GAME THEORETIC APPLICATIONS IN SUPPLY CHAIN MANAGEMENT: A REVIEW[1,2]

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ABSTRACT

Recent emphasis on competition and cooperation in supply chains has resulted in the resurgence of game theory as a relevant tool for analyzing such interactions in a supply chain. This paper presents a review of more than 130 papers concerned with game theoretical applications in supply chain management (SCM). We first give a brief summary of the basic solution concepts in non-cooperative and cooperative games such as Nash and Stackelberg equilibria, Nash arbitration scheme and cooperation with sidepayments, the core, the Shapley value and nucleolus. Our review of supply chain-related game theoretical applications is presented in five areas: (i) Inventory games with fixed unit purchase cost, (ii) Inventory games with quantity discounts, (iii) Production and pricing competition, (iv) Games with other attributes, (v) Games with joint decisions on inventory, production/pricing and other attributes. The paper concludes with a summary of our review, suggestions for potential applications of game theory in SCM and an alternative classification of all reviewed papers.

Keywords: Supply chain management, non-cooperative and cooperative games.

1. INTRODUCTION

Game theory is concerned with the analysis of situations involving conflict and cooperation. Since its development in the early 1940s game theory has found applications in diverse areas such as anthropology, auctions, biology, business, economics, management-labour arbitration, philosophy, politics, sports and warfare. After the initial excitement generated by its potential applications, interest in game theory by operations research/management science specialists seemed to have waned during the 1960s and the 1970s. However, the last two decades have witnessed a renewed interest by academics and practitioners in the management of supply chains and a new emphasis on the interactions among the decision makers ("players") constituting a supply chain. This has resulted in the proliferation of publications in scattered journals dealing with the use of game theory in the analysis of supply chain-related problems. The purpose of this paper is to provide a wide-ranging survey (of more than 130 papers) focusing on game theoretic applications in different areas of supply chain management (SCM).

A supply chain can be defined as "a system of suppliers, manufacturers, distributors, retailers, and customers where materials flow downstream from suppliers to customers and information flows in both directions" (Ganeshan et al. [57]). Supply chain *management*, on the other hand,

1 The authors thank two anonymous referees for their thorough and thoughtful comments that improved the paper.
2 Research supported by the Natural Sciences and Engineering Research Council of Canada.
3 Research conducted while the first author was a Ph.D. student in Management Science/Systems at McMaster University.

Recd. Aug. 2003, Rev. Sept. 2004/Feb. 2005, Acc. Feb. 2005 INFOR vol. 43, no. 3 Aug. 2005

is defined by some researchers as a set of management *processes*. For example, LaLonde [94] defines SCM as "the process of managing relationships, information, and materials flow across enterprise borders to deliver enhanced customer service and economic value through synchronized management of the flow of physical goods and associated information from sourcing to consumption." (See Mentzer et al. [118] for a collection of competing definitions.) Adopting LaLonde's definition, one observes that most SCM-related research has features that are common to operations management and marketing problems, e.g., inventory control, production and pricing competition, capacity investments, service and product quality competition, advertising and new product introduction.

Several survey papers related to SCM have appeared in the literature. For example, Tayur et al. [154] have edited a book emphasizing quantitative models for SCM. Ganeshan et al. [57] proposed a taxonomic review and framework that help both practitioners and academic researchers better understand the up-to-date state of SCM research. Wilcox et al. [172] presented a brief survey of the papers on the price-quantity discount. McAlister [113] reviewed a model of distribution channels incorporating behavior dimensions. Goyal and Gupta [64] provided a survey of literature that treated buyer-vendor coordination with integrated inventory models.

In addition to the above, some reviews focussing on the application of game theory in economics or management science have appeared in the last five decades. An early survey of game theoretic applications in management science was given by Shubik [148]. Feichtinger and Jørgensen [53] published a review that was restricted to differential game applications in management science and operations research. More recently, Wang and Parlar [167] presented a survey of the static game theory applications in management science problems. A review of applications of differential games in advertising was given by Jørgensen [78]. Li and Whang [105] provided a survey of game theoretic models applied in operations management and information systems where the SCM-related literature focusing on information sharing and manufacturing/marketing incentives was also discussed. In addition, several books (e.g., Chatterjee and Samuelson [31], Gautschi [59], Kuhn and Szego [91] and Sheth et al. [147]) partially reviewed some specific game-related topics in SCM.

In the last few years two important reviews focussing on game theoretical applications in supply chain management were published. In [27] Cachon and Netessine outlined game-theoretic concepts and surveyed applications of game theory in supply chain management. Cachon and Netessine classified games that were developed for SCM into four categories based on game-theoretical techniques: (i) Non-cooperative static games, (ii) dynamic games, (iii) cooperative games, and (iv) signaling, screening and Bayesian games. In each category, the authors presented the major techniques that are commonly used in the existing papers and those that could be applied in future research. Our paper differs from Cachon and Netessine [27] because we review about 130 papers based on a classification of SCM topics (rather than game-theoretical techniques). In [20] Cachon reviewed the literature on supply chain collaboration with contracts. Our paper differs from [20] as we review game models concerned with coordination and competition in supply chains. Moreover, we review several very recent papers which were not mentioned in [20] or [27].

Most significant – and interesting – topics arising in SCM emphasize the coordination/co-operation and competition among supply chain's channel members. In a centralized supply chain the "central" decision maker may coordinate the members' activities to increase the competitive capability of the supply chain. In other words, the single decision maker determines the optimal solution that globally improves the supply chain performance; thus, in these type of centralized problems game theory is not used. However, for a decentralized supply chain where each supply chain member is an independent decision maker, there arise two issues: (i) Supply chain members compete to improve their individual performance. For example, several agents at the same echelon of a supply chain may compete for limited resources or compete for demand from the same group of customers. As a result, various competitive game-related issues arise in the analysis of the decentralized supply chains with competition, (ii) Supply chain members may agree to have a contract to coordinate their strategies in order to improve the global performance of the system as well as their individual profits. For this type of decentralized supply chains with cooperation/coordination, channel members may not only achieve supply chain-wide optimization but also they would have no incentives to deviate from the global optimal solution. Naturally, a prime methodological tool for dealing with these problems is non-cooperative and cooperative game theory that focuses on the simultaneous or sequential decision-making of multiple-players under complete or incomplete information.

This literature review is organized as follows. Section 2 presents a description of important game theoretic concepts used in the solution of non-cooperative and cooperative games. These include Nash and Stackelberg equilibria, the Nash arbitration scheme and cooperation with side-payments, the core, the Shapley value and the nucleolus. In this section we also mention subgame-perfection and trigger strategy that are commonly used in multi-stage (dynamic) and repeated games which are becoming more relevent in supply chain applications. Section 2 also includes a classification of five categories where supply chain-related game theoretical applications are found: (i) Inventory games with fixed unit purchase cost, (ii) Inventory games with quantity discounts, (iii) Production and pricing competition, (iv) Games with other attributes, (v) Games with joint decisions on inventory, production/pricing and other attributes. Review of papers in these five areas is presented in the five subsequent sections, i.e., Section 3 covers category (i), i.e., inventory games with fixed unit purchase cost, Section 4 discusses category (ii), Section 5 deals with category (iii), Section 6 covers category (iv), and Section 7 reviews category (v). The final section presents our concluding remarks and some suggestions for potential applications of game theory in SCM. Finally, in Appendix A we categorize the reviewed papers according to an alternative classification scheme based on their game theoretic nature, i.e., non-cooperative vs. cooperative game. In Appendix B we present a summary distribution of the reviewed papers in all five classes.

2. GAME THEORY AND SUPPLY CHAIN MANAGEMENT

As discussed in Section 1, game theory has become a primary methodology used in SCM-related problems. The goal of this section is to provide a concise framework of game theoretic models and their applications to various SCM issues classified into different categories.

2.1 Brief Review of Some Solution Concepts in Game Theory

Game theoretic models can be classified as non-cooperative or cooperative depending on the nature of interaction among the players. In this subsection we describe some of the standard approaches in each category.

2.1.1 Non-cooperative Games

Nash and *Stackelberg equilibria* are two important solution concepts used in many non-cooperative games. In a game, the feasible actions that could be adopted by the players are called their *strategies*. For a player, all possible strategies form the player's *strategy set*. When each player in a game chooses a feasible strategy, an outcome appears as the specific *payoffs* to all players. When players in a game choose their strategies simultaneously, Nash equilibrium applies. But in a leader-following scenario where one player can act before the other, the strategy for each player can be determined by finding the Stackelberg solution. Both Nash and Stackelberg strategies require the analysis of the "best response functions." We illustrate these ideas by presenting a simple two-person non-cooperative nonzero-sum game.

Best Response Functions: Consider a two-person nonzero-sum game with $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$ as the objective functions of the two non-cooperating players where $x_1 \in X_1$ and

$x_2 \in X_2$ represent the strategies chosen by player 1 (P1) and player 2 (P2) over their respective feasible regions X_1 and X_2. We assume that each player's objective is to maximize his/her objective function. Suppose P2 chooses the strategy as $x_2 = \hat{x}_2$ and announces it to P1. The best response $x_1^R(\hat{x}_2)$ of P1 is obtained as the solution of the optimization problem $x_1^R(\hat{x}_2) \equiv \arg\max_{x_1 \in X_1} f_1(x_1, \hat{x}_2)$. Performing this optimization for all $x_2 \in X_2$ we obtain the best response $x_1^R(x_2)$ of P1 given as a function of x_2. Similarly, the best response $x_2^R(x_1)$ of P2 can be found as a function of x_1.

Example 1
Consider a two-person nonzero-sum game where players 1 and 2 attempt to maximize their respective objective functions $f_1(x_1, x_2) = -2x_1^2 + 5x_1x_2$ and $f_2(x_1, x_2) = -3x_2^2 + 2x_1x_2 + x_2$ for $x_1, x_2 \geq 0$. Note that for any $x_2 = \hat{x}_2$, P1's objective function $f_1(x_1, \hat{x}_2)$ is concave in x_1, and for any $x_1 = \hat{x}_1$, P2's objective function $f_2(\hat{x}_1, x_2)$ is concave in x_2. Differentiating $f_1(x_1, \hat{x}_2)$ with respect to x_1 we find $\partial f_1(x_1, \hat{x}_2)/\partial x_1 = -4x_1 + 5\hat{x}_2$. Solving $\partial f_1(x_1, \hat{x}_2)/\partial x_1 = 0$ for x_1 gives $x_1^R(x_2) = \frac{5}{4}x_2$ as the best response function for P1. For P2 the best response function is found as $x_2^R(x_1) = \frac{1}{6}(2x_1 + 1)$. ◀

We now describe the computation of Nash and Stackelberg equilibria using the best response functions for each player.

Nash Equilibrium: This concept applies when the players announce their decisions simultaneously (as in the children's game known as "rock, paper and scissors" (Kreps [88]). It is also applicable when the players cannot communicate (as in the game known as "prisoners' dilemma" (Shubik [149]). The following definition formalizes the concept of Nash equilibrium (Nash [124]).

Definition 1
A pair of strategies (x_1^N, x_2^N) *is said to constitute a* Nash equilibrium *if the following pair of inequalities is satisfied for all* $x_1 \in X_1$ *and for all* $x_2 \in X_2$:

$$f_1(x_1^N, x_2^N) \geq f_1(x_1, x_2^N) \quad \text{and} \quad f_2(x_1^N, x_2^N) \geq f_2(x_1^N, x_2).$$

That is, x_1^N *and* x_2^N *solve* $\max_{x_1 \in X_1} f_1(x_1, x_2^N)$ *and* $\max_{x_2 \in X_2} f_2(x_1^N, x_2)$, *respectively. (See, for example, Başar and Olsder [11] and Gibbons [61, p. 8].)*

Assuming continuity, differentiability and $(x_1, x_2) \in \mathbf{R}^2$, this definition implies that if the pair (x_1^N, x_2^N) is to be a Nash equilibrium, the players' decisions must satisfy

$$\left.\frac{\partial f_1(x_1, x_2^N)}{\partial x_1}\right|_{x_1 = x_1^N} = 0 \quad \text{and} \quad \left.\frac{\partial f_2(x_1^N, x_2)}{\partial x_2}\right|_{x_2 = x_2^N} = 0.$$

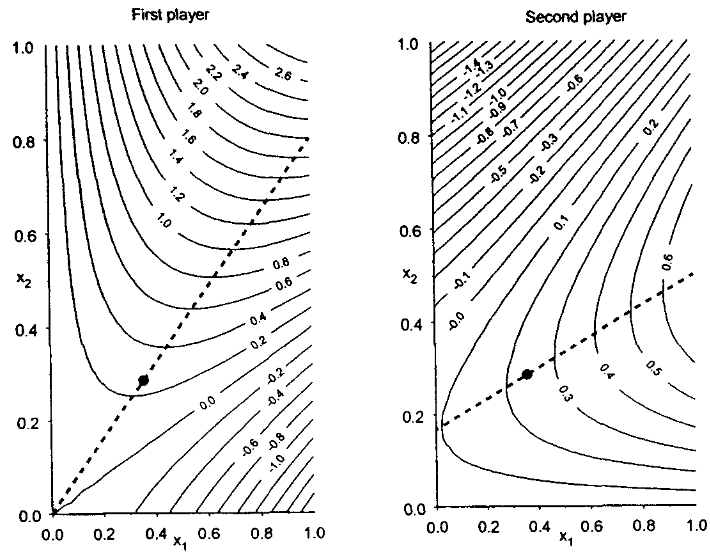
Equivalently, the Nash equilibrium is obtained by solving the (nonlinear) system of equations $x_1 = x_1^R(x_2)$ and $x_2 = x_2^R(x_1)$.

