Managing On-Demand Ridesharing Operations: Optimal Pricing Decisions for a Ridesharing Platform

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Abstract

We investigate an on-demand ridesharing system consisting of a ridesharing platform, multiple drivers, and multiple passengers. We begin by analyzing a choice problem for a passenger who chooses either a ridesharing or taxi service and also studying a choice problem for a driver who decides on whether or not to serve. Then, we obtain the ridesharing platform's optimal service price charged to passengers and its optimal wage paid to drivers. We perform sensitivity analysis to draw a number of managerial implications. An increase in the number of potential passengers (drivers) usually results in an increase (a decrease) in both the price and wage. We also find that the platform is better off when both the number of passengers and the number of drivers are higher, and an increase in the passengers' mental costs for the taxi service can help increase the platform's profit. Moreover, the dynamic pricing strategy can not only improve the platform's profit but also generate larger surpluses to passengers and drivers.

Key words: optimization; decision analysis; ridesharing operations.

1 Introduction

In practice, many persons need delivery services (e.g., food delivery and passenger transport) but cannot confirm an exact start time point for the services in advance. As a result, these persons expect to obtain instant services which are available immediately when they need. Such demands cannot be satisfied by extant online booking platforms where customers have to make appointments with service providers prior to obtaining delivery services. To meet the increasing demands for instant delivery services, a number of online platforms have appeared in recent years to provide on-demand services that allow service demand and supply to quickly match one another. As Taylor (2018) summarized, popular on-demand services include restaurant food delivery (e.g., Caviar, DoorDash, and Meituan), consumer goods delivery (e.g., UberRush and Go-Mart), and taxi-style transportation (e.g., Lyft, Uber, and Didi). Since more and more on-demand platforms serve customers with a need for immediate services, one may observe the emergence of the "on-demand economy" in today's society (Williams 2015).

In this paper, we consider an on-demand ridesharing platform that provides passengers with instant transport service. Hereafter, we simply call the platform a "ridesharing platform," which, as in practice, does not employ any staff to provide driving services but instead allows qualified drivers to register at the platform and deliver services only at times when they are willing to serve (Chan and Shaheen 2012). Since the registered drivers can independently decide on whether and when to work, the business model operated by the platform is viewed as a "sharing" one. This is different from traditional business models in which a firm can schedule employees' work times and determine salary payments to employees (Taylor 2018). The ridesharing platform provides an App that is available for ridesharing drivers to download. Every ridesharing driver uses his or her own vehicle to serve passengers. If a driver decides to serve at a time point, then the driver turns on the App to indicate his or her availability. Otherwise, the driver turns off the App and joins another activity that could be more important to the driver than the on-demand transport service. Since the ridesharing platform does not enforce any job assignment to any registered driver but operates in a very flexible mode, each driver can treat the ridesharing service as a "part-time" job that does not influence the driver's other business activities. This operational model provides qualified drivers with a new way to improve the utilization of their times and vehicle resources, thereby having attracted many drivers to register at ridesharing platforms and serve a large number of passengers. For instance, in 2017, the number of daily ridesharing transactions completed at the Didi Express had been 25 millions in China; for more information, see the Didi Chuxing Corporate Citizenship Report (2017).

In the ridesharing model, the platform needs to pay a wage to each registered driver who has completed a ridesharing service, and passengers make their payments for ridesharing services to the platform rather than the drivers. The wage is based on the trip length of passenger(s) served by the driver and the unit wage. That is, the platform usually calculates the wage as a unit (per kilometer) wage times the trip length completed in the ridesharing service. Naturally, the platform should make a decision on the unit wage. When the unit wage increases, more registered drivers may have an incentive to deliver transport service, more passengers can enjoy the services, and the platform may achieve a higher revenue. However, increasing the platform's payments to drivers reduces the platform's unit profit. When the unit wage decreases, less drivers are willing to serve and some passengers' requests may be lost, which may reduce the platform's revenue although the platform bears a lower wage payment. Since the unit wage plays an important role in the platform's ridesharing model, it behooves us to find an optimal unit wage for the platform.

Since the platform incurs an operational cost and makes wage payments to drivers, it should charge the ridesharing price to passengers for profitability. A higher price can increase the platform's per transaction revenue but is likely to discourage passengers from requesting ridesharing services (due to the competition of traditional taxi services), whereas a lower price can entice passengers to take ridesharing services but reduces the platform's per transaction revenue. Therefore, it is important for the platform to charge an optimal price. In this paper, we jointly investigate the platform's optimal decisions on both the unit wage and the service price. In practice, each passenger can choose either a ridesharing service or a taxi service, and each ridesharing driver can decide to either serve passengers or indicate his or her unavailability. Accordingly, we begin by analyzing a passenger's and a driver's choice models to find the expected number of passengers for ridesharing services and the expected number of ridesharing drivers available for a passenger's request, respectively. Using the analytic results derived from our choice models, we construct the platform's profit function and maximize it to obtain its optimal unit wage and price. In addition, we compute the optimal payout ratio, which is defined as the ratio of the wage to the price and can be used to measure the platform's capability of profiting from the ridesharing service.

Since March 2016, the Didi Express platform has adopted a new operational model (i.e., an order-assigning model), under which, for a new transport request, the platform assigns the order to the nearest driver whose status on the App is "available." The driver should not reject the order assignment; otherwise, the platform lowers the driver's service score, which then reduces the driver's chance to obtain new orders in future. Nonetheless, a driver may be assigned to serve a passenger in a location far away from the driver, if there is no other driver closer to the passenger. This induces the driver to necessarily estimate his expected financial benefit prior to deciding to turn its App on or off. Therefore, the driver's decision on whether or not to serve is important and thus worth investigating. Accordingly, we perform a choice analysis for ridesharing drivers to provide an one-time ridesharing service, which is different from Taylor (2018) and Bai et al. (2019) but somewhat similar to Cachon, Daniels, and Lobel (2017). In this regard, the driver's waiting time only influences his chance to obtain financial benefits from other sources and thus, it can be viewed as a part of the driver's opportunity cost.

Different cities differ in urban size, petrol price, taxi service price, population density, residents' average income as well as their information technology qualifications and habits, etc. Therefore, the ridesharing platform may set different prices and wages when it serves different cities. An interesting question is about how the factors characterizing different service environments influence the platform's optimal decisions and payout ratio as well as its maximum profit. Therefore, we perform sensitivity analysis to explore the impact of some important parameters on the ridesharing platform. These important parameters include passengers' average trip distance, ridesharing drivers' unit (per kilometer) running cost, passengers' mental cost from taking a taxi, unit (per kilometer) distance fare for the taxi service, the parameters for ridesharing drivers' opportunity gains when they are unavailable for ridesharing service, the number of total potential passengers, and the number of total potential drivers. From our sensitivity analysis we draw a number of managerial implications.

In a city, the number of potential passengers and unit running cost of ridesharing drivers usually vary from non-peak times to peak times. Thus, should the ridesharing platform adopt a dynamic pricing strategy (by setting a higher price and wage in peak times) or use a static pricing strategy (by setting a price and wage that do not differ in non-peak and peak times)? Most ridesharing platforms prefer to adopt the dynamic pricing strategy, whereas some consumer protection organizations have argued that increasing the service price may decrease consumers' interests in the ridesharing service. Accordingly, we perform numerical experiments and reveal that the dynamic pricing strategy not only results in a higher profit to the ridesharing platform but also generates higher surpluses to passengers and drivers. That is, the dynamic pricing strategy is basically better than the static pricing strategy for all the stakeholders.

The remainder of this paper is organized as follows. In Section 2, we review the existing relevant papers. In Section 3, we conduct choice analyses for passengers and drivers. In Section 4, we construct the ridesharing platform's profit function, and maximize it to find optimal decisions. We also calculate the optimal payout ratio and maximum profit for the platform. In Section 5 we perform sensitivity analysis to expose managerial implications. Section 6 presents numerical experiments to address the question of whether or not the dynamic pricing strategy is better than the static one. This paper ends with a summary of managerial insights in Section 7. In addition, we relegate the proofs of a lemma and all theorems to Appendix A, where the proofs are given in order that they appear in the main body of our paper.

2 Literature Review

Our paper belongs to a rapidly growing research stream that is concerned with online ridesharing platforms' operational issues arising in the on-demand economy. Chan and Shaheen (2012) reviewed the history of ridesharing-related operations, found that mobile phone and Internet technology had greatly promoted ridesharing, and also revealed that relevant research activities are needed to better understand the impact of ridesharing on infrastructure, congestion, and energy/emissions. Furuhata et al. (2013) presented a framework that can help identify key challenges in the widespread use of ridesharing and can thus foster the development of effective ridesharing mechanisms that would overcome those challenges and promote massification.

2.1 Ridesharing Operations with No On-Demand (Scheduled Demand) Mode

We review some representative publications regarding ridesharing operations that do not use the on-demand mode but instead the scheduled demand mode. Agatz et al. (2011) developed optimization-based approaches to find a match between drivers and passengers, which can minimize the system-wide vehicle miles incurred by system users. The authors also performed a simulation study based on the

2008 travel demand data obtained from the metropolitan Atlanta, and showed that the use of sophisticated optimization methods instead of simple greedy matching rules can substantially improve the performance of ridesharing systems. Zha, Yin, and Yang (2016) analyzed a ride-sourcing market using an aggregate model where the match between passengers and drivers are captured by an exogenous matching function. They found that without any regulatory intervention, a monopoly ride-sourcing platform can maximize the profit jointly achieved together with its drivers. Stiglic et al. (2016) conducted an extensive computational study to quantify the impact of different types of participants' flexibility on the performance of a single-driver, single-rider ridesharing system. They showed that small increases in flexibility, e.g., in terms of desired departure time or maximum detour time, can significantly increase the expected matching rate, especially when the number of trip announcements in the system is small. Stiglic et al. (2016) provided a basis for the design of information campaigns and incentives schemes, with an aim to increase the performance and success of ridesharing systems.

In the above publications, the ridesharing platform does not implement any on-demand mode but a scheduled demand mode. As a result, passengers and drivers put up their itineraries, and the platform should immediately assign a driver to each passenger. The passengers who cannot immediately obtain a driver's service will choose other transport tools such as taxi and bus. Because the platform cannot modify the service price and the wage for a special moment with ex-ante observing the unbalance between demand and supply, the pricing analysis in the on-demand mode is more complex than that in the scheduled demand mode.

2.2 On-Demand Ridesharing Operations with No Pricing Decisions

Recent developments of various on-demand service platforms such as Uber and DoorDash (Kokalitcheva 2015) have motivated researchers to explore various ridesharing operational issues in the on-demand mode. We first review several empirical study-related publications. Cramer and Krueger (2016) examined the efficiency of ridesharing services vis-à-vis taxis by comparing the capacity utilization rate of UberX drivers with that of traditional taxi drivers in five cities. The authors found that UberX drivers had spent a significantly higher fraction of their times, and had driven a substantially higher share of miles with passenger(s) in their cars than what taxi drivers did. Four factors that were likely to contribute to the higher capacity utilization rate of UberX drivers were identified as follows: (i) UberX's more efficient driver-passenger matching technology, (ii) the larger scale of UberX than taxi companies, (iii) inefficient taxi regulations, and (iv) UberX's flexible labor supply model and surge pricing model that can better match supply with demand throughout the day. To address the debates on social justice, equity, and improvements of taxi service, Leng et al. (2016) collected 37-day trip data of over 9000 taxis in Beijing to study the influence of the promotion battle between Didi and Kuaidadi, which are two leading ridesharing platforms in China. The paper quantitatively demonstrated how several important service indices (e.g., travelling distances and idle time lengths) of taxi drivers had been changed.

In a publication that delivers analytic results for on-demand, ridesharing operations, Gabel (2016) studied the persistence of market power in the taxi industry, and found that despite large scale entry and low barriers to entry, monopoly power persists, which illustrates that new technologies may not quickly eviscerate monopoly power, and Uber experiences a few risks in his entry into the taxi industry. Shi and Lian (2016) developed a doubled queueing model to study a passenger–taxi problem, analyzed the strategic behavior of passengers in both observable and unobservable cases, and investigated the problem of how a government controls the number of taxis by subsidizing taxi or levying a tax on taxi. Shi and Lian (2016) have contributed to the literature by maximizing the social welfare and optimizing the allocation of taxi market resources.

The publications reviewed above have addressed various operational problems for on-demand ridesharing platform. However, none of them investigates the ridesharing platform's decision-making problems for the service price and the unit wage, although Shi and Lian(2016) have investigated a government's optimal decision. Different with them, we focus on the optimal pricing policy for a platform, which is important as mentioned in Section 1.

2.3 On-Demand Ridesharing Operations with Pricing Decisions

Banerjee, Riquelme, and Johari (2015) and Cachon, Daniels, and Lobel (2017) compared the impact of static versus dynamic prices and wages. Assuming that the potential demand is constant, the real demand depends on the service price, the availability of drivers is independent of wages, Banerjee, Riquelme, and Johari (2015) revealed that the platform's revenue under the static pricing strategy is not inferior than that under the dynamic pricing strategy (under which the service price varies with the number of available drivers), and the payout ratio is constant. Cachon, Daniels, and Lobel (2017) assumed that the potential demand has two possible status (i.e., high and low), and the number of potential drivers are constant. They showed that a surging price can bring a higher profit than a static price (which is a constant price whenever the potential demand is high or low). The two publications above did not provide any closed-form price function but mainly used numerical approaches to obtain optimal decisions.

Zha, Yin, and Du (2017) proposed a time-expanded network to delineate possible work schedules for drivers. Based on the proposed network, they provided formulations and algorithms for both neoclassical and income targeting hypotheses to characterize the labor supply. The authors also built a bi-level programming framework, and proved that the dynamic pricing policy can generate a higher revenue to the platform and drivers than the static pricing policy. Taylor (2018) investigated the impact of uncertainty and time-delay sensitiveness on the service price and the unit wage, and drew the following insights. First, the delay sensitivity increases the optimal price when passenger valuation uncertainty is moderate. Secondly, the delay sensitivity decreases the optimal unit wage when the uncertainty in drivers' opportunity cost is high and their expected opportunity cost is moderate. Thirdly, when drivers' opportunity cost is uncertain, the independence among drivers decreases the service price. Fourthly, under passenger valuation uncertainty, the independence increases the service price if and only if the valuation uncertainty is sufficiently high. Although Zha, Yin, and Du (2017) and Taylor (2018) considered the impact of waiting time on the real request of passengers, they could not find any closed-form price. In fact, they mainly aimed to prove that surging pricing strategy is better than static pricing strategy.

Yu et al. (2017) developed a two-period dynamic game that captures the strategic interactions of multiple stakeholders, and found that the on-demand ride service platform can make the traditional taxi industry out of the market if there is no government regulation. Besbes et al.(2018) and Afeche et al.(2018) studied a spatial price problem, and they derived a price function in quasi-closed form. Besbes et al.(2018) showed that the platform should set differential prices to induce drivers' movements to a region where is beneficial to the platform; and Afeche et al.(2018) exposed that strategically rejecting a demand in a low-demand location may be optimal, because drivers usually aim at serving a high demand location. Although the above three publications have presented optimal prices, they did not show how the price, wage, and payout-ratio change with the outer environment (e.g., average trip distance and taxi service price).

Another recent publication related to our paper was proposed by Bai et al. (2019), who analyzed the on-demand service platform by using a queueing model with endogenous supply and endogenous demand. To coordinate endogenous demand with endogenous supply, they used the steady state performance in equilibrium to characterize the optimal service price, optimal unit wage, and optimal payout ratio. The authors found that the increment of potential passenger demand can raise the optimal prices. Our paper differs from Bai et al. (2019) because of the following facts. First, similar to the difference between ours and Taylor (2018), Bai et al. (2019) considered a queueing system to study the decision problem in a time period, whereas we investigated the decision problem when passengers' orders can be assigned in a short time, as mentioned in Section 1. Secondly, Bai et al. (2019) did not consider the competition from the taxi service, whereas our paper includes a passenger's choice model to character the competition between taxi and on-demand ridesharing services. Thirdly, Bai et al. (2019) did not involve ridesharing drivers' running costs but only involved their opportunity costs. Different with them, we consider both running costs and opportunity costs. As a result of the second and third differences, we can perform sensitivity for some parameters to study their impacts on optimal decisions.

Among all relevant publications, Bai et al. (2019) and Taylor (2018) constructed queueing models to find the optimal price and the optimal wage, and computed the optimal payout ratio. Different from these two papers, we do not use queueing theory but investigate the choice models for both passengers and drivers. We justify our approach as follows: on a number of occasions, when there is no nearby driver immediately available to serve a passenger who requests the ridesharing service, the passenger is unlikely to stick to the ridesharing service any longer but makes his option open to any other transportation tool. That is, if another transportation tool (e.g., a transient taxi) appears prior to a ridesharing vehicle, then the passenger does not insist on waiting for the ridesharing service but chooses to take the available transportation tool. For these occasions, the passenger does not stay on any queue for the ridesharing service but accepts another transportation choice, which means that the choice model is suitable for some cases. In fact, the taxi service providers are still playing an important role in any local transport system, and are thus competing with the ridesharing platform for passengers. This indicates the importance of the choice analysis for passengers.

3 The Choice Models for Passengers and Drivers

An online platform with n_d registered drivers is serving a certain area in which there are n_p potential passengers. When a passenger needs a driving service, he or she may call for an on-demand ride-sharing service at the online platform or may request a taxi service. The number of potential passengers for the platform is dependent on the popularity of electronic business and the service promotion of the platform in this area. If the residents are more likely to choose online service and/or the platform increases its promotion effort to attract passengers, then the value of n_p is larger; otherwise, the value of n_p is smaller. In a busy time period, there is usually a large number of passengers who need a trip service, whereas in other time periods, there are less potential passengers.

A paradigm best serving to illustrate the platform is the Didi, which has been providing passengers with the Didi Express service. As in practice, the platform needs to determine base fare p_{r0} and distance fare p_{r1}. Base fare p_{r0} denotes the fare that a passenger pays to the platform when his/her trip distance is no longer than base distance L_0, and distance fare p_{r1} represents the fare that a passenger pays to the platform for each extra kilometer service beyond base distance L_0. The platform also needs to determine base wage w_0 and distance wage w_1. Base wage w_0 means the payment that the platform makes to a registered driver who successfully completes a ride-sharing service with a trip distance no longer than base distance L_0, and distance wage w_1 is the per kilometer payment that the platform makes to the driver for an extra distance longer than base distance L_0. In addition, a number of taxi drivers compete with the platform for passengers, and they also charge passengers base fare p_{t0} when a trip distance is no longer than base distance L_0; and when distance is longer than base distance L_0, they will charge distance fare p_{t1} for each extra kilometer. We note that taxi providers and ridesharing platforms usually set an identical base distance, for example, $L_0 = 3km$, in Hangzhou, China. We consider a single platform, similar to Banerjee, Riquelme, and Johari (2015), Cachon, Daniels, and Lobel (2017), and Taylor (2018). In addition to the express service above, the Didi platform also provides the Special Car and Carpooling services. However, we note that the actual demands for those services are significantly small as against that for the Didi Express service. Accordingly, in this paper we do not consider the competition between the express service and any other service by the Didi.

To simplify the on-demand ridesharing analysis, Banerjee, Riquelme, and Johari (2015), Cachon, Daniels, and Lobel (2017) and Taylor (2018) set a distance-independent price; that is, they assumed that all trip distances are equal to 1. Moreover, Bai et al. (2019) supposed that all passengers demand an identical trip distance. Similar to extant relevant publications, we denote each passenger's trip distance by L. Intuitively, the value of L is greater in a larger city, whereas the value of L is less in a smaller city.

3.1 The Choice Model for a Passenger

When a passenger plans to request a driving service for a trip of distance L, the passenger needs to decide on whether to call for a ridesharing service at the platform or to look for a taxi. The passenger's choice model is thus with respect to the comparison between his or her expense for the platform ridesharing service and that for the taxi service. We can analyze the choice model to derive the condition under which the passenger prefers the platform's ridesharing service to the taxi service. When the passenger takes the ridesharing service from the platform, he or she incurs the total cost $C_r \equiv p_{r0} + p_{r1} \times (L - L_0)$, where p_{r0} and p_{r1} are base fare and distance fare, respectively, as defined previously.

