Subsidizing Purchases of Public Interest Products: A Duopoly Analysis under a Subsidy Scheme

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Abstract: We investigate a symmetric duopoly setting in which two manufacturers produce the traditional and public interest (PI) products under a government's subsidy scheme. A higher subsidy can increase the sales of the PI product but reduce the sales of the traditional product. Then, we study an asymmetric setting in which a manufacturer produces one of the two products and the other manufacturer produces both products. The government's optimal subsidy is increasing in the marginal externality of the PI product.

Keywords: public interest product; subsidy; duopoly; competition.

1 Introduction

In today's markets, there exist a variety of public interest (PI) products that possess functional attributes similar to traditional products. The PI products have a substantial social impact, as the consumption of these products can result in some direct outcomes as expected, and also generate indirect, or involuntary, benefits to the persons or firms who do not use the products. For example, an energy efficient ("green") air conditioner is a typical PI product, because the air conditioner can directly serve those persons who use it, and also benefits the society by consuming less energy than traditional air conditioners. An electric vehicle is another PI product, because it can help reduce carbon emissions compared with traditional vehicles. The electric vehicle benefits not only its driver but also all the persons around the vehicle. As Ovchinnikov and Raz [7] mentioned, other PI products include the energy efficient appliances (e.g., water-saving toilets and energy-saving lamp), the eco-consumables (e.g., organic fertilizer), etc.

Observing the substantial social value of the PI products, some governments have implemented incentive schemes to stimulate purchases of the PI products. The most often used one is the consumer subsidy scheme under which a government awards a subsidy to each consumer who buys a PI product. The subsidy scheme mainly aims at improving the affordability of PI products and maximizing the social welfare. Some governments have subsidized consumers, encouraging them to buy electric vehicles. For instance, Desk [2] reported that in September 2011, the Swedish government approved a subsidy program to provide a subsidy of 40,000 kr per car for purchases of electric cars and other "super green cars" with ultra-low carbon emissions since January 2012. An *interesting* research question arises as follows: Can a subsidy scheme help reduce the selling price and increase the sales of the PI products? It thus behooves us to

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investigate manufacturers' pricing decisions and profits as well as the sales of the PI products under a government's subsidy scheme.

In practice, a manufacturer may produce only the traditional product, only the PI product, or both products. For example, some automobile manufacturers (e.g., Mazda and Chrysler) only make the fuel vehicles (traditional products), some manufacturers (e.g., Fisker Automotive and Tesla Motors) only focus on the production of electric vehicles (PI products), and the others (e.g., Toyota and Honda) produce both fuel vehicles and electric vehicles. First, in Section 2, we consider a symmetric duopoly setting in which two manufacturers make both the traditional and the PI products. For the game, we obtain the two manufacturers' pricing decisions in Nash equilibrium, construct a social welfare function for the government, and maximize the social welfare to find the optimal subsidy. For the symmetric duopoly setting, we also analyze the case in which each manufacturer produces only the PI product. Then, in Section 3, we investigate an asymmetric duopoly setting in which a manufacturer produces either the traditional product or the PI product, whereas the other manufacturer produces both the traditional and the PI products. For the setting, we also obtain the pricing decisions in Nash equilibrium, and maximize the social welfare to find the optimal subsidy for the government.

Some relevant publications are concerned with a government's subsidy scheme for the adoption of green technologies. For example, Huang et al. [5] investigated an automobile scrappage program under which a government awards a subsidy to each consumer who trades in his or her used automobile with a new fuel-efficient automobile. Chan, Leng, and Liang [1] examined the impact of a government's tax reduction policies on the sales of new automobiles and the profits of manufacturers. Luo et al. [6] investigated a government's price-discount incentive scheme that involves a price discount rate and a subsidy ceiling. Those publications mainly focus on the impact of a government's subsidy without considering the competition between traditional and energy-efficient products. Moreover, the relevant publications did not consider the externality of the PI products, which captures the most important feature of such products. Different from them, we incorporate the externality into the social welfare function. A future research direction is suggested in Section 4, and the proofs of propositions are relegated to online Appendix A.

2 Game-Theoretic Analysis and Social Welfare in a Symmetric Duopoly Setting

We consider a symmetric duopoly setting in which two manufacturers (i.e., manufacturer i, $i = 1, 2$) make and sell the same or similar products to consumers in a market. That is, in the setting, both manufacturers produce the traditional and the PI products. The two manufacturers hold different brands for their products, which possess similar quality, function, and other attributes. A government implements a subsidy scheme to promote the PI products made by the two manufacturers. Since this paper is concerned with the PI products, we focus on the two-product case in which each manufacturer makes the two types of products and the PI-product case in which only the PI products are produced.

For each case, we analyze a "simultaneous-move" game in which the two manufacturers make their pricing decisions under the subsidy scheme. Then, we compute the social welfare that is generated by implementation of the subsidy scheme and maximize it for the government's optimal subsidy.

2.1 The Duopoly Analysis with Two Products

We consider a symmetric duopoly setting in which the two manufacturers sell their traditional and PI products to compete for consumers.