Example 2
Consider again the problem discussed in Example 1. To compute the Nash equilibrium we solve

$$x_1 = \tfrac{5}{4}x_2 \quad \text{and} \quad x_2 = \tfrac{1}{6}(2x_1 + 1)$$

and obtain $(x_1^N, x_2^N) = (0.36, 0.29)$, see Figure 1. Substituting this result in the players' objective functions gives $f_1(x_1^N, x_2^N) = 0.2628$ and $f_2(x_1^N, x_2^N) = 0.2465$. The solution found is the equilibrium since a unilateral move by any of the players results in an inferior solution

Figure 1: These graphs display the contour curves for each player's objective function $J_1(x_1, x_2)$ and $J_2(x_1, x_2)$. The best response functions for player 1 and player 2 are given by the straight lines $x_1 = \frac{5}{4}x_2$ and $x_2 = \frac{1}{6}(2x_1 + 1)$, respectively. Solving the linear system of the best response functions gives the unique Nash equilibrium as $(x_1^N, x_2^N) = (0.36, 0.29)$. Graphically, the Nash equilibrium is found by superimposing the two figures and finding the intersection of the two lines (which is denoted by a circle •).



for that player. For example, if P2 moves away from $x_2^N = 0.29$ while P1 still plays $x_1^N = 0.36$, we find that P2's objective deteriorates. Similarly, if P1 moves away from $x_1^N = 0.36$ while P2 still plays $x_2^N = 0.29$, then P1's objective deteriorates. Hence, in this non-cooperative game rational players must choose $(x_1^N, x_2^N) = (0.36, 0.29)$ as their Nash solution. ◀

Stackelberg Equilibrium: This equilibrium concept – due to von Stackelberg [161] – applies when one of the players can move before the other player(s) and assumes the role of the leader. For example, a company may complete its R&D activities and launch a new product before the others thus assuming the leadership position in the market. In a macroeconomic setting, the government (leader) sets its fiscal and monetary policy and the firms follow by choosing their price and employment levels. In a leader-follower environment, the follower chooses her best response to the leader's decision; and the leader optimizes his objective function subject to the follower's response. In some SCM problems Stackelberg solution concept is more realistic than Nash equilibrium as a channel member sometimes plays the role of the leader by first announcing his strategy to the other channel member(s). For instance, in a quantity discount problem involving a seller and a buyer, the seller (leader) may first announce his discount policy to the buyer, and the buyer (follower) makes her purchase decision in response to seller's decision.

Consider again a two-person game where, say, P1 is the leader and P2 is the follower with the respective objective functions $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$. For any x_1 that P1 chooses, P2 uses her best response function to determine her response $x_2 = x_2^R(x_1)$. Since the leader can determine the follower's response to his decision (assuming, of course, that the game is played under complete information), he then optimizes his objective $f_1(x_1, x_2)$ subject to the constraint $x_2 = x_2^R(x_1)$. We now formalize the concept of Stackelberg equilibrium with the following definition.

Definition 2

In a two-person game with P1 as the leader and P2 as the follower, the strategy $x_1^S \in X_1$ is called a Stackelberg equilibrium *for the leader if, for all x_1,*

$$f_1(x_1^S, x_2 | x_2 = x_2^R(x_1^S)) \geq f_1(x_1, x_2 | x_2 = x_2^R(x_1)),$$

where $x_2^R(x_1)$ is the best response function of the follower. (See, Başar and Olsder [11].)

Example 3

Consider again the problem discussed in Example 1 with $f_1(x_1, x_2) = -2x_1^2 + 5x_1x_2$ and $f_2(x_1, x_2) = -3x_2^2 + 2x_1x_2 + x_2$ and the follower's best response function as $x_2 = x_2^R(x_1) = \frac{1}{6}(2x_1 + 1)$. To determine the leader's Stackelberg strategy, we maximize his objective $f_1(x_1, x_2)$ subject to the constraint $x_2 = x_2^R(x_1)$ for $x_1 \geq 0$. (Graphically, in Figure 1, this corresponds to maximizing the first player's objective on the line representing the best response function of the second player.) Thus,

$$\begin{aligned} x_1^S &= \arg\max_{x_1 \geq 0} f_1(x_1, x_2^R(x_1)) \\ &= \arg\max_{x_1 \geq 0} \left\{-2x_1^2 + 5x_1 \left[\frac{1}{6}(2x_1 + 1)\right]\right\} = \arg\max_{x_1 \geq 0} \left(-\frac{1}{3}x_1^2 + \frac{5}{6}x_1\right) \\ &= \frac{5}{4}. \end{aligned}$$

The follower's Stackelberg solution is then found as $x_2^S = x_2^R(x_1^S) = \frac{1}{6}(2x_1^S + 1) = \frac{7}{12}$. Substituting the solution $(x_1^S, x_2^S) = (\frac{5}{4}, \frac{7}{12})$ into the two players' objective functions gives $f_1(x_1^S, x_2^S) = \frac{25}{48} \approx 0.52$ and $f_2(x_1^S, x_2^S) = \frac{49}{48} \approx 1.02$. ◀

Comparing the Nash and Stackelberg solutions found in Examples 2 and 3, we see that both players improve their objective functions and the follower does even better than the leader. This result is sometimes observed in practical situations where a high-cost leader loses market share to a low-cost follower who imitates cheaper copies of the product without investing in costly R&D activities. For other interesting aspects of Stackelberg solution, we refer the reader to the excellent text by Başar and Olsder [11].

We should also mention two other solution concepts (subgame-perfection and trigger strategy) which are becoming relevant in supply chain applications. *Subgame-perfection* is an important concept used in the solution of dynamic games which are represented in extensive form. In a dynamic game consisting of subgames, a Nash equilibrium is defined as subgame-perfect if the players' strategies constitute a Nash equilibrium in each subgame. In a repeated game which is played infinitely many times, a player i may cooperate with player j until j stops cooperating which *triggers* player i to switch to non-cooperation. For a detailed description of these concepts and illustrative examples, see Gibbons [61, Ch. 2].

In the above paragraphs we have presented a very brief review of some of the solution concepts associated with non-cooperative games as most existing papers dealing with a variety of SCM problems focus on finding Nash and Stackelberg solutions. Although most papers on SCM use the Nash and Stackelberg equilibria to determine the channel members' decisions, there are also *cooperative* solution concepts that are used in the analysis of supply chain problems as we describe below.

2.1.2 Cooperative games

In a cooperative game, communication between players is allowed (or, possible) so that they could agree to implement an outcome better than the Nash or Stackelberg equilibrium. Since the aim of cooperation between channel members in a supply chain is to improve their (and the supply chain's) profitability, it is important to understand the concepts used in cooperative game theory. Most cooperative games with three or more players are formulated using the characteristic function form which specifies the payoffs to each coalition; such games are solved using concepts such as the Shapley value [145] and nucleolus [142]. Cooperative games that involve only two players are usually analyzed by using the Nash arbitration scheme [123] which is not given in terms of characteristic functions.

Cooperative Games not in Characteristic Function Form: For cooperative games with two players which are not stated in characteristic function form, Nash arbitration scheme [123], or cooperation with side-payments (where a system-wide objective function is optimized) may provide an acceptable solution.

The Nash Arbitration Scheme: Since this scheme is determined as the solution of a bargaining game, it is also called Nash Bargaining Solution (see, Nash [123]). This scheme is based on (i) the concept of undominated Pareto optimal solutions that make up the efficient frontier of payoff values for the two players, and (ii) the *status quo* point corresponding to the players' "security" levels, i.e., the payoffs (f_1^0, f_2^0) guaranteed to each player even when they do not cooperate. An arbitrated solution to a non-zero sum game is (i) Pareto optimal and, (ii) at or above the security levels for both players.

One way of determining the Pareto optimal solutions is by solving a nonlinear programming problem which maximizes P1's objective $f_1(x_1, x_2)$ subject to the constraint that P2 receives b, i.e., $f_2(x_1, x_2) = b$. This problem is solved for each value of b and a parametric solution (a nonlinear curve) is obtained for the optimal (f_1^*, f_2^*). The Pareto optimal solutions $\mathcal{P}$ on this curve are those points which are not dominated by any other point on the curve.

Nash's arbitration scheme depends on four axioms: (i) Rationality, (ii) linear invariance, (iii) symmetry, and (iv) independence of irrelevant alternatives. With these axioms Nash shows that there is a unique arbitration solution found by solving the optimization problem

$$\max_{f_1 \geq f_1^0, f_2 \geq f_2^0} (f_1 - f_1^0)(f_2 - f_2^0) \quad \text{s.t. } (f_1, f_2) \in \mathcal{P}$$

where (f_1^0, f_2^0) is the status quo point. For an application of Nash's arbitration scheme to product quality competition, see Reyniers and Tapiero [136].

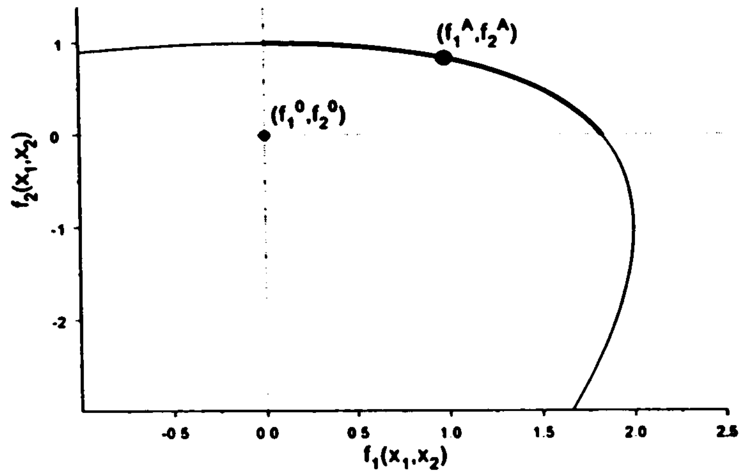
Example 4

Consider a two-person non-zero sum cooperative game with the objective functions $f_1(x_1, x_2) = 2 - [(x_1 - 1)^2 + (x_2 - 1)^2]$ and $f_2(x_1, x_2) = 1 - [(x_1 - 2)^2 + (x_2 - 2)^2]$. Here, the decision variables (x_1, x_2) could correspond to production levels chosen by each firm whose profit functions are given by $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$. To determine the Pareto optimal solutions for this game we maximize $f_1(x_1, x_2)$ subject to $f_2(x_1, x_2) = b$. [Since the global maximum value of $f_2(x_1, x_2)$ is 1, we must have $b \leq 1$.] This is achieved by forming the Lagrangian as $L(x_1, x_2, \lambda) = f_1(x_1, x_2) + \lambda[b - f_2(x_1, x_2)]$. Partially differentiating $L(x_1, x_2, \lambda)$, equating the derivatives to zero and solving the resulting system of three nonlinear equations we find a set of two solutions as $x_1 = 2 \pm \frac{1}{2}\sqrt{2(1-b)}$, $x_2 = 2 \pm \frac{1}{2}\sqrt{2(1-b)}$, and $\lambda = 1 \pm \sqrt{2(1-b)}/(1-b)$. Substituting these in $f_1(x_1, x_2)$ and $f_2(x_1, x_2)$ we have (in parametric form)

$$[f_1(x_1, x_2), f_2(x_1, x_2)] = \left[2 - 2\left(1 \pm \frac{1}{2}\sqrt{2(1-b)}\right)^2, b\right] \text{ for } b \leq 1.$$

