end{aligned}$$

Next, we consider period 1 in which the retailer chooses p_1 to optimize his total discounted profits: $\max_{p_1}\pi_{r_1}^M+\delta\pi_{r_2}^M$. Using the values of w_2, τ_m, p_2, and Q_2^M, we have

$$p_1=\frac{[8(\beta+\gamma)-\delta\beta]\alpha_1-\delta\beta\alpha_2+\delta\beta(\beta+\gamma)c_2-\delta\gamma\beta\Delta_m+8\beta(\beta+\gamma)w_1}{\beta[16(\beta+\gamma)-\delta\beta]} \text{ and } q_1=\alpha_1-\beta p_1.$$

Then, the manufacturer chooses an optimal value of w_1 to maximize her total discounted profits $\pi_{m_1}^M+\delta\pi_{m_2}^M$. Using $w_2(w_1)$, $\tau_m(w_1)$, and $p_1(w_1)$, we find w_1^{*M} as given in this proposition. ■

Proof of Proposition 2. In Scenario R in which the retailer operates the trade-in service, the two firms' optimization problems are given as follows:

$$\begin{aligned}\max_{w_1,w_2,p_r} \Pi_m^R &= q_1(w_1-c_1)+\delta Q_2^R(w_2-c_2)+\delta\hat{q}_2((\Delta_m-p_r+g)),\\ \max_{p_1,p_2,\tau_r} \Pi_r^R &= q_1(p_1-w_1)+\delta Q_2^R(p_2-w_2)+\delta\hat{q}_2(p_r-g-\tau_r p_1).\end{aligned}$$

First, given the manufacturer's decisions, the retailer optimizes his second-period profit to determine the retail price p_2 and rebate rate τ_r. The Hessian matrix is

$$H=\begin{pmatrix}\partial^2\pi_{r_2}^R/\partial p_2^2 & \partial^2\pi_{r_2}^R/(\partial p_2\partial\tau_r)\\ \partial^2\pi_{r_2}^R/(\partial\tau_r\partial p_2) & \partial^2\pi_{r_2}^R/\partial\tau_r^2\end{pmatrix}=\begin{pmatrix}-2(\beta+\gamma) & 2\gamma p_1\\ 2\gamma p_1 & -2\gamma p_1^2\end{pmatrix},$$

which is negative definite and thus, the retailer's profit function is strictly concave in p_2 and τ_r. We then have unique solutions as

$$p_2=\frac{\alpha_2+\beta w_2}{2\beta}>0 \text{ and } \tau_r=\frac{\gamma\alpha_2+\gamma\beta(p_r-g)-\beta q_1}{2\gamma\beta p_1}>0, \text{ where } \gamma>\frac{\beta q_1}{\alpha_2-\beta(p_r-g)}.$$

Then, the manufacturer optimizes her second-period profit $\pi_{m_2}^R$ to determine the wholesale price w_2 and the buy-back price p_r. The Hessian matrix is

$$H=\begin{pmatrix}\partial^2\pi_{m_2}^R/\partial w_2^2 & \partial^2\pi_{m_2}^R/(\partial w_2\partial p_r)\\ \partial^2\pi_{m_2}^R/(\partial p_r\partial w_2) & \partial^2\pi_{m_2}^R/\partial p_r^2\end{pmatrix}=\begin{pmatrix}-(\beta+\gamma) & \gamma\\ \gamma & -\gamma\end{pmatrix},$$

which is negative definite and thus, the retailer's profit function is strictly concave in w_2 and p_r. We then have unique solutions as

$$p_2=\frac{\alpha_2+\beta c_2}{2\beta}>0 \text{ and } p_r=\frac{\gamma\alpha_2+\gamma\beta(\Delta_m+2g)-\beta q_1}{2\gamma\beta}>0.$$

Substituting the above into p_2, τ_r and Q_2^R gives

$$\begin{aligned}p_2 &= \frac{3\alpha_2+\beta c_2}{4\beta},\ \tau_r=\frac{3\gamma\alpha_2-3\beta q_1+\beta\gamma\Delta_m}{4\beta\gamma p_1},\\ Q_2^R &= q_2+\hat{q}_2=\frac{1}{4}[\alpha_2+q_1-(\beta+\gamma)c_2+\gamma\Delta_m].\end{aligned}$$