2.1.1 The Sales Functions

Under a subsidy scheme, the government provides a consumer with a subsidy \$$s$ when the consumer purchases a PI product. If the consumer decides to buy a traditional product from manufacturer i ($i = 1, 2$), then he or she needs to pay \$$p_i$; otherwise, the consumer should make his or her own payment \$$(\hat{p}_i - s)$ to buy a PI product from manufacturer i. Hereafter, to simply distinguish the two product types, we use "ˆ" to indicate the PI product.

We consider the following inverse sales curve for each product made by manufacturer i:

$$\begin{cases} p_i = \theta - (q_i + \beta q_{3-i}) - \gamma(\hat{q}_i + \beta \hat{q}_{3-i}), \\ \hat{p}_i - s = \theta - (\hat{q}_i + \beta \hat{q}_{3-i}) - \tilde{\gamma}(q_i + \beta q_{3-i}), \end{cases} \tag{1}$$

where θ denotes the possible highest price; p_i ($\hat{p}_i$) and q_i ($\hat{q}_i$) are the price and the sales for manufacturer i's traditional (PI) product, respectively; β reflects the substitutability (competition) between the two manufacturers; and γ ($\tilde{\gamma}$) represents the degree of substituting the PI (traditional) product for the traditional (PI) product. As the value of γ ($\tilde{\gamma}$) increases, a consumer is more willing to accept the PI (traditional) product. Similar inverse sales/demand functions have been widely used by many scholars in the economics and operations management areas to analyze various competition-related issues with two or more products; see, e.g., Dobson and Waterson ([3], [4]). Solving equations in (1) for q_i and $\hat{q}_i$ ($i = 1, 2$) yields the sales functions as

$$\begin{cases} q_i = \dfrac{(1-\gamma)(1-\beta)\theta - p_i + \gamma(\hat{p}_i - s) + \beta[p_{3-i} - \gamma(\hat{p}_{3-i} - s)]}{(1-\beta^2)(1-\gamma\tilde{\gamma})}, \\ \hat{q}_i = \dfrac{(1-\tilde{\gamma})(1-\beta)\theta - (\hat{p}_i - s) + \tilde{\gamma}p_i + \beta[(\hat{p}_{3-i} - s) - \tilde{\gamma}p_{3-i}]}{(1-\beta^2)(1-\gamma\tilde{\gamma})}. \end{cases} \tag{2}$$

2.1.2 Game-Theoretic Analysis under a Subsidy Scheme

Since the two manufacturers' products possess similar attributes, they incur an identical unit acquisition cost for each product. Accordingly, we denote each manufacturer's unit cost of the traditional product and that of the PI product by c and $\hat{c}$, respectively. Manufacturer i's profit drawn from selling a traditional (PI) product is $p_i - c$ ($\hat{p}_i - \hat{c}$). The manufacturer's total profit generated from the sales of both products can be calculated as $\pi_i = (p_i - c)q_i + (\hat{p}_i - \hat{c})\hat{q}_i$, which, using (2), can be specified as

$$\begin{aligned} \pi_i \;=\; & \frac{1}{J}\{(p_i - c)[(1-\gamma)(1-\beta)\theta - p_i + \gamma(\hat{p}_i - s) + \beta p_{3-i} - \beta\gamma(\hat{p}_{3-i} - s)] \\ & +(\hat{p}_i - \hat{c})[(1-\tilde{\gamma})(1-\beta)\theta - (\hat{p}_i - s) + \tilde{\gamma}p_i + \beta(\hat{p}_{3-i} - s) - \beta\tilde{\gamma}p_{3-i}]\}, \end{aligned} \tag{3}$$

where $J \equiv (1-\beta^2)(1-\gamma\tilde{\gamma})$.

Proposition 1 When the two manufacturers sell their traditional and PI products under the government's subsidy scheme, then, in Nash equilibrium, manufacturer i's ($i = 1, 2$) price for his traditional product is

$$p_{1i} = \frac{c[2-\beta-(1-\beta)\gamma^2-\gamma\tilde{\gamma}] + (1-\beta)\{(s-\hat{c})(\tilde{\gamma}-\gamma) + [(1+\gamma)(2-\gamma-\tilde{\gamma}) - \beta(1-\gamma\tilde{\gamma})]\theta\}}{(1-\beta)[4-(\gamma+\tilde{\gamma})^2] + \beta^2(1-\gamma\tilde{\gamma})},$$

and that for the PI product is

$$\begin{aligned} \hat{p}_{1i} &= \frac{\hat{c}[2-\beta-(1-\beta)\tilde{\gamma}^2-\gamma\tilde{\gamma}]}{(1-\beta)[4-(\gamma+\tilde{\gamma})^2] + \beta^2(1-\gamma\tilde{\gamma})} \\ &\quad + \frac{(1-\beta)\{c(\tilde{\gamma}-\gamma) + s(2-\beta-\gamma^2-\gamma\tilde{\gamma}+\beta\tilde{\gamma}^2) + [(1+\gamma)(2-\gamma-\tilde{\gamma}) - \beta(1-\gamma\tilde{\gamma})]\theta\}}{(1-\beta)[4-(\gamma+\tilde{\gamma})^2] + \beta^2(1-\gamma\tilde{\gamma})}. \end{aligned}$$