When $b = -1$, we see that $f_1(x_1, x_2)$ reaches its highest value of 2. Thus, the efficient frontier is obtained in terms of (f_1, f_2) as $f_1 = g(f_2) = 2 - 2(1 - \frac{1}{2}\sqrt{2(1 - f_2)})^2$ for $0 \leq f_2 \leq 1$. Assuming

Figure 2: The Pareto optimal solutions $\mathcal{P}$ are the collection of points on the thick curve. The Nash arbitrated solution is $(f_1^A, f_2^A) \in \mathcal{P}$ denoted by the circle • on the curve.



the status quo to be $(f_1^0, f_2^0) = (0,0)$, Nash's arbitrated solution (f_1^A, f_2^A) is found by maximizing $f_1 \times f_2 = [2 - 2(1 - \frac{1}{2}\sqrt{2(1-f_2)})^2] \times f_2$ subject to $f_1, f_2 \geq 0$. Performing the optimization gives $(f_1^A, f_2^A) = (1.21, 0.72)$ which correspond to production levels of $(x_1, x_2) = (1.62, 1.62)$. With this solution the total system-wide objective is found as $f^A = f_1^A + f_2^A = 1.21 + 0.72 = 1.93$. ◀

Cooperation with Side-payments: Now assume that it is possible for one player to make *side-payments* to the other player. The players can then cooperate by maximizing the system-wide objective function $f(x_1, x_2) = f_1(x_1, x_2) + f_2(x_1, x_2)$ and agree to split the extra profit resulting from this cooperation. In the case of Example 4 this corresponds to maximizing the objective function $f(x_1, x_2) = 3 - (x_1 - 1)^2 - (x_2 - 1)^2 - (x_1 - 2)^2 - (x_2 - 2)^2$ which results in the optimal solution $(x_1^*, x_2^*) = (1.5, 1.5)$ with $f^* \equiv f(x_1^*, x_2^*) = 2$. Jointly optimizing the system-wide objective function results in a higher profit ($f^* = 2$) than the total profit obtained under the arbitrated solution ($f^A = 1.93$) – the difference being 0.07. Now, player 1 can make a side-payment of, say, 0.03 to player 2 thus making them both better off than at the arbitrated solution.

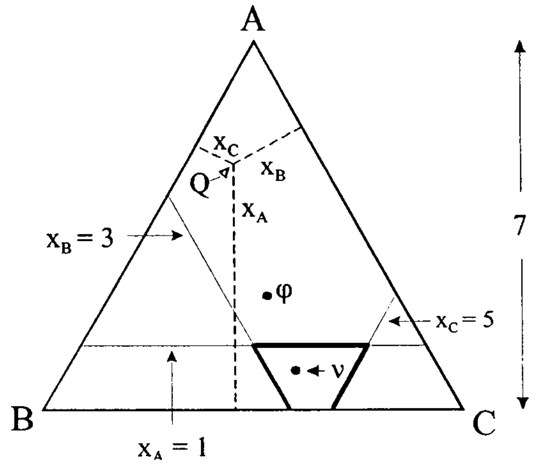
Cooperative Games in Characteristic Function Form: Consider a game with multiple players who can communicate and (perhaps) improve their payoff by cooperation. Many such games can be analyzed by casting them in characteristic function form defined as follows.

Definition 3
A game $\mathcal{G} = (N, v)$ in characteristic function form is a set of N players and a function v which assigns a number $v(S)$ to any subset $S \subseteq N$.

The number $v(S)$ assigned to the *coalition* S is interpreted as the amount that players in set S could win if they formed a coalition. A game in characteristic form is said to be *superadditive* when $v(S \cup T) \geq v(S) + v(T)$ for any two disjoint coalitions S and T. A superadditive N-person game is *inessential* if $\sum_{i \in N} v(i) = v(N)$. Otherwise, the game is *essential*.

For an N-person game in characteristic form, the payoff to each player is expressed as an n-tuple of numbers $\mathbf{x} = (x_1, x_2, \ldots, x_n)$. A payoff n-tuple, which satisfies individual rationality

Figure 3: The core of the 3-person game with $v(\emptyset) = v(A) = v(B) = v(C) = 0$, $v(AB) = 2$, $v(AC) = 4$, $v(BC) = 6$ and $v(ABC) = 7$ is the area indicated by thick lines. The Shapley value φ is outside the core, but the nucleolus ν is inside it.



[i.e., $x_i \geq v(i)$ for each player i] and collective rationality [i.e., $\sum_{i \in N} x_i = v(N)$], is called an *imputation* for the game (N, v).

Example 5
As an example, consider a 3-person game with $N = \{A, B, C\}$ where the characteristic functions are given as $v(\emptyset) = v(A) = v(B) = v(C) = 0$, $v(AB) = 2$, $v(AC) = 4$, $v(BC) = 6$ and $v(ABC) = 7$. Here, individually, none of the players can receive any payoff. But if they cooperate, different coalitions result in a positive payoff for each coalition. If they all cooperate, then the "grand coalition" receives an amount $v(ABC)$ higher than any other coalition. ◀

In the last 50 years more than a dozen solution concepts have been introduced to find a "fair" allocation for cooperative games. Here we briefly describe the three most important cooperative solution concepts commonly encountered in the literature.

The Core: This concept arises from the argument that the total payoff to the members of any coalition S should be at least as much as their coalition could provide them, i.e., the imputations should be undominated. That is, the *core* of a game in characteristic form is defined as the set of all imputations $(x_1, x_2, \ldots, x_n)$ such that for all $S \subseteq N$, $\sum_{i \in S} x_i \geq v(S)$; see Owen [128] and Rapaport [134].

In Example 5, the core is the set of all (x_A, x_B, x_C) satisfying $x_A + x_B \geq v(AB) = 2$, $x_A + x_C \geq v(AC) = 4$, $x_B + x_C \geq v(BC) = 6$ and $x_A + x_B + x_C = v(ABC) = 7$. This is a non-empty set which includes, for example, $(x_A, x_B, x_C) = (0, 2, 5)$ and $(x_A, x_B, x_C) = (0.3, 3.0, 3.7)$, among infinitely many others. [In this example the core would be empty, if, we had $v(AB) = 5$.]

The set of imputations in this game can be represented by an equilateral triangle with height equal to $v(ABC) = 7$. For any point $\mathbf{Q} = (x_A, x_B, x_C)$ in the triangle, x_i is the distance to side of the opposite corner $i = A, B, C$ as indicated in Figure 3. (Thus, player i prefers imputations that are close to corner i.) Since $x_i + x_j \geq v(ij) \Leftrightarrow x_k \leq v(ABC) - v(ij)$ for $i \neq j \neq k$, the latter inequalities can be drawn to obtain the core – provided that it is nonempty.The core in this game is obtained by drawing the regions $x_A \leq 1$, $x_B \leq 3$ and $x_C \leq 5$; these give rise to the area indicated by thick lines in Figure 3.

The Shapley Value: Shapley [145] suggested a solution concept for cooperative games which provides a unique imputation and represents payoffs distributed "fairly" by an outside arbitrator. The Shapley value $\varphi = (\varphi_1, \ldots, \varphi_n)$ is determined based on three axioms: (i) the symmetries in v, (ii) irrelevance of a "dummy" player, and (iii) the sum of two games. Axiom (i) implies that if some players have symmetric roles in v, then the Shapley values to these players should be the same. From Axiom (ii), the Shapley value to the player who adds nothing to any coalition should be determined as zero. Axiom (iii) says that if two games have the same player set, then the characteristic value of the sum game for any coalition should be the sum of the characteristic values of two games. For example, consider two games respectively denoted by (N, v) and (N, w). For a coalition S, we have $(v + w)(S) = v(S) + w(S)$.

Based on the three axioms, Shapley determines the unique values $\varphi_i = \sum_{i \in S}(|S| - 1)!(n - |S|)![v(S) - v(S - i)]/n!$ where S denotes a coalition and $|S|$ is the size of S. For Example 5, the Shapley values for the three players are found as $\varphi = (\varphi_A, \varphi_B, \varphi_C) = (1\frac{1}{3}, 2\frac{1}{3}, 3\frac{1}{3})$ reflecting the importance of C's contribution to the coalitions of which she is a member. Note, however, that in this example the Shapley value is *not* in the core.

The Nucleolus: This solution concept, proposed by Schmeidler [142], minimizes the "unhappiness" of the most unhappy coalition. Let $e_S(\mathbf{x}) = v(S) - \sum_{i \in S} x_i$ denote the excess (unhappiness) of a coalition S with an imputation $\mathbf{x}$. With this definition, nucleolus can be found as follows: (i) First consider those coalitions S whose excess $e_S(\mathbf{x})$ is the largest for a given imputation $\mathbf{x}$, (ii) If possible, vary $\mathbf{x}$ to make this largest excess smaller, (iii) When the largest excess is made as small as possible, consider the next largest excess and vary $\mathbf{x}$ to make it as small as possible, etc. Although for small problems with a few players this approach works efficiently, large problems are normally solved using a series of linear programming problems; see Wang [163] and Carter [30]. For Example 5 the nucleolus solution is found as $\boldsymbol{\nu} = (\nu_A, \nu_B, \nu_C) = (0.50, 2.25, 4.25)$ with the corresponding excesses $e_A(\boldsymbol{\nu}) = -0.50$, $e_B(\boldsymbol{\nu}) = -2.25$, $e_C(\boldsymbol{\nu}) = -4.25$, $e_{AB}(\boldsymbol{\nu}) = -0.75$, $e_{AC}(\boldsymbol{\nu}) = -0.75$, $e_{BC}(\boldsymbol{\nu}) = -0.50$. Since the excesses are all negative, their absolute values could be considered as the level of *happiness* for each coalition.

2.2 Topical Classification of SCM-related Problems

Our classification of game-theoretical applications in SCM is based on five application areas. Better control and maintenance of inventory systems can result in significant benefits for a supply chain in the form of lower costs, higher profits and more satisfactory service quality. However, in a supply chain one member's inventory decision may impact the upstream and/or downstream members. Thus, game theoretic analysis can provide important insights into the co-operative/competitive nature of inventory-related supply chain decisions. Since a large number of publications have focused on game theoretical applications in inventory management, we divide our review of inventory-related games into two separate classes: (I) INVENTORY GAMES WITH FIXED UNIT PURCHASE COST, and (II) INVENTORY GAMES WITH QUANTITY DISCOUNTS. Even before game theory was rigorously formalized in the 1940s, some early applications involving competitive behavior of decision makers in production/pricing were made by Bertrand [14] and Cournot [47] in the 19th century. Many papers in this area which we label (III) PRODUCTION AND PRICING COMPETITION focused on the vertical competition between a manufacturer and a retailer, or horizontal competition between two manufacturers or two retailers. In addition to the game theoretical applications in inventory management and production/pricing competition, there exist a considerable number of papers concerned with attributes such as capacity, service/product quality, advertising and new product introduction. We categorize and review papers with these attributes into a single class labelled (IV) GAMES WITH OTHER ATTRIBUTES. In some game-theoretic models the supply chain members make joint decisions involving some of the attributes indicated in the last four classes. Papers concerned with such issues (e.g., jointly made inventory and pricing decisions of supply chain members) are included in this class labelled (V) GAMES WITH JOINT DECISIONS ON INVENTORY, PRODUCTION/PRICING AND OTHER ATTRIBUTES.