Prior to computing the passenger's expense for the taxi service, we discuss the advantages of the platform service as against the taxi service. Because the platform can provide passengers with the information regarding exact positions of available cars, it can largely reduce the uncertainty of passengers' waiting times. Moreover, when a registered driver at the platform finishes a ridesharing service for a passenger, the passenger can comment on the driver's performance, which helps maintain and improve the service quality of registered drivers. This is also a major reason why the registered drivers usually appear to be patient and friendly when they serve passengers. When a passenger plans to take a taxi, he or she has no knowledge about when an available taxi appears and may spend a long time for looking for a taxi. Moreover, a taxi driver may somehow refuse to serve a passenger or may make a detour to increase the passenger's expense (see, `http://cq.people.com.cn/n2/2017/1229/c365405-31086483.html`). Therefore, the passenger may have some unhappiness from taking the taxi, which is called a "mental cost" θ (Burstein 2018).

However, a number of passengers may dislike the ridesharing service but prefer to take the taxi service, mainly because there are several sexual assault news about the Didi and the Uber, and some past passengers have reported their difficulties in using the Didi's App. Accordingly, mental cost θ can be either positive or negative. A passenger's negative mental cost means the passenger's happiness with the taxi service. For an application of the mental cost in the operations management literature, see, for example, Feng and Zhang's publication (2017). We can obtain the value/distribution of the mental cost by conducting a survey of passengers, in which each passenger indicates the monetary value of his or her utility of taking a taxi.

Since passengers may differ in their sensitivity to waiting time and preference on the taxi service, they may possess different values of mental cost θ, which is consistent with Moreno and Terwiesch's argument (2013) that buyers are usually heterogeneous in a bid market. Similar to Bai et al. (2019) who assumed that customers' valuations on a service are uniformly distributed in the range $[0, 1]$, we consider mental cost θ as an uniformly-distributed variable in the range $[\underline{\theta}, \overline{\theta}]$, where $\underline{\theta}$ and $\overline{\theta}$ denote the minimum and maximum mental costs incurred by a passenger from the taxi service, respectively. Since some passengers may prefer the ridesharing service to the taxi service, the value of $\overline{\theta}$ should be positive, i.e., $\overline{\theta} > 0$. However, the other passengers may prefer to choose the traditional taxi service, which implies that the value of $\underline{\theta}$ should be negative, i.e., $\underline{\theta} < 0$. As a result, the mean of θ is $(\overline{\theta}+\underline{\theta})/2$, which may not be equal to 0. In this paper, we do not simply assume that $\overline{\theta} = 1$ and $\underline{\theta} = -1$, because we expect to examine the impacts of $\overline{\theta}$ and $\underline{\theta}$ on the platform's decisions and performance, thereby learning how the platform responds to different types of passenger valuations on the taxi service.

Except for the mental cost, the passenger should pay the service fare $p_{t0} + p_{t1}(L - L_0)$ to the taxi driver. We thus calculate his/her total cost as $C_t \equiv p_{t0} + p_{t1} \times (L - L_0) + \theta$, where p_{t0} and p_{t1} are base and distance fares for taxi service. If $C_r \leq C_t$, then the passenger should choose the platform's ridesharing service; otherwise, if $C_r > C_t$, then the passenger prefers to take a taxi. Therefore, the ridesharing is a preferable choice when mental cost θ satisfies the following condition:

$$\theta \geq (p_{r0} - p_{t0}) + (p_{r1} - p_{t1}) \times (L - L_0). \tag{1}$$

Since there are n_p potential passengers, we can compute the expected number of passengers who intend to choose the platform's ridesharing service as

$$Q_p = n_p \int_{(p_{r0}-p_{t0})+(p_{r1}-p_{t1})\times(L-L_0)}^{\overline{\theta}} g(\theta)d\theta = \frac{n_p[\overline{\theta} - (p_{r0} - p_{t0}) - (p_{r1} - p_{t1}) \times (L - L_0)]}{\overline{\theta} - \underline{\theta}}. \tag{2}$$

Naturally, Q_p is no greater than n_p. Accordingly, the distance fare p_{r1} should be determined such that $p_{r1} \geq p_{t1} + \left[\underline{\theta} - (p_{r0} - p_{t0})\right] / (L - L_0)$.

3.2 The Choice Model for a Ridesharing Driver

In the area where the platform provides its ridesharing service, there are n_d registered drivers who are free to turn on the platform's App to indicate their availability or turn off the App to be unavailable for the ridesharing service. As in practice, the platform cannot "force" any driver to provide a ridesharing service. That is, a driver may be available in a certain time but may be unavailable in another time. Since a driver's unavailability usually occurs when the driver is more interested in another business matter than the ridesharing service, we use parameter γ to denote the driver's opportunity gain resulting from his or her unavailability to serve any ridesharing passenger. Hereafter, we simply call γ the driver's "opportunity gain from unavailability." If the value of γ is small, i.e., the driver cannot enjoy a satisfactory gain from any other business, then the driver has a high incentive to deliver the ridesharing service. Otherwise, if the value of γ is large, then the driver has a low incentive to serve passengers with the ridesharing service. This implies that the driver's opportunity gain from unavailability may vary in different times.

The driver's opportunity cost is irrelevant with any service request, similar to Cachon, Daniels, and Lobel (2017) who argued that all drivers are homogenous, and their opportunity gains satisfy an identical probability distribution. Bai et al. (2019) assumed that the drivers' reservation rates are uniformly distributed in $[0, 1]$. In our analysis, we treat parameter γ as a uniformly-distributed variable in the range $[\gamma_1 - \gamma_2, \gamma_1 + \gamma_2]$, where γ_1 is the mean of the driver's opportunity gains, and γ_2 is the difference between the mean and the maximum (or, the minimum) value. Therefore, $\gamma_1 + \gamma_2$ and $\gamma_1 - \gamma_2$ represent the maximum and minimum of the driver's opportunity gains, respectively. Since $\gamma_1 - \gamma_2 > 0$, γ_1 and γ_2 are given such that $\gamma_1 > \gamma_2$. The probability distribution function (p.d.f.) of parameter γ is $\varphi(\gamma) = 1/(2\gamma_2)$.

When the driver turns on the platform's app to indicate his or her availability, there is neither an income nor a cost if the platform does not assign any order to the driver. For this case, there is no difference between the driver's availability and unavailability. However, if the driver receives an order, then he or she has an income $I_d = w_0 + w_1(L - L_0)$ and incurs a running cost $C_d^T = C_d^v L$, where C_d^v denotes the unit (per kilometer) running cost including oil consumption, car depreciation, and others. Apparently, for each taxi or ridesharing driver, the service fare charged to a passenger should be greater than the unit running cost incurred by the driver, i.e., $p_{r1}, p_{t1} > C_d^v$. We compute the ridesharing driver's net income as $\bar{I}_d \equiv I_d - C_d^T = w_0 + w_1(L - L_0) - C_d^v L$. If net income $\bar{I}_d$ is smaller than a driver's opportunity gain (i.e., γ), then the driver turns off the app to indicate his or her unavailability. Therefore, the driver is willing to deliver the ridesharing service if and only if $\gamma \leq \bar{I}_d$. This means that, in a certain time, the probability for the driver to willingly serve is $\Pr(\gamma \leq \bar{I}_d) = \int_{\gamma_1-\gamma_2}^{\bar{I}_d} \varphi(\gamma) d\gamma = [w_0 + w_1(L - L_0) - C_d^v L - (\gamma_1 - \gamma_2)]/(2\gamma_2)$.

Since all registered drivers are homogenous in their opportunity gains from unavailability, we can view $\Pr(\gamma \leq \bar{I}_d)$ as the percentage of registered drivers who are available to deliver the ridesharing service when a passenger request arrives. In a certain time, the expected number of available drivers at the platform is

$$Q_d = n_d \Pr(\gamma \leq \bar{I}_d) = \frac{n_d[w_0 + w_1(L - L_0) - C_d^v L - (\gamma_1 - \gamma_2)]}{2\gamma_2} \tag{3}$$

As Q_d is naturally no greater than n_d, wage w should be determined such that $w_0 + w_1(L - L_0) \leq \gamma_1 + \gamma_2 + C_d^v L$.

Lemma 1 *If and only if each passenger's maximum mental cost for taxi service $\overline{\theta}$ is no less than $\theta_1 \equiv (\gamma_1 - \gamma_2 + C_d^v L) - [p_{t0} + p_{t1}(L - L_0)]$, the platform can obtain a profit and has its incentive to operate the ridesharing service.*

We learn from Lemma 1 that if inhabitants in a city are more likely to take the taxi service (i.e., $\overline{\theta}$ is smaller) and the ridesharing drivers are more likely to take break in their free times rather than to earn money by providing services (i.e., $\gamma_1 - \gamma_2$ is larger), then the chance for the ridesharing platform to obtain a profit in such a city is smaller.

4 The Platform's Optimal Pricing Decisions

In this section, we compute the platform's optimal pricing decisions that maximize its profit. The platform's profit $\pi(p_{r0}, p_{r1}, w_0, w_1)$ is calculated as the service fare paid by passengers to the platform minus total wages that the platform pays to all registered drivers. A service delivery is successful if and only if there is an available driver and a passenger requesting the ridesharing service. That is, the number of successful services is the minimum of the number of available drivers and the number of passengers. Recalling from Section 3 that there are Q_p passengers and Q_d drivers, we obtain the platform's profit as $\pi(p_{r0}, p_{r1}, w_0, w_1) = \min(Q_p, Q_d)[(p_{r0} - w_0) + (p_{r1} - w_1)(L - L_0)]$. In order to find optimal decisions p_{r0}^*, p_{r1}^*, w_0^* and w_1^*, we need to solve the following constrained optimization problem: $\max \pi(p_{r0}, p_{r1}, w_0, w_1)$, subject to $p_{r1} \geq p_{t1} + [\underline{\theta} - (p_{r0} - p_{t0})] / (L - L_0)$ and $w_0 + w_1(L - L_0) \leq \gamma_1 + \gamma_2 + C_d^v L$, where the two constraints are discussed in Section 3.

Theorem 1 *The platform's optimal base fare and base wage are*

$$p_{r0}^* = \frac{n_d\left(\overline{\theta}-\underline{\theta}\right)\left[\left(\overline{\theta}+p_{t0}\right)+\left(\gamma_1-\gamma_2+C_d^v L_0\right)\right]+4n_p\gamma_2\left(\overline{\theta}+p_{t0}\right)}{2n_d\left(\overline{\theta}-\underline{\theta}\right)+4n_p\gamma_2}, \tag{4}$$

and

$$w_0^* = \left(\gamma_1-\gamma_2+C_d^v L_0\right)+\frac{n_p\gamma_2\left[\left(\overline{\theta}+p_{t0}\right)-\left(\gamma_1-\gamma_2+C_d^v L_0\right)\right]}{n_d\left(\overline{\theta}-\underline{\theta}\right)+2n_p\gamma_2}, \tag{5}$$

respectively, and optimal distance fare and wage are obtained as follows:

1. *If $L \leq \hat{L}$, where*

$$\hat{L} = L_0 + \frac{2[n_d\left(\overline{\theta}-\underline{\theta}\right)+2n_p\gamma_2]}{\max(n_p,n_d)\left(p_{t1}-C_d^v\right)} - \frac{\left(\overline{\theta}+p_{t0}\right)-\left(\gamma_1-\gamma_2+C_d^v L_0\right)}{p_{t1}-C_d^v}, \tag{6}$$

then

$$p_{r1}^* = \tilde{p}_{r1} \equiv \frac{4n_p\gamma_2 p_{t1}+n_d\left(\overline{\theta}-\underline{\theta}\right)\left(p_{t1}+C_d^v\right)}{2n_d\left(\overline{\theta}-\underline{\theta}\right)+4n_p\gamma_2} \text{ and } w_1^* = \tilde{w}_1 \equiv C_d^v + \frac{n_p\gamma_2\left(p_{t1}-C_d^v\right)}{n_d\left(\overline{\theta}-\underline{\theta}\right)+2n_p\gamma_2}. \tag{7}$$

2. *If $L > \hat{L}$, then*

$$\begin{cases} p_{r1}^* \equiv p_{t1} + \dfrac{\left[\left(\overline{\theta}+p_{t0}\right)-\left(\gamma_1-\gamma_2+C_d^v L_0\right)\right]n_d\left(\overline{\theta}-\underline{\theta}\right)}{\left[2n_d\left(\overline{\theta}-\underline{\theta}\right)+4n_p\gamma_2\right](L-L_0)} - \dfrac{\min(n_p,n_d)\left(\overline{\theta}-\underline{\theta}\right)}{n_p(L-L_0)}, \\ w_1^* \equiv C_d^v - \dfrac{n_p\gamma_2\left[\left(\overline{\theta}+p_{t0}\right)-\left(\gamma_1-\gamma_2+C_d^v L_0\right)\right]}{[n_d\left(\overline{\theta}-\underline{\theta}\right)+2n_p\gamma_2](L-L_0)} + \dfrac{2\min(n_p,n_d)\gamma_2}{n_d(L-L_0)}. \end{cases} \tag{8}$$

In the above theorem, we find that when trip distance L is sufficiently small such that $L \leq \hat{L}$, $p_{r1}^* = \tilde{p}_{r1}$ and $w_1^* = \tilde{w}_1$. However, when $L > \hat{L}$, $\tilde{p}_{r1} < p_{t1} + [\underline{\theta} - (p_{r0}-p_{t0})]/(L-L_0)$ (while $n_p \leq n_d$) or $w_0 + w_1(L-L_0) > \gamma_1+\gamma_2+C_d^v L$ (while $n_p > n_d$), which means that the decision $(\tilde{p}_{r1}, \tilde{w}_1)$ does not satisfy the constraints $p_{r1} \geq p_{t1} + [\underline{\theta}-(p_{r0}-p_{t0})]/(L-L_0)$ or $w_0+w_1(L-L_0) \leq \gamma_1+\gamma_2+C_d^v L$. Therefore, when trip distance L is greater than cutoff level $\hat{L}$, we can derive the results for p_{r1}^* and w_1^* as shown in (8). Since the platform receives $p_{r0}+p_{r1}(L-L_0)$ for each successful service but pays $w_0+w_1(L-L_0)$ to each driver, we can view $p_{r0}+p_{r1}(L-L_0)$ and $w_0+w_1(L-L_0)$ as the platform's cash "inflow" and "outflow," respectively. Defining payout ratio r as the ratio of $w_0+w_1(L-L_0)$ to $p_{r0}+p_{r1}(L-L_0)$, we obtain the optimal payout ratio as $r^* \equiv [w_0^*+w_1^*(L-L_0)]/[p_{r0}^*+p_{r1}^*(L-L_0)]$, which reflects the platform's capability of profiting from the ridesharing service. A smaller value of r^* implies that the platform is capable of achieving a higher profit. Moreover, the platform's maximum profit is $\pi^* \equiv \pi(p_{r0}^*, p_{r1}^*, w_0^*, w_1^*)$. Using Theorem 1, we compute r^* and π^* as shown below.

1. If $L \leq \hat{L}$, then

$$\begin{cases} r^* \equiv \dfrac{2n_d\left(\overline{\theta}-\underline{\theta}\right)\left(\gamma_1-\gamma_2+C_d^v L\right)+2n_p\gamma_2\left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}(L-L_0)+\left(\gamma_1-\gamma_2+C_d^v L\right)\right]}{n_d\left(\overline{\theta}-\underline{\theta}\right)\left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}(L-L_0)+\left(\gamma_1-\gamma_2+C_d^v L\right)\right]+4n_p\gamma_2\left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}(L-L_0)\right]}, \\ \pi^* \equiv \dfrac{n_p n_d\left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}(L-L_0)-\left(\gamma_1-\gamma_2+C_d^v L\right)\right]^2}{4[n_d\left(\overline{\theta}-\underline{\theta}\right)+2n_p\gamma_2]}. \end{cases} \tag{9}$$

2. If $L > \hat{L}$, then

$$\begin{cases} r^* \equiv \dfrac{n_p n_d\left[(\gamma_1-\gamma_2)+C_d^v L\right]+2n_p\gamma_2\min(n_p,n_d)}{n_p n_d\left[(\overline{\theta}+p_{t0})+p_{t1}(L-L_0)\right]-\min(n_p,n_d)n_d\left(\overline{\theta}-\underline{\theta}\right)}, \\ \pi^* \equiv \min(n_p,n_d)\left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}(L-L_0)-(\gamma_1-\gamma_2+C_d^v L)-\dfrac{n_d\left(\overline{\theta}-\underline{\theta}\right)+2n_p\gamma_2}{\max(n_p,n_d)}\right]. \end{cases} \tag{10}$$

5 Sensitivity Analyses with Managerial Implications

We perform sensitivity analyses to investigate the impacts of some parameters on the ridesharing platform's optimal decisions p_{r1}^* and w_1^* as well as the platform's maximum profit π^* and optimal payout ratio r^*. In our sensitivity analyses, the parameters include passengers' average trip distance (i.e., L), ridesharing drivers' unit (per kilometer) running cost (i.e., C_d^v), the upper and lower bounds of passengers' mental costs for taking the taxi service (i.e., $\overline{\theta}$ and $\underline{\theta}$), distance fare for the taxi service (i.e., p_{t1}), the mean and dispersion of ridesharing drivers' opportunity gains when they are unavailable for ridesharing service (i.e., γ_1 and γ_2), and the number of total potential passengers and drivers (i.e., n_p and n_d).

5.1 Sensitivity Analysis of Trip Distance L

We begin by studying the influences of trip distance L on p_{r1}^*, w_1^*, r^*, and π^*.

Theorem 2 *We summarize the impacts of L on p_{r1}^*, w_1^*, r^* and π^* as in Table 1, in which*

$$\begin{cases} \theta_2 \equiv \overline{\theta}+\dfrac{2n_p\gamma_2}{n_d}-\dfrac{\left[\left(\overline{\theta}+p_{t0}\right)-(\gamma_1-\gamma_2+C_d^v L_0)\right]\max(n_p,n_d)}{2n_d}, \\ C_1 \equiv \dfrac{(\gamma_1-\gamma_2+C_d^v L_0)\,p_{t1}}{\overline{\theta}+p_{t0}}. \end{cases} \tag{11}$$

Conditions	$L \le \hat{L}$	$L > \hat{L}$
p_{r1}^*	$-$	$\begin{cases} \downarrow \text{ if } \underline{\theta} \le \theta_2, \\ \uparrow \text{ if } \underline{\theta} > \theta_2. \end{cases}$
w_1^*	$-$	$\begin{cases} \uparrow \text{ if } \underline{\theta} \le \theta_2, \\ \downarrow \text{ if } \underline{\theta} > \theta_2. \end{cases}$
r^*	$\begin{cases} \downarrow \text{ if } C_d^v \le C_1, \\ \uparrow \text{ if } C_d^v > C_1. \end{cases}$	$\downarrow$
π^*	$\uparrow$	$\uparrow$

Table 1: The impacts of L on p_{r1}^*, w_1^*, r^*, and π^*. Note that the marks "↑" , "–" and "↓" indicate that optimal decisions p_{r1}^* and w_1^*, optimal payout ratio r^*, and maximum profit π^* are "increasing in," "independent of," and "decreasing in" passengers' trip distance L, respectively.

We learn from Theorem 2 that passengers' trip distance L has a small influence on the platform's distance fare. When $L \le \hat{L}$, L has no influence on the distance fare because passengers' mental cost θ and drivers' opportunity gain γ only affect base fare p_{r0} and wage w_0. However, payout ratio r and profit π depend on not only price (p_{r0}, p_{r1}) and wage $(w_0,\ w_1)$ but also the number of served passengers $(Q_p = Q_d)$. Since both Q_p and Q_d are influenced by trip distance L, r^* and π^* are all dependent on L. Profit π^* is increasing in L, i.e. when a city is larger, passengers' trip distance becomes longer and the ridesharing platform can obtain a higher profit. The impact of L on payout ratio r^* depends on drivers' running cost C_d^v.