As a result, the sales for the traditional and PI products are computed as

$$q_{1i} = \frac{(\theta-c)(2-\beta) - \theta[(1-\beta)\gamma+\tilde{\gamma}] - (s-\hat{c})[\gamma+(1-\beta)\tilde{\gamma}]}{(1+\beta)\{(1-\beta)[4-(\gamma+\tilde{\gamma})^2] + \beta^2(1-\gamma\tilde{\gamma})\}}, \tag{4}$$

and

$$\hat{q}_{1i} = \frac{(2-\beta-\gamma-\tilde{\gamma}+\beta\tilde{\gamma})\theta - (\hat{c}-s)(2-\beta) + c(\gamma+\tilde{\gamma}-\beta\gamma)}{(1+\beta)\{(1-\beta)[4-(\gamma+\tilde{\gamma})^2] + \beta^2(1-\gamma\tilde{\gamma})\}}. \quad \blacksquare \tag{5}$$

We learn from the above proposition that the prices for traditional products may increase or decrease as the government subsidizes the purchases of the PI product, which depends on the substitutability between the two products (i.e., γ and $\tilde{\gamma}$). Specifically, if the degree of substituting the traditional products for the PI products (i.e., $\tilde{\gamma}$) is larger than that of substituting the PI products for the traditional products (i.e., γ), then the price for each manufacturer's traditional product is increasing in subsidy s. The price for each PI product is increasing in s. We also compute a consumer's net payment for the purchase of a PI product as

$$\begin{aligned} \hat{p}_{1i} - s &= \frac{\hat{c}(2-\beta-\tilde{\gamma}^2-\gamma\tilde{\gamma}+\beta\tilde{\gamma}^2)}{(1-\beta)[4-(\gamma+\tilde{\gamma})^2] + \beta^2(1-\gamma\tilde{\gamma})} \\ &\quad + \frac{(1-\beta)\{c(\tilde{\gamma}-\gamma) - s[(1-\gamma\tilde{\gamma}) + (1-\beta)(1-\tilde{\gamma}^2)]\}}{(1-\beta)[4-(\gamma+\tilde{\gamma})^2] + \beta^2(1-\gamma\tilde{\gamma})} \\ &\quad + \frac{(1-\beta)[\theta(1+\gamma)(2-\gamma-\tilde{\gamma}) - \beta(1-\gamma\tilde{\gamma})]}{(1-\beta)[4-(\gamma+\tilde{\gamma})^2] + \beta^2(1-\gamma\tilde{\gamma})}, \end{aligned}$$

which is decreasing in s. This means that as the government raises its subsidy, each consumer pays less. The price for each traditional (PI) product is increasing in the product's unit cost c ($\hat{c}$), but may be increasing or decreasing in the substitute's unit cost $\hat{c}$ (c), which depends on the substitutability between the two products.

Using (4) and (5) we find that the sales of a PI product are increasing in subsidy s, whereas the sales of a traditional product are decreasing in s. The results imply that as the government executes an incentive scheme to encourage consumers to buy PI products, it not only promotes the sales of the PI product but also hinders the sales of the traditional product. The impact of s on the sales of the PI product is greater when the substitutability between the two products is higher, because, according to (5), we find that $\partial\hat{q}_{1i}/\partial s = (2-\beta)/\{(1+\beta)\{(1-\beta)[4-(\gamma+\tilde{\gamma})^2]+\beta^2(1-\gamma\tilde{\gamma})\}\}$ is increasing in γ and $\tilde{\gamma}$.

2.1.3 The Social Welfare

Since the two sales functions in (2) are linearly dependent on prices, the consumer surplus can be calculated as $CS_1 \equiv \sum_{i=1}^{2}[(p_i^{\max} - p_{1i})q_{1i} + (\hat{p}_i^{\max} - \hat{p}_{1i})\hat{q}_{1i}]/2$, where $p_i^{\max} \equiv \theta$ and $\hat{p}_i^{\max} \equiv \theta + s$ are the highest selling prices of the traditional and PI products under the the subsidy scheme, respectively, as $p_i = \theta - (q_i + \beta q_{3-i}) - \gamma(q_i + \beta q_{3-i})$ and $\hat{p}_i = \theta + s - (q_i + \beta q_{3-i}) - \tilde{\gamma}(\hat{q}_i + \beta\hat{q}_{3-i})$. For similar models, see Tan et al. ([10], [9]).

In practice, the externality—i.e., an influence of an economic activity on unrelated third parties—can be regarded as a major feature of the PI products, which distinguishes these products from others. Every PI product can generate an externality benefit, which is explained as the positive impact of the PI product on the parties that are not related to the PI product. For example, the externality benefits of an energy efficient air conditioner and an electric vehicle are the energy savings and the reduction in the CO_2 emission compared with their corresponding traditional products, respectively. For the PI product, we compute the externality benefit as $EB_1 \equiv \alpha\sum_{i=1}^{2}\hat{q}_{1i}$, where α is the marginal externality for one unit of the PI product. Assuming that the PI product is an electric vehicle, we can explain α as the social benefit resulting from the CO_2 emission reduction achieved by the electric vehicle compared with the average emission level by traditional vehicles. As Ovchinnikov and Raz [7] roughly calculated, the value of α is around \$840.