3. INVENTORY GAMES WITH FIXED UNIT PURCHASE COST

Inventory management problems involving competition arise in either horizontal or vertical channels. First, consider examples of competition in horizontal channels. In one of the early papers in this area Parlar [129] developed a single-period context game theoretic model of competition between two players. In his model the products sold by two retailers are substitutable and the retailers simultaneously choose their order quantities u and v to maximize their expected profits $J_1(u,v)$ and $J_2(u,v)$, respectively. The first retailer's objective is given as

$$J_1(u,v) = (s_1+p_1)\left[\int_0^u xf(x)\,dx + u\int_u^\infty f(x)\,dx\right] - p_1E(X) + q_1\int_0^u (u-x)f(x)\,dx$$
$$+ (s_1-q_1)\int_0^u \left[\int_v^B b(y-v)g(y)\,dy + \int_B^\infty (u-x)g(y)\,dy\right] f(x)\,dx - c_1u,$$

where $f(x)$ and $g(y)$ are the demand densities faced by each retailer, a and b $(0 \le a,b \le 1)$ are the substitution rates of the retailer's products when they are sold out; s_1, c_1, q_1 and p_1 are the unit selling price, purchase cost, salvage value and and shortage penalty cost for first player's product, and $B = [(u-x)/b] + v$ and $A = [(v-y)/a] + u$. For this model Parlar proved the existence and uniqueness of the Nash equilibrium and showed that cooperation between two players can increase their profits. Wang and Parlar [168] extended the model to describe a three-person game in the same context (i.e., a single-period inventory competition with substitutable products). They also investigated the cooperation of retailers when switching excess inventory between the three players (side-payment) is and is not allowed. They showed that Nash equilibrium exists in both cases and cooperation reduces inventory. Furthermore, they used the concept of core to study the cooperation model and presented the conditions for non-empty core. More recently, Avşar and Baykal-Gürsoy [5] extended Parlar's model in [129] to the infinite horizon and lost-sales case and examined a two-person nonzero-sum stochastic game under the discounted payoff criterion.

In another early work on single-period models, Nti [126] examined an inventory procurement model with n competitive organizations (countries). In random demand setting, Nti proved that a unique Nash equilibrium exists. Lippman and McCardle [109] analyzed a competitive newsboy model in both oligopoly and duopoly contexts. They started the duopoly case with two aspects of demand allocation: the initial allocation and the reallocation. With the initial allocation, they specified several rules to split demands to various firms. The reallocation is the same cooperative scheme (side-payment) as in Wang and Parlar [168]. In Mahajan and van Ryzin [111], a more general model with n-firm inventory competition was analyzed with dynamic choice behavior of heterogeneous consumers and its effect on firms' inventory and profit. Anupindi, Bassok and Zemel [4] developed a general framework for the analysis of a two-stage decentralized distribution systems where N retailers face stochastic demands. More specifically, in the first (non-cooperative) stage, each retailer decides on his order quantity to satisfy his own demand. In the second (cooperative) stage, the retailers transship products for the residual demands and allocate the corresponding additional profits. The authors derived the sufficient conditions for existence of a Nash equilibrium in the first stage, and in the second stage used the concept of core for the allocation of profit and also presented the sufficient conditions for the existence of the core. Granot and Sosic [65] extended the results in Anupindi et al. [4] to a three-stage model where the first and third stages are the same as the first and second stages in [4], and in the second stage each retailer decides how much of his residual supply/demand he wants to share with the other retailers.

A few papers have also been published emphasizing cooperative inventory system. Gerchak and Gupta [60] examined the allocation of joint inventory control costs among multiple (N) customers of a single supplier. They first proved that centralization is always beneficial in this model, i.e.,

$$C(\hat{Q},\hat{r}) \le \sum_{i=1}^{N} C_i(Q_i^*, r_i^*),$$

where $C(Q,r)$ denotes the inventory relevant costs containing ordering, holding and shortage costs; Q_i^* the customer i's EOQ-like quantity; r_i^* the customer i's optimal reorder point; and $\hat{Q} = \sum_{i=1}^{N} Q_i^*$ and $\hat{r} = \sum_{i=1}^{N} r_i^*$. These authors also showed that the control costs for the model have the superadditive feature. As an extension of Gerchak and Gupta's work Robinson [137] showed that the best of allocation approaches in the preceding work is unstable, i.e., not in the core of an associated game. Robinson also pointed out that the Shapley value as an allocation scheme satisfies stability, where the Shapley value sets the costs allocated to customer i as

$$X_i = t^{-1}\left[C_{\{i\}} + \sum_{S\subseteq T\setminus\{i\}} \frac{C_{S\cup\{i\}} - C_S}{\binom{t-1}{|S|}}\right]. \tag{1}$$

Here, the index set of the t customers is denoted by $T = \{1,\ldots,t\}$; $S \subset T$ denotes a nonempty subset of T; C_S is the joint control costs for the subset S; and $|S|$ is the cardinality of the subset S. Referring to our description of the Shapley value in Section 2.1.2 we see that the second term of (1) represents a Shapley value to customer i, where C_S can be thought of as characteristic value of coalition S. Hartman and Dror [68] re-examined the cost allocation scheme for the centralized and continuous-review inventory system. In their work three criteria (stability, justifiability and polynomial computability) are proposed to evaluate seven allocation methods including the Shapley value discussed in Robinson [137] and the nucleolus scheme. Hartman and Dror [69] analyzed the problem of minimizing the cost of inventory centralization as a function of the covariance matrix for the single period inventory models with normally distributed, correlated individual demands. They developed a three-step algorithm to find an optimal centralization solution for which the conditions of the nonempty core are always satisfied. As another work of cooperation in inventory systems, Rudi, Kapur and Pyke [139] investigated a two-location inventory problem with transshipment.

As we indicated, the papers reviewed above have focused on the horizontal channel in a supply chain. Now we restrict our attention to the vertical competition issues related to inventory control. Cachon [18] considered a two echelon competitive supply chain inventory problem with a single supplier and a single retailer that faces stochastic demand. In his model, the two firms implement base stock policies while holding and backorder cost as well as the positive lead times between stages exist. Cachon used the response function procedure described in Section 2.1 to analyze the game and find the Nash equilibrium. The response function $r_1(s_2)$ for the retailer is expressed as $r_1(s_2) = \{s_1 \in \sigma | H_1(s_1,s_2) = \min_{x\in\sigma} H_1(x,s_2)\}$ where s_1 and s_2 denote the base stock levels determined respectively by the retailer and supplier with strategy space $\sigma = [0,S]$ and $H_1(s_1,s_2)$ is defined to be retailer's expected cost per period. The supplier's response function $r_2(s_1)$ has an analogous pattern. The retailer's objective function $H_1(s_1,s_2)$ is given as $H_1(s_1,s_2) = \Phi^{L_2}(s_2)G_1(s_1) + \int_{s_2}^{\infty} \phi^{L_2}(x)G_1(s_1+s_2-x)\,dx$ where L_2 (L_1) represents the lead time for shipments from the source (supplier) to the supplier (retailer); Φ^{L_2} and ϕ^{L_2} the distribution and density functions of demand over L_2 periods, and $G_1(s_1)$ is the retailer's expected cost in period $(1+L_1)$ with the inventory position s_1 at the reorder time t. Cachon showed that there is a pair of unique Nash equilibria (s_1^*, s_2^*), where $s_1^* \in r_1(s_2^*)$ and $s_2^* \in r_2(s_1^*)$. Furthermore, Cachon also showed that the equilibrium is not optimal solution for global supply chain performance. When shortages result in lost sales (instead of backorders), Cachon [17] obtained a similar competitive equilibrium and optimal policy.

In another work [28] by Cachon and Zipkin, a two-stage serial supply chain with stationary stochastic demand and fixed transportation times was investigated. The authors provided two different games under two tracking methods for firms specified as a supplier and a retailer. In a competitive setting either game has a unique Nash equilibrium. Under conditions of cooperation with simple linear transfer payments (side-payment) it was also claimed that global supply chain optimal solution can be achieved as a Nash equilibrium. Further, the Stackelberg solution was also discussed. Cachon [19] also extended the above models to analyze the competitive and cooperative inventory issue in a two echelon supply chain with one supplier and N retailers. Wang, Guo and Efstathiou [162] extended the model in Cachon and Zipkin [28] to a one-supplier and n-retailer situation where the supply from the supplier might not satisfy the demand of multiple retailers. In their model, the authors separated sufficient supply from the supplier and insufficient supplies from the supplier. Moreover, several Nash equilibrium contracts were designed for the system-wide optimal cooperation.

Raghunathan [132] considered a one manufacturer, N-retailer supply chain with the correlated demand at retailers and applied the Shapley value concept to analyze the expected manufacturer and retailer shares of the surplus incurred due to information sharing. In this paper, the author examined the impact of demand correlation on the value of information sharing and the relative incentives of manufacturers and retailers to form information sharing partnerships. Continuing our focus on vertical competition and cooperation in a supply chain, we refer the reader to the papers by Anupindi and Bassok [2], Anupindi and Bassok [3] and Axsäter [6]. Another paper in this area is by Corbett [43] who studied the well-known (Q, r) model in a supplier-buyer supply chain with conflicting objectives and asymmetric information.

4. INVENTORY GAMES WITH QUANTITY DISCOUNTS

Quantity discount policy is a common marketing scheme adopted in many industries. With this policy the buyer has an incentive to increase her purchase quantity to obtain a lower unit price. In recent years several reviews focussing on quantity discounts have been published including Chiang et al. [35] and Wilcox et al. [172]. Since the quantity discount scheme plays an important role in the analysis of two-stage vertical supply chains, we review this topic in the present section.

In one of the early papers in this area, Monahan [119] developed and analyzed a quantity discount model to determine the optimal quantity discount schedule for a vendor. The paper considered the scenario in which a vendor and a buyer are involved in a sequential-move (Stackelberg) game model. Monahan assumed that the vendor requests the buyer to increase her order size by a factor of K and performed the analysis to determine the buyer's response. Monahan defined D_1 as the total annual demand faced by the buyer, S_1 and S_2 as the buyer's and vendor's fixed order processing costs, Q_1 as the buyer's current order size, H_1 as the buyer's annual inventory holding cost as a percentage of the value of the item and P_1 as the current delivered unit price paid by the buyer. The vendor's yearly net profits, denoted by YNP, was given as

$$\mathrm{YNP} = D_1(M_2 P_1 - d_K) - \left(\frac{D_1}{Q_1 K}\right) S_2,$$

where M_2 denotes the vendor's gross profit on sales, expressed as a percent, and d_K is "break even price discount" given as $d_K = (K-1)^2\sqrt{2S_1H_1P_1/D_1}/(2K)$. The vendor's optimal value for factor K was then found as

$$K^* = \sqrt{\frac{2D_1S_2}{Q_1\sqrt{2D_1S_1H_1P_1}} + 1}.$$

As one of the early works on quantity-discount decisions, Monahan's paper [119] is an important contribution to the literature. However, Joglekar [77] pointed out some shortcomings of [119] as well as its contribution. These shortcomings are due to several implicit assumptions which make Monahan's results unpractical. In response to these comments, Monahan [120] argued that the principal purpose in [119] is to provide an introductory model in this area. Another note on Monahan's model in 1984 was published by Banerjee [8] who presented an extension by incorporating vendor's inventory carrying costs to obtain a general version. These papers opened up a significant direction of quantity discount research with game theory applications in the field.

Lee and Rosenblatt [98] also extended Monahan's model [119] by addressing two important issues: (i) Impose some constraints on the amount of price discount so as to make it less than the selling price of the product; (ii) revise the order-to-order assumption in [119] to the situation for supplier to order a larger quantity than buyer's order amount. With these assumptions, Lee and Rosenblatt found the optimal discount schedule for the supplier in the general context. Considering again this model, Goyal [63] presented a much simpler approach. Rosenblatt and Lee [138] studied another extension of Monahan's model [119]. They developed different objective functions for vendor and retailer and simultaneous-move (Nash) game. In addition, Lal and Staelin [93] investigated the same problem in [119] respectively under the cooperative and competitive environment.

Extending Lal and Staelin's work, Kohli and Park [87] examined a cooperative game theory model of quantity discounts to analyze a transaction-efficiency rationale for quantity discounts offered in a bargaining context. In this model, a buyer and a seller negotiate over lot size orders and the average unit prices. The authors used the Pareto optimal approach to investigate the Pareto efficient transactions. Kim and Hwang [85] studied the effects of quantity discount on supplier's profit and buyer's cost in the competitive and cooperative contexts. They explored how the supplier decides the discount schedule given the assumption that the buyer always behaves optimally by using the classical EOQ inventory decision. Chiang et al. [35] investigated the game theoretic discount problem in both two-stage competition and cooperative contexts. For the non-cooperative game, a Stackelberg solution was obtained, and for the cooperative game the Pareto optimal criterion was utilized to find multiple optimal strategies. They concluded that quantity discounts is a mechanism of coordinating channel members. A similar result was found earlier by Jeuland and Shugan [76] who paid attention to the simultaneous-move competitive and cooperative behaviors between a manufacturer and a buyer. More discussion on this issue was provided by Jeuland and Shugan [75], Rao [133], Sabavala [141] and Sen [143].