In period 1, the retailer optimally chooses p_1 to maximize his total discounted profit $\pi_{r_1}^M+\delta\pi_{r_2}^M$. Using the values of w_2, p_2, τ_r, and q_2, we have

$$p_1=\frac{(8\gamma-\delta\beta)\alpha_1+\delta\beta\gamma(\Delta_m-c_2)+8\beta\gamma w_1}{\beta(16\gamma-\delta\beta)},$$

and $q_1=\alpha_1-\beta p_1$. The manufacturer then chooses an optimal wholesale price to maximize her total discounted profits $\pi_{m_1}^M+\delta\pi_{m_2}^M$. Using $w_2(w_1)$, $p_r(w_1)$, $\tau_r(w_1)$, and $p_1(w_1)$, we obtain w_1^{*R} as in this proposition. ■

Proof of Proposition 3. Using Proposition 1 and Proposition 2, we compute the first-order derivatives of w_1^M, w_2^M, p_1^M, p_2^M and w_1^{*R}, w_2^{*R}, p_1^{*R}, p_2^{*R} w.r.t. Δ_m as:

$$\begin{aligned}
\frac{\partial w_1^{*M}}{\partial \Delta_m} &= \frac{-\delta[16\gamma\beta(4\beta^2+5\gamma\beta+\gamma^2)+\delta\gamma^2\beta^2]}{32\beta(\beta+\gamma)^2(8\gamma-\delta\beta)} < 0,\ \frac{\partial w_2^{*M}}{\partial \Delta_m} = 0\\
\frac{\partial p_1^{*M}}{\partial \Delta_m} &= \frac{-\delta\gamma\beta-8\delta\beta(\beta+\gamma)[16\gamma\beta(4\beta^2+5\gamma\beta+\gamma^2)+\delta\gamma^2\beta^2]}{\beta[16(\beta+\gamma)-\delta\beta]} < 0,\\
\frac{\partial p_2^{*M}}{\partial \Delta_m} &= \frac{\beta(64-\delta)}{64\beta(8-\delta)} > 0.\\
\frac{\partial w_1^{*R}}{\partial \Delta_m} &= -\frac{\delta(16\gamma+\delta\beta)}{32(8\gamma-\delta\beta)} < 0,\ \frac{\partial w_2^{*R}}{\partial \Delta_m} = 0\\
\frac{\partial p_1^{*R}}{\partial \Delta_m} &= -\frac{3\delta\beta\gamma}{4\beta(8\gamma-\delta\beta)} < 0,\ \frac{\partial p_2^{*R}}{\partial \Delta_m} = 0.
\end{aligned}$$

This proposition is thus proved. ■

Proof of Proposition 4. Setting $\delta = 1$ and $c_i = 0$ in τ_m^* and τ_r^*, we have

$$\begin{aligned}
\frac{\partial \tau_m^*}{\partial \Delta_m} &= \frac{2(8\gamma^2+7\gamma\beta-\beta^2)[\alpha_1(24\gamma^2+17\gamma\beta-8\beta^2)+\gamma\alpha_2(3\gamma+5\beta)]}{[4\alpha_1(\beta^2-5\beta\gamma-6\gamma^2)-\gamma\beta\alpha_2+\gamma\beta(3\gamma+4)\Delta_m^2]^2} > 0;\\
\frac{\partial \tau_r^*}{\partial \Delta_m} &= \frac{\beta(8\gamma-\beta)[(24\gamma-13\beta)\alpha_1+9\gamma\beta\alpha_2}{[4(\beta-6\gamma)\alpha_1+3\gamma\beta\Delta_m]^2} > 0.
\end{aligned}$$