We then compute the government's cost for its subsidy scheme as $GC_1 \equiv g + s\sum_{i=1}^{2}\hat{q}_{1i}$, where g represents the fixed cost incurred by the government. It then follows that the social welfare is calculated as

$$SW_1 \equiv \sum_{i=1}^{2}\pi_{1i} + CS_1 + EB_1 - GC_1. \tag{6}$$

Proposition 2 When each manufacturer produces both the traditional and the PI products, the government's optimal subsidy is

$$s_1^* = \alpha(2-\beta) + (\theta - \hat{c})(1-\beta) - \frac{(\theta - c)[\gamma + (1-2\beta)\tilde{\gamma}]}{2}.$$

The above proposition indicates that the optimal subsidy is increasing in the marginal externality (i.e., α) but decreasing in the competition between the two manufacturers (i.e., β). This means that as the marginal externality of the PI product is greater, the government should award a larger subsidy to consumers. But, as a response to a higher competition between the two manufacturers, the government should reduce its subsidy, because, otherwise, the manufacturers may intend to compete more for consumers, which cannot help improve the social welfare.

We also learn from Proposition 2 that the optimal subsidy s_1^* is decreasing in the degree of substituting the PI product for the traditional product (i.e., γ), which is mainly ascribed to the fact that when consumers are more willing to accept the PI product, the government does not need to increase the subsidy but can reduce the subsidy for the improvement of the social welfare. Moreover, if the competition between the two manufacturers is sufficiently small such that $\beta \le 1/2$, then s_1^* is decreasing in the degree of substituting the traditional product for the PI product (i.e., $\tilde{\gamma}$); otherwise, if $\beta > 1/2$, then s_1^* is increasing in $\tilde{\gamma}$. This could be justified as follows: When the manufacturers do not intensely compete, any manufacturer cannot significantly influence the other manufacturer's sales, which means that each manufacturer can "independently" manage the sales of his traditional and PI products. As consumers are more willing to accept the traditional product, each manufacturer may prefer to sell his traditional product rather than his PI product, and the government's subsidy scheme may be less attractive to the manufacturer. Accordingly, the government may reduce its subsidy from the perspective of social welfare. But, when the competition between the manufacturers is sufficiently high, the manufacturers may intend to take advantage of the subsidy scheme to promote the sales of their PI products and increase their competitiveness in the market. To assure the manufacturers' incentives to sell their PI products, the government needs to award a larger subsidy when consumers are more willing to accept the manufacturers' traditional products.

Moreover, the optimal subsidy s^* is decreasing in the unit cost of the PI product (i.e., $\hat{c}$) but increasing in the unit cost of the traditional product (i.e., c). This reflects the following fact: If the value of $\hat{c}$ is higher, then the manufacturers may raise the prices for their PI products, which reduces the impact of the subsidy on the improvement of the social welfare. As a response, the government should reduce the subsidy, because, otherwise, a higher subsidy may further increase the prices of the PI products and reduce the social welfare.

2.2 The Duopoly Analysis with Only the PI Product

We consider a symmetric duopoly setting in which the two manufacturers only sell their PI products in the market. Similar to Section 2.1, using the inverse demand curve $\hat{p}_i - s = \theta - \hat{q}_i - \beta\hat{q}_{3-i}$, for $i = 1, 2$, we derive the sales function $\hat{q}_i = [(1-\beta)\theta - (\hat{p}_i - s) + \beta(\hat{p}_{3-i} - s)]/(1-\beta^2)$, for $i = 1, 2$, which can be regarded as a form of the classical competition model proposed by Singh and Vives [8].

We then find that, in Nash equilibrium, the two manufacturers' prices for their PI products are $\hat{p}_{21} = \hat{p}_{22} = [(1-\beta)(\theta+s)+\hat{c}]/(2-\beta)$, and the sales of the two manufacturers' PI products are $\hat{q}_{21} = \hat{q}_{22} = (\theta+s-\hat{c})/(2+\beta-\beta^2)$. Moreover, we can obtain manufacturer i's ($i = 1, 2$) expected profit as $\pi_{21} = \pi_{22} = [(1+\beta)(\theta+s-\hat{c})^2]/[(2-\beta)^2(1-\beta)]$, and compute the social welfare as $SW_2 = \sum_{i=1}^{2} \pi_{2i} + CS_2 + EB_2 - GC_2$, where $CS_2 \equiv \sum_{i=1}^{2}[(\hat{p}_i^{\max} - \hat{p}_{2i})\hat{q}_{2i}]$, $EB_2 \equiv \alpha \sum_{i=1}^{2} \hat{q}_{2i}$, and $GC_2 \equiv g + s\sum_{i=1}^{2} \hat{q}_{2i}$.