Similar to the papers by Chiang et al., [35] and Jeuland and Shugan [75], [76], Parlar and Wang [130] investigated the discounting scheme of the seller and a linear ordering decision of a group of homogeneous customers in a game framework. However, in this paper Parlar and Wang assumed that the seller's discount influences the buyer's demand. The authors started with a Stackelberg model of the problem and reached two important conclusions: (i) Gains from the discount schedule motivate the seller to set up a discount schedule such that the buyer orders more than EOQ, and (ii) benefit from the discount policy comes from decreasing the inventory-related costs and increasing the market demand. The effect of discount scheme on joint maximum gain for seller and buyer was also examined. An extension of this model was also studied by Parlar and Wang [131] with incomplete information. Another similar work was from Corbett and Groote [45].

In a paper on cooperation Weng [171] presented a model for analyzing the impact of joint decision policies on channel coordination in a supply chain including a supplier and a group of homogeneous buyers. In this model, the supplier's and the buyer's annual profit functions were given as $G_s(p; x, Q) = (p - c)D(x) - S_s D(x)/Q - \frac{1}{2} h_S Q$ and $G_b(x, Q; p) = (x - p)D(x) - S_b D(x)/Q - \frac{1}{2} h_b Q$, respectively, where p denotes the unit purchase price charged by supplier, x and Q are the buyer's selling price and order quantity, $D(x)$ is the annual demand rate, h_s and h_b are the supplier's and buyer's unit inventory holding cost per year, and

S_s and S_b are the supplier's and buyer's fixed cost per order. Weng showed that: (i) Quantity discounts alone are not sufficient to guarantee joint profit maximization, and (ii) the all-unit and incremental discount policies have the same effect on coordination under complete information. The problem regarding cooperation between seller and buyer was also addressed by Li and Huang [106].

By utilizing the uniform quantity-discount policy in a Stackelberg game system, Wang [164] also investigated the coordinating issue between a vendor (supplier) and a group of independent buyers. Chen, Federgruen and Zheng [32] adopted a power-of-two policy to coordinate the replenishments within a decentralized supply chain with one supplier and multiple retailers. Wang [166] considered a similar decentralized supply chain and developed a coordination strategy that combines integer-ratio time coordination and uniform quantity discounts. Wang showed that the integer-ratio time coordination provides a better coordination mechanism than the power-of-two time coordination used in [32]. Further, Wang [165] and Wang and Wu [169] proposed the optimal quantity discount schedule for supplier with different (heterogeneous) buyers.

5. PRODUCTION AND PRICING COMPETITION

Some of the earliest applications of game theoretical ideas were in production and pricing competition and they can be traced back to the 19th century. Since production and pricing decisions play an important role in the profitable operation of a supply chain we now review some papers on this topic.

Earliest publications dealing with production/pricing competition are due to Cournot [47] and Bertrand [14]. In [47], Cournot derived the production equilibrium in a market where two producers supply similar products to the same market while Bertrand [14] focused on pricing equilibrium. In the Cournot model, q_1 and q_2 denote the production quantities chosen by firms 1 and 2, respectively with $Q = q_1 + q_2$ as the aggregate demand. Firm i's total cost of producing q_i units is $C_i(q_i) = cq_i$, $i = 1, 2$ (with c as the marginal cost) and $p(Q) = a - Q$ (for $Q < a$ and $c < a$) is the price charged when Q units are produced. Thus, the profit functions of each firm are given as

$$\pi_i(q_i, q_j) = q_i[p(q_i + q_j) - c] = q_i[a - (q_i + q_j) - c].$$

For this model, the two firms' best response functions are obtained as

$$q_1 = \tfrac{1}{2}(a - c - q_2), \quad \text{and} \quad q_2 = \tfrac{1}{2}(a - c - q_1).$$

Solving these two equations, the Nash equilibrium (q_1^N, q_2^N) is $q_1^N = q_2^N = \frac{1}{3}(a - c)$.

In Bertrand's model two firms simultaneously and independently choose their prices p_1 and p_2, and the market demand q is allocated to the firm who provides the lower price. It is assumed that demand is a linear function of the prices, i.e., $q = a - p_1 - p_2$ where $a \le p_1 + p_2$, $c \le p_1$ and $c \le p_2$. Thus the profit function for each firm is expressed in terms of the prices p_1 and p_2 as

$$\pi_i(p_i, p_j) = \begin{cases} (p_i - c)(a - p_1 - p_2), & \text{if } p_i < p_j, \\ \frac{1}{2}[(p_i - c)(a - p_1 - p_2)], & \text{if } p_i = p_j, \\ 0, & \text{if } p_i > p_j. \end{cases}$$

For this model, the Nash equilibrium (p_1^N, p_2^N) is found as $p_1^N = p_2^N = c$. In this equilibrium solution, both firms obtain a zero profit. However, since in real world, firms compete in prices and can make positive profits, this result is known as the "Bertrand paradox."

The Austrian economist von Stackelberg [161] extended the Cournot model by assuming that Firm 1 acts as the leader and Firm 2 as the follower. In the leader-follower game, Firm 1 determines the production quantity q_1 by solving

$$\max_{q_1} \pi_1(q_1, q_2^R(q_1)) = \max_{q_1} \tfrac{1}{2}[q_1(a - q_1 - c)].$$

Table 1: Summary of some papers related to production/pricing with constraints.

Year	Author	Brief Review of the Game Model
1972	Levitan and Shubik [100]	Cournot's and Bertrand's equilibria under capacity restraints for firms that face a linear demand.
1991	Hviid [73]	Bertrand's equilibrium for capacity-constrained firms under random demand in a duopoly market.
1991	Gal-Or [56]	A game where a manufacturer imposes the pricing constraints on his retailers.
1997	Butz [15]	A game of a manufacturer who controls his vertical relationship with retailers by using many levers (e.g., vertical integration, buyback).

This yields the Stackelberg solutions as $q_1^S = \frac{1}{2}(a-c)$ and $q_2^S = \frac{1}{4}(a-c)$. For detailed discussion on Cournot, Stackelberg and Bertrand games, see Kreps [89, Ch. 10], Osborne [127, Ch. 3] and Tirole [155, Ch. 5].

A large number of papers extending Cournot and Bertrand's results have appeared in economics and management science literature. Shapley and Shubik [144] applied game theory to study a monopolistic price competition among firms (sellers) with differentiated products, under the assumptions of a linear demand, constant average costs and given capacities for the firms. When demand was assumed random, Levitan and Shubik [99] studied the price variation and duopoly (oligopoly) with differentiated products. Jain and Kannan [74] proposed a model for the pricing problem of an online information product. In their paper, they examined the conditions under which the most commonly used pricing schemes – connect-time-based pricing, search-based pricing, and subscription-fee pricing – are optimal. For a two-level supply chain involving a seller and a buyer, Banks, Hutchinson and Meyer [10] investigated the impacts of the firms' reputations on their pricing equilibrium strategies.

Joint production and pricing strategies were also considered: Klemperer and Meyer [86] analyzed the Nash equilibrium prices and quantities as strategic variables in a one-stage duopolistic game with differentiated products. By using a differential game approach, Jø rgensen [79] considered a continuous-time game problem to compute optimal production, purchasing and pricing policies in a two-stage vertical channel involving one manufacturer and one retailer. In [46], Corbett and Karmarkar developed an explicit game model of entry (Nash-characterized) and post-entry (Cournot) competition in serial multi-tier supply chains with price-sensitive linear deterministic demand. The authors derived expressions for prices and production quantities as functions of the number of entrants at each level.

There are other papers focusing on different forms of constrains (such as price constraints) which we summarize in Table 1.

The first publication emphasizing the channel cooperation in this category was by Zusman and Etgar [175] with a combined application of economic contract theory and Nash bargaining theory. Individual contracts involving payment schedules between members of a three-level channel were investigated and the equilibrium set of contracts was obtained. Later, a large number of papers appeared investigating channel coordination/cooperation. In McGuire and Staelin [115], four industry structures induced by two types of channel system consisting of two manufacturers were studied. Under the assumption that one seller (retailer) carries the product of only one manufacturer, they derived the Nash equilibrium prices, quantities and profits for each of four different structures. An extension of the cooperative game model in [115] was again proposed by McGuire and Staelin [117]. By extending the non-cooperative model in [115], McGuire and Staelin [116] also studied the effect of product substitutability on Nash equilibrium distribution structures in a duopoly (two-manufacturer) competitive system.

For the decentralized competitive problem mentioned in [116], Moorthy [122] studied the effect of strategic interaction (complements or substitutability) on Nash equilibrium strategy. Dong and Rudi [51] proposed a game model for supply chain interaction between a manufacturer and a number of retailers with transshipment scheme.

Some recent papers have investigated the pricing policy used as a means for coordinating supply chains. Zhao and Wang [173] developed a Stackelberg game for a two-level supply chain where a manufacturer acts as leader and a distributor/retailer acts as follower. In the game, both parties make pricing and production/ordering decisions over a finite-time horizon. It was shown that there exists a manufacturer's price schedule that induces the distributor to adopt decisions to achieve the performance of a centralized supply chain. Under the e-commerce environment, Chiang, Chhajed and Hess [34] developed a price-setting game for a two-level supply chain where a manufacturer directly sells a single product to online customers rather than via his independent retailers. It was shown that the direct marketing can indirectly increase the flow of profits through the retail channel and help the manufacturer improve overall profitability.

Choi [36] studied the effect of existence of channel intermediary on the intensity of horizontal competition between two manufacturers. He considered three non-cooperative structures (two Stackelberg games and one Nash game) between the two manufacturers and one common retailer. In these three structures, manufacturer i's and the retailer's profit functions (Π_{M_i} and Π_R) were respectively given as

$$\Pi_{M_i} = (w_i - c_i)q_i, \; i = 1,2, \quad \text{and} \quad \Pi_R = \sum_{i=1}^{2} m_i q_i, \tag{2}$$

where w_i denotes manufacturer i's wholesale price; m_i is the retailer's margin on product i; c_i is manufacturer i's variable cost of producing its product; and q_i is the demand for brand i at price p_i given that the price of the other brand j is p_j. As in McGuire and Staelin [116], q_i in (2) was expressed by the linear duopoly demand function $q_i = a - bp_i + \gamma p_j$ that captures product differentiation where the parameters a, b and γ satisfy $a > 0$ and $b > \gamma > 0$. For the model with linear demand, Choi assumed equal costs for manufacturing (i.e., $c_1 = c_2 = c$) and obtained a Nash equilibrium as

$$w_1 = w_2 = \frac{a + 2bc}{3b - \gamma} \quad \text{and} \quad p_1 = p_2 = \frac{a(2b - \gamma)}{(3b - \gamma)(b - \gamma)} + \frac{bc}{3b - \gamma}.$$

The Stackelberg equilibria were also found explicitly in terms of the model parameters. With the linear demand function, Choi [36] reached the conclusion that a manufacturer is better off by maintaining exclusive retailers while a retailer prefers to have several manufacturers. Another counter-intuitive result was found which indicated that all channel members' prices and profits increase as products are less differentiated. When the demand function is assumed nonlinear, an exclusive retailer channel provides higher profits to all members. As an extension of Choi [36], Trivedi [156] analyzed three channel structures dealing with competition at both two manufacturer and two retailer levels. Kadiyali, Chintagunta and Vilcassim [82] also extended Choi's work [36] by allowing a continuum of possible channel integration between manufacturers and a retailer instead of three channel interaction games.

As the channel members in a supply chain (should) attempt to cooperate to increase their profits, they may have incentives to share information about the market. Thus, we review papers concerned with information sharing in Cournot and Bertrand competition. In the context of an N-player Bayesian Cournot game, Clarke [41] examined incentives for firms to share private information in a stochastic market. In a similar setting Gal-Or [54] investigated an oligopolistic market with uncertain demand. Vives [160] developed a symmetric differentiated duopoly model in which two firms have private information on market data on the uncertain and linear demand. Gal-Or [55] examined the incentives of two duopolists to share information in Bertrand or Cournot competition under unknown private costs. Li [101] extended the papers by Clarke [41], Gal-Or [54] and Vives [160] under common demand uncertainty and the private cost uncertainty. In both cases a unique Bayesian Nash equilibrium was derived for the second-stage game (information sharing followed by Cournot or Bertrand competition).