However, when $L > \hat{L}$, trip distance L influences distance fare p_{r1} and wage w_1, and such influences depend on the lower bound of mental cost (i.e., $\underline{\theta}$). When passengers are less (more) likely to take the taxi service, i.e. $\underline{\theta} > \theta_2 (\underline{\theta} \leq \theta_2)$, then optimal distance fare p_{r1}^* is increasing (decreasing) in L but wage w_1^* is decreasing (increasing) in L. For the case that $L > \hat{L}$, a longer trip distance decreases payout ratio r^* but increases the platform's profit π^*.

5.2 Sensitivity Analysis of Unit Running Cost C_d^v

We perform sensitivity analysis of ridesharing drivers' unit running cost C_d^v to investigate its impacts on optimal decisions p_{r1}^* and w_1^*, optimal payout ratio r^*, and the platform's maximum profit π^*.

Theorem 3 *The platform's optimal distance wage w_1^* and optimal payout ratio r^* are both increasing in C_d^v, and the platform's maximum profit π^* is decreasing in C_d^v. However, the impact of C_d^v on the platform's optimal distance fare p_{r1}^* depends on passengers' trip distance L. Specifically, if L is sufficiently small such that $L \leq \hat{L}$, then p_{r1}^* is increasing in C_d^v; otherwise, if $L > \hat{L}$, p_{r1}^* is decreasing in C_d^v.*

We learn from Theorem 3 that the running cost plays a role in the platform's optimal decisions. Usually, in the area served by the platform, if the running cost is increased because, for example, the gas oil is more costly or there is a traffic jam, then the platform should increase the price and wage simultaneously. Moreover, the optimal payout ratio becomes greater, which implies that the increment of w_1^* is larger than that of p_{r1}^*, viz., an increase in the running cost can always reduce the platform's profit. However, in a big city where $L > \hat{L}$, an increase in the running cost may result in a lower distance fare.

5.3 Sensitivity Analysis of Mental Cost θ

We perform sensitivity analyses of $\bar{\theta}$ and $\underline{\theta}$ to examine the influences of the bounds of passengers' mental cost θ on the platform.

Theorem 4 *We summarize the impacts of $\overline{\theta}$ and $\underline{\theta}$ on p_{r1}^*, w_1^*, r^*, and π^* as in Table 2, in which ξ_1, ξ_2, θ_3, and θ_4 are defined as the following*

$$\begin{cases} \xi_1 \equiv (n_p - 2n_d)\, n_d \left(\overline{\theta} - \underline{\theta}\right)^2 + 4n_p\gamma_2 \left(n_p - 2n_d\right) \left(\overline{\theta} - \underline{\theta}\right) + 2n_p^2\gamma_2[(\underline{\theta} + p_{t0}) - (\gamma_1 + 2\gamma_2 + C_d^v L_0)], \\ \xi_2 \equiv 2n_p\gamma_2 - n_d\left[(\underline{\theta} + p_{t0}) - (\gamma_1 - \gamma_2 + C_d^v L_0)\right], \\ \theta_3 \equiv \underline{\theta} + \{\sqrt{2n_d n_p\gamma_2[(\underline{\theta} + p_{t0}) - (\gamma_1 - \gamma_2 + C_d^v L_0)] - 4(n_p\gamma_2)^2} - 2n_p\gamma_2\}/n_d, \\ \theta_4 \equiv \overline{\theta} + \dfrac{n_p}{n_d} \left\{ 2\gamma_2 - \sqrt{\dfrac{n_d\gamma_2\left[(\overline{\theta} + p_{t0}) - (\gamma_1 - \gamma_2 + C_d^v L_0)\right]}{\min(n_p, n_d)}} \right\}. \end{cases} \tag{12}$$

<table>
<tr><td rowspan="3">Conditions</td><td colspan="3">$\overline{\theta}$</td><td colspan="2">$\underline{\theta}$</td></tr>
<tr><td rowspan="2">$L \leq \hat{L}$</td><td colspan="2">$L > \hat{L}$</td><td rowspan="2">$L \leq \hat{L}$</td><td>$L > \hat{L}$</td></tr>
<tr><td>$n_p \leq n_d$</td><td>$n_p > n_d$</td><td></td></tr>
<tr><td>p_{r1}^*</td><td>$\downarrow$</td><td>$\begin{cases} \uparrow \text{ if } \overline{\theta} \leq \theta_3, \\ \downarrow \text{ if } \overline{\theta} > \theta_3. \end{cases}$</td><td>$\begin{cases} \uparrow \text{ if } \xi_1 \geq 0, \\ \downarrow \text{ if } \xi_1 < 0. \end{cases}$</td><td>$\uparrow$</td><td>$\begin{cases} \uparrow \text{ if } \underline{\theta} \leq \theta_4, \\ \downarrow \text{ if } \underline{\theta} > \theta_4. \end{cases}$</td></tr>
<tr><td>w_1^*</td><td>$\downarrow$</td><td colspan="2">$\begin{cases} \downarrow \text{ if } \xi_2 \geq 0, \\ \uparrow \text{ if } \xi_2 < 0. \end{cases}$</td><td>$\uparrow$</td><td>$\downarrow$</td></tr>
<tr><td>r^*</td><td>$\downarrow$</td><td>$-$</td><td>$\downarrow$</td><td>$\uparrow$</td><td>$\downarrow$</td></tr>
<tr><td>π^*</td><td>$\uparrow$</td><td>$-$</td><td>$\uparrow$</td><td>$\uparrow$</td><td>$\uparrow$</td></tr>
</table>

Table 2: The impacts of $\bar{\theta}$ and $\underline{\theta}$ on p_{r1}^*, w_1^*, r^*, and π^*. Note that the marks "$\uparrow$" , "$-$" and "$\downarrow$" indicate that optimal decisions p_{r1}^* and w_1^*, optimal payout ratio r^*, and maximum profit π^* are "increasing in," "independent of," and "decreasing in" passengers' maximum(minimum) mental cost for taxi service $\bar{\theta}(\underline{\theta})$, respectively.

The above theorem shows that the upper and lower bounds of the mental cost usually have opposite impacts on the platform's decisions. When $L \leq \hat{L}$, an increase in the lower bound of the mental cost (i.e., $\underline{\theta}$) mostly raises distance fare p_{r1}^* and wage w_1^*. This means that if passengers are more likely to decline the Taxi service or accept the ridesharing service, then the platform should raise its distance fare and pays a higher distance wage to attract more drivers. As a result, the platform can obtain a higher profit. An increase in the upper bound of the mental cost (i.e., $\overline{\theta}$) decreases distance fare p_{r1}^* and wage w_1^* but increases profit π^*. Thus, the popularization of the ridesharing service is good to the platform. When $L > \hat{L}$, an increase in lower bound $\underline{\theta}$ always decreases (raises) the platform's payout ratio r^*(profit π^*), whereas payout ratio r^* and profit π^* do not vary with the value of $\overline{\theta}$ when $n_p \leq n_d$, which means upper bound $\overline{\theta}$ impacts the platform only when $n_p > n_d$.

5.4 Sensitivity Analyses of Unit Distance Fare for Taxi Service p_{t1}

We conduct sensitivity analyses of p_{t1} to explore the influences of the unit distance fare for taxi service on the platform's optimal decisions, optimal payout ratio, and maximum profit.

Theorem 5 *Both the platform's optimal distance fare p_{r1}^* and its maximum profit π^* are increasing in p_{t1}. However, the impacts of p_{t1} on the platform's optimal unit distance wage w_1^* and payout ratio r^* depend on passengers' trip distance L. Specifically, if L is sufficiently small, then w_1^* is increasing in p_{t1} (when $L \leq \hat{L}$) and r^* is increasing in p_{t1} (when $L \leq \overline{L} \equiv (2n_p\gamma_2)^2/\{[n_d(\overline{\theta} - \underline{\theta}) + 2n_p\gamma_2]^2 + [n_d(\overline{\theta} - \underline{\theta})]^2\}$). Otherwise, w_1^* is independent of p_{t1} (when $L > \hat{L}$) and r^* is decreasing in p_{t1} (when $L > \overline{L}$).*

We learn from Theorem 5 that an increase in taxi distance fare p_{t1} results in a greater ridesharing distance fare p_{r1}^* and generates a higher profit for the platform; but, it cannot always raise the ridesharing drivers' distance wage w_1^*, which depends on passengers' trip distance. When L is sufficiently long, the platform does not raise the wage as a response to a higher unit distance fare for the taxi service.

5.5 Sensitivity Analyses of Opportunity Gain γ

We recall from Section 3.2 that the drivers who are not available for ridesharing service can gain γ, whose mean is γ_1, and the difference between the mean and the maximum (minimum) is γ_2. We perform sensitive analyses of γ_1 and γ_2 to learn managerial implications regarding their impacts on the platform's operations.

Theorem 6 *We summarize the impacts of γ_1 and γ_2 on p_{r1}^*, w_1^*, r^*, and π^* as in Table 2, in which ξ_3, ξ_4, γ_3, γ_4, and γ_5 are defined as*

$$\begin{cases} \xi_3 \equiv & 2n_p\left\{n_d\left(\bar{\theta}-\underline{\theta}\right)-2n_p\left[\left(\bar{\theta}+p_{t0}\right)+p_{t1}\left(L-L_0\right)\right]\right\}\gamma_2^2-4n_pn_d\left(\bar{\theta}-\underline{\theta}\right)\left(\gamma_1+C_d^vL\right)\gamma_2+2n_pn_d \\ & \times\left(\bar{\theta}-\underline{\theta}\right)\left[\left(\bar{\theta}+p_{t0}\right)+p_{t1}\left(L-L_0\right)-\left(\gamma_1+C_d^vL\right)\right]^2-n_d^2\left(\bar{\theta}-\underline{\theta}\right)^2\left[\left(\bar{\theta}+p_{t0}\right)+p_{t1}\left(L-L_0\right)\right], \\ \xi_4 \equiv & \left(8n_p-2n_d\right)n_p\gamma_2\left[n_p\gamma_2+n_d\left(\bar{\theta}-\underline{\theta}\right)\right]+n_pn_d^2\left(\bar{\theta}-\underline{\theta}\right)\left\{\left[2\left(\bar{\theta}-\underline{\theta}\right)-\left(\bar{\theta}+p_{t0}\right)+\left(\gamma_1+C_d^vL_0\right)\right]\right\}, \\ \gamma_3 \equiv & \left[1+\left(\gamma_1+C_d^vL\right)-\left(\bar{\theta}+p_{t0}\right)-p_{t1}\left(L-L_0\right)\right] \\ & +\sqrt{\left[1+2\left(\gamma_1+C_d^vL\right)-2\left(\bar{\theta}+p_{t0}\right)-2p_{t1}\left(L-L_0\right)\right]n_p^2+n_pn_d\left(\bar{\theta}-\underline{\theta}\right)}/n_p, \\ \gamma_4 \equiv & \left(\bar{\theta}+p_{t0}-C_d^vL_0\right)-n_d\left(\bar{\theta}-\underline{\theta}\right)/\left(2n_p\right), \\ \gamma_5 \equiv & \left\{n_p\sqrt{6n_pn_d\left(\bar{\theta}-\underline{\theta}\right)\left[\left(\bar{\theta}+p_{t0}\right)-\left(\gamma_1+C_d^vL_0\right)\right]-3n_d^2\left(\bar{\theta}-\underline{\theta}\right)^2}-3n_d\left(\bar{\theta}-\underline{\theta}\right)\right\}/\left(6n_p^2\right). \end{cases} \tag{13}$$

Conditions	γ_1: $L \le \hat{L}$	γ_1: $L > \hat{L}$	γ_2: $L \le \hat{L}$	γ_2: $L > \hat{L}$, $n_p \le n_d$	γ_2: $L > \hat{L}$, $n_p > n_d$
p_{r1}^*	–	↓	↑	↓ if $\gamma_1 \le \gamma_4$, ↑ if $\gamma_1 > \gamma_4$.	
w_1^*	–	↓	↑	↑ if $\xi_4 \ge 0$, ↓ if $\xi_4 < 0$.	↓ if $\gamma_2 \le \gamma_5$, ↑ if $\gamma_2 > \gamma_5$.
r^*	↑	↑	↑ if $\xi_3 \ge 0$, ↓ if $\xi_3 < 0$.	↓ if $n_p \le n_d/2$, ↑ if $n_p > n_d/2$.	
π^*	↓	↓	↑ if $\gamma_2 \le \gamma_3$, ↓ if $\gamma_2 > \gamma_3$.	↑ if $n_p \le n_d/2$, ↓ if $n_p > n_d/2$.	

Table 3: The impacts of γ_1 and γ_2 on p_{r1}^*, w_1^*, r^*, and π^*. Note that the marks "↑" , "–" and "↓" indicate that optimal decisions p_{r1}^* and w_1^*, optimal payout ratio r^*, and maximum profit π^* are "increasing in," "independent of," and "decreasing in" mean(radius) of drivers' opportunity cost $\gamma_1(\gamma_2)$, respectively.

Theorem 6 indicates that ridesharing drivers' opportunity cost significantly influences the platform's performance. If, in a served region, the drivers can gain higher benefits from non-ridesharing chances than the ridesharing service delivery (i.e. γ_1 is larger), then the platform should respond by making a higher payment ratio to the drivers for attracting them to willingly deliver the ridesharing service. This increases the platform's operational cost and then reduces its profit. However, the mean of the opportunity cost has no impact on the distance fare and wage when the trip distance is sufficiently short.

The dispersion degree of the opportunity cost has complex influences on the optimal decision and performance. Nonetheless, we find that, for a sufficiently short distance, when the ridesharing drivers are more diverse in their opportunity gains, the platform should reduce distance wage and also distance fare. However, the platform can profit from the increase of dispersion degree when it is sufficiently small.

5.6 Sensitivity Analyses of the Number of Total Potential Passengers n_p and Drivers n_d

We investigate how the market size in the served region affects the platform's optimal decisions and profit. We begin by studying the influences of the number of total potential passengers (i.e., n_p) and the number of total potential drivers (i.e., n_d) on optimal decisions p_{r1}^* and w_1^* as well as optimal payout ratio r^*. Then, we explore the impacts of n_p and n_d on the platform's maximum profit π^*.

Theorem 7 *We summarize the impacts of n_p and n_d on p_{r1}^*, w_1^*, and r^* as in Table 4, in which ξ_6 and ξ_7 are defined as*

$$\begin{cases} \xi_6 \equiv 2[n_d\left(\overline{\theta}-\underline{\theta}\right)+2n_p\gamma_2]^2 - n_d^2\left(\overline{\theta}-\underline{\theta}\right)\left[\left(\overline{\theta}+p_{t0}\right)-\left(\gamma_1-\gamma_2+C_d^vL_0\right)\right], \\ \xi_7 \equiv \left[n_d\left(\overline{\theta}-\underline{\theta}\right)+2n_p\gamma_2\right]^2 - n_p^2\gamma_2\left[\left(\overline{\theta}+p_{t0}\right)-\left(\gamma_1-\gamma_2+C_d^vL_0\right)\right]. \end{cases} \tag{14}$$

<table>
<tr><th rowspan="3">Conditions</th><th colspan="3">n_p</th><th colspan="3">n_d</th></tr>
<tr><th rowspan="2">$L \le \hat{L}$</th><th colspan="2">$L > \hat{L}$</th><th rowspan="2">$L \le \hat{L}$</th><th colspan="2">$L > \hat{L}$</th></tr>
<tr><th>$n_p \le n_d$</th><th>$n_p > n_d$</th><th>$n_p \le n_d$</th><th>$n_p > n_d$</th></tr>
<tr><td>p_{r1}^*</td><td>↑</td><td>↓</td><td>$\begin{cases}\uparrow \text{ if } \xi_7 \ge 0,\\ \downarrow \text{ if } \xi_7 < 0.\end{cases}$</td><td>↓</td><td>↑</td><td>$\begin{cases}\downarrow \text{ if } \xi_7 \ge 0,\\ \uparrow \text{ if } \xi_7 < 0.\end{cases}$</td></tr>
<tr><td>w_1^*</td><td>↑</td><td>$\begin{cases}\uparrow \text{ if } \xi_6 \ge 0,\\ \downarrow \text{ if } \xi_6 < 0.\end{cases}$</td><td>↓</td><td>↓</td><td>$\begin{cases}\downarrow \text{ if } \xi_6 \ge 0,\\ \uparrow \text{ if } \xi_6 < 0.\end{cases}$</td><td>↑</td></tr>
<tr><td>r^*</td><td>↑</td><td>↑</td><td>↓</td><td>↓</td><td>↓</td><td>↑</td></tr>
</table>

Table 4: The impacts of n_p and n_d on p_{r1}^*, w_1^* and r^*. Note that the marks "↑" , "− " and "↓" indicate that optimal decisions p_{r1}^* and w_1^*, and optimal payout ratio r^* are "increasing in," "independent of," and "decreasing in" the number of total passengers n_p and drivers n_d, respectively.

When $L > \hat{L}$ and $n_p > n_d$, all potential drivers have an incentive to provide ridesharing service; and when $L > \hat{L}$ and $n_p \le n_d$, all potential passengers prefer the ridesharing service to the Taxi service. This means that the scenario "$L > \hat{L}$" rarely occurs in practice. Accordingly, we concentrate our discussion to the scenario "$L \le \hat{L}$" under which p_{r1}^* and w_1^* are increasing in n_p and they are decreasing in n_d.

In practice, the number of potential passengers varies over time. During the peak time interval (e.g., 7:30-9:30), the number of potential passengers surges. Theorem 7 indicates that the platform should raise the distance fare and distance wage simultaneously for the service in peak times when $L \le \hat{L}$. The number decreases during the non-peak time interval (e.g., 9:30-23:00). In order to attract more passengers, the platform lowers the distance fare and also reduces the distance wage to offset the loss. For example, in Hangzhou (the capital city of Zhejiang province, China), Didi (the largest on-demand, ridesharing service platform in China) sets the distance fare as ¥2.2/km in non-peak times, and increases the distance fare to ¥2.5/km in peak times. To ensure the fixed payout ratio 80%, Didi also raises the distance wage in peak times.

The number of potential drivers varies over time. For example, in the weekends, drivers are usually available for providing the ridesharing service on the platform. As Theorem 7 indicates, the distance fare and distance wage should decrease simultaneously with the increase in the number of drivers who turn on the App to indicate his/her willingness to serve while $L \leq \hat{L}$. For some cases in which, for example, bad weather makes it difficult to drive, less drivers agree to provide the service, the platform would respond by increasing the distance fare and distance wage simultaneously. Moreover, the payout ratio also varies with the number of potential passengers and drivers. As usual, when there are more potential passengers, drivers should receive a higher allocation of the total fare from the platform. However, when there are more potential drivers, the platform should obtain a higher share. In recent years, the platform encourages passengers to pay red packets to the ridesharing drivers in bad weather conditions, which, in fact, is an obscure measure to raise the price and reduce the platform's share in the total fare.

Theorem 8 *The platform's maximum profit π^* is an increasing, concave function of n_p and n_d. Moreover, the values of n_d and n_p are complementary to each other in influencing π^*, as $\partial^2\pi^*/\partial n_p \partial n_d \geq 0$.*

Theorem 8 indicates that the increases in both the number of potential passengers (i.e., a larger value of n_p) and the number of potential drivers (i.e., a larger value of n_d) can help increase the platform's profit, which reflects the fact that a higher market "demand" and/or "supply" are beneficial to the platform. That is, in order to obtain a greater profit, the platform may need to entice more passengers or more drivers or both.

We also learn from Theorem 8 that if we continuously increase either the number of potential passengers or the number of drivers, the impact on profit may be weaker. This means that when there are more potential passengers (or drivers) but the platform does not also entice more drivers (or passengers), the profit does not increase as significantly as when the value of n_p (or n_d) is sufficiently small. According to Theorem 8, we find that the values of n_p and n_d are complementary to each other when they affect the platform's profit. The result occurs mainly because the platform's profit is dependent on the match between the market "demand" and "supply." This also exposes the fact that the platform should not only entice more passengers to use its App for the ridesharing service but also induce more drivers to join the ridesharing service network.