which imply the results in this proposition. ■

Proof of Proposition 5. Setting $\delta = 1$ and $c_i = 0$, we find

$$\begin{aligned}
\lim_{\alpha_1,\alpha_2\to\alpha} \frac{\partial w_1^{*M}}{\partial \gamma} &= \frac{\alpha(101\beta^3+419\gamma\beta^2+16\gamma^2\beta-320\gamma^3)+2\beta\Delta_m(32\beta^3+49\gamma\beta^2+212\gamma^2\beta+204\gamma^3)}{32(\beta-8\gamma)^2(\beta+\gamma)^3} > 0;\\
\lim_{\alpha_1,\alpha_2\to\alpha} \frac{\partial p_1^{*M}}{\partial \gamma} &= \frac{\alpha\beta(7+16\gamma)+\Delta_m(4\beta^2+6\gamma\beta+11\gamma^2)}{4(\beta-8\gamma)^2(\beta+\gamma)^2} > 0,\ \frac{\partial w_2^{*M}}{\partial \gamma} = 0;\\
\lim_{\alpha_1,\alpha_2\to\alpha} \frac{\partial p_2^{*M}}{\partial \gamma} &= \frac{-\alpha(11\beta^3-51\gamma\beta^2+240\gamma^2\beta+320\gamma^3)+2\beta\Delta_m\gamma(-31\beta^2+76\gamma\beta+116\gamma^2)}{16(\beta-8\gamma)^2(\beta+\gamma)^2} < 0.
\end{aligned}$$

when α_1 and α_2 are in the neighbourhood of α because of the properties of continuous function. We also compute

$$\frac{\partial w_1^{*R}}{\partial \gamma} = \frac{8\alpha_1+3\beta\Delta_m}{4(\beta-8\gamma)^2} > 0,\ \frac{\partial p_1^{*R}}{\partial \gamma} = \frac{8\alpha_1+3\beta\Delta_m}{4(\beta-8\gamma)^2} > 0,\ \frac{\partial w_2^{*R}}{\partial \gamma} = 0,\ \frac{\partial w_2^{*R}}{\partial \gamma} = 0.$$

Thus,

$$\begin{aligned}
\frac{\partial w_1^{*M}}{\partial \gamma} &> 0,\ \frac{\partial p_1^{*M}}{\partial \gamma} > 0,\ \frac{\partial p_2^{*M}}{\partial \gamma} < 0,\ \frac{\partial w_1^{*R}}{\partial \gamma} > 0,\\
\frac{\partial p_1^{*R}}{\partial \gamma} &> 0,\ \frac{\partial w_2^{*M}}{\partial \gamma} = 0,\ \frac{\partial w_2^{*R}}{\partial \gamma} = 0 \text{ and } \frac{\partial p_2^{*R}}{\partial \gamma} = 0.
\end{aligned}$$

■

Proof of Proposition 6. Setting $\delta = 1$ and $c_i = 0$, we compute

$$\begin{aligned}\lim_{\alpha_1,\alpha_2\to\alpha}\frac{\partial\tau_m^*}{\partial\gamma} &= \frac{\beta\alpha^2(192\gamma^2+272\gamma\beta+151\beta^2)+2\alpha\beta^2\Delta_m(2\gamma^2+31\gamma\beta+120\beta^2)}{2[\alpha(4\beta^2-21\gamma\beta-24\gamma^2)+\beta\gamma\Delta_m(4+3\gamma)]^2}\\ &\quad -\frac{\beta^2\Delta_m^2(53\gamma^2+48\gamma\beta+32\beta^2)}{2[\alpha(4\beta^2-21\gamma\beta-24\gamma^2)+\beta\gamma\Delta_m(4+3\gamma)]^2} > 0,\\ \lim_{\alpha_1,\alpha_2\to\alpha}\frac{\partial\tau_r^*}{\partial\gamma} &= \frac{\beta(8\alpha+3\beta\Delta_m)(60\alpha-13\beta\Delta_m)}{4[4(\beta-6\gamma)\alpha+3\gamma\beta\Delta_m]^2} > 0.\end{aligned}$$

It thus follows that $\partial\tau_m^*/\partial\gamma > 0$ and $\partial\tau_r^*/\partial\gamma > 0$ when α_1 and α_2 are in the neighbourhood of α because of the properties of continuous function. ■

Proof of Proposition 7. Setting $\delta = 1$ and $c_i = 0$, we have

$$\frac{\partial p_r^*}{\partial\Delta_m} = \frac{32\beta\gamma-7\beta^2}{8\beta(8\gamma-\beta)} > 0, \text{ when } \gamma > \frac{7\beta}{32}; \text{ and } \frac{\partial p_r^*}{\partial\gamma} = \frac{8\alpha_1+3\beta\Delta_m}{(\beta-8\gamma)^2} > 0.$$