In a recent publication, Li [103] has examined the incentives for firms to share information vertically for improving the performance of a single manufacturer, N retailer supply chain. In the supply chain, the retailers are engaged in Cournot competition and the manufacturer determines the wholesale price. The conditions under which information can be shared were derived in the paper. In the context of information transparency in a B2B electronic market, Zhu [174] developed a game-theoretic model to examine whether the incentives to join a B2B exchange would be different under different competition modes (quantity and price), different information structures, and by varying the nature of the products (substitutes and complements).

In this topical category there are two more important papers focusing on the contract structure in a coordinated supply chain. Lariviere [95] considered the supply chain coordination issues with random demand under several contract schemes such as price-only contracts, buyback contracts and quantity-flexibility contracts. Corbett and DeCroix [44] developed shared-savings contracts for indirect materials in a supply chain containing a supplier and a buyer (customer).

6. GAMES WITH OTHER ATTRIBUTES

In the preceding three sections, we reviewed inventory game models with fixed unit purchase cost, with quantity discounts and games with production and price competition. There are also papers that are concerned with a variety of topics such as capacity decisions, service quality, product quality and advertising and new product introduction. We now review papers belonging to each of these subclasses.

6.1 Capacity Decisions

Cachon and Lariviere [25] conducted an equilibrium analysis on a capacity-constrained system where a supplier utilizes linear, proportional and uniform allocation schedules. Additionally, Cachon and Lariviere [24] applied the manipulable and truth-inducing capacity allocation schemes to study the retailers' order behaviors and supplier's capacity choice problem. Further, one recent paper was associated with forecast sharing issues: Cachon and Lariviere [26] investigated a forecast sharing model of a manufacturer and a supplier. The forecast sharing procedure between the two channel members is given as follows: (i) The manufacturer provides her initial forecast to the supplier; (ii) if supplier accepts the forecast, he sets up capacity; otherwise (iii) the manufacturer receives the updated forecast and submits the final order. The paper showed that in the specified setting firm commitments are not useful for aligning incentives but useful for communicating information. Motivated by the experiences of a major US-based semiconductor manufacturer, Mallik and Harker [112] developed a game model involving multiple product managers and multiple manufacturing managers who forecast the means of their respective demand and capacity distributions. A central coordinator decides on the allocation of the capacities to product lines. The authors designed a truth-eliciting bonus mechanism and an allocation rule for the supply chain.

Hall and Porteus [67] considered a game where firms compete on the capacity investment for market share. Hall and Porteus assumed that market share of either firm depends on the prior realized level of customer service that is considered as the capacity per customer. Based on this assumption with two firms i and j the expected market share of firm i in month $t+1$ is

$$E(\lambda_{i,t+1}|\lambda_{it}, \lambda_{jt}) = \lambda_{it} - \lambda_{it}\gamma_i h_i(y_{it}) + \lambda_{jt}\gamma_j h_i(y_{jt}), \tag{3}$$

where λ_{it} denotes the fractional market share for firm i in month t; y_{it} is the normalized capacity of firm i in month t; γ_i is the switching rate of customers experiencing service failure from firm

i to firm j ($0 \leq \gamma_i \leq 1$). Hall and Porteus denoted by μ_{it} the capacity selected by firm i in month t which is expressed as $\mu_{it} \equiv y_{it}\lambda_{it}$. Defining $h(y_{it})$ as the customer service, $\lambda_{it}h(y_{it})$ is the expected number of firm i's customers that experience service failure in month t when firm i has a normalized capacity of y_{it}. The term $\lambda_{jt}\gamma_j h_i(y_{jt})$ in (3) refers to the expected number of firm i's customers that switch to firm j in month $t+1$. The authors then derived an optimal capacity choice (Nash equilibrium) and the conditions under which the Nash equilibrium capacity levels scale directly and linearly in the number of customers being served. The model developed in the paper was also applied in two contexts: competition between Internet service providers and inventory availability competition.

6.2 Service Quality

The eventual goal of a supply chain is to deliver goods to a consuming market with the satisfaction of ultimate consumers. Consumers usually pay attention not only to the sale price but also to product and service quality. Product quality is an easily understood concept; service quality may involve issues such as a firm's response time to customer demand, waiting time of customers, post-sale service, etc. In order to build up the loyalty of existing customers and attract more demand and new customers, channel members might strengthen their market power by improving product and service quality. Therefore, the appropriate trade-off between expenditure and benefits are considered by competing firms. We restrict our attention to game theoretic approaches for service quality in this subsection and for product quality competition in the next subsection.

A firm's service speed (response time) to customer demand is an important factor implicitly affecting the profitability of a firm. Game theory has also been applied to service speed decisions of firms. Kalai, Kamien and Rubinovitch [83] proposed a two-server game theoretic model with exponential service time and Poisson arrival of customers. In [58], Gans developed a model of m suppliers competing on service quality for customers whose choices respond to random variation of quality. The author obtained a closed-form expression for a customer's choice as the long-run purchase fraction. Based on the expression, the suppliers seek to maximize their long-run average profits. The paper shows that (i) the consumer's switching behavior forces suppliers to maintain an industry norm that increases with the number of competitive suppliers and (ii) a competitor with cost advantage can increase investment for quality improvement that induces higher market share.

The following papers examined other models associated with service quality. Cohen and Whang [42] developed a Stackelberg game model of product life cycle. In this sequential-game framework, there is vertical competition for the provision of after-sales service quality in a channel consisting of a manufacturer and an independent service operator. Chu and Desai [40] proposed a game model to describe a manufacturer motivating a retailer with two incentive schedules, i.e., CS (Consumer Satisfaction) assistance and CSI (Consumer Satisfaction Index) bonus. From the viewpoint of customer, Kulkarni [92] considered a queuing system with one single server station and two types of customers.

6.3 Product Quality

If we restrict our attention to the literature related to product quality competition in supply chain management, we find a limited number of papers in this area. As one of the first papers emphasizing the contract design for product quality, Reyniers and Tapiero [136] determined the effect of contract parameters on the quality of the end product in a vertical channel including a supplier and a producer. In this contract the supplier and producer negotiate the price rebates and after-sale warranty for the delivered materials or parts from the supplier. The game in this paper corresponds to a bimatrix $(\mathbf{A}, \mathbf{B})$ with entries (a_{ij}, b_{ij}), where i refers to the quality (1 for low quality and 2 for high quality) and j is producer's decision on whether or not to test the incoming parts (1 for test and 2 for no test). In this bimatrix, a_{ij} and b_{ij} respectively denote a risk-neutral producer's and a supplier's expected payoffs such that

$$(a_{ij}, b_{ij}) = \begin{cases} (\theta - m - [\pi - p_i\Delta\pi], \pi - p_i(\Delta\pi + C) - T_i), & j = 1, \\ (\theta - [\pi + p_i(1-\alpha)R], \pi - p_i\alpha R - T_i), & j = 2, \end{cases} \quad \text{for } i = 1, 2,$$

where θ denotes the producer's selling profit (net of manufacturing costs), p_1 and p_2 the probabilities of a defective part with technologies 1 and 2, respectively. Additionally, m is the cost of testing an incoming part, π is the producer's unit sale price, $\Delta\pi$ is the reduction in sale price incurred when a unit is defective, C is the producer's repair cost, R is the post-sales failure cost, α is a parameter in sharing R between producer and supplier, and T_i is the unit cost of production borne by the supplier such that $T_1 < T_2$. For different values of these above parameters, the authors found different Nash solutions containing one mixed strategy. Extending Reyniers and Tapiero's model [136], Lim [108] designed producer-supplier contracts with incomplete information.

A paper emphasizing the product quality signaling mechanism was published by Chu and Chu [39] who analyzed a game theoretical model of a manufacturer selling a product through a reputable retailer to signal its product quality. It was shown that, in equilibrium, manufacturers of high quality distribute product through strongly reputable retailers while in turn manufacturers of low quality distribute products through retailers without reputation.

6.4 Advertising and New Product Introduction

Game theoretic applications in advertising-related SCM problems date back to the 1970s. One of the earliest game theory models for an oligopolistic market with advertising competition is Balch [7]. In this paper each firm in the competitive market decides on the advertising outlay to maximize its individual profit and market share in the next production/marketing period. With this assumption, the kth firm's expected profit for the next day is

$$\pi_k(\mathbf{x}) = \beta_k \varphi_k(\mathbf{x}) - x_k, \tag{4}$$

where x_k is defined as the firm k's decision on advertising outlay and $\mathbf{x} = (x_1, x_2, \ldots, x_n)'$ is the strategy vector for n firms. The β_k term in (4) is given as $\beta_k = (p - c_k)D$ where p denotes the unit price that is cooperatively set for next day, c_k is the kth firm's average production cost per unit, and D is given in term of p represents the cooperative expectation at p for the next day's total demand. The $\varphi_k(\mathbf{x})$ term is defined as the kth component in an expected market share vector $\varphi(\mathbf{x})$ for the next day, i.e., $\varphi_k(\mathbf{x}) = (1-\theta)\Phi_k + \theta\alpha_k x_k/\boldsymbol{\alpha}\mathbf{x}$, where $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ is an n-tuple of positive weights reflecting firmwise current advertising appeal and $(\alpha_k x_k)/(\boldsymbol{\alpha}\mathbf{x})$ is the purchase from firm k with (conditional) probability Φ_k and θ is the fraction of all consumers of the previous day who differentiated product primarily on the appeal of a particularly current advertising campaign. For this model a Nash equilibrium for the firms was characterized. Another early paper by Deal [49] determines the optimal time of advertising expenditure over a finite planning horizon in a dynamic duopoly competitive situation. A few other papers focusing on advertising-related decisions are summarized in Table 2.

There are a few other papers associated with new product introduction. In Chu's work [38], the channel members (manufacturers and retailers) dealt with asymmetric information in two ways: (i) demand signalling by manufacturers through advertising and wholesale price, (ii) demand screening by retailers through slotting allowance. In [1], Amaldoss et al. examined three types of strategic alliances that may help participants to compete: (i) Same-function alliances, (ii) parallel development of new products, (iii) cross-functional alliances. They modeled the interaction within an alliance as a noncooperative game where each firm invests part of its resources to increase the utility of a new product offering. Desai [50] studied how a high-demand manufacturer uses advertising, slotting allowances, and wholesale prices to signal its

Table 2: Summary of some papers related to advertising decisions.

Year	Author	Brief Review of Game Model
1984	Karnani [84]	A dynamic game model of marketing competition in an oligopoly with differentiated products.
1989	Hauser and Wernerfelt [70]	A supply chain game where consumers chooses a brand based on advertising and price.
2001	Wang and Wu [170]	A differential game model of competitive advertising decisions by extending Deal's model [49].
2001	Huang and Li [71]	Two noncooperative and one cooperative advertising game models for a vertical channel.
2002	Li et al. [107]	Three Stackelberg games for the supply chain analyzed in [71].
2002	Huang, Li and Mahajan [72]	A co-op advertising game in the supply chain with one manufacturer and multiple retailers.
2003	Jørgensen, Taboubi and Zaccour [81]	A supply chain game where a manufacturer shares the brand promotion costs with a retailer.

high new product demand to retailers. The author also investigated the impact of retailer's uncertainty on the effectiveness of the manufacturer's advertising.

7. GAMES WITH JOINT DECISIONS ON INVENTORY, PRODUCTION/PRICING AND OTHER ATTRIBUTES

In many realistic problems, supply chain members encounter problems involving two or more decisions that must be made simultaneously. For example, a supply chain member may have to make joint decisions on inventory and pricing problems. In the section, we review the papers concerned with joint decisions on inventory, production/pricing and other attributes.

7.1 Joint Inventory and Production/Pricing Decisions

In an early paper [52], Eliashberg and Steinberg considered a Stackelberg game in a vertical channel consisting of a manufacturer and a distributor. Jørgensen and Kort [80] analyzed a two-step inventory and pricing decision problem with one store and one central warehouse and investigated both non-cooperative and cooperative games. Bylka [16] considered a game model for the decentralized dynamic production–distribution control where a vendor produces a product using batch production and supplies it to a buyer under deterministic conditions.