6 Dynamic Pricing vs. Static Pricing Methods: Numerical Experiments with Real Data-Based Parameter Estimates

We learn from Sections 3 and 5.6 that the total number of potential passengers (i.e., n_p) impacts the platform's optimal pricing decisions. The pricing strategy under which the ridesharing price and the driver wage vary with n_p is called "dynamic pricing strategy," and the pricing strategy under which the ridesharing price and the driver's wage are independent of the value of n_p is called "static pricing strategy." Our analysis in Section 5 exposes that the platform's optimal distance fare and distance wage should vary when the values of some parameters—such as total potential demand, the number of drivers, mental cost, running cost, and others—change, which implies that dynamic pricing strategy could help the platform enjoy a higher profit than static pricing strategy. However, many researchers and practitioners have commented that increasing the service price in peak hours is unfair to passengers. Cachon, Daniels, and Lobel (2017) argued that whether or not dynamic pricing strategy is better than static pricing strategy should base on not only the platform's profit but also passengers' surpluses. Accordingly, we perform numerical experiments to examine if dynamic pricing strategy always generates more profits to the platform and also results in a higher surplus/profit to passengers and drivers.

6.1 Estimated Values of Major Parameters

We begin by estimating the values of major parameters based on the real evidences provided by Bai et al. (2019). To verify analytic findings regarding the impact of potential passenger demand on the ridesharing service price, the wage to drivers, and the payout ratio, Bai et al. (2019) performed numerical experiments using the data from the Didi in Hangzhou on September 7-13 and November 1-30 in 2015. Since Hangzhou is a large city in which many passengers may not easily access drivers, Bai et al. (2019) divided the city into 20 zones, and found that in each zone, the average demand request (i.e., passengers who choose the Didi Express rather than the Taxi service) rate is equal to 100 persons per hour in peak times. Bai et al. (2019) also estimated the average customer (i.e., potential passengers) rate as 200 persons per hour in peak times; and the average demand request rate and customer rate are 50 persons per hour and 100 persons per hour, respectively, in non-peak times. Thus, in our numerical experiments, we can reasonably assume that the total number of potential passengers n_p varies from 100 to 200.

	Peak-times	**Non-peak times**
average demand request rate	100	50
average customer rate	200	100

Table 5: The demand request rates and passenger rates.

Both Bai et al (2019) and Taylor (2018) revealed that the service capacity should be higher than the potential demand in any ridesharing system. Following Bai et al (2019), we set the total number of potential drivers n_d to be 300, and the average trip length $L = 6km$. In peak and non-peak times, drivers' running speeds are $19km/h$ and $26km/h$, respectively. It is thereby reasonable for us to assume that the running cost C_d^v equals ¥1.3 per km in peak times and ¥1 per km in non-peak times, respectively.

According to the pricing policies for Didi and Taxi in Hangzhou, we learn that $L_0 = 3$km, $p_{t0} =$ ¥13, and $p_{t1} =$ ¥$2.6/km$. After comparing the policies for Didi and those for Taxi in Hangzhou, we find that Didi's total price is a bit cheaper than Taxi in non-peak times but a little more expensive than Taxi in peak times. Thus, we believe that some passengers are more likely to choose Didi rather than Taxi, and we set the upper bound of mental cost as ¥5, i.e., $\bar{\theta}$ = ¥5; and there are a few passengers who are more likely to choose Taxi rather than Didi in non-peak times, and we thus set $\underline{\theta}$ = −¥2. In addition, as a trip needs about 20 minutes on average, a ridesharing driver's willingness to complete a trip requires that the driver should earn at least ¥5 to ¥15, which means that the mean of ridesharing drivers' opportunity gain (i.e., γ_1) is ¥10, and the difference between the mean and the maximum (or, the minimum) value (i.e., γ_2) is ¥5. The parameters' values are summarized as in Table 6.

Parameter	n_d	L	p_{t0}	p_{t1}	L_0	$\bar{\theta}$	$\underline{\theta}$	γ_1	γ_2
Value	300	$6km$	¥13	¥2.6/km	$3km$	¥5	−¥2	¥10	¥5

Table 6: The values of parameters.

6.2 Numerical Experiments

We use the estimated parameter values in Section 6.1 to compute the ridesharing platform's static and dynamic pricing decisions, and compare the profits under the two pricing strategies to find whether or not dynamic pricing strategy always results in a higher profit for the platform. This can help address the question of whether the platform prefers the dynamic pricing strategy to the static pricing strategy. In addition, we consider the impacts of the platform's pricing strategy on the passengers' and drivers' surpluses, which can help explore the externalities of the ridesharing service.

We start with the computation of the static service price and driver wage, using the formula in Theorem 1 with $n_p = 150$ and C_d^v = ¥1.15/km, which are obtained as the average of their values in peak times and those in non-peak times. We find that under the optimal static pricing policy, p_{r0}^{S*} = ¥15.21 and p_{r1}^{S*} = ¥2.18/km; and, under the optimal static wage policy, w_0^{S*} = ¥10.44 and w_1^{S*} = ¥1.45/km.

6.2.1 Sensitivity Analyses for the Optimal Pricing Decisions and Profit

Using the parameter values given in Section 6.1, we perform numerical experiments to expose the impacts of n_p and C_d^v on the ridesharing platform's optimal (dynamic) pricing decisions (i.e., p_{r1}^* and w_1^*) and its profit in the dynamic pricing setting (i.e., π^*). For the sensitivity analyses, we increase the value of n_p in steps of 10 from 100 to 200, and raise the value of C_d^v in steps of 0.03 from 1 to 1.3. For any combination of the values of n_p and C_d^v, we use Theorem 1 to compute the corresponding optimal distance fare p_{r1}^* and optimal distance wage w_1^*, as shown in Figure 1.

We learn from Figure 1 that both the optimal distance fare p_{r1}^* and distance wage w_1^* are increasing in both n_p and C_d^v under the dynamic pricing strategy. This is consistent with Theorems 3 and 7. In order to investigate whether the ridesharing platform prefers to adopt the dynamic pricing strategy or to use the static pricing strategy, we compute the difference between the platform's maximum profits under the two strategies, as shown in Figure 2. Figure 2 indicates that the platform's maximum profit under the dynamic pricing strategy (i.e., π^{D*}) is always higher than that under the static pricing strategy (i.e., π^{S*}), except for the case that $n_p = 150$ and C_d^v = ¥1.15/km (for which $\pi^{D*} = \pi^{S*}$).

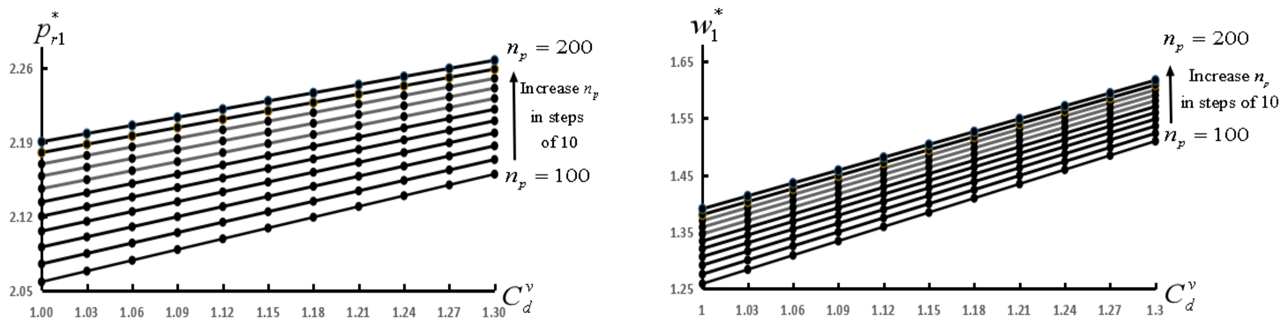

Figure 1: The impacts of n_p and C_d^v on optimal distance price p_{r1}^* and optimal distance wage to ridesharing drivers w_1^*.

We also find that, on average, π^{D*} is 12.3% higher than π^{S*}, which means that the dynamic pricing strategy generates a higher profit to the platform than the static one. Moreover, we note from Figure 2 that, when the per kilometer running cost (i.e., C_d^v) is sufficiently high (e.g., C_d^v = ¥1.30/km), the dynamic pricing strategy helps the platform to obtain a greater extra profit—i.e., the difference between the profit under the dynamic pricing strategy and that under the static strategy—from a larger number of potential passengers (i.e., n_p). However, if the value of C_d^v is sufficiently small (e.g., C_d^v = ¥1.00/km), then the dynamic pricing strategy generates a greater extra profit when the value of n_p is smaller. This result shows that the dynamic pricing strategy makes a greater extra profit when the number of potential passengers and the ridesharing drivers' per kilometer running cost increase or decrease simultaneously. This is actually a prevailing issue in practice.

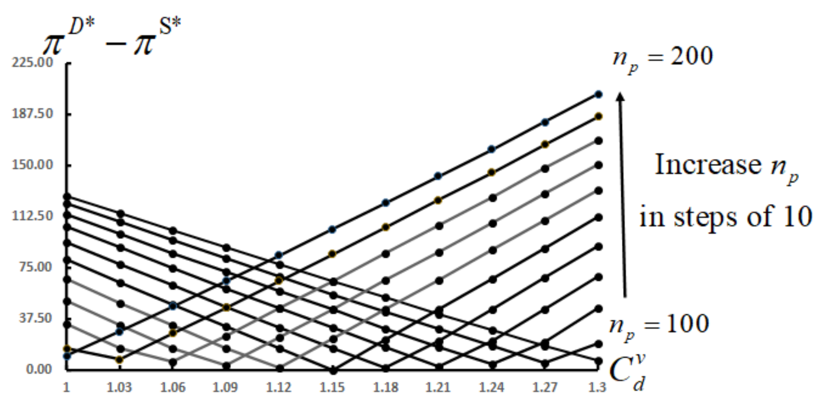

Figure 2: The differences between the platform's maximum profits in the dynamic and static pricing strategies, i.e., $\pi^{D*} - \pi^{S*}$.

6.2.2 Sensitivity Analyses for the Passengers' and the Drivers' Surpluses

We perform numerical experiments to investigate how the ridesharing platform's optimal pricing strategy (i.e., its optimal pricing decisions under the dynamic/static pricing strategy) influences the passengers' and the drivers' benefits in terms of their surpluses. This is important because our results can help the platform to observe the responses of passengers and drivers to its ridesharing operations. We also perform our sensitivity analyses for the surpluses when the values of C_d^v and n_p change.

Analysis of the Total Expected Surplus of Passengers We analyze the influence of the ridesharing platform's optimal pricing strategy on the passengers, starting with the computation of a passenger's surplus. A passenger with trip length L needs to pay $p_{r0} + p_{r1}(L - L_0)$ if he/she chooses the ridesharing service, whereas the passenger absorbs cost $\theta + p_{t0} + p_{t1}(L - L_0)$ if he/she chooses a taxi service. Therefore, the cost savings that the passenger can achieve from the ridesharing service are $\theta - [(p_{r0} - p_{t0}) + (p_{r1} - p_{t1})(L - L_0)]$, which can be simply called the passenger's "surplus." If the surplus is greater than zero, or, mental cost θ is larger than $[(p_{r0} - p_{t0}) + (p_{r1} - p_{t1})(L - L_0)]$, then the passenger prefers to choose the ridesharing service instead of a taxi service. As θ is uniformly distributed in $[\underline{\theta}, \overline{\theta}]$, we compute the total expected surplus of passengers as

$$\begin{aligned} S_p &= n_p \int_{[(p_{r0}-p_{t0})+(p_{r1}-p_{t1})(L-L_0)]}^{\overline{\theta}} \{\theta - [(p_{r0} - p_{t0}) + (p_{r1} - p_{t1})(L - L_0)]\} g(\theta) d\theta \\ &= \frac{n_p}{2\left(\overline{\theta} - \underline{\theta}\right)} \left\{\overline{\theta} - [(p_{r0} - p_{t0}) + (p_{r1} - p_{t1})(L - L_0)]\right\}^2 . \end{aligned} \tag{15}$$

Next, we compute surpluses S_p under the dynamic and static pricing strategies. According to Theorem 1, under the dynamic pricing strategy, when the platform makes its optimal pricing decisions, the number of attracted passengers and that of drivers are identical (i.e., $Q_p = Q_d$). This means that each passenger who chooses the platform's service can be served by a driver, and each driver who is available at the platform can be assigned to a passenger. Using parameter values in Section 6.1 and assuming that optimal pricing policy $(p_{r0}^*,\ p_{r1}^*)$ for each value of n_p in the range of [100, 200] and each value of C_d^v in [1, 1.3], we can compute S_p^D in (15) for each combination of n_p and C_d^v, where S_p^D denotes the total expected surplus of passengers under the dynamic pricing strategy.

Under the static pricing strategy, the service price and the drivers' wage are constant when the values of n_p and C_d^v vary. In addition, there are more passengers than drivers (i.e. $Q_p > Q_d$) when the values of n_p and C_d^v are sufficiently large. This means that a passenger who places an order at the platform can be served only with probability Q_d/Q_p. Then, according to (15), the total expected surplus of passengers under the static pricing strategy is $S_p^S = n_p Q_d \{\overline{\theta} - [(p_{r0} - p_{t0}) + (p_{r1} - p_{t1})(L - L_0)]\}^2 / [2Q_p(\overline{\theta} - \underline{\theta})]$, where Q_d and Q_p are given by (3) and (2), respectively. When the values of n_p and C_d^v are sufficiently small, under the platform's static pricing strategy, there are more drivers than passengers (i.e. $Q_d > Q_p$). As a result, each passenger can receive a ridesharing service, and the total expected surplus of passengers is $S_p^S = n_p \{\overline{\theta} - [(p_{r0} - p_{t0}) + (p_{r1} - p_{t1})(L - L_0)]\}^2 / [2(\overline{\theta} - \underline{\theta})]$. Using the above results, we compute surpluses S_p^S when the value of n_p increases in steps of 10 from 100 to 200 and the value of C_d^v increases in steps of 0.03 from 1 to 1.3.

To facilitate the comparison between S_p^D and S_p^S, we plot ratio S_p^D/S_p^S as in Figure 3, from which we learn that in most scenarios, $S_p^D/S_p^S > 1$. However, when the value of n_p is sufficiently large (e.g., $n_p \geq 160$) and the value of C_d^v is sufficiently small (e.g., $C_d^v \leq 1.12$), S_p^D is sometimes smaller than S_p^S. The smallest value of ratio S_p^D/S_p^S is 0.889 when $n_p = 200$ and $C_d^v = 1.00$. This result exposes that passengers benefit more from the static pricing strategy than the dynamic one, when the number of passengers is sufficiently large and the running cost is sufficiently small. Nevertheless, a large value of n_p and a small value of C_d^v are unlikely to occur in practice, because of the following fact: when the number of potential passengers (i.e., n_p) increases, the traffic would be busier and the running cost C_d^v are likely to rise. It thus follows that in general, S_p^D is larger than S_p^S; that is, passengers always benefit more from the dynamic pricing strategy than from the static one.

We also reveal an interesting, and somewhat surprising, result as follows: in peak hours ($n_p = 200$, and $C_d^v = 1.3$), although passengers have to pay for a higher fare under the dynamic pricing strategy than under the static pricing strategy, the dynamic strategy can still make a greater surplus to passengers than the static strategy. According to our analysis regarding the impact of the number of expected total passengers and the number of drivers, we find that, for a ridesharing driver, a higher wage resulting from the dynamic pricing strategy can offset the driver's loss generated by an increase in the unit running cost C_d^v. This can thereby attract more drivers to provide service, or mitigate the effect of a reduction in the number of drivers. In addition, a higher fare could prevent the passengers with a smaller mental cost θ from requesting service, which leads to the result that the surplus that any passenger enjoys from each successful ridesharing deal does not decrease.

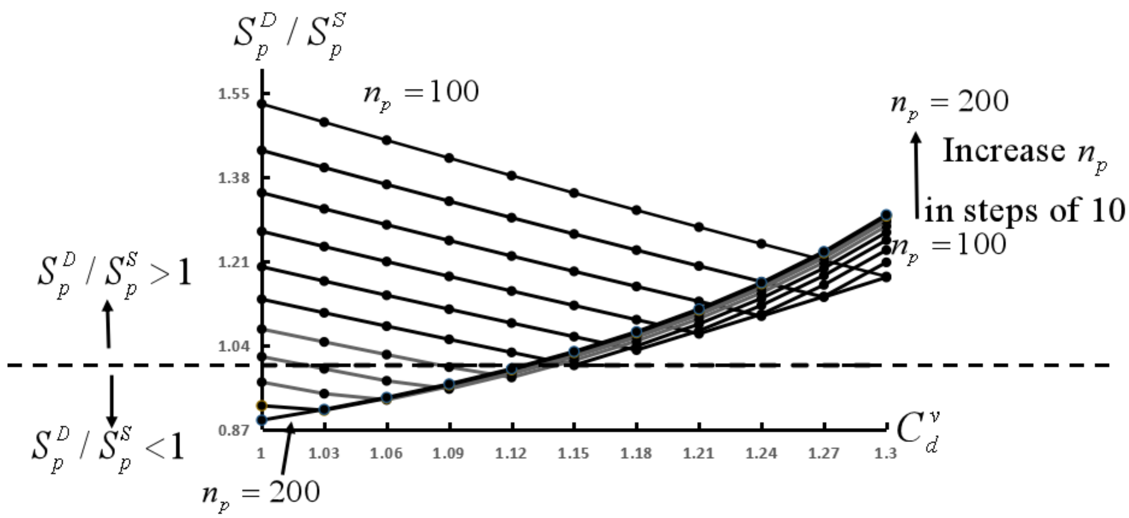

Figure 3: The ratios of S_p^D to S_p^S when the values of n_p and C_d^v vary.

Analysis of the Total Expected Surplus of Drivers We examine the impact of the platform's optimal pricing strategy on the ridesharing drivers, whose total expected surplus is computed as follows. If a driver serves a passenger for a trip with distance L, then the driver can obtain a payment $w_0 + w_1(L - L_0)$ from the platform. However, for the trip, the driver absorbs a running cost $C_d^v L$, and he or she has an opportunity cost γ. Therefore, the driver's surplus (net income) is $w_0 + w_1(L - L_0) - (C_d^v L + \gamma)$, which indicates that only the drivers with opportunity cost $\gamma \leq w_0 + w_1(L - L_0) - C_d^v L$ are willing to serve passengers. Recall from Section 3.2 that γ is uniformly distributed over $[\gamma_1 - \gamma_2, \gamma_1 + \gamma_2]$, and there are n_d potential drivers in total. We can compute the total expected surplus of drivers as

$$
\begin{aligned}
S_d &= n_d \int_{\gamma_1-\gamma_2}^{w_0+w_1(L-L_0)-C_d^v L} [w_0 + w_1(L-L_0) - C_d^v L - \gamma]\varphi(\gamma)d\gamma \\
&= \frac{n_d}{4\gamma_2}[w_0 + w_1(L-L_0) - C_d^v L - (\gamma_1-\gamma_2)]^2. \qquad (16)
\end{aligned}
$$

Similar to the computation of passengers' total expected surplus, we can calculate drivers' total expected surplus under the dynamic pricing strategy and that under the static pricing strategy, which are denoted by S_d^D and S_d^S, respectively.

Using parameter values in Section 6.1 as well as optimal service pricing policy (p_{r0}^*, p_{r1}^*) and optimal wage (w_0^*, w_1^*) for $n_p \in [100, 200]$ and $C_d^v \in [1, 1.3]$, we can compute S_d^D by using (16) for different values of n_p and C_d^v. For the static pricing strategy, when the values of n_p and C_d^v are sufficiently large, there are more passengers than drivers and each driver can serve a passenger. As a result, the total expected surplus of drivers is given by (16). However, when the values of n_p and C_d^v are sufficiently small, under the platform's static pricing strategy, there are more drivers than passengers (i.e. $Q_d > Q_p$), which means that a driver who is available at the platform can serve a passenger only with probability Q_p/Q_d. Then, according to (16), the total expected surplus of drivers is $S_d^S = n_d[w_0 + w_1(L-L_0) - C_d^v L - (\gamma_1-\gamma_2)]^2 Q_p/(4\gamma_2 Q_d)$.