Proposition 3 If each manufacturer only produces the PI product, then the government's optimal subsidy is $s_2^* = \alpha + (1-\beta)[\theta + \alpha - \hat{c}]$. ■

The above proposition implies that the optimal subsidy is increasing in the marginal externality but decreasing in the competition between the two manufacturers and the unit cost of the PI product.

3 Game-Theoretic Analysis and Social Welfare in an Asymmetric Duopoly Setting

We analyze our problem in an asymmetric duopoly setting in which a manufacturer produces either the traditional product or the PI product, whereas the other manufacturer produces both the traditional and the PI products.

3.1 The Duopoly Analysis when a Manufacturer Produces Only the Traditional Product

Without loss of generality, we consider the setting that manufacturer 1 (e.g., Mazda and Chrysler) produces only the traditional product and manufacturer 2 (e.g., Toyota and Honda) makes two products. Setting $\hat{q}_1 = 0$ in (1), we can obtain the reduced inverse sales curves and then derive the sales functions as

$$\begin{cases} q_1 = \dfrac{(1-\beta)\theta - p_1 - \beta p_2}{1-\beta^2}, \\ q_2 = \dfrac{(1-\gamma)\theta - p_2 + \gamma(\hat{p}_2 - s)}{1-\gamma\tilde{\gamma}} + \dfrac{\beta[p_1 - \beta p_2 - (1-\beta)\theta]}{1-\beta^2}, \\ \hat{q}_2 = \dfrac{(1-\tilde{\gamma})\theta - (\hat{p}_2 - s) + \tilde{\gamma} p_2}{1-\gamma\tilde{\gamma}}. \end{cases} \tag{7}$$

Then, we find that in Nash equilibrium, the manufacturers' prices for their traditional products are

$$\begin{aligned} p_{31} \;=\; & \frac{c(4-\gamma^2+\beta^3\gamma(\gamma-\tilde{\gamma})+\beta^2(\gamma-\tilde{\gamma})^2-2\gamma\tilde{\gamma}-\tilde{\gamma}^2-\beta(-2+\gamma^2+\gamma\tilde{\gamma})}{2(4-\gamma^2-2\gamma\tilde{\gamma}-\tilde{\gamma}^2+\beta^2(1-\gamma^2+\gamma\tilde{\gamma}-\tilde{\gamma}^2))} \\ & + \frac{(1-\beta)(\hat{c}-s)\beta(1+\beta)(\gamma-\tilde{\gamma})}{2(4-\gamma^2-2\gamma\tilde{\gamma}-\tilde{\gamma}^2+\beta^2(1-\gamma^2+\gamma\tilde{\gamma}-\tilde{\gamma}^2))} \\ & + \frac{\theta(1-\beta)[4+\beta^2(1-\gamma)(\tilde{\gamma}-\gamma)-(\gamma+\tilde{\gamma})^2-\beta(1+\tilde{\gamma})(-2+\gamma-\tilde{\gamma})]}{2(4-\gamma^2-2\gamma\tilde{\gamma}-\tilde{\gamma}^2+\beta^2(1-\gamma^2+\gamma\tilde{\gamma}-\tilde{\gamma}^2))}, \end{aligned}$$

and

$$\begin{aligned} p_{32} \;=\; & \frac{(1-\beta)\{(\hat{c}-s)(1+\beta)(\gamma-\tilde{\gamma})-[(1+\tilde{\gamma})(-2+\gamma+\tilde{\gamma})+\beta(-1+\gamma-\tilde{\gamma}+\tilde{\gamma}^2)]\theta\}}{4-\gamma^2-2\gamma\tilde{\gamma}-\tilde{\gamma}^2+\beta^2(1-\gamma^2+\gamma\tilde{\gamma}-\tilde{\gamma}^2)} \\ & + \frac{c[2+\beta-\gamma^2+\beta^2\gamma(\gamma-\tilde{\gamma})-(1+\beta)\gamma\tilde{\gamma}]}{4-\gamma^2-2\gamma\tilde{\gamma}-\tilde{\gamma}^2+\beta^2(1-\gamma^2+\gamma\tilde{\gamma}-\tilde{\gamma}^2)}; \end{aligned}$$