■

Proof of Proposition 8. Setting $\delta = 1$ and $c_i = 0$, we find that, in period 1,

$$w_1^{*M} - w_1^{*R} = \frac{(48\gamma^2+43\gamma\beta-4\beta^2)\alpha_2 - 8\beta(\beta+\gamma)\alpha_1 - (48\gamma^2+46\gamma\beta-\beta^2)\beta\Delta_m}{32(8\gamma-\beta)(\beta+\gamma)^2} > 0,$$

where

$$\gamma > \frac{\beta}{8} \text{ and } \alpha_2 > \frac{8\beta(\beta+\gamma)\alpha_1 + (48\gamma^2+46\gamma\beta-\beta^2)\beta\Delta_m}{48\gamma^2+43\gamma\beta-4\beta^2};$$

and

$$p_1^{*M} - p_1^{*R} = \frac{(\alpha_2-\beta\Delta_m)\gamma}{4(8\gamma-\beta)(\beta+\gamma)} > 0, \text{ when } \gamma > \frac{\beta}{8}.$$

In addition,

$$(p_1^{*R} - w_1^{*R}) - (p_1^{*M} - w_1^{*M}) = \frac{(40\gamma^2+35\gamma\beta-4\beta^2)\alpha_2 - 8\beta(1+\gamma)\alpha_1 - (40\gamma^2+38\gamma\beta-\beta^2)\beta\Delta_m}{32(8\gamma-\beta)(\beta+\gamma)^2} > 0,$$

where

$$\gamma > \frac{\beta}{8} \text{ and } \alpha_2 > \frac{8\beta(1+\gamma)\alpha_1 + (40\gamma^2+38\gamma\beta-\beta^2)\beta\Delta_m}{40\gamma^2+35\gamma\beta-4\beta^2}.$$

In period 2, we compute

$$p_2^{*M} - p_2^{*R} = \frac{\gamma[8\beta(1+\gamma)\alpha_1 + (32\gamma+31\beta)\gamma\beta\Delta_m - (32\gamma^2+28\gamma\beta-3\beta^2)\alpha_2]}{16(8\gamma-\beta)(\beta+r)^2} < 0,$$

where

$$\gamma > \frac{\beta}{8} \text{ and } \alpha_2 > \frac{8\beta(\beta+\gamma)\alpha_1 + (32\gamma+31\beta)\gamma\beta\Delta_m}{32\gamma^2+28\gamma\beta-3\beta^2}; \text{ and } w_2^{*M} = w_2^{*R}.$$

Moreover,

$$p_2^{*M} - \tau_m^* p_1^{*M} - (p_2^{*R} - \tau_r^* p_1^{*R}) = \frac{\gamma[(32\gamma^2 + 25\gamma\beta - 6\beta^2)\alpha_2 - 8\beta(1+\gamma)\alpha_1 - (32\gamma^2 + 28\beta\gamma - 3\beta^2)\beta\Delta_m]}{16(8\gamma - \beta)(\beta + r)^2} > 0,$$

where

$$\gamma > \frac{\beta}{8} \text{ and } \alpha_2 > \frac{8\beta(\beta+\gamma)\alpha_1 + (32\gamma^2 + 28\beta\gamma - 3\beta^2)\beta\Delta_m}{32\gamma^2 + 25\gamma\beta - 6\beta^2}.$$

■

Proof of Proposition 9. Setting $\delta = 1$ and $c_i = 0$, we compute

$$q_1^{*M} - q_1^{*R} = -\frac{\gamma\beta(\alpha_2 - \beta\Delta_m)}{4(8\gamma - \beta)(\beta + r)} < 0, \text{ when } \gamma > \frac{\beta}{8},$$

and

$$\hat{q}_2^{*M} - \hat{q}_2^{*R} = -\frac{\gamma[(32\gamma^2 + 29\gamma\beta - 2\beta^2)\alpha_2 - 8\beta(\beta+\gamma)\alpha_1 - (32\gamma^2 + 32\gamma\beta + \beta^2)\beta\Delta_m]}{16(8\gamma - \beta)(\beta + r)^2} < 0,$$

where

$$\gamma > \frac{\beta}{8} \text{ and } \alpha_2 > \frac{8\beta(\beta+\gamma)\alpha_1 + (32\gamma^2 + 32\gamma\beta + \beta^2)\beta\Delta_m}{32\gamma^2 + 28\gamma\beta - 3\beta^2}.$$