To compare the difference of the total expected surplus of drivers under the dynamic pricing strategy (i.e., S_d^D) and that under the static pricing strategy (i.e., S_d^S), we compute the ratios of S_d^D/S_d^S for each combination of n_p and C_d^v and plot it in Figure 4. We note from Figure 4 that $S_d^D/S_d^S < 1$ when C_d^v is sufficiently small (e.g., $C_d^v < 1.03$). That is, when the unit running cost is sufficiently small, the dynamic pricing strategy results in a lower wage than the static pricing strategy, which thus makes a higher surplus to drivers than the dynamic pricing strategy. In addition, comparing the average values of S_d^D and S_d^S, we find that the dynamic pricing strategy, on average, generates a 10.4% higher surplus to drivers than the static one, similar to our analysis for passengers.

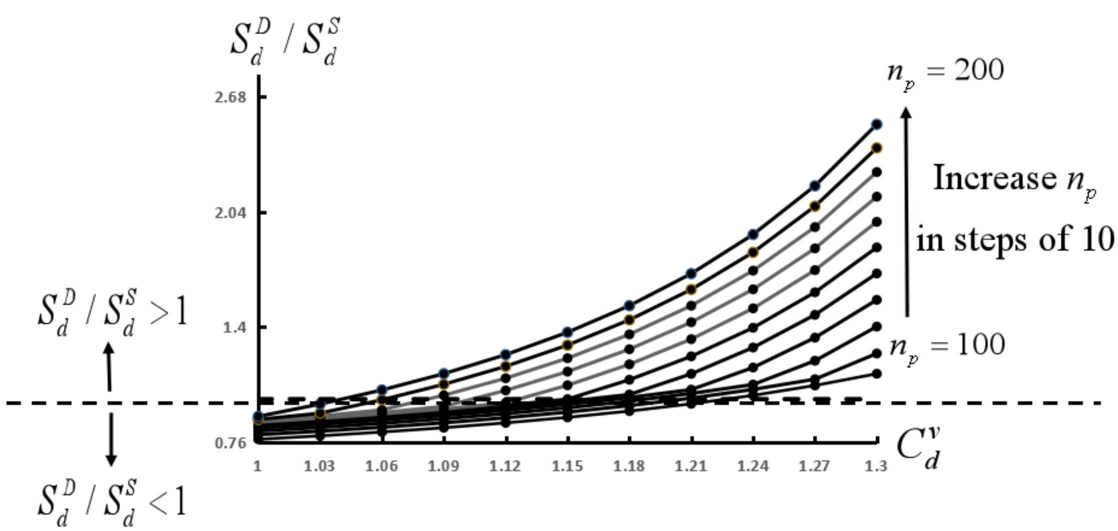

Figure 4: The ratios of S_d^D to S_d^S when the values of n_p and C_d^v vary.

Analysis of the Sum of Passengers' and Drivers' Surpluses Summing the total expected surplus of passengers and that of drivers, we obtain the total expected surplus of passengers and drivers, which is denoted by S_{pd}^{D} under the dynamic pricing strategy and by S_{pd}^{S} under the static pricing strategy. To facilitate our comparison between S_{pd}^{D} and S_{pd}^{S}, we calculate ratio S_{pd}^{D}/S_{pd}^{S} for each combination of n_p and C_d^v, and show our results in Figure 5, which indicates that S_{pd}^{D} is smaller than S_{pd}^{S} with the probability of about 15.7%, which takes place when n_p is sufficiently large and C_d^v is sufficiently small. Note from Section 6.2.2 that the situation of a large value of n_p and a small value of C_d^v is unlikely to happen. Thus, we conclude that the dynamic pricing strategy can generate a higher total expected surplus to passengers and drivers than static one.

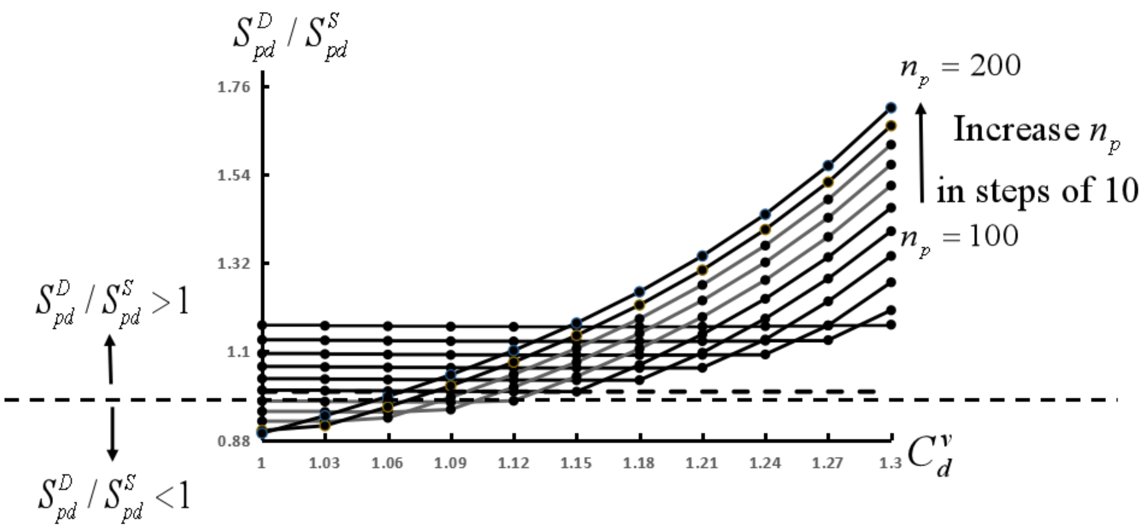

Figure 5: The ratios of S_{pd}^{D} to S_{pd}^{S} when the values of n_p and C_d^v vary.

According to our numerical experiments, the dynamic pricing strategy not only generates a greater profit to the platform but also usually results in higher surpluses to passengers and drivers. We also observe that even when the dynamic strategy reduces the total expected surplus of drivers, the passengers can still obtain a higher surplus. This implies that the dynamic pricing strategy is basically better than the static pricing strategy.

7 Summary and Concluding Remarks

We consider an optimal pricing problem for a ridesharing platform who determines both the ridesharing service fare charged to passengers and the wage paid to drivers registered on the platform. As in practice, a passenger can choose either a ridesharing or a taxi service for a trip and a driver can decide to either turn on the App to serve passengers or switch off the App to indicate his or her unavailability. We accordingly begin by analyzing passengers' problems for their choices between ridesharing and taxi services, and investigating drivers' problems for their choices between willingness and unwillingness to serve passengers. Then, using our analytic results for the two choice problems, we construct a profit function for a ridesharing platform and obtain the platform's optimal pricing decisions.

In order to expose managerial implications regarding the ridesharing platform's optimal pricing decisions as well as its maximum profit and optimal payout ratio, we perform sensitivity analyses of some important parameters, which include passengers' average trip distance, ridesharing drivers' unit running cost, passengers' mental cost for taking a taxi service, distance fare for the taxi service, parameters for ridesharing drivers' opportunity gains when they are unavailable for ridesharing service, the number of total potential passengers and drivers. Our analytic results are summarized in Table 7. Moreover, we conduct real data-based numerical experiments to show that comparing with the static pricing strategy, the dynamic pricing strategy generates a higher profit to the ridesharing platform, and provides a larger surplus to passengers and drivers.

According to the analytic results from our sensitivity analyses and the numerical experiments, we draw a number of managerial implications as below.

1. When the number of passengers increases, the platform should raise the distance fare and wage simultaneously, and the platform should raise the payout ratio to attract more drivers, i.e. drivers obtain a higher benefit from the increase in the number of passengers than the platform. When the number of drivers increases, the platform should decrease the distance fare and wage simultaneously, and it can reduce the payout ratio to obtain a higher profit. Thus, the platform should change not only the fare and wage but also the payout ratio when the number of passengers or drivers varies.
2. Both an increase in the driver number and an increase in passenger number can help increase the platform's profits, if the numbers are smaller than their cutoff levels. When the number of passengers (drivers) increases above its cutoff level, the platform's profit still rises, but the impact becomes weaker if the number of drivers (passengers) does not increase. The result suggests that the platform is better off when both the number of passengers and the number of drivers are higher.
3. Regardless of whether the upper or lower bound of mental cost increases, the platform's profit rises. Therefore, to achieve a higher profit, the platform should strive to improve its social position, induce passengers to more likely choose the ridesharing service, and increase their mental costs for the taxi service.
4. The dynamic pricing strategy is beneficial to all stakeholders. First, the dynamic pricing strategy can generate a higher profit to the ridesharing platform than the static one. Second, the dynamic strategy can, by and large, generate greater surpluses to both drivers and passengers than the static strategy. It thus follows that for the platform, the dynamic pricing strategy dominates the static strategy.
5. The platform's payout ratio—which can measure the platform's profit-making capability—is increasing in the ridesharing drivers' running cost, passengers' minimum mental cost and mean opportunity cost as well as the number of total potential passengers.

Parameters	p_{r1}^*			w_1^*		
	$L \le \hat{L}$	$L > \hat{L}$		$L \le \hat{L}$	$L > \hat{L}$	
		$n_p \le n_d$	$n_p > n_d$		$n_p \le n_d$	$n_p > n_d$
L	$-$	$\begin{cases}\downarrow \text{ if } \underline{\theta} \le \theta_2, \\ \uparrow \text{ if } \underline{\theta} > \theta_2.\end{cases}$		$-$	$\begin{cases}\uparrow \text{ if } \underline{\theta} \le \theta_2, \\ \downarrow \text{ if } \underline{\theta} > \theta_2.\end{cases}$	
C_d^v	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
$\overline{\theta}$	$\downarrow$	$\begin{cases}\uparrow \text{ if } \overline{\theta} \le \theta_3, \\ \downarrow \text{ if } \overline{\theta} > \theta_3.\end{cases}$	$\begin{cases}\uparrow \text{ if } \xi_1 \ge 0, \\ \downarrow \text{ if } \xi_1 < 0.\end{cases}$	$\downarrow$	$\begin{cases}\downarrow \text{ if } \xi_2 \ge 0, \\ \uparrow \text{ if } \xi_2 < 0.\end{cases}$	
$\underline{\theta}$	$\uparrow$	$\begin{cases}\uparrow \text{ if } \underline{\theta} \le \theta_4, \\ \downarrow \text{ if } \underline{\theta} > \theta_4.\end{cases}$		$\uparrow$	$\downarrow$	$\downarrow$
p_{t1}	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$-$	$-$
γ_1	$-$	$\downarrow$	$\downarrow$	$-$	$\downarrow$	$\downarrow$
γ_2	$\uparrow$	$\begin{cases}\downarrow \text{ if } \gamma_1 \le \gamma_4, \\ \uparrow \text{ if } \gamma_1 > \gamma_4.\end{cases}$		$\uparrow$	$\begin{cases}\uparrow \text{ if } \xi_4 \ge 0, \\ \downarrow \text{ if } \xi_4 < 0.\end{cases}$	$\begin{cases}\downarrow \text{ if } \gamma_2 \le \gamma_5, \\ \uparrow \text{ if } \gamma_2 > \gamma_5.\end{cases}$
n_p	$\uparrow$	$\downarrow$	$\begin{cases}\uparrow \text{ if } \xi_7 \ge 0, \\ \downarrow \text{ if } \xi_7 < 0.\end{cases}$	$\uparrow$	$\begin{cases}\uparrow \text{ if } \xi_6 \ge 0, \\ \downarrow \text{ if } \xi_6 < 0.\end{cases}$	$\downarrow$
n_d	$\downarrow$	$\uparrow$	$\begin{cases}\downarrow \text{ if } \xi_7 \ge 0, \\ \uparrow \text{ if } \xi_7 < 0.\end{cases}$	$\downarrow$	$\begin{cases}\downarrow \text{ if } \xi_6 \ge 0, \\ \uparrow \text{ if } \xi_6 < 0.\end{cases}$	$\uparrow$

Parameters	r^*			π^*		
	$L \le \hat{L}$	$L > \hat{L}$		$L \le \hat{L}$	$L > \hat{L}$	
		$n_p \le n_d$	$n_p > n_d$		$n_p \le n_d$	$n_p > n_d$
L	$\begin{cases}\downarrow \text{ if } C_d^v \le C_1, \\ \uparrow \text{ if } C_d^v > C_1.\end{cases}$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
C_d^v	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$
$\overline{\theta}$	$\downarrow$	$-$	$\downarrow$	$\uparrow$	$-$	$\uparrow$
$\underline{\theta}$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
p_{t1}	$\begin{cases}\uparrow \text{ if } L \le \overline{L}, \\ \downarrow \text{ if } L > \overline{L}.\end{cases}$	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
γ_1	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$
γ_2	$\begin{cases}\uparrow \text{ if } \xi_3 \ge 0, \\ \downarrow \text{ if } \xi_3 < 0.\end{cases}$	$\begin{cases}\downarrow \text{ if } n_p \le n_d/2, \\ \uparrow \text{ if } n_p > n_d/2.\end{cases}$		$\begin{cases}\uparrow \text{ if } \gamma_2 \le \gamma_3, \\ \downarrow \text{ if } \gamma_2 > \gamma_3.\end{cases}$	$\begin{cases}\uparrow \text{ if } n_p \le n_d/2, \\ \downarrow \text{ if } n_p > n_d/2.\end{cases}$	
n_p	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
n_d	$\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$

Table 7: A summary of analytic results obtained from our sensitivity analysis. Note that the marks "↑," "−," and "↓" indicate that optimal decisions p_{r1}^* and w_1^*, optimal payout ratio r^* and optimal profit π^* are "increasing in," "independent of," and "decreasing in" those parameters, respectively.

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Appendix A Proofs

Proof of Lemma 1. We learn from (1) that a passenger's willingness to choose the ridesharing service requires the inequality $p_{r0} + p_{r1}(L - L_0) \le \left(\overline{\theta} + p_{t0}\right) + p_{t1}(L - L_0)$. As the function in (3) implies, a driver is willing to deliver the ridesharing service, if $w_0 + w_1(L - L_0) \ge C_d^v L + (\gamma_1 - \gamma_2)$. Noting that the platform should set $p_{r0} \ge w_0$ and $p_{r1} \ge w_1$ in order to avoid any loss, we have

$$\overline{\theta} + p_{t0} + p_{t1}(L - L_0) \ge p_{r0} + p_{r1}(L - L_0) \ge w_0 + w_1(L - L_0) \ge (\gamma_1 - \gamma_2) + C_d^v L, \tag{17}$$

which can be rewritten as $\overline{\theta} \ge \theta_1$, where θ_1 is defined as in this lemma. ■

Proof of Theorem 1. Prior to solving the constrained maximization problem, we compare Q_p and Q_d. We learn from (2) that Q_p is decreasing in the platform's decision variables p_{r0} and p_{r1} but independent of the decision variables w_0 and w_1, and also find from (3) that Q_d is independent of p_{r0} and p_{r1} but increasing in w_0 and w_1. We discuss the following two cases.

1. If the expected number of passenger Q_p is larger than the expected number of drivers Q_d, i.e., $Q_p > Q_d$, then the expected number of successful services is $\min(Q_p, Q_d) = Q_d$. Noting that Q_p is decreasing both in p_{r0} and p_{r1}, we find that if the platform raises its prices (p_{r0}, p_{r1}) by a sufficiently small value such that $Q_p \ge Q_d$, then the expected number of successful services (i.e., Q_d) does not change and the profit margin (i.e., $p_{r0} - w_0 + (p_{r1} - w_1)(L - L_0)$) increases. As a result, the platform can obtain a higher profit by increasing (p_{r0}, p_{r1}). That is, for this case, the optimal prices (p_{r0}^*, p_{r1}^*) (maximizing the platform's profit) are obtained when $Q_p \le Q_d$.
2. If $Q_p < Q_d$, then $\min(Q_p, Q_d) = Q_p$ and the platform can raise its profit if it reduces unit wages (w_0, w_1) to decrease Q_d by a sufficiently small value such that $Q_p \le Q_d$. That is, for this case, we can obtain the optimal solutions (w_0^*, w_1^*) when $Q_p \ge Q_d$.

Our discussions above indicate that $Q_p = Q_d$ when the platform chooses optimal price and wages.

Next, we compute base fare p_{r0} and base wage w_0. When passengers' trip distance is L_0, according to equations in (2) and (3), we rewrite the equation $Q_p = Q_d$ as

$$\frac{n_p[(\overline{\theta} + p_{t0}) - p_{r0}]}{\overline{\theta} - \underline{\theta}} = \frac{n_d[w_0 - (\gamma_1 - \gamma_2 + C_d^v L_0)]}{2\gamma_2}. \tag{18}$$

Solve the above equation for w_0 yields

$$w_0 = (\gamma_1 - \gamma_2 + C_d^v L_0) + \frac{2n_p\gamma_2[(\overline{\theta} + p_{t0}) - p_{r0}]}{n_d\left(\overline{\theta} - \underline{\theta}\right)}. \tag{19}$$

When the trip distance is L_0, the platform's profit is

$$\pi(L_0) = \frac{n_p[(\overline{\theta} + p_{t0}) - p_{r0}]}{\overline{\theta} - \underline{\theta}} \left\{ p_{r0} - (\gamma_1 - \gamma_2 + C_d^v L_0) - \frac{2n_p\gamma_2[(\overline{\theta} + p_{t0}) - p_{r0}]}{n_d\left(\overline{\theta} - \underline{\theta}\right)} \right\}, \tag{20}$$

which only depends on the decision variable p_{r0}. Calculating its first-order derivative w.r.t. p_{r0}, we have

$$\frac{\partial \pi(L_0)}{\partial p_{r0}} = \frac{-n_p}{\overline{\theta}-\underline{\theta}}\left\{p_{r0} - C_d^v L_0 - (\gamma_1-\gamma_2) - \frac{2n_p\gamma_2[\overline{\theta}-(p_{r0}-p_{t0})]}{n_d\left(\overline{\theta}-\underline{\theta}\right)}\right\} + \frac{n_p[\overline{\theta}-(p_{r0}-p_{t0})]}{\overline{\theta}-\underline{\theta}}\left[1+\frac{2n_p\gamma_2}{n_d\left(\overline{\theta}-\underline{\theta}\right)}\right];$$

and we obtain its second-order derivative as $\partial^2\pi(L_0)/\partial p_{r0}^2 = -2n_p[n_d(\overline{\theta}-\underline{\theta})+2n_p\gamma_2]/[n_d(\overline{\theta}-\underline{\theta})^2] < 0$. We can thereby solve the first order condition (i.e., $\partial\pi(L_0)/\partial p_{r0} = 0$) to find

$$p_{r0}^* = \frac{n_d\left(\overline{\theta}-\underline{\theta}\right)\left[\left(\overline{\theta}+p_{t0}\right)+\left(\gamma_1-\gamma_2+C_d^v L_0\right)\right]+4n_p\gamma_2\left(\overline{\theta}+p_{t0}\right)}{2n_d\left(\overline{\theta}-\underline{\theta}\right)+4n_p\gamma_2}. \tag{21}$$

Substituting p_{r0}^* in (21) into (19) yields

$$w_0^* = (\gamma_1-\gamma_2+C_d^v L_0) + \frac{n_p\gamma_2\left[\left(\overline{\theta}+p_{t0}\right)-\left(\gamma_1-\gamma_2+C_d^v L_0\right)\right]}{\left[n_d\left(\overline{\theta}-\underline{\theta}\right)+2n_p\gamma_2\right]}. \tag{22}$$

We then analyze distance fare p_{r1} and distance wage w_1. When passengers' trip distance is L, we use (2) and (3) to rewrite the equation $Q_p = Q_d$ as

$$\frac{n_p[\overline{\theta}-(p_{r0}^*-p_{t0})-(p_{r1}-p_{t1})\times(L-L_0)]}{\overline{\theta}-\underline{\theta}} = \frac{n_d[w_0^*-w_1L_0+\left(w_1-C_d^v\right)L-(\gamma_1-\gamma_2)]}{2\gamma_2}. \tag{23}$$

Solving the above equation for w_1 yields

$$w_1 = \frac{(\gamma_1-\gamma_2)-w_0^*+C_d^v L}{L-L_0} + \frac{2n_p\gamma_2[\overline{\theta}-(p_{r0}^*-p_{t0})]}{n_d\left(\overline{\theta}-\underline{\theta}\right)(L-L_0)} - \frac{2n_p\gamma_2\left(p_{r1}-p_{t1}\right)}{n_d\left(\overline{\theta}-\underline{\theta}\right)}. \tag{24}$$

Then, when the trip distance is L, the platform's profit is

$$\pi(L) = \frac{n_p[\overline{\theta}-(p_{r0}^*-p_{t0})-(p_{r1}-p_{t1})(L-L_0)]}{\overline{\theta}-\underline{\theta}}\Bigg\{(p_{r0}^*-w_0^*) + \left\{p_{r1} - \frac{(\gamma_1-\gamma_2)-w_0^*+C_d^v L}{L-L_0} - \frac{2n_p\gamma_2[\overline{\theta}-(p_{r0}^*-p_{t0})]}{n_d\left(\overline{\theta}-\underline{\theta}\right)(L-L_0)} + \frac{2n_p\gamma_2\left(p_{r1}-p_{t1}\right)}{n_d\left(\overline{\theta}-\underline{\theta}\right)}\right\}(L-L_0)\Bigg\}, \tag{25}$$

which is only dependent on the decision variable p_{r1}.