and manufacturer 2's price for the PI product is

$$\begin{aligned}\hat{p}_{32} &= \frac{\theta[4-\beta^2+(2-\beta-\beta^2)\gamma-2\gamma^2(1-\beta^2)-(2+\beta+2\gamma)\tilde{\gamma}-\beta\gamma\tilde{\gamma}(\beta-\gamma-\beta\gamma)]}{2(4-\gamma^2-2\gamma\tilde{\gamma}-\tilde{\gamma}^2+\beta^2(1-\gamma^2+\gamma\tilde{\gamma}-\tilde{\gamma}^2))} \\ &\quad+\frac{\theta\beta\gamma\tilde{\gamma}^2(2+\beta)+2\hat{c}\gamma+c\beta\gamma(1+\beta)+(2+\beta)c\tilde{\gamma}-c\beta\gamma\tilde{\gamma}(\gamma-\beta\gamma+\tilde{\gamma}+2\beta\tilde{\gamma})}{2(4-\gamma^2-2\gamma\tilde{\gamma}-\tilde{\gamma}^2+\beta^2(1-\gamma^2+\gamma\tilde{\gamma}-\tilde{\gamma}^2))} \\ &\quad+\frac{s[2(2-\gamma^2-\gamma\tilde{\gamma})-\beta^2(1-2\gamma^2+\gamma\tilde{\gamma})]}{2(4-\gamma^2-2\gamma\tilde{\gamma}-\tilde{\gamma}^2+\beta^2(1-\gamma^2+\gamma\tilde{\gamma}-\tilde{\gamma}^2))} \\ &\quad-\frac{\hat{c}[\beta^2(1+\gamma\tilde{\gamma}-2\tilde{\gamma}^2)-2(2-\gamma\tilde{\gamma}-\tilde{\gamma}^2)]}{2(4-\gamma^2-2\gamma\tilde{\gamma}-\tilde{\gamma}^2+\beta^2(1-\gamma^2+\gamma\tilde{\gamma}-\tilde{\gamma}^2))}.\end{aligned}$$

Substituting the Nash equilibrium-characterized prices into the sales functions in (7), we can obtain the sales of the two manufacturers' traditional products and the sales of manufacturer 2's PI product, which are too complicated to indicate any meaning insight.

Assuming that $\gamma=\tilde{\gamma}$, we can reduce the optimal prices as $p_{31}|_{\gamma=\tilde{\gamma}}=p_{32}|_{\gamma=\tilde{\gamma}}=[(1-\beta)\theta+c]/(2-\beta)$ and $\hat{p}_{32}|_{\gamma=\tilde{\gamma}}=[(2-\beta-\beta\gamma)\theta+c\beta\gamma+(2-\beta)(\hat{c}+s)]/[2(2-\beta)]$, and also find the sales as $q_{31}|_{\gamma=\tilde{\gamma}}=(\theta-c)/[(2-\beta)(1+\beta)]$, $\hat{q}_{32}|_{\gamma=\tilde{\gamma}}=[(1-\gamma)\theta+c\gamma+s-\hat{c}]/[2(1-\gamma^2)]$, and

$$\begin{aligned}q_{32}|_{\gamma=\tilde{\gamma}} &= \frac{(2-2\gamma-\beta\gamma+\beta^2\gamma+\beta\gamma^2-\beta^2\gamma^2)\theta-c(2+\beta\gamma^2-\beta^2\gamma^2)}{2(2-\beta)(1+\beta)(1-\gamma^2)} \\ &\quad+\frac{\hat{c}(2+\beta-\beta^2)\gamma-s(2\gamma+\beta\gamma-\beta^2\gamma)}{2(2-\beta)(1+\beta)(1-\gamma^2)}.\end{aligned}$$

We learn from the above that, as $\gamma=\tilde{\gamma}$, the two manufacturers' prices for their traditional products are identical. The sales of the PI product is increasing in subsidy s, and manufacturer 2's sales for the traditional product is decreasing in s whereas manufacturer 1's sales for the traditional product are independent of s.

When $\gamma=\tilde{\gamma}$, we compute the social welfare as $SW_3\equiv\sum_{i=1}^{2}\pi_{3i}+CS_3+EB_3-GC_3$, where $\pi_{31}=(p_{31}|_{\gamma=\tilde{\gamma}}-c)q_{31}|_{\gamma=\tilde{\gamma}}$ and $\pi_{32}=(p_{32}|_{\gamma=\tilde{\gamma}}-c)q_{32}|_{\gamma=\tilde{\gamma}}+(\hat{p}_{32}|_{\gamma=\tilde{\gamma}}-\hat{c})\hat{q}_{32}|_{\gamma=\tilde{\gamma}}$; $CS_3\equiv[(p_1^{\max}-p_{31}|_{\gamma=\tilde{\gamma}})q_{32}|_{\gamma=\tilde{\gamma}}]/2+[(p_2^{\max}-p_{32}|_{\gamma=\tilde{\gamma}})q_{32}|_{\gamma=\tilde{\gamma}}+(\hat{p}_2^{\max}-\hat{p}_{32}|_{\gamma=\tilde{\gamma}})\hat{q}_{32}|_{\gamma=\tilde{\gamma}}]/2$; $EB_3\equiv\alpha\hat{q}_{32}|_{\gamma=\tilde{\gamma}}$; and $GC_3\equiv g+s\hat{q}_{32}|_{\gamma=\tilde{\gamma}}$. Maximizing the social welfare yields the optimal subsidy $s_3^*=(1-\gamma)\theta+2\alpha-\hat{c}+\gamma c$, which is increasing in the marginal externality but decreasing in the substitutability between the two products and the unit cost of the PI product.