In addition,

$$Q_2^{*M} - Q_2^{*R} = -\frac{\gamma\beta(\alpha_2 - \beta\Delta_m)}{16(8\gamma - \beta)(\beta + r)} < 0, \text{ when } \gamma > \frac{\beta}{8}.$$

■

Proof of Proposition 10. Setting $\delta = 1$ and $c_i = 0$, we find that, if $f(\alpha_1, \alpha_2) = \Omega_4^2\alpha_1^2 + 2\Omega_8\alpha_1 - 2\Omega_2\Omega_4\alpha_1\alpha_2 - 2\Omega_7\alpha_2 + +\Omega_5\alpha_2^2 + \Omega_6 > 0$, where

$$\begin{aligned}
\Omega_1 &= 16(\beta+\gamma)^2(8\gamma-\beta)^2 > 0,\ \Omega_2 = 48\gamma^2 + 43\gamma\beta - 4\beta^2 > 0,\\
\Omega_3 &= 48\gamma^2 + 46\gamma\beta - \beta^2 > 0,\ \Omega_4 = \beta(\beta+\gamma) > 0,\ \Omega_5 = (\Omega_1 + \Omega_2)\beta^2 > 0,\\
\Omega_6 &= (\Omega_1 + 6\gamma\Omega_3 + 327\beta\gamma^2 - 271\beta^2\gamma - 19\beta^3)\beta^3\Delta_m^2 > 0,\\
\Omega_7 &= \Omega_1 + 3\Omega_2 + 192\beta\gamma + 172\beta^2 > 0,\ \Omega_8 = \beta\Delta_m\Omega_3\Omega_4 > 0,
\end{aligned}$$

then

$$\Pi^{*M} - \Pi^{*R} = \frac{\gamma(\Omega_4^2\alpha_1^2 + 2\Omega_8\alpha_1 - 2\Omega_2\Omega_4\alpha_1\alpha_2 - 2\Omega_7\alpha_2 + \Omega_5\alpha_2^2 + \Omega_6)}{256\beta(\beta - 8\gamma)^2(\beta+\gamma)^3} > 0.$$

When $\gamma \to \beta$, $f(\alpha_1, \alpha_2)|_{\gamma\to\beta} = 256\alpha_1^2 + 32(93\beta\Delta_m - 87\alpha_2)\alpha_1 - 7490\beta\Delta_m\alpha_2 + 3223\alpha_2^2 + 4303\beta^2\Delta_m^2$ and $\lim_{\gamma\to\beta}(\Pi^{*M} - \Pi^{*R}) > 0$, when $f(\alpha_1, \alpha_2)|_{\gamma\to\beta} > 0$.

Similarly, we compute

$$\begin{aligned}
u(\alpha_1, \alpha_2)|_{\gamma\to\beta} &= -256\alpha_1^2 + 3164\alpha_2^2 - 1088\beta\Delta_m\alpha_1 - 3192\beta\Delta_m\alpha_2 - 344\beta^2\Delta_m^2,\\
v(\alpha_1, \alpha_2)|_{\gamma\to\beta} &= -768\alpha_1^2 + 3105\alpha_2^2 + 32(87\alpha_2 - 161\beta\Delta_m)\alpha_1 + 1106\beta\Delta_m\alpha_2 - 4991\beta^2\Delta_m^2,
\end{aligned}$$

and

$$\lim_{\gamma\to\beta}(\Pi_m^{*M} - \Pi_m^{*R}) > 0 \text{ when } u(\alpha_1, \alpha_2)|_{\gamma\to\beta} > 0\text{; and } \lim_{\gamma\to\beta}(\Pi_r^{*M} - \Pi_r^{*R}) < 0 \text{ when } v(\alpha_1, \alpha_2)|_{\gamma\to\beta} > 0.$$

■