We first ignore the constraints $p_{r1} \geq p_{t1} + [\underline{\theta}-(p_{r0}^*-p_{t0})]/(L-L_0)$ and $w_0^* + w_1(L-L_0) \leq \gamma_1+\gamma_2+C_d^v L$. Without the constraints, the platform maximizes its profit $\pi(L)$ in (25) to determine optimal distance fare p_{r1}^*. Partially differentiating $\pi(L)$ in (25) once with respect to p_{r1} gives

$$\begin{aligned}\frac{\partial \pi(L)}{\partial p_{r1}} &= \frac{-n_p(L-L_0)}{\overline{\theta}-\underline{\theta}}\left\{(p_{r0}^*-w_0^*)+\left\{p_{r1}-\left\{\frac{(\gamma_1-\gamma_2)-w_0^*+C_d^v L}{L-L_0}\right.\right.\right.\\ &\left.\left.\left.+\frac{2n_p\gamma_2[\overline{\theta}-(p_{r0}^*-p_{t0})]}{n_d\left(\overline{\theta}-\underline{\theta}\right)(L-L_0)}-\frac{2n_p\gamma_2\left(p_{r1}-p_{t1}\right)}{n_d\left(\overline{\theta}-\underline{\theta}\right)}\right\}\right\}(L-L_0)\right\}\\ &+\frac{n_p[\overline{\theta}-(p_{r0}^*-p_{t0})-(p_{r1}-p_{t1})(L-L_0)]}{\overline{\theta}-\underline{\theta}}\left\{\left[1+\frac{2n_p\gamma_2}{n_d\left(\overline{\theta}-\underline{\theta}\right)}\right](L-L_0)\right\}.\end{aligned}$$

We also compute its second-order derivative as $\partial^2\pi(L)/\partial p_{r1}=(L-L_0)^2\partial^2\pi(L_0)/\partial p_{r0}<0$. We can thereby solve the first order condition (i.e., $\partial\pi(L)/\partial p_{r1}=0$) to find

$$\begin{aligned}p_{r1}^* &= \frac{4n_p\gamma_2[\overline{\theta}+p_{t0}+p_{t1}(L-L_0)]+[(\overline{\theta}+p_{t0})+p_{t1}(L-L_0)}{\left[2n_d\left(\overline{\theta}-\underline{\theta}\right)+4n_p\gamma_2\right](L-L_0)}\\ &\quad\frac{+(\gamma_1-\gamma_2+C_d^v L)]n_d\left(\overline{\theta}-\underline{\theta}\right)}{\left[2n_d\left(\overline{\theta}-\underline{\theta}\right)+4n_p\gamma_2\right](L-L_0)}-\frac{p_{r0}^*}{L-L_0}.\end{aligned} \tag{26}$$

Substituting p_{r0}^* in (21) into (26) yields $p_{r1}^*=\tilde{p}_{r1}$, as shown in (7), and substituting (21), (22) and (26) into (24) gives $w_1^*=\tilde{w}_1$, as shown in (7).

Next, we consider the constraints $p_{r1}\geq p_{t1}+[\underline{\theta}-(p_{r0}^*-p_{t0})]/(L-L_0)$ and $w_0^*+w_1(L-L_0)\leq \gamma_1+\gamma_2+C_d^v L$. We find that (i) if and only if $L\leq L_1=\hat{L}|_{n_p\leq n_d}\equiv L_0+2[n_d(\overline{\theta}-\underline{\theta})+2n_p\gamma_2]/[n_d(p_{t1}-C_d^v)]-[(\overline{\theta}+p_{t0})-(\gamma_1-\gamma_2+C_d^v L_0)]/(p_{t1}-C_d^v)$, $\tilde{p}_{r1}\geq p_{t1}+[\underline{\theta}-(p_{r0}^*-p_{t0})]/(L-L_0)$; and (ii) if and only if $L\leq L_2=\hat{L}|_{n_p>n_d}\equiv L_0+2[n_d(\overline{\theta}-\underline{\theta})+2n_p\gamma_2]/[n_p(p_{t1}-C_d^v)]-[(\overline{\theta}+p_{t0})-(\gamma_1-\gamma_2+C_d^v L_0)]/(p_{t1}-C_d^v)$, $w_0^*+\tilde{w}_1(L-L_0)\leq\gamma_1+\gamma_2+C_d^v L$. We calculate $L_1-L_2=2(n_p-n_d)[2n_p\gamma_2+n_d(\overline{\theta}-\underline{\theta})]/[n_pn_d(p_{t1}-C_d^v)]$, which means that if $n_p\leq n_d$, then $L_1\leq L_2$; otherwise, $L_1>L_2$. Comparing n_p and n_d, we obtain optimal decisions for the following two scenarios:

1. When $n_p\leq n_d$, we find that, if $L\leq\hat{L}|_{n_p\leq n_d}=L_1\leq L_2$, then $\tilde{p}_{r1}\geq p_{t1}+[\underline{\theta}-(p_{r0}^*-p_{t0})]/(L-L_0)$ and $w_0^*+\tilde{w}_1(L-L_0)\leq\gamma_1+\gamma_2+C_d^v L$. That is, $\tilde{p}_{r1}$ and $\tilde{w}_1$ satisfy the constraints, and optimal decisions are $p_{r1}^*=\tilde{p}_{r1}$ and $w_1^*=\tilde{w}_1$. Otherwise, if $L>\hat{L}|_{n_p\leq n_d}=L_1$, then $\tilde{p}_{r1}<p_{t1}+[\underline{\theta}-(p_{r0}^*-p_{t0})]/(L-L_0)$ and optimal distance fare is $p_{r1}^*=p_{t1}+[\underline{\theta}-(p_{r0}^*-p_{t0})]/(L-L_0)$. According to (24), we find that $w_1^*=[(\gamma_1-\gamma_2)-w_0^*+C_d^v L]/(L-L_0)+2n_p\gamma_2/[n_d(L-L_0)]$. Using (21) and (22), we have

$$p_{r1}^*=p_{t1}+\frac{\left[(\overline{\theta}+p_{t0})-(\gamma_1-\gamma_2+C_d^v L_0)\right]n_d\left(\overline{\theta}-\underline{\theta}\right)}{\left[2n_d\left(\overline{\theta}-\underline{\theta}\right)+4n_p\gamma_2\right](L-L_0)}-\frac{\overline{\theta}-\underline{\theta}}{L-L_0},$$

   and

$$w_1^*=C_d^v-\frac{n_p\gamma_2\left[(\overline{\theta}+p_{t0})-(\gamma_1-\gamma_2+C_d^v L_0)\right]}{[n_d\left(\overline{\theta}-\underline{\theta}\right)+2n_p\gamma_2](L-L_0)}+\frac{2n_p\gamma_2}{n_d\left(L-L_0\right)}.$$

2. When $n_p>n_d$, if $L\leq\hat{L}|_{n_p>n_d}=L_2<L_1$, then $\tilde{p}_{r1}\geq p_{t1}+[\underline{\theta}-(p_{r0}^*-p_{t0})]/(L-L_0)$ and $w_0^*+\tilde{w}_1(L-L_0)\leq\gamma_1+\gamma_2+C_d^v L$. Then, the optimal decisions are $p_{r1}^*=\tilde{p}_{r1}$ and $w_1^*=\tilde{w}_1$. Otherwise, if $L>\hat{L}|_{n_p>n_d}=L_2$, then $w_0^*+\tilde{w}_1(L-L_0)>\gamma_1+\gamma_2+C_d^v L$, and the optimal

distance wage is $w_1^* = [(\gamma_1 - \gamma_2) - w_0^* + C_d^v L]/(L - L_0) + 2\gamma_2/(L - L_0)$. We can use (23) to find $p_{r1}^* = p_{t1} + \{n_p[\overline{\theta} - (p_{r0}^* - p_{t0})] - n_d(\overline{\theta} - \underline{\theta})\}/[n_p(L - L_0)]$. We use (21) and (22) to find that

$$p_{r1}^* = p_{t1} + \frac{\left[\left(\overline{\theta} + p_{t0}\right) - \left(\gamma_1 - \gamma_2 + C_d^v L_0\right)\right] n_d \left(\overline{\theta} - \underline{\theta}\right)}{\left[2n_d \left(\overline{\theta} - \underline{\theta}\right) + 4n_p\gamma_2\right] (L - L_0)} - \frac{n_d \left(\overline{\theta} - \underline{\theta}\right)}{n_p (L - L_0)},$$

and

$$w_1^* = C_d^v - \frac{n_p\gamma_2 \left[\left(\overline{\theta} + p_{t0}\right) - \left(\gamma_1 - \gamma_2 + C_d^v L_0\right)\right]}{[n_d \left(\overline{\theta} - \underline{\theta}\right) + 2n_p\gamma_2](L - L_0)} + \frac{2\gamma_2}{L - L_0}.$$

Summarizing the above, we obtain this theorem. ■

Proof of Theorem 2. We first investigate the impact of L on optimal decisions p_{r1}^* and w_1^*. From Theorem 1, we find that when $L \le \hat{L}$, where $\hat{L}$ is defined as in (6), optimal distance fare p_{r1}^* and optimal distance wage w_1^* are $p_{r1}^* = [4n_p\gamma_2 p_{t1} + n_d(\overline{\theta} - \underline{\theta})(p_{t1} + C_d^v)]/[4n_p\gamma_2 + 2n_d(\overline{\theta} - \underline{\theta})]$ and $w_1^* = C_d^v + n_p\gamma_2(p_{t1} - C_d^v)/[n_d(\overline{\theta} - \underline{\theta}) + 2n_p\gamma_2]$, respectively, which are obviously all independent of L.

When $L > \hat{L}$, we use Theorem 1 to compute the optimal decisions as

$$p_{r1}^* = p_{t1} + \left\{ \frac{\left[\left(\overline{\theta} + p_{t0}\right) - \left(\gamma_1 - \gamma_2 + C_d^v L_0\right)\right] n_d}{2\left[n_d \left(\overline{\theta} - \underline{\theta}\right) + 2n_p\gamma_2\right]} - \frac{\min(n_p, n_d)}{n_p} \right\} \frac{\left(\overline{\theta} - \underline{\theta}\right)}{(L - L_0)},$$

and

$$w_1^* = C_d^v + 2\left\{ \frac{\min(n_p, n_d)}{n_d} - \frac{n_p\left[\left(\overline{\theta} + p_{t0}\right) - \left(\gamma_1 - \gamma_2 + C_d^v L_0\right)\right]}{n_d \left(\overline{\theta} - \underline{\theta}\right) + 2n_p\gamma_2} \right\} \frac{\gamma_2}{(L - L_0)}.$$

If $\underline{\theta} \le \theta_2$, where θ_2 is shown in (11), then we have $n_d[(\overline{\theta} + p_{t0}) - (\gamma_1 - \gamma_2 + C_d^v L_0)]/[2n_d(\overline{\theta} - \underline{\theta}) + 4n_p\gamma_2] - \min(n_p, n_d)/n_p \le 0$. As a result, p_{r1}^* (w_1^*) is decreasing (increasing) in L. Otherwise, p_{r1}^* (w_1^*) is increasing (decreasing) in L.

Then, we analyze how trip distance L influences optimal payout ratio r^*. When $L \le \hat{L}$, we find r^* as given in (9), and compute the first-order partial derivative of r^* w.r.t. L as

$$\frac{\partial r^*}{\partial L} = \frac{2\left[\left(\overline{\theta} + p_{t0}\right) C_d^v - \left(\gamma_1 - \gamma_2 + C_d^v L_0\right) p_{t1}\right] \left[n_d \left(\overline{\theta} - \underline{\theta}\right) + 2n_p\gamma_2\right]^2}{\left\{n_d \left(\overline{\theta} - \underline{\theta}\right) \left[\left(\overline{\theta} + p_{t0}\right) + p_{t1} (L - L_0) + \left(\gamma_1 - \gamma_2 + C_d^v L\right)\right] + 4n_p\gamma_2 \left[\left(\overline{\theta} + p_{t0}\right) + p_{t1} (L - L_0)\right]\right\}^2}.$$

If $C_d^v \le C_1$, where C_1 is defined in (11), then $\partial r^*/\partial L \le 0$, which means that r^* is decreasing in L. Otherwise, r^* is increasing in L. When $L > \hat{L}$, we obtain r^* as given in (10), and compute its first-order partial derivative w.r.t. L as

$$\frac{\partial r^*}{\partial L} = -\frac{n_p n_d C_d^v \left[\left(\overline{\theta} + p_{t0}\right) + p_{t1}(L - L_0)\right] (p_{t1} - C_d^v) + \min(n_p, n_d) \left[n_d \left(\overline{\theta} - \underline{\theta}\right) C_d^v + 2n_p\gamma_2 p_{t1}\right]}{\left\{n_p n_d \left[\left(\overline{\theta} + p_{t0}\right) + p_{t1}(L - L_0)\right] - \min(n_p, n_d) n_d \left(\overline{\theta} - \underline{\theta}\right)\right\}^2}.$$

Recalling from Section 3.2 that p_{t1} should be larger than C_d^v, i.e. $p_{t1} > C_d^v$, we obtain $\partial r^*/\partial L < 0$ regardless of whether $n_p \le n_d$ or $n_p > n_d$. That is, r^* is always decreasing in L when $L > \hat{L}$.

Lastly, we analyze how trip distance L influences optimal profit π^*. When $L \le \hat{L}$, differentiating

π^* as shown in (9) once w.r.t. L yields

$$\frac{\partial \pi^*}{\partial L} = \frac{n_p n_d \left[\left(\overline{\theta} + p_{t0}\right) + p_{t1}\left(L - L_0\right) - \left(\gamma_1 - \gamma_2 + C_d^v L\right)\right]\left(p_{t1} - C_d^v\right)}{2[n_d\left(\overline{\theta} - \underline{\theta}\right) + 2n_p\gamma_2]}.$$

Note that $p_{t1} > C_d^v$ and the condition in (17) shows that $(\overline{\theta} + p_{t0}) + p_{t1}(L - L_0) - (\gamma_1 - \gamma_2 + C_d^v L) \geq 0$. Thus, $\partial\pi^*/\partial L \geq 0$; this means that π^* is always increasing in L when $L \leq \hat{L}$. And when $L > \hat{L}$, we find π^* as given in (10), and compute the first-order partial derivative of π^* w.r.t. L as $\partial\pi^*/\partial L = \min(n_p, n_d)(p_{t1} - C_d^v) > 0$, i.e. when $L > \hat{L}$, π^* is also increasing in L wether $n_p \leq n_d$ or $n_p > n_d$.

Summarizing the above we have our results as shown in Table 1. ■

Proof of Theorem 3. If $L \leq \hat{L}$, where $\hat{L}$ is defined as in (6), then, using (7), we find $p_{r1}^* = [4n_p\gamma_2 p_{t1} + n_d(\overline{\theta} - \underline{\theta})(p_{t1} + C_d^v)]/[4n_p\gamma_2 + 2n_d(\overline{\theta} - \underline{\theta})]$ and

$$w_1^* = \frac{n_p\gamma_2 p_{t1}}{n_d\left(\overline{\theta} - \underline{\theta}\right) + 2n_p\gamma_2} + \frac{n_d\left(\overline{\theta} - \underline{\theta}\right) + n_p\gamma_2}{n_d\left(\overline{\theta} - \underline{\theta}\right) + 2n_p\gamma_2} C_d^v.$$

Therefore, both p_{r1}^* and w_1^* are increasing in C_d^v at constant slopes $[n_d(\overline{\theta} - \underline{\theta})]/[4n_p\gamma_2 + 2n_d(\overline{\theta} - \underline{\theta})] \in (0, 1)$ and $[n_d(\overline{\theta} - \underline{\theta}) + n_p\gamma_2]/[n_d(\overline{\theta} - \underline{\theta}) + 2n_p\gamma_2] \in (0, 1)$, respectively.

We rewrite r^* in (9) as

$$r^* = \frac{2n_d\left(\overline{\theta} - \underline{\theta}\right)\left(\gamma_1 - \gamma_2\right) + 2n_p\gamma_2\left[\left(\overline{\theta} + p_{t0}\right) + p_{t1}\left(L - L_0\right) + \left(\gamma_1 - \gamma_2\right)\right] + \left[2n_d\left(\overline{\theta} - \underline{\theta}\right) + 2n_p\gamma_2\right] C_d^v L}{n_d\left(\overline{\theta} - \underline{\theta}\right)\left[\left(\overline{\theta} + p_{t0}\right) + p_{t1}\left(L - L_0\right) + \left(\gamma_1 - \gamma_2 + C_d^v L\right)\right] + 4n_p\gamma_2\left[\left(\overline{\theta} + p_{t0}\right) + p_{t1}\left(L - L_0\right)\right]},$$

which indicates that r^* is increasing in C_d^v.

Next, we calculate the first-order partial derivative of optimal profit π^* in (9) w.r.t. C_d^v as

$$\frac{\partial \pi^*}{\partial C_d^v} = -\frac{n_p n_d[\left(\overline{\theta} + p_{t0}\right) + p_{t1}(L - L_0) - (\gamma_1 - \gamma_2 + C_d^v L)]}{2[n_d\left(\overline{\theta} - \underline{\theta}\right) + 2n_p\gamma_2]}.$$

Lemma 1 shows that $\left(\overline{\theta} + p_{t0}\right) + p_{t1}(L - L_0) - (\gamma_1 - \gamma_2 + C_d^v L)] \geq 0$; so, π^* is decreasing in C_d^v.

When $L > \hat{L}$, optimal decisions p_{r1}^* and w_1^* are shown in (8), from which we can observe that p_{r1}^* is decreasing in C_d^v, whereas w_1^* is increasing in C_d^v. Using the formula in (10), we find that r^* is increasing in C_d^v, whereas π^* is decreasing in C_d^v. ■

Proof of Theorem 4. When $L \leq \hat{L}$, p_r^* is shown in (7). Differentiating it once w.r.t. $\overline{\theta}$ yields $\partial p_{r1}^*/\partial\overline{\theta} = 4n_p\gamma_2 n_d(C_d^v - p_t)/[4n_p\gamma_2 + 2n_d(\overline{\theta} - \underline{\theta})]^2$. Recalling from Section 3.2 that $p_{t1} > C_d^v$, we obtain $\partial p_{r1}^*/\partial\overline{\theta} < 0$, which means that p_{r1}^* is decreasing in $\overline{\theta}$. Thus, $\partial p_{r1}^*/\partial\underline{\theta} = -\partial p_{r1}^*/\partial\overline{\theta} > 0$, i.e., p_{r1}^* is increasing in $\underline{\theta}$. According to (7), $w_1^* = C_d^v + n_p\gamma_2(p_{t1} - C_d^v)/[n_d(\overline{\theta} - \underline{\theta}) + 2n_p\gamma_2]$. As $p_{t1} - C_d^v > 0$, w_1^* is decreasing in $\overline{\theta}$ but increasing in $\underline{\theta}$.