Bernstein and Federgruen [12] considered a two-echelon supply chain where a supplier distributes a single product to N competing retailers, each of which facing a deterministic demand rate dependent on all retailers' prices. In this paper, the authors first characterized the solution to a centralized supply chain. Then, assuming linear wholesale pricing schemes by the supplier, the paper investigated the decentralized systems under Cournot and Bertrand competition, respectively. In the retailer game, retailer i's profit function $\pi_i(p_i|\mathbf{p}_{-i}, w_i)$ with his optimal EOQ replenishment policy is given as

$$\pi_i(p_i|\mathbf{p}_{-i}, w_i) = (p_i - c_i - w_i)d_i(\mathbf{p}) - \sqrt{2d_i(\mathbf{p})\bar{h}_i K_i^r}, \tag{5}$$

where p_i denotes retailer i's price, $\mathbf{p}_{-i} \equiv (p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_N)$, w_i is the constant per-unit wholesale price charged by the supplier to retailer i, c_i is the unit transportation cost from the supplier to retailer i, $\bar{h}_i$ is the annual holding cost per unit inventory at retailer i, K_i^r is the per-delivery fixed cost incurred by retailer i and $d_i(\mathbf{p}) \equiv a_i - b_i p_i + \sum_{j \neq i} \beta_{ij} p_j$ is the demand function for retailer i where the parameters a_i and b_i are both positive and $\beta_{ij} \geq 0$. Under Bertrand price competition, it was shown that if $[d_i(\mathbf{p})]^{\frac{3}{2}} \geq \frac{1}{8} b_i \sqrt{2\bar{h}_i K_i^r}$, then the retailer game has a Nash equilibrium $\mathbf{p}^*$. The authors also found a similar result for the Cournot quantity competition. Bernstein and Federgruen [13] extended Bernstein and Federgruen [12] to a periodic review, infinite-horizon model with stochastic demand faced by retailers.

There are two recent papers focusing on the allocation problems. Cachon [21] analyzed the problem of allocating inventory risk between a supplier and a retailer via three types of wholesale price contracts: (i) Push, (ii) pull, and (iii) advance-purchase discount. It was shown that the efficiency of a single wholesale price contract (i.e., push or pull contract) is considerably high. By applying the concept of Nash bargaining solution, Gjerdrum, Shah and Papageorgiou [62] found optimal multi-partner profit levels subject to given minimum echelon profit requirements,and presented a mixed-integer programming formulation for fairly allocating optimized profits between echelons in a general multi-enterprise supply chain.

In a recent paper Su and Shi [153] developed a game model involving quantity discounts and buyback pricing decisions. The authors incorporated return (buyback) contracts into the traditional quantity discount problems in a two-stage game with a manufacturer and a retailer. In the first stage two supply chain members determine the inventory level cooperatively as $Q^* = F^{-1}\{(p+s-m)/(p+s)\}$, where p, s and m denote unit retail price, unit goodwill loss and unit production cost, respectively, and $F(\cdot)$ is the distribution function of the market demand D. In the second stage the manufacturer bargains with the retailer for a quantity discount and return schemes to maintain channel efficiency. The quantity discount Δw was given as

$$\Delta w = (w_0 - w^l) - u\left[\frac{1}{Q^*} E(Q^* - D)^+\right], \tag{6}$$

where w_0 denotes the baseline wholesale price per item, u is the unit buyback price, and

$$\begin{aligned} w^l &\equiv \frac{1}{Q^*}\{pE\,[\min(Q^*, D)] - sE(D - Q^*)^+ - \pi_r(w_0, \hat{Q})\}, \\ \pi_r(w_0, \hat{Q}) &= p \min(\hat{Q}, D) - w_0\hat{Q} - (D - \hat{Q})^+ \\ \hat{Q} &= F^{-1}\{(p+s-w_0)/(p+s)\}. \end{aligned}$$

It was shown that all feasible set $(\Delta w, u)$ combinations in equation (6) satisfy the Pareto efficiency.

7.2 Joint Inventory and Capacity Decisions

We now focus on the review of game models with joint inventory and capacity decisions. Cachon and Lariviere [23] considered a supply chain comprising of one supplier and multiple retailers. When the sum of the retailers' orders exceeds the supplier's fixed capacity, the supplier uses a turn-and-earn capacity allocation scheme which allocates capacity for a retailer in one period equal to the retailer's sale volume in the last period. Mahajan, Radas and Vakharia [110] examined a supply chain where a supplier distributes two independent products through multiple retailers. For the unlimited or limited capacity of the supplier, respectively, the authors determined the optimal stocking policies for the retailers. Chen and Wan [33] analyzed the competition between two make-to-order firms each provides a value of service and a service rate (capacity) and has a firm-dependent unit costs of waiting.

Caldentey and Wein [29] developed a supply chain game where a supplier chooses the production capacity v and a risk-neutral retailer adopts an $(s-1, s)$ base-stock replenishment policy. Assuming that the retailer faces a Poisson demand process, the authors derived the retailer's and the supplier's expected cost functions respectively as

$$C_R(s,\nu) = s - \frac{1-e^{-\nu s}}{\nu} + \frac{\alpha b e^{-\nu s}}{\nu} \quad \text{and} \quad C_S(s,\nu) = (1-\alpha)\frac{be^{-\nu s}}{\nu} + c\nu,$$

where b denotes per unit backorder cost, α the retailer's backorder cost share, c the supplier's capacity cost per unit of product. Caldentey and Wein presented a unique Nash equilibrium solution as well as a centralized solution to system-wide cost $C_R(s,\nu) + C_S(s,\nu)$. It was found from comparison of the two solutions that the Nash equilibrium is inefficient. Thus the two authors designed a linear transfer payment contract for coordinating the supply chain, i.e., the game model with the linear transfer payment is

$$\tilde{C}_R(s,\nu) \equiv C_R(s,\nu) - \tau(s,\nu) \quad \text{and} \quad \tilde{C}_S(s,\nu) \equiv C_S(s,\nu) + \tau(s,\nu),$$

where the linear transfer payment function $\tau(s,\nu)$ is defined by $\tau(s,\nu) \equiv \gamma C_R(s,\nu) - (1-\gamma)C_S(s,\nu)$. The paper also examined a Stackelberg game with the supplier as leader and the retailer as follower, and compared system-wide costs, the agents' decision variables and the customer service level of the Nash, centralized and Stackelberg solutions.

7.3 Joint Production/Pricing and Capacity Decisions

In the subsection we discuss some papers concerned with joint production/pricing and capacity decisions. Kreps and Scheinkman [90] considered a two-stage game where, in the first stage, two firms simultaneously and independently determine their production capacities, and in the second stage they engage in Bertrand-like price competition. The two authors showed that the unique equilibrium production capacities for both firms are the Cournot solutions. (For a detailed discussion of [90], see Tirole [155].) Van Mieghem [157] investigated the channel coordination between a manufacturer and a subcontractor for decisions on capacity investment, production and sales.

Similar to [157], Van Mieghem and Dada [158] provided a two-stage decision model of postponing strategies, where firms make three decisions: capacity investment, production (inventory) quantity and price. There were six strategies in the model: (1) No postponement, (2) production postponement, (3) price postponement with clearance, (4) price postponement with hold-back, (5) price and production postponement, and (6) full postponement. For Strategies 1 and 2, the value functions of each firm involve joint decisions on capacity investment and price, i.e.,

$$V(K,p) = \begin{cases} -(c_K + c_q + c_h)K + pE\min(K, (\varepsilon - p)^+), & \text{for Strategy 1,} \\ -c_K K + (p - c_q)E\min(K, (\varepsilon - p)^+), & \text{for Strategy 2,} \end{cases}$$

where K and p are production capacity and price set by the firm, c_K, c_q and c_h respectively denote unit capacity investment cost, constant marginal production cost and constant marginal inventory holding cost rate of ex-ante production, and the random variable ε represents the uncertainty in the market demand D. The authors showed how competition, uncertainty and the timing of operational decisions influence the strategic investment decision of the firm and its value.

7.4 Joint Production/Pricing and Service/Product Quality Decisions

We first briefly review two early papers emphasizing the joint production/pricing and product quality decisions. In an early paper [121], Moorthy considered a duopolistic game model comprising of two horizontal firms who compete on product quality and pricing strategy for consumer. Another early paper focusing on competition between firms in a price competitive market with differentiation of product quality was by Reitman [135].

There are several papers focusing on the joint production/pricing and service quality decisions. Combining the service quality and pricing competitions, McGahan and Ghemawat [114] developed a game model in a duopoly system with two-stage competitions. Cohen and Whang [42] developed a Stackelberg game involving product life cycles where the product price and the service quality were characterized. Rump and Stidham [140] studied the dynamic behavior of an input-pricing mechanism for a service facility in which decisions of heterogeneous self-optimizing customers are based on their previous experience.

A few other models have considered the delivery time decision as service quality. When the pricing, production, scheduling and delivery-time components were jointly considered in the competition between firms that produce goods or services for customers sensitive to delay time, Lederer and Li [97] analyzed the game model in two cases and found that a unique Nash solution exists in either case. Assuming that demands in a market are sensitive to both the price and delivery time guarantees, So [151] considered a supply chain involving multiple firms who compete for customers in the market. In the horizontal competition, Firm i chooses an optimal price and delivery time guarantee to maximize his operating profit function given as

$$\max_{p_i,t_i} \Pi_i(p_i,t_i) = \frac{(p_i-\gamma_i)\lambda p_i^{-a}t_i^{-b}}{p_i^{-a}t_i^{-b}+\beta_i}, \quad \text{s.t.} \quad \left(\mu_i - \frac{\lambda p_i^{-a}t_i^{-b}}{p_i^{-a}t_i^{-b}+\beta_i}\right)t_i \geq k, \tag{7}$$

where p_i and t_i denote Firm i's price and delivery time guarantee, respectively, γ_i is the unit operating cost for Firm i, μ_i is the capacity of Firm i, λ is the fixed market size, a and b are two nonnegative constants denoting the price and time attraction factors of the market. Furthermore, k and β_i in (7) are defined as $k \equiv -\log(1-\alpha)$, and $\beta_i \equiv (1/L_i)\sum_{j\neq i} L_j p_j^{-a}t_j^{-b}$, where α represents the service reliability for each firm, and the term $L_i p_i^{-a}t_i^{-b}$ refers to the attraction of Firm i with the parameter $L_i > 0$. Through a numerical study, the author illustrated how the different firm and market characteristics would affect the price and delivery equilibrium solutions in the market.

Cachon and Harker [22] considered a model of two firms facing scale economies (i.e., each firm's cost per unit of demand is decreasing in demand). The general framework, which was used in this paper, involved a queuing game (i.e., competition between two service providers with price- and time-sensitive demand) and an economic order quantity game (i.e., competition between two retailers with fixed-ordering costs and price-sensitive consumers). In a supply chain where two suppliers compete for supply to a customer, Ha, Li and Ng [66] analyzed pricing and delivery-frequency decision by developing two three-stage games with different decision rights designated to the parties.

A paper for joint decisions on production/pricing , product and service quantities is the following: Li and Lee [104] investigated a game model with the duopolistic competition when customer preferences are concerned with not only the price and product quality but also service quality (i.e., delivery time).

7.5 Joint Production/Pricing and Advertising/New Product Introduction Decisions

In this subsection we review three papers concerned with the joint decisions on production/pricing and advertising/new product introduction. Hauser and Wernerfelt [70] explored the interaction between price and advertising decisions. Lariviere and Porteus [96] analyzed the game model of a manufacturer who decides on the wholesale price when a new product is introduced in a distribution channel. Banerjee and Bandyopadhyay [9] constructed a multi-stage game-theoretical model of advertising and price competition in a differentiated products duopoly, where proportions of consumers exhibit latent inertia in favor of repeat purchase. The authors characterized the nature of equilibria under symmetry and showed that when a large proportion of consumers exhibit inertial tendencies, then a multiplicity of equilibria exists. Marketing implications and comparative statics were also discussed.

8. CONCLUSION AND FURTHER DISCUSSION

In this paper we presented a brief description of some of the important solution concepts used in non-cooperative and cooperative games. We also reviewed more than 130 papers that focused on game theoretic applications in SCM. The papers reviewed were presented using a topical classification scheme consisting of five classes: (i) Inventory games with fixed unit purchase cost, (ii) inventory games with quantity discounts, (iii) production and pricing competition, (iv) games with other attributes and (v) games with joint decisions on inventory, production/pricing and other attributes. We conclude from this survey that game theory has been found useful in solving a variety of competitive and cooperative problems in this field.