3.2 The Duopoly Analysis when a Manufacturer Produces Only the PI Product

In the duopoly setting, manufacturer 1 (e.g., Fisker Automotive and Tesla Motors) produces only the PI product and the manufacturer 2 produces both products. Setting $q_1=0$ in (1), we derive the sales functions as

$$\begin{cases}\hat{q}_1=\dfrac{(1-\beta)\theta-(\hat{p}_1-s)+\beta(\hat{p}_2-s)}{1-\beta^2},\\ q_2=\dfrac{(1-\gamma)\theta-p_2+\gamma(\hat{p}_2-s)}{1-\gamma\tilde{\gamma}},\\ \hat{q}_2=\dfrac{(1-\tilde{\gamma})\theta-(\hat{p}_2-s)+\tilde{\gamma}p_2}{1-\gamma\tilde{\gamma}}-\beta\dfrac{(1-\beta)\theta-(\hat{p}_1-s)+\beta(\hat{p}_2-s)}{1-\beta^2}.\end{cases} \tag{8}$$

We can then find the prices in Nash equilibrium and compute the resulting sales. Similar to Section 3.1, to find meaningful insights, we consider the special case that $\gamma = \tilde{\gamma}$, for which in Nash equilibrium, the two manufacturers' prices for their PI products are identical, i.e., $\hat{p}_{41}|_{\gamma=\tilde{\gamma}} = \hat{p}_{42}|_{\gamma=\tilde{\gamma}} = [(1-\beta)(\theta+s)+c_2]/2-\beta$, and manufacturer 2's price for the traditional product is $p_{42}|_{\gamma=\tilde{\gamma}} = [(2-\beta-\beta\gamma)\theta+c(2-\beta)+(\hat{c}-s)\beta\gamma]/[2(2-\beta)]$. As a result, for manufacturer 1, the sales of the PI product are $\hat{q}_{41}|_{\gamma=\tilde{\gamma}} = (\theta+s-\hat{c})/[(2-\beta)(1+\beta)]$; and for manufacturer 2, the sales of the traditional and the PI products are $q_{42}|_{\gamma=\tilde{\gamma}} = [(1-\gamma)\theta-c+\hat{c}\gamma-s\gamma]/[2(1-\gamma^2)]$ and

$$\begin{aligned}\hat{q}_{42}|_{\gamma=\tilde{\gamma}} &= \frac{(2-2\gamma-\beta\gamma+\beta^2\gamma+\beta\gamma^2-\beta^2\gamma^2)\theta+c\gamma(2+\beta-\beta^2)}{2(2-\beta)(1+\beta)(1-\gamma^2)} \\ &\quad+\frac{s(2+\beta\gamma^2-\beta^2\gamma^2)-\hat{c}(2+\beta\gamma^2-\beta^2\gamma^2)}{2(2-\beta)(1+\beta)(1-\gamma^2)},\end{aligned}$$

respectively. We find from the above that for each manufacturer, the sales of the PI product are increasing in subsidy s, whereas the sales of the traditional product are decreasing in s.

The social welfare is $SW_4 \equiv \sum_{i=1}^{2}\pi_{4i}+CS_4+EB_4-GC_4$, where $\pi_{41} = (p_{41}|_{\gamma=\tilde{\gamma}}-c)q_{41}|_{\gamma=\tilde{\gamma}}$ and $\pi_{42} = (p_{42}|_{\gamma=\tilde{\gamma}}-c)q_{42}|_{\gamma=\tilde{\gamma}} + (\hat{p}_{42}|_{\gamma=\tilde{\gamma}}-\hat{c})\hat{q}_{42}|_{\gamma=\tilde{\gamma}}$; $CS_4 \equiv [(\hat{p}_1^{\max}-\hat{p}_{42}|_{\gamma=\tilde{\gamma}})\hat{q}_{42}|_{\gamma=\tilde{\gamma}}]/2 + [(p_2^{\max}-p_{42}|_{\gamma=\tilde{\gamma}})q_{42}|_{\gamma=\tilde{\gamma}} + (\hat{p}_2^{\max}-\hat{p}_{42}|_{\gamma=\tilde{\gamma}})\hat{q}_{42}|_{\gamma=\tilde{\gamma}}]/2$; $EB_4 \equiv \alpha\sum_{i=1}^{2}\hat{q}_{4i}|_{\gamma=\tilde{\gamma}}$; and $GC_4 \equiv g + s\sum_{i=1}^{2}\hat{q}_{4i}|_{\gamma=\tilde{\gamma}}$. Maximizing SW_4 gives the optimal subsidy as

$$s_4^* = \frac{c(2-\beta)^2(1+\beta)\gamma-\hat{c}(8-8\beta-4\gamma^2+8\beta\gamma^2-3\beta^2\gamma^2+\beta^3\gamma^2)+Z}{8+(-4-3\beta^2+\beta^3)\gamma^2},$$

where

$$Z \equiv 2\alpha(2-\beta)[4-(2-\beta+\beta^2)\gamma^2]+[(3\beta^2-\beta^3)(1-\gamma)\gamma-8\beta(1-\gamma^2)+4(2-\gamma-\gamma^2)]\theta.$$

The optimal subsidy is increasing in the marginal externality but decreasing in the unit cost of the PI product.