To investigate the impact on r^*, we differentiate r^* in (9) once w.r.t. $\overline{\theta}$ and $\underline{\theta}$, and find

$$\frac{\partial r^*}{\partial\overline{\theta}} = -\frac{2n_d n_p\gamma_2\left[\left(\overline{\theta} + p_{t0}\right) + p_{t1}\left(L - L_0\right) - \left(\gamma_1 - \gamma_2 + C_d^v L\right)\right]^2 + 2(\gamma_1 - \gamma_2 + C_d^v L)\left[n_d\left(\overline{\theta} - \underline{\theta}\right) + 2n_p\gamma_2\right]^2}{\left\{n_d\left(\overline{\theta} - \underline{\theta}\right)\left[\left(\overline{\theta} + p_{t0}\right) + p_{t1}\left(L - L_0\right) + \left(\gamma_1 - \gamma_2 + C_d^v L\right)\right] + 4n_p\gamma_2\left[\left(\overline{\theta} + p_{t0}\right) + p_{t1}\left(L - L_0\right)\right]\right\}^2} < 0$$

and

$$\frac{\partial r^*}{\partial \underline{\theta}} = \frac{2n_d n_p \gamma_2 \left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}\left(L-L_0\right)-\left(\gamma_1-\gamma_2+C_d^v L\right)\right]^2}{\left\{n_d\left(\overline{\theta}-\underline{\theta}\right)\left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}\left(L-L_0\right)+\left(\gamma_1-\gamma_2+C_d^v L\right)\right]+4n_p\gamma_2\left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}\left(L-L_0\right)\right]\right\}^2} > 0$$

respectively. It thus follows that r^* is always decreasing (increasing) in $\overline{\theta}$ ($\underline{\theta}$).

We then examine the impacts of $\overline{\theta}$ and $\underline{\theta}$ on π^*. To this end, we differentiate π^* in (9) once w.r.t. $\overline{\theta}$, and obtain $\partial\pi^*/\partial\overline{\theta} = n_p n_d AB/\{4[n_d(\overline{\theta}-\underline{\theta})+2n_p\gamma_2]^2\}$, where $A = (\overline{\theta}+p_{t0})+p_{t1}(L-L_0)-(\gamma_1-\gamma_2+C_d^v L)$ and $B = 2[n_d(\overline{\theta}-\underline{\theta})+2n_p\gamma_2]-n_d[(\overline{\theta}+p_{t0})+p_{t1}(L-L_0)-(\gamma_1-\gamma_2+C_d^v L)]$. Noting from (17) that $A \geq 0$, we can reduce $L \leq \hat{L}$ to $B \geq 0$. Thus, $\partial\pi^*/\partial\overline{\theta} \geq 0$, i.e., π^* is always increasing in $\overline{\theta}$. Using (9), we find that $\pi^* = n_p n_d[(\overline{\theta}+p_{t0})+p_{t1}(L-L_0)-(\gamma_1-\gamma_2+C_d^v L)]^2/\{4[n_d(\overline{\theta}-\underline{\theta})+2n_p\gamma_2]\}$ is increasing in $\underline{\theta}$.

When $L > \hat{L}$, we analyze the impact of $\overline{\theta}$ on p_{r1}^*. Differentiating p_{r1}^* in (8) once w.r.t. $\overline{\theta}$ yields

$$\begin{aligned}\frac{\partial p_{r1}^*}{\partial\overline{\theta}} &= \frac{n_d\left(\overline{\theta}-\underline{\theta}\right)}{\left[2n_d\left(\overline{\theta}-\underline{\theta}\right)+4n_p\gamma_2\right]\left(L-L_0\right)} \\ &\quad + \frac{n_d\left\{4n_p\gamma_2\left[\left(\overline{\theta}+p_{t0}\right)-\left(\gamma_1-\gamma_2+C_d^v L_0\right)\right]\right\}}{\left[2n_d\left(\overline{\theta}-\underline{\theta}\right)+4n_p\gamma_2\right]^2\left(L-L_0\right)} - \frac{\min(n_p,n_d)}{n_p\left(L-L_0\right)}. \end{aligned} \tag{27}$$

Then, we study the impact for the following two cases.

1. When $n_p \leq n_d$, we rewrite (27) as

$$\frac{\partial p_{r1}^*}{\partial\overline{\theta}} = \frac{-2\left[n_d\left(\overline{\theta}-\underline{\theta}\right)+2n_p\gamma_2\right]^2-8\left(n_p\gamma_2\right)^2+4n_d n_p\gamma_2\left[\left(\underline{\theta}+p_{t0}\right)-\left(\gamma_1-\gamma_2+C_d^v L_0\right)\right]}{\left[2n_d\left(\overline{\theta}-\underline{\theta}\right)+4n_p\gamma_2\right]^2\left(L-L_0\right)}.$$

   Then, solving the equation $\partial p_{r1}^*/\partial\overline{\theta} = 0$ yields two solutions

$$\overline{\theta} = \underline{\theta} + \frac{\pm\sqrt{2n_d n_p\gamma_2\left[\left(\underline{\theta}+p_{t0}\right)-\left(\gamma_1-\gamma_2+C_d^v L_0\right)\right]-4\left(n_p\gamma_2\right)^2}-2n_p\gamma_2}{n_d}.$$

   So, when $\overline{\theta} \leq \theta_3$, as shown in (12), $\partial p_{r1}^*/\partial\overline{\theta} > 0$, which means that p_{r1}^* is increasing in $\overline{\theta}$. When $\overline{\theta} > \theta_3$, p_{r1}^* is decreasing in $\overline{\theta}$.

2. When $n_p > n_d$, we rewrite (27) as

$$\frac{\partial p_{r1}^*}{\partial\overline{\theta}} = \frac{2n_d\left\{\left(n_p-2n_d\right)n_d\left(\overline{\theta}-\underline{\theta}\right)^2+4n_p\gamma_2\left(n_p-2n_d\right)\left(\overline{\theta}-\underline{\theta}\right)+2n_p^2\gamma_2\left[\left(\underline{\theta}+p_{t0}\right)-\left(\gamma_1+2\gamma_2+C_d^v L_0\right)\right]\right\}}{n_p\left[2n_d\left(\overline{\theta}-\underline{\theta}\right)+4n_p\gamma_2\right]^2\left(L-L_0\right)}.$$

   Therefore, if $\xi_1 \geq 0$, where ξ_1 is defined in (12), then $\partial p_{r1}^*/\partial\overline{\theta} \geq 0$; that is, p_{r1}^* is increasing in $\overline{\theta}$. Otherwise, p_{r1}^* is decreasing in $\overline{\theta}$.

We then analyze the impact of $\underline{\theta}$ on p_{r1}^*. Differentiating p_{r1}^* in (8) once w.r.t. $\underline{\theta}$ yields

$$\frac{\partial p_{r1}^*}{\partial \underline{\theta}} = \frac{-n_d n_p \gamma_2 \left[\left(\overline{\theta} + p_{t0}\right) - \left(\gamma_1 - \gamma_2 + C_d^v L_0\right)\right]}{\left[n_d \left(\overline{\theta} - \underline{\theta}\right) + 2n_p\gamma_2\right]^2 (L - L_0)} + \frac{\min(n_p, n_d)}{n_p (L - L_0)}.$$

So, if $\underline{\theta} \le \theta_4$, as shown in (12), then $\partial p_{r1}^*/\partial \underline{\theta} \ge 0$, i.e., p_{r1}^* is increasing in $\underline{\theta}$. Otherwise, p_{r1}^* is decreasing in $\underline{\theta}$. To study the impact on w_1^*, we partially differentiate w_1^* in (8) once w.r.t. $\overline{\theta}$ yields

$$\frac{\partial w_1^*}{\partial \overline{\theta}} = -\frac{n_p\gamma_2 \left\{2n_p\gamma_2 - n_d \left[\left(\underline{\theta} + p_{t0}\right) - \left(\gamma_1 - \gamma_2 + C_d^v L_0\right)\right]\right\}}{(L - L_0)[n_d \left(\overline{\theta} - \underline{\theta}\right) + 2n_p\gamma_2]^2}.$$

Hence, if $\xi_2 \ge 0$, where ξ_2 is defined in (12), then $\partial w_1^*/\partial \overline{\theta} \le 0$, which means that w_1^* is decreasing in $\overline{\theta}$. Otherwise, w_1^* is increasing in $\overline{\theta}$. We can observe that w_1^* is always decreasing in $\underline{\theta}$.

To study the impacts of $\overline{\theta}$ and $\underline{\theta}$ on r^* and π^*, we use (10) to rewrite r^* and π^* as

$$\frac{1}{r^*} = \frac{\left[n_p - \min(n_p, n_d)\right] n_d\overline{\theta} + \min(n_p, n_d) n_d\underline{\theta} + n_p n_d \left[p_{t0} + p_{t1}(L - L_0)\right]}{n_p n_d \left[(\gamma_1 - \gamma_2) + C_d^v L\right] + 2n_p\gamma_2 \min(n_p, n_d)},$$

and

$$\pi^* = \min(n_p, n_d) \left\{\frac{\left[\max(n_p, n_d) - n_d\right]\overline{\theta} + n_d\underline{\theta} + 2n_p\gamma_2}{\max(n_p, n_d)} + p_{t0} - (\gamma_1 - \gamma_2 + C_d^v L) + p_{t1}(L - L_0)\right\}.$$

So, both $1/r^*$ and π^* are increasing in $\underline{\theta}$. Noting that payout ratio $r^* > 0$, we can conclude that r^* is decreasing in $\underline{\theta}$. When $n_p \le n_d$, both $1/r^*$ and π^* are independent of $\overline{\theta}$ and thus, r^* is also independent of $\overline{\theta}$. When $n_p > n_d$, r^* is decreasing in $\overline{\theta}$, whereas π^* is increasing in $\overline{\theta}$. ■

Proof of Theorem 5. When $L \le \hat{L}$, according to (7), we find that $p_{r1}^* = [4n_p\gamma_2 p_{t1} + n_d(\overline{\theta} - \underline{\theta})(p_{t1} + C_d^v)]/[4n_p\gamma_2 + 2n_d(\overline{\theta} - \underline{\theta})]$ and $w_1^* = C_d^v + n_p\gamma_2(p_{t1} - C_d^v)/[n_d(\overline{\theta} - \underline{\theta}) + 2n_p\gamma_2]$, which are both increasing in p_{t1}. Then, we differentiate r^* in (9) once w.r.t. p_{t1}, and obtain

$$\frac{\partial r^*}{\partial p_{t1}} = \frac{(L - L_0)\left\{(2n_p\gamma_2)^2 - \left\{\left[n_d\left(\overline{\theta} - \underline{\theta}\right) + 2n_p\gamma_2\right]^2 + \left[n_d\left(\overline{\theta} - \underline{\theta}\right)\right]^2\right\}(\gamma_1 - \gamma_2 + C_d^v L)\right\}}{\left\{n_d\left(\overline{\theta} - \underline{\theta}\right)\left[\left(\overline{\theta} + p_{t0}\right) + p_{t1}(L - L_0) + \left(\gamma_1 - \gamma_2 + C_d^v L\right)\right] + 4n_p\gamma_2\left[\left(\overline{\theta} + p_{t0}\right) + p_{t1}(L - L_0)\right]\right\}^2}.$$

So, if $L \le \overline{L} = (2n_p\gamma_2)^2/\{[n_d(\overline{\theta} - \underline{\theta}) + 2n_p\gamma_2]^2 + [n_d(\overline{\theta} - \underline{\theta})]^2\}$, then $\partial r^*/\partial p_{t1} \ge 0$, i.e., r^* is increasing in p_{t1}; Otherwise, r^* is decreasing in p_{t1}. Differentiating π^* in (9) once w.r.t. p_{t1} yields

$$\frac{\partial \pi^*}{\partial p_{t1}} = \frac{2n_p n_d (L - L_0)\left[\left(\overline{\theta} + p_{t0}\right) + p_{t1}(L - L_0) - (\gamma_1 - \gamma_2 + C_d^v L)\right]}{4[n_d\left(\overline{\theta} - \underline{\theta}\right) + 2n_p\gamma_2]}.$$

Note from (17) that $(\overline{\theta} + p_{t0}) + p_{t1}(L - L_0) - (\gamma_1 - \gamma_2 + C_d^v L) > 0$; so, $\partial \pi^*/\partial p_{t1} \ge 0$, which means that π^* is increasing in p_{t1}.

When $L > \hat{L}$, we use (8) to find that p_{r1}^* is increasing in p_{t1} and w_1^* is independent of p_{t1}. In addition, according to (10), we reveal that r^* is decreasing in p_{t1} and π^* is increasing in p_{t1}. ■

Proof of Theorem 6. When $L \le \hat{L}$, we find from (7) that both p_{r1}^* and w_1^* are independent of

γ_1, and w_1^* is increasing in γ_2. Then, partially differentiating p_{r1}^* once w.r.t. γ_2 yields $\partial p_{r1}^*/\partial\gamma_2 = 4n_p n_d(\overline{\theta}-\underline{\theta})(p_{t1}-C_d^v)/[4n_p\gamma_2+2n_d(\overline{\theta}-\underline{\theta})]^2>0$, which means that p_{r1}^* is increasing in γ_2. Partially differentiating r^* and π^* in (9) once w.r.t. γ_1 gives

$$\frac{\partial r^*}{\partial\gamma_1}=\frac{2\left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}\left(L-L_0\right)\right]\left[n_d\left(\overline{\theta}-\underline{\theta}\right)+2n_p\gamma_2\right]^2}{\left\{n_d\left(\overline{\theta}-\underline{\theta}\right)\left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}\left(L-L_0\right)+\left(\gamma_1-\gamma_2+C_d^vL\right)\right]+4n_p\gamma_2\left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}\left(L-L_0\right)\right]\right\}^2}>0,$$

and

$$\frac{\partial\pi^*}{\partial\gamma_1}=\frac{-n_pn_d\left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}\left(L-L_0\right)-\left(\gamma_1-\gamma_2+C_d^vL\right)\right]}{2\left[n_d\left(\overline{\theta}-\underline{\theta}\right)+2n_p\gamma_2\right]}<0.$$

respectively. We can find that r^* (π^*) is always increasing (decreasing) in γ_1. We differentiate r^* in (9) once w.r.t. γ_2 and find

$$\frac{\partial r^*}{\partial\gamma_2}=\frac{\xi_3}{\left\{n_d\left(\overline{\theta}-\underline{\theta}\right)\left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}\left(L-L_0\right)+\left(\gamma_1-\gamma_2+C_d^vL\right)\right]+4n_p\gamma_2\left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}\left(L-L_0\right)\right]\right\}^2},$$

where ξ_3 is defined in (13). If $\xi_3\geq 0$, then r^* is increasing in γ_2; otherwise, r^* is decreasing in γ_2.

Next, we differentiate π^* in (9) once w.r.t. γ_2, and have

$$\frac{\partial\pi^*}{\partial\gamma_2}=\frac{2n_pn_d\left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}\left(L-L_0\right)-\left(\gamma_1-\gamma_2+C_d^vL\right)\right]\xi_5}{4[n_d\left(\overline{\theta}-\underline{\theta}\right)+2n_p\gamma_2]^2},$$

where

$$\begin{aligned}\xi_5 \;=\;& -n_p\gamma_2^2-2n_p\left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}\left(L-L_0\right)-\left(\gamma_1+C_d^vL\right)-1\right]\gamma_2\\ &+n_d\left(\overline{\theta}-\underline{\theta}\right)-n_p\left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}\left(L-L_0\right)-\left(\gamma_1+C_d^vL\right)-1\right]^2.\end{aligned}$$

Noting from (17) that $(\overline{\theta}+p_{t0})+p_{t1}(L-L_0)-(\gamma_1-\gamma_2+C_d^vL)>0$, we find that the sign of $\partial\pi^*/\partial\gamma_2$ is the same as the sign of ξ_5. Solving the equation $\xi_5=0$ yields

$$\begin{aligned}\gamma_2 \;=\;& \left[1+\left(\gamma_1+C_d^vL\right)-\left(\overline{\theta}+p_{t0}\right)-p_{t1}\left(L-L_0\right)\right]\\ &\pm\sqrt{\left[1+2\left(\gamma_1+C_d^vL\right)-2\left(\overline{\theta}+p_{t0}\right)-2p_{t1}\left(L-L_0\right)\right]n_p^2+n_pn_d\left(\overline{\theta}-\underline{\theta}\right)}/n_p.\end{aligned}$$

If $\gamma_2\leq\gamma_3$, as shown in (13), then $\partial\pi^*/\partial\gamma_2\geq 0$, i.e., π^* is increasing in γ; Otherwise, π^* is decreasing in γ_2.

When $L>\hat{L}$, we find from (8) that both p_{r1}^* and w_1^* are decreasing in γ_1. Partially differentiating p_{r1}^* once w.r.t. γ_2 yields

$$\frac{\partial p_{r1}^*}{\partial\gamma_2}=\frac{n_d\left(\overline{\theta}-\underline{\theta}\right)\left[2n_d\left(\overline{\theta}-\underline{\theta}\right)-4n_p\left(\overline{\theta}+p_{t0}-C_d^vL_0\right)+4n_p\gamma_1\right]}{\left[2n_d\left(\overline{\theta}-\underline{\theta}\right)+4n_p\gamma_2\right]^2\left(L-L_0\right)}.$$

So, if $\gamma_1\leq\gamma_4$, as shown in (13), then $\partial p_{r1}^*/\partial\gamma_2\leq 0$, that is, p_{r1}^* is decreasing in γ_2; Otherwise, p_{r1}^* is increasing in γ_2.

We calculate first-order partially derivative of w_1^* w.r.t. γ_2 as

$$\begin{aligned}\frac{\partial w_1^*}{\partial \gamma_2} &= \frac{[8\min(n_p,n_d)-2n_d]\,(n_p\gamma_2)^2+[8\min(n_p,n_d)-2n_d]\,n_d\left(\overline{\theta}-\underline{\theta}\right)n_p\gamma_2}{n_d\,(L-L_0)\,[n_d\left(\overline{\theta}-\underline{\theta}\right)+2n_p\gamma_2]^2}\\ &\quad+\frac{n_d^2\left(\overline{\theta}-\underline{\theta}\right)\left\{2\min(n_p,n_d)[\left(\overline{\theta}-\underline{\theta}\right)]-n_p\left[\left(\overline{\theta}+p_{t0}\right)-\left(\gamma_1+C_d^vL_0\right)\right]\right\}}{n_d\,(L-L_0)\,[n_d\left(\overline{\theta}-\underline{\theta}\right)+2n_p\gamma_2]^2}.\end{aligned} \tag{28}$$

As the denominator of $\partial w_1^*/\partial\gamma_2$ in (28) is positive, the sign of $\partial w_1^*/\partial\gamma_2$ depends on the numerator of $\partial w_1^*/\partial\gamma_2$. We accordingly discuss the following two cases.