Table 3 in Appendix B indicates the number of papers we reviewed for each class during each five-year period. Furthermore, Figure 4 (in Appendix B) shows that around 58% of all our reviewed papers were published in the past decade, and about 40% papers of all papers appeared in the period of from 2000 to 2004. We note in Figure 5 (in Appendix B) that during the most recent period of 2000–2004, many researchers have focused on problems in Classes (i), (iv) and (v). We predict that as supply chain members usually face multiple decision problems, the number of published papers in Class (v) will continue to grow rapidly. We also note that most of models developed during 2000–2004 that fall in Class (iii) have focused on pricing decisions in the context of information sharing and/or eBusiness. We believe that production/inventory and pricing decisions in the eBusiness setting should be a significant direction in the field of supply chain-related games.

We now present some suggestions for further directions in SCM research with game theoretic applications. As discussed at the beginning of Section 2, Nash and Stackelberg equilibria are two most-frequently used solution concepts in non-cooperative games. Although more non-cooperative game models have appeared than cooperative ones, some researchers have also examined coordination and cooperation issues in SCM. Based on our observation, we find that most theoretical analyses in cooperative game models have applied the side-payment or side-payment-like concept. Only a few authors have made use of the other cooperative game solution concepts involving the characteristic function form (such as the core, Shapley value, nucleolus, etc.).

The objective of every supply chain is to maximize the overall value (i.e., profitability) generated. Supply chain profitability is the total profit to be shared across all supply chain stages; see Chopra and Meindl [37, pp. 5–6]. Naturally, such profitability can be achieved only if the decision makers in each stage of a supply chain agree to cooperate. For supply chain researchers interested in applying game theory, this should be an exciting observation. We have here the basic ingredients of a cooperative game, i.e., a group of decision makers having different objectives; and if they cooperate then they can improve their well-being as a whole. Since our survey has revealed that very few researchers have looked at problems in SCM involving cooperative games in characteristic function form one may develop a cooperative game of a supply chain involving the major "players" of a supply chain, i.e., the supplier, manufacturer, distributor, retailer or even the customer. The problem of sharing fairly the increased profit in a supply chain may be analyzed by using the solution concepts of cooperative game theory such as the Shapley value or nucleolus.

We now suggest, based on our topical classification, some potential research topics in SCM that can be analyzed using game theoretic tools.

Inventory Games with Fixed Unit Purchase Cost: With regard to inventory control problems with fixed purchase cost, we have encountered more papers that focus on the decentralized channel than on the centralized channel. Since coordination/cooperation is a critical issue in centralized system, we feel that more attention should be paid to investigating models involving centralization settings. Some papers in this class considered game models for substitutable products. One can also analyze some competitive problems with complementary products. For example, consider a game problem where suppliers of two different components (e.g., memory chips vs. monitors) serve the same manufacturer. In this situation, the suppliers' products are complementary since the manufacturer must use both components to produce the final product. When a supplier holds more inventory than the other not only would the former carry excess stock resulting in holding cost but also the manufacturer may be unable to complete the assembly of his products resulting in lost sales. Thus, when the suppliers do not communicate and the manufacturer faces a random demand the resulting 3-person game can be analyzed using cooperative game theory.

Inventory Games with Quantity Discounts: As most of the publications in this class deal with vertical supply chains, future research in this area may pay more attention to horizontal supply chains. For example, we may consider a supply chain involving two retailers who choose their respective optimal quantity discount policies to compete for customers in a market. Both competitive and cooperative games could be developed for the horizontal supply chain.

Production and Pricing Competition: This category of problems have been analyzed in a wide variety of SCM contexts. However, production/pricing competition in the eBusiness context may also attract the interest of researchers such as Jain and Kannan [74] who considered a vertical supply chain problem involving the pricing decisions. We could consider a horizontal supply chain where two firms determine prices of their substitutable information products to compete for customers in a online market.

Games with Other Attributes: This category involved a variety of games with other attributes (e.g., capacity, service/product quality, advertising and new product introduction) and we predict that there may be more research opportunities in this class. For example, service competition in vertical channels and product competition in horizontal channels have not attracted much attention from SCM academics which can be explored further. We could consider a vertical channel where a manufacturer and a retailer offer an after-sale service (e.g., free repair) to their customers. The service quality (level) is defined in terms of service availability when a customer calls for repair to either manufacturer or retailer. Each member in the system employs professional workers to set up service capacity, and the service quality of the system can be considered in terms of total workforce. Given a certain service quality (i.e., the total number of workers), the two channel members compete on the labor force hiring decisions.

Game with Joint Decisions on Inventory, Production/Pricing and Other Attributes: As in the topics discussed above one could examine many new topics in this class, for example, one may analyze a problem where the firms not only determine their Cournot quantities but also the contract parameters on the quality of the end product.

A. ALTERNATIVE CLASSIFICATION

Our survey reveals that although many papers have examined the problems of competition in supply chains, some have also considered cooperation among channel members using side-payments (or, in the case of a few, using the characteristic function form). In this Appendix we present an alternative classifications based on the nature of interaction among the players, i.e., (i) non-cooperative games, (ii) cooperative games.

A.1 Non-cooperative Game Models

Amaldoss, Meyer, Raju & Rapoport [1], Anupindi & Bassok [2], Anupindi & Bassok [3], Avşar & Baykal-Gürsoy [5], Axsäter [6], Banerjee [8], Banerjee & Bandyopadhyay [9], Banks, Hutchinson & Meyer [10], Bernstein & Federgruen [12], Bernstein & Federgruen [13], Bertrand [14], Butz [15], Bylka [16], Cachon [17], Cachon [18], Cachon [19], Cachon [21], Cachon &

Harker [22], Cachon & Larivere [23], Cachon & Larivere [24], Cachon & Larivere [25], Cachon & Lariviere [26], Cachon & Zipkin [28], Caldentey & Wein [29], Chen & Wan [33], Chiang, Chhajed & Hess [34], Chiang, Fitzsimmons, Huang & Li [35], Choi [36], Chu [38], Chu & Chu [39], Chu & Desai [40], Clarke [41], Cohen & Whang [42], Corbett [43], Corbett & DeCroix [44], Corbett & Groote [45], Corbett & Karmarkar [46], Cournot [47], Cvsa & Gilbert [48], Deal [49], Desai [50], Eliashberg & Steinberg [52], Gal-Or [54], Gal-Or [55], Gal-Or [56], Gans [58], Gjerdrum, Shah & Papageorgiou [62], Goyal [63], Granot & Sosic [65], Ha, Li & Ng [66], Hall & Porteus [67], Hauser & Wernerfelt [70], Huang & Li [71], Hviid [73], Jain & Kannan [74], Joglekar [77], Jørgensen [79], Jørgensen & Kort [80], Kadiyali, Chintagunta & Vilcassim [82], Kalai, Kamien & Rubinovitch [83], Karnani [84], Kim & Hwang [85], Klemperer & Meyer [86], Kreps & Scheinkman [90], Kulkarni [92], Lal & Staelin [93], Lariviere & Porteus [96], Lederer & Li [97], Lee & Rosenblatt [98], Levitan & Shubik [99], Levitan & Shubik [100], Li [101], Li [102], Li [103], Li & Lee [104], Lim [108], Mahajan, Radas & Vakharia [110], Mahajan & van Ryzin [111], McGahan & Ghemawat [114], McGuire & Staelin [115], McGuire & Staelin [116], Monahan [119], Monahan [120], Moorthy [121], Moorthy [122], Netessine & Rudi [125], Nti [126], Parlar [129], Parlar & Wang [130], Parlar & Wang [131], Reitman [135], Reyniers & Tapiero [136], Rosenblatt & Lee [138], Rump & Stidham [140], Shapley & Shubik [144], Shugan [150], So [152], Su & Shi [153], Trivedi [156], Van Mieghem & Dada [158], van Ryzin & Mahajan [159], Vives [160], Wang [165], Wang & Wu [169], Wang & Wu [170], Zhu [174].

A.2 Cooperative Game Models

Anupindi & Bassok [2], Anupindi, Bassok & Zemel [4], Balch [7], Bernstein & Federgruen [13], Bylka[16], Cachon [21], Cachon & Lariviere [26], Cachon & Zipkin [28], Chen, Federgruen & Zheng [32], Chiang, Fitzsimmons, Huang & Li [35], Corbett & Groote [45], Dong & Rudi [51], Gerchak & Gupta [60], Hartman & Dror [68], Hartman & Dror [69], Huang & Li [71], Huang, Li & Mahajan [72], Jeuland & Shugan [75], Jeuland & Shugan [76], Jørgensen & Kort [80], Jørgensen, Taboubi & Zaccour [81], Kim & Hwang [85], Kohli & Park [87], Lal & Staelin [93], Lariviere [95], Li [101], Li & Huang [106], Li, Huang, Zhu & Chau [107], Lippman & McCardle [109], Mallik & Harker [112], McGuire & Staelin [115], McGuire & Staelin [117], Parlar & Wang [130], Raghunathan [132], Rao [133], Reyniers & Tapiero [136], Robinson [137], Rudi, Kapur & Pyke [139], Sabavala [141], Sen [143], Sherali & Rajan [146], Van Mieghem [157], Wang [164], Wang [166], Wang, Guo & Efstathiou [162], Wang & Parlar [168], Weng [171], Zhao & Wang [173], Zusman & Etgar [175].

B. DISTRIBUTION OF THE REVIEWED PAPERS

Table 3: Distribution of the papers reviewed in the survey. Here, the five classes are defined as follows: (i) Inventory games with fixed unit purchase cost, (ii) Inventory games with quantity discounts, (iii) Production and pricing competition, (iv) Games with other attributes and (v) Games with joint decisions on inventory, production/pricing and other attributes.

	Class (i)	Class (ii)	Class (iii)	Class (iv)	Class (v)	Total
2000–2004	12	6	10	12	14	54
1995–1999	8	3	3	5	7	26
1990–1994	3	2	3	3	3	14
1985–1989	2	8	6	1	3	20
1980–1984	0	7	4	2	1	15
Before 1980	0	0	6	2	0	8
Total	25	26	33	25	28	137

Figure 4: Percentages of the reviewed papers published during the six five-year periods.

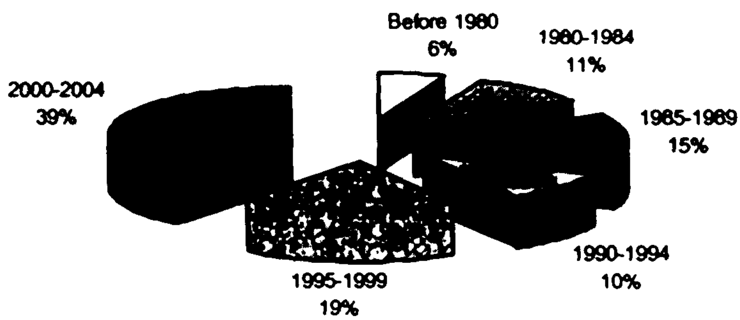
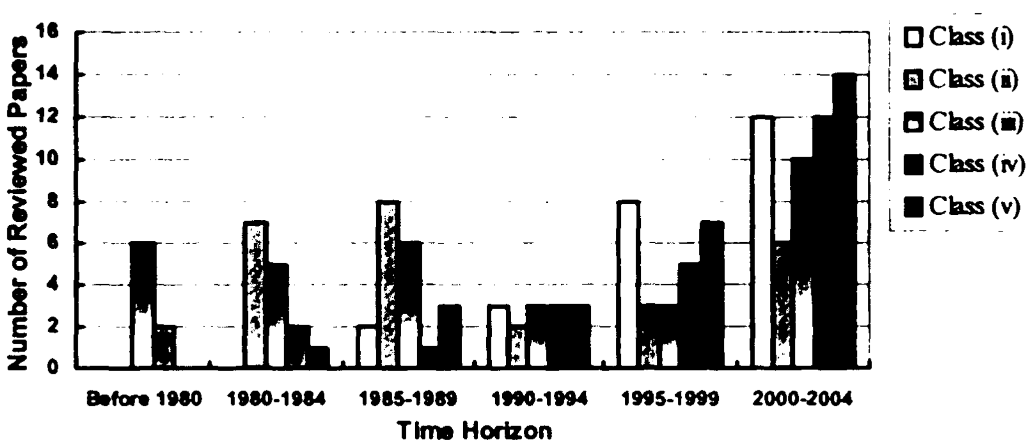


Figure 5: Distribution of the reviewed papers in the five classes. The five classes are: (i) Inventory games with fixed unit purchase cost, (ii) Inventory with quantity discounts, (iii) Production and pricing competition, (iv) Games with other attributes and (v) Games with joint decisions on inventory/pricing and other attributes.



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