4 Future Research Direction

To characterize the competition between the traditional and the PI products and also that between the two manufacturers, we choose a linear demand function, which help us derive meaningful results. It is more interesting and realistic to investigate our problem with a nonlinear demand function. However, we find that it is difficult to solve our problem with a nonlinear function. Nevertheless, we believe that the game-theoretic analysis of a similar problem with a nonlinear demand function is a research direction in the future.

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References

[1] K. H. Chan, M. Leng, and L. Liang. Impact of tax reduction policies on consumer purchase of new automobiles: An analytical investigation with real data-based experiments. *Naval Research Logistics*, 61(12):577–598, 2014.

[2] Desk Lifestyle. Sweden follows suit with electric car subsidy, 2011. The Global Herald, 2011-09-08, `http://theglobalherald.com/sweden-follows-suit-with-electric-car-subsidy/24444/` (URL last accessed on August 3, 2017).

[3] P. Dobson and M. Waterson. Countervailing power and consumer prices. *Economic Journal*, 107:418–430, 1997.

[4] P. Dobson and M. Waterson. The competition effects of industry-wide vertical price fixing in bilateral oligopoly. *International Journal of Industrial Organization*, 25:935–962, 2007.

[5] J. Huang, M. Leng, L. Liang, and C. Luo. Qualifying for a governments scrappage program to stimulate consumers trade-in transactions? analysis of an automobile supply chain involving a manufacturer and a retailer. *European Journal of Operational Research*, 239:363–376, 2014.

[6] C. Luo, M. Leng, J. Huang, and L. Liang. Supply chain analysis under a price-discount incentive scheme for electric vehicles. *European Journal of Operational Research*, 235:329–333, 2014.

[7] A. Ovchinnikov and G. Raz. Coordinating pricing and supply of public interest goods using rebates and subsidies. *IEEE Transactions on Engineering Management*, 62(1):65–79, 2015.

[8] N. Singh and X. Vives. Price and quantity competition in a dfferentiated duopoly. *RAND Journal of Economics*, 15(4):546–554, 1984.

[9] Y. Tan and J. E. Carrillo. Strategic analysis of the agency model for digital goods. *Production and Operations Management*, 26(4):724–741, 2017.

[10] Y. Tan, J. E. Carrillo, and H. K. Cheng. The agency model for digital goods. *Decision Science*, 47(4):628–660, 2016.

Online Supplements

"Subsidizing Purchases of Public Interest Products: A Duopoly Analysis under a Subsidy Scheme"

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Appendix A Proofs

Proof of Proposition 1. Under the subsidy scheme, we differentiate manufacturer i's ($i = 1, 2$) profit π_i once w.r.t. p_i and $\hat{p}_i$, equate them to 0, and have

$$\begin{cases} 2p_i - (\gamma + \tilde{\gamma})\hat{p}_i - \beta p_{3-i} + \beta\gamma\hat{p}_{3-i} = (1-\gamma)(1-\beta)\theta - \gamma s + \beta\gamma s + c_i - \tilde{\gamma}\hat{c}_i, \\ 2\hat{p}_i - (\gamma + \tilde{\gamma})p_i - \beta\hat{p}_{3-i} + \beta\tilde{\gamma}p_{3-i} = (1-\gamma)(1-\beta)\theta + s - \beta s + \hat{c}_i - \gamma c. \end{cases} \tag{9}$$

Next, to show the concavity of π_i, we compute the second partial derivatives of π_i w.r.t. p_i and $\hat{p}$ as well as the mixed partial derivatives as

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = -2/J, \quad \frac{\partial^2 \pi_i}{\partial \hat{p}_i^2} = -2/J, \quad \text{and } \frac{\partial^2 \pi_i}{\partial p_i \partial \hat{p}_i} = (\gamma + \tilde{\gamma})/J.$$

The Hessian matrix is thus obtained as

$$\begin{bmatrix} -2/J & (\gamma + \tilde{\gamma})/J \\ (\gamma + \tilde{\gamma})/J & -2/J \end{bmatrix},$$

which is negative definite because its first- and second-order principle minors are $-2/J < 0$ and $[4 - (\gamma + \gamma)^2]/J^2 > 0$, respectively.

Solving the equations in (9), we obtain manufacturer i's prices for his traditional and PI products as shown in this proposition. Substituting the Nash equilibrium-characterized prices into the sales functions in (2), we obtain the sales for manufacturer i's traditional and PI products as in (4) and (5), respectively. ■

Proof of Proposition 2. When the manufacturers make both the traditional and the PI products, the social welfare is SW_1 as given in (6). Differentiating SW_1 once w.r.t. s, equating it to 0, and solving it, we obtain s_1^* as in this proposition. Since

$$\left.\frac{dSW_1}{ds}\right|_{s<s_1^*} > 0 \quad \text{and} \quad \left.\frac{dSW_1}{ds}\right|_{s>s_1^*} < 0,$$

s_1^* is the optimal subsidy for the government. ■

Proof of Proposition 3. Similar to the proof of Proposition 2, we can maximize SW_2 to find the optimal subsidy s_2^* as given in this proposition. ■