1. When $n_p \le n_d$, the numerator is ξ_4, where ξ_4 is defined in (13). If $\xi_4 \ge 0$, then w_1^* is increasing in γ_2; otherwise, w_1^* is decreasing in γ_2.
2. While $n_p > n_d$, we compute the numerator as $n_d\{6(n_p\gamma_2)^2+6n_dn_p\gamma_2(\overline{\theta}-\underline{\theta})+n_d(\overline{\theta}-\underline{\theta})\{2n_d(\overline{\theta}-\underline{\theta})-n_p[(\overline{\theta}+p_{t0})-(\gamma_1+C_d^vL_0)]\}\}$, equate the numerator to zero, and solve the resulting equation to find the following two solutions:

$$\gamma_2=\frac{-3n_d\left(\overline{\theta}-\underline{\theta}\right)\pm n_p\sqrt{6n_pn_d\left(\overline{\theta}-\underline{\theta}\right)\left[\left(\overline{\theta}+p_{t0}\right)-\left(\gamma_1+C_d^vL_0\right)\right]-3n_d^2\left(\overline{\theta}-\underline{\theta}\right)^2}}{6n_p^2}.$$

So, if $\gamma_2 \le \gamma_5$, as shown in (13), then w_1^* is decreasing in γ_2; otherwise, w_1^* is increasing in γ_2. To study the impact on r^*, we rewrite r^* in (10) as

$$r^*=\frac{n_pn_d(\gamma_1+C_d^vL)+[2\min(n_p,n_d)-n_d]\,n_p\gamma_2}{n_pn_d\,[p_{t0}+p_{t1}(L-L_0)]+\min(n_p,n_d)n_d\underline{\theta}+n_d\overline{\theta}\,[n_p-\min(n_p,n_d)]}.$$

Because the denominator of r^* is positive, r^* is increasing in γ_1. If $n_p \le n_d/2$, then r^* is decreasing in γ_2; Otherwise, r^* is increasing in γ_2. We rewrite π^* in (10) as

$$\pi^*=\min(n_p,n_d)\left[\left(\overline{\theta}+p_{t0}\right)-(\gamma_1+C_d^vL)+p_{t1}(L-L_0)+\frac{\max(n_p,n_d)-2n_p}{\max(n_p,n_d)}\gamma_2\right].$$

So, π^* is decreasing in γ_1. If $n_p \le n_d/2$, then π^* is increasing in γ_2; Otherwise, π^* is decreasing in γ_2.

■

Proof of Theorem 7. We consider the following three cases.

1. When $L \le \hat{L}$, we rewrite p_{r1}^* and w_1^* in (7) as

$$p_{r1}^*=p_{t1}-\frac{(p_{t1}-C_d^v)\left(\overline{\theta}-\underline{\theta}\right)}{\frac{4n_p\gamma_2}{n_d}+2\left(\overline{\theta}-\underline{\theta}\right)}\ \text{and}\ w_1^*=C_d^v+\frac{p_{t1}-C_d^v}{\frac{n_d(\overline{\theta}-\underline{\theta})}{n_p\gamma_2}+2}.$$

As $p_{t1}-C_d^v>0$, both p_{r1}^* and w_1^* are increasing (decreasing) in n_p (n_d). Then, partially differentiating r^* in (9) once w.r.t. n_p and n_d yield

$$\frac{\partial r^*}{\partial n_p} = \frac{2\gamma_2 n_d\left(\overline{\theta}-\underline{\theta}\right)\left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}\left(L-L_0\right)-\left(\gamma_1-\gamma_2+C_d^v L\right)\right]^2}{\left\{n_d\left(\overline{\theta}-\underline{\theta}\right)\left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}\left(L-L_0\right)+\left(\gamma_1-\gamma_2+C_d^v L\right)\right]+4n_p\gamma_2\left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}\left(L-L_0\right)\right]\right\}^2} > 0,$$

and

$$\frac{\partial r^*}{\partial n_d} = \frac{-2n_p\gamma_2\left(\overline{\theta}-\underline{\theta}\right)\left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}\left(L-L_0\right)-\left(\gamma_1-\gamma_2+C_d^v L\right)\right]^2}{\left\{n_d\left(\overline{\theta}-\underline{\theta}\right)\left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}\left(L-L_0\right)+\left(\gamma_1-\gamma_2+C_d^v L\right)\right]+4n_p\gamma_2\left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}\left(L-L_0\right)\right]\right\}^2} < 0.$$

Thus, r^* is always increasing (decreasing) in n_p (n_d).

2. When $L > \hat{L}$ and $n_p \le n_d$, we rewrite p_{r1}^* in (8) as

$$p_{r1}^* = p_{t1} + \frac{\left[\left(\overline{\theta}+p_{t0}\right)-\left(\gamma_1-\gamma_2+C_d^v L_0\right)\right]\left(\overline{\theta}-\underline{\theta}\right)}{\left[2\left(\overline{\theta}-\underline{\theta}\right)+\frac{4n_p\gamma_2}{n_d}\right]\left(L-L_0\right)} - \frac{\left(\overline{\theta}-\underline{\theta}\right)}{\left(L-L_0\right)}.$$

Obviously, p_{r1}^* is always decreasing (increasing) in n_p (n_d). Then, differentiating w_1^* in (8) once w.r.t. n_p and n_d yield $\partial w_1^*/\partial n_p = \gamma_2\xi_6/\{n_d(L-L_0)[n_d(\overline{\theta}-\underline{\theta})+2n_p\gamma_2]^2\}$ and $\partial w_1^*/\partial n_d = -n_p\gamma_2\xi_6/\{n_d^2(L-L_0)[n_d(\overline{\theta}-\underline{\theta})+2n_p\gamma_2]^2\}$, respectively, where ξ_6 is defined as in (14). So, if $\xi_6 \ge 0$, then w_1^* is increasing (decreasing) in n_p (n_d); otherwise, w_1^* is decreasing (increasing) in n_p (n_d). Next, we rewrite r^* in (10) as

$$r^* = \frac{[(\gamma_1-\gamma_2)+C_d^v L]}{[(\underline{\theta}+p_{t0})+p_{t1}(L-L_0)]} + \frac{2n_p\gamma_2}{n_d\left[(\underline{\theta}+p_{t0})+p_{t1}(L-L_0)\right]},$$

which indicates that r^* is always increasing (decreasing) in n_p (n_d).

3. When $L > \hat{L}$ and $n_p > n_d$, we partially differentiate p_{r1}^* in (8) once w.r.t. n_p and n_d and find

$$\frac{\partial p_{r1}^*}{\partial n_p} = \frac{n_d\left(\overline{\theta}-\underline{\theta}\right)\xi_7}{n_p^2\left(L-L_0\right)\left[n_d\left(\overline{\theta}-\underline{\theta}\right)+2n_p\gamma_2\right]^2} \text{ and } \frac{\partial p_{r1}^*}{\partial n_d} = \frac{-\left(\overline{\theta}-\underline{\theta}\right)\xi_7}{n_p\left(L-L_0\right)\left[n_d\left(\overline{\theta}-\underline{\theta}\right)+2n_p\gamma_2\right]^2},$$

respectively, where ξ_7 is defined as in (14). So, if $\xi_7 \ge 0$, then $\partial p_{r1}^*/\partial n_p \ge 0$ and $\partial p_{r1}^*/\partial n_d \le 0$, i.e., p_{r1}^* is increasing (decreasing) in n_p (n_d); otherwise, p_{r1}^* is decreasing (increasing) in n_p (n_d). Next, we rewrite w_1^* in (8) as

$$w_1^* = C_d^v + \frac{2\gamma_2}{(L-L_0)} - \frac{\gamma_2\left[\left(\overline{\theta}+p_{t0}\right)-\left(\gamma_1-\gamma_2+C_d^v L_0\right)\right]}{[n_d\left(\overline{\theta}-\underline{\theta}\right)/n_p+2\gamma_2](L-L_0)},$$

which indicates that w_1^* is decreasing (increasing) in n_p (n_d). We rewrite r^* in (10) as

$$r^* = \frac{[(\gamma_1+\gamma_2)+C_d^v L]}{\left[\left(\overline{\theta}+p_{t0}\right)+p_{t1}(L-L_0)\right]-\frac{n_d\left(\overline{\theta}-\underline{\theta}\right)}{n_p}},$$

which shows that r^* is always decreasing (increasing) in n_p (n_d).

Summarizing the above, we obtain this theorem. ■

Proof of Theorem 8.

We conduct our analyses in the following three cases.

1. If $L \leq \hat{L}$, then the platform's maximum profit π^* is given as in (9), and the first-order partial derivatives of π^* w.r.t. n_p and n_d are computed as $\partial\pi^*/\partial n_p = (\overline{\theta}-\underline{\theta})\{n_d[(\overline{\theta}+p_{t0})+p_{t1}(L-L_0)-(\gamma_1-\gamma_2+C_d^v L)]\}^2/\{4[n_d(\overline{\theta}-\underline{\theta})+2n_p\gamma_2]^2\} \geq 0$ and $\partial\pi^*/\partial n_d = \gamma_2 n_p^2[(\overline{\theta}+p_{t0})+p_{t1}(L-L_0)-(\gamma_1-\gamma_2+C_d^v L)]^2/\{2[n_d(\overline{\theta}-\underline{\theta})+2n_p\gamma_2]^2\} \geq 0$, respectively. Moreover, we partially differentiate π^* twice w.r.t. n_p and n_d as $\partial^2\pi^*/\partial(n_p)^2 = -n_d^2\gamma_2(\overline{\theta}-\underline{\theta})[(\overline{\theta}+p_{t0})+p_{t1}(L-L_0)-(\gamma_1-\gamma_2+C_d^v L)]^2/[n_d(\overline{\theta}-\underline{\theta})+2n_p\gamma_2]^3 \leq 0$ and $\partial^2\pi^*/\partial(n_d)^2 = -n_p^2\gamma_2(\overline{\theta}-\underline{\theta})[(\overline{\theta}+p_{t0})+p_{t1}(L-L_0)-(\gamma_1-\gamma_2+C_d^v L)]^2/[n_d(\overline{\theta}-\underline{\theta})+2n_p\gamma_2]^3 \leq 0$, respectively. We also calculate the second-order cross-partial derivative of π^* w.r.t. n_p and n_d as $\partial^2\pi^*/(\partial n_p\partial n_d) = n_d n_p\gamma_2(\overline{\theta}-\underline{\theta})[(\overline{\theta}+p_{t0})+p_{t1}(L-L_0)-(\gamma_1-\gamma_2+C_d^v L)]^2/[n_d(\overline{\theta}-\underline{\theta})+2n_p\gamma_2]^3 \geq 0$. Using the above we obtain the result when $L \leq \hat{L}$ as shown in this theorem.
2. If $L > \hat{L}$ when $n_p \leq n_d$, then $\pi^* = n_p\{(p_t - C_d^v)L + \underline{\theta} - (\gamma_1-\gamma_2) - 2n_p\gamma_2/n_d\}$. Partially differentiating π^* once w.r.t. n_p yields $\partial\pi^*/\partial n_p = [(\underline{\theta}+p_{t0})+p_{t1}(L-L_0)-(\gamma_1-\gamma_2+C_d^v L)] - 4n_p\gamma_2/n_d$. Recalling from Section 3.2 that $p_{t1} > C_d^v$ and $\hat{L}|_{n_p\leq n_d} = L_0 + 2[n_d(\overline{\theta}-\underline{\theta})+2n_p\gamma_2]/[n_d(p_{t1}-C_d^v)] - [(\overline{\theta}+p_{t0})-(\gamma_1-\gamma_2+C_d^v L_0)]/(p_{t1}-C_d^v)$, we find that $\partial\pi^*/\partial n_p > \overline{\theta}-\underline{\theta} \geq 0$; that is, π^* is increasing in n_p. The first-order partial derivative of π^* w.r.t. n_d is $\partial\pi^*/\partial n_d = 2n_p^2\gamma_2/n_d^2 > 0$, which means that π^* is increasing in n_d. In addition, the second-order derivatives of π^* w.r.t. n_p and n_d are $\partial^2\pi^*/\partial n_p^2 = -4\gamma_2/n_d < 0$ and $\partial^2\pi^*/\partial n_d^2 = -4n_p^2\gamma_2/n_d^3 < 0$, and the second-order, cross-partial derivative is $\partial^2\pi^*/\partial n_p\partial n_d = 4n_p\gamma_2/n_d^2 > 0$.
3. If $L > \hat{L}$ when $n_p > n_d$, then $\pi^* = n_d[(\overline{\theta}+p_{t0})-(\gamma_1+\gamma_2+C_d^v L)+p_{t1}(L-L_0)] - n_d^2(\overline{\theta}-\underline{\theta})/n_p$, according to (10). We partially differentiate π^* once w.r.t. n_p, and find $\partial\pi^*/\partial n_p = (n_d/n_p)^2(\overline{\theta}-\underline{\theta}) > 0$, i.e., π^* is increasing in n_p. The first-order partial derivative of π^* w.r.t. n_d is $\partial\pi^*/\partial n_d = [(\overline{\theta}+p_{t0})-(\gamma_1+\gamma_2+C_d^v L)+p_{t1}(L-L_0)] - 2n_d(\overline{\theta}-\underline{\theta})/n_p$. Since $L > \hat{L}|_{n_p>n_d} = L_0 + 2[n_d(\overline{\theta}-\underline{\theta})+2n_p\gamma_2]/[n_p(p_{t1}-C_d^v)] - [(\overline{\theta}+p_{t0})-(\gamma_1-\gamma_2+C_d^v L_0)]/(p_{t1}-C_d^v)$ and $p_{t1} > C_d^v$, $\partial\pi^*/\partial n_d > 2\gamma_2 > 0$, which means that π^* is increasing in n_d. In addition, partially differentiating π^* twice w.r.t. n_p and n_d yields $\partial^2\pi^*/\partial n_p^2 = -2n_d^2(\overline{\theta}-\underline{\theta})/n_p^3 < 0$ and $\partial^2\pi^*/\partial n_d^2 = -2(\overline{\theta}-\underline{\theta})/n_p < 0$, and the cross-partial derivative is $\partial^2\pi^*/(\partial n_p\partial n_d) = 2n_d(\overline{\theta}-\underline{\theta})/n_p^2 > 0$.

Summarizing the above, we obtain this theorem. ■

Appendix B Robustness Tests

In order to test the robustness of the results regarding the impacts of the number of potential passengers n_p, we increase the value of n_p in steps of 5, and plot the results in Figures 6-10. We can find that Figures 6, 7, 8, 9, and 10 are similar to Figures 1, 2, 3, 4, and 5, respectively. This means that the results about the impact of n_p are robust.

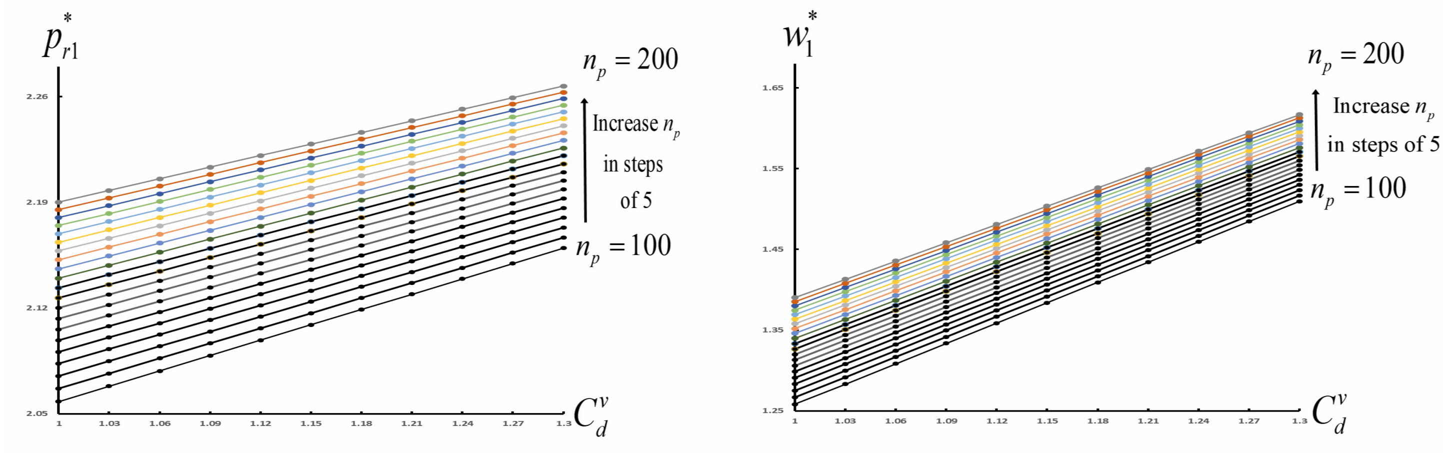

Figure 6: The impacts of n_p and C_d^v on the optimal distance price p_{r1}^* and the optimal distance wage to ridesharing drivers w_1^*, when we increase the value of n_p in steps of 5.

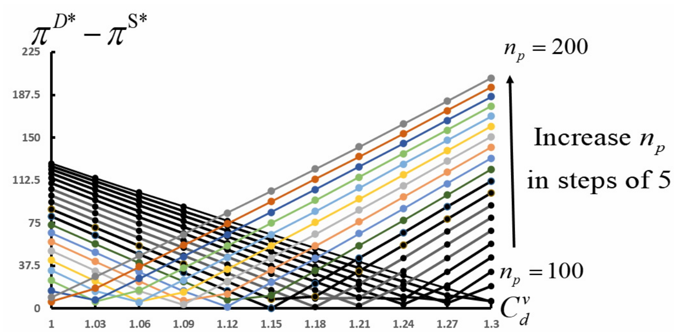

Figure 7: The differences between the platform's maximum profits in the dynamic and static pricing strategies, i.e., $\pi^{D*} - \pi^{S*}$, when we increase the value of n_p in steps of 5.

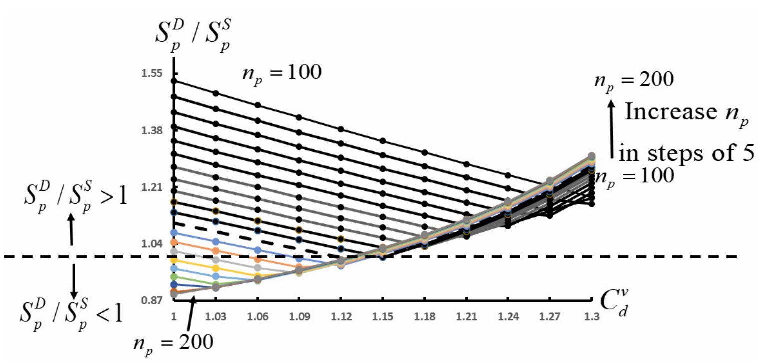

Figure 8: The ratios of S_p^D to S_p^S when the values of n_p and C_d^v vary, when we increase the value of n_p in steps of 5.

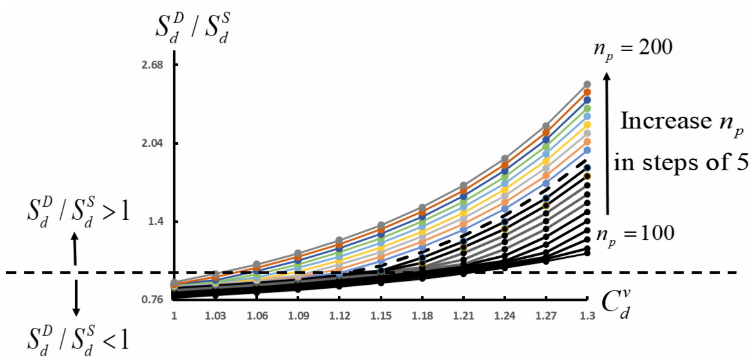

Figure 9: The ratios of S_d^D to S_d^S when the values of n_p and C_d^v vary, when we increase the value of n_p in steps of 5.

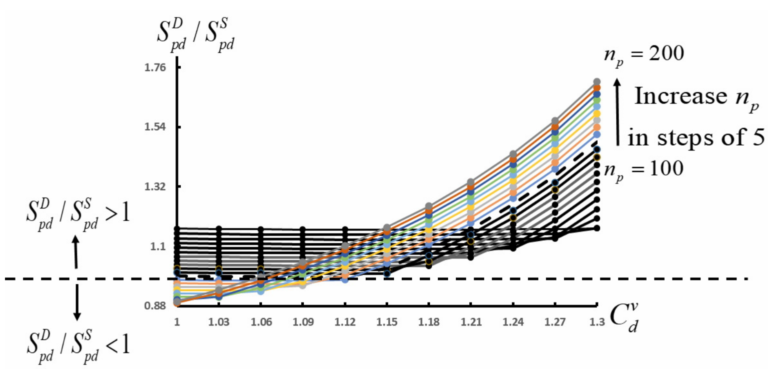

Figure 10: The ratios of S_{pd}^D to S_{pd}^S when the values of n_p and C_d^v vary, when we increase the value of n_p in steps of 5.