

A Real-Time, Personalized Consumption-Based Pricing Scheme for the Consumptions of Traditional and Renewable Energies*

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Abstract:

A real-time, personalized consumption-based pricing scheme can induce electricity users to change their purchase behaviors, thus becoming an important issue in exploring the management mechanisms of electricity markets. To stabilize electricity prices, increase operators' revenues, and balance market demands, we consider the pricing scheme in a smart grid market where traditional and renewable energies are available for sales. Under the scheme, we develop a leader-follower game to characterize the strategic interactions between a demand side management center and residential users, and show that there exists a unique Stackelberg equilibrium. Our numerical analysis indicates that the real-time pricing scheme makes the electricity price difference between valley and peak times within 0.4 cents, thereby achieving the goal of mitigating peak loads and stabilizing electricity prices. We reveal that the renewable energy loads dominate traditional energy loads even when the price of renewable energy is higher than that of traditional energy. We also perform sensitivity analysis and find that an increase in a user's dissatisfaction with the electricity supply can raise electricity prices for the user and two different electricity loads. Moreover, the demand side management center's revenue changes with a concave appearance.

Keywords: Real-time Pricing; Leader-follower Game; Smart Grid; Personalized Consumption

1. Introduction

In the globalization process, the energy sustainability transition has been a core issue that attracts great attention from international communities. Nonetheless, the oil continues to hold the largest share of the energy mix (33.1%) and the coal is the second largest fuel to account for 27.0% in the shares of global primary energy [1]. This behooves us to examine the issues regarding energy consumption and environmental protection. It is of vital importance to choose a cleaner way of development while following the laws of nature [2]. As a necessary part of sustainable development, renewable energy tends to be more and more popular in energy

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market, which has risen to the record highs of 5.0% in the shares of global primary energy in 2019 since it can result in a visible change for energy market from the high-carbon to the low-carbon. Qualified as an important energy source for future power grid, renewable energy has intrigued numerous scholars [1, 3-6].

In recent years, renewable energy is transforming its own role from a non-conventional energy to a main energy and also from an alternative energy to a leading energy in the energy supply of power grid. That is, the power system needs to be upgraded with the integration of traditional and renewable energies. Therefore, the traditional power grid has been transformed into a smart and decentralized power system—smart grid [7, 8]. Smart grid, as a novel power grid integrating renewable energy and advanced information technology, tends to play an important role in the process of developing a low-carbon economy [9]. At present, more and more scholars focus on different areas of smart grid from different perspectives such as fundamental equipment, privacy protection and demand response (DR) [10, 11]. As one of the key issues of demand side management (DSM) in smart grid, DR maintains the strategic interaction between power grid and users, so as to improve the operation efficiency of power grid, reduce the electricity investment, and shave the peak load as well [12]. Therefore, to achieve the goal of sustainable development, it is necessary to design an effective DR mechanism integrating renewable energy in smart grid [13].

DR mechanism integrating renewable energy can be defined as the electricity usage change of users in response to various electricity prices or incentive payoffs by using electricity from traditional and renewable energy generation [14, 15]. The current researches mainly focus on price-based DR including Critical Peak Pricing (CPP), Time-of-Use (TOU) pricing, Peak Load Pricing (PLP) and Real-time Pricing (RTP) [16], which the RTP is the hottest area of DR mechanism and an ideal method to adjust loads of users in recent years [17].

Many researches about RTP based DR mechanism have been done in the existing works. From the perspective of public goods, DR mechanism maximizing social welfare is adopted by some researchers. For example, Dong et al. [16] proposed a smart DR mechanism based on maximizing social welfare and designed a new solving algorithm. Tan [18] constructed a multi-scenario operation optimization model for park integrated energy system based on multi-energy demand response. Using the similar method, Zhu et al. [17] broke through the single-stage limitation in Dong et al. [16]. Considering the effect of random fluctuation of electricity consumption, Tao et al. [19] also proposed a RTP scheme based on expectation bilevel programming. The above works mainly reflect the user's direct DR to electricity price, and aim to maximize the social welfare to regulate the power load depicted by utility function, but renewable energy is ignored in these works.

Relative researches for modelling the RTP problems in smart grid have also been performed increasingly with game theory in recent years. Dipti et al. [20] evaluated and compared game theory based dynamic pricing strategy for Singapore electricity market, and the results demonstrated that the RTP scheme was the best in reducing the peak load and increasing the profit. Similarly, Abapour et al. [21] designed a Bayesian pricing game model to depict the interaction among DR aggregators. Tao and Gao [22] studied the smart grid system with shortage device and distributed renewable energy based on dual decomposition theory. Most of the above works construct the game model among the electricity suppliers or among users, however, the interaction between the suppliers and users is also an important subject of smart grid. Given the typical hierarchical structure of the electricity market, it is a new trend in recent years that establish a leader-follower Game between the electricity suppliers and users for RTP research. Dai et al. [23] established a DR model with incentive factor based on leader-follower Game to adjust the price information from electricity retailers for guaranteeing the grid operation and assuring supply-demand balance. Yu and Hong [24] proposed a RTP algorithm for power load management of residential electrical appliances, so as to obtain an optimal household appliance management solution in the condition of real-time electricity price change. Luo et al. [25] constructed a hierarchical leader-follower Game to study the energy scheduling problem of a three-level integrated energy system. Nevertheless, the renewable energy was still not fully considered in these literatures.

Note that most of the existing RTP researches based on game models focus on day-ahead electricity price, and each user must follow the optimal price in the next day once the game equilibrium is determined. However, there always exists a large gap between day-ahead prices and real-time prices. In addition, considering the dynamic real-time characteristics of market changes and the informationization of smart devices, there is a continuous two-way information interaction between the electricity suppliers and users. Thus how to ensure the balance between supply and demand of electricity is still the key of electricity market. To date, this kind of information interaction is based on the premise that the electricity suppliers and the users are rational partners considering the fairness preference [26, 27], but the personalized electricity consumption behavior of users is not considered by the authors of existing literatures. The above problems are exactly the key consideration objects in our paper.

In conclusion, a leader-follower Game is developed to model the strategic interaction between the DSM center of power supplier and residential users in the presence of rapidly updated real-time prices to operate the optimal power load management [28]. On that basis, we propose a RTP scheme by considering electricity users' personalized electricity consumption for the traditional and renewable energies. The users may consume both renewable and traditional energies at the same time, but consuming the latter incurs a guilt cost [29, 30].

The DSM center allocates the electricity price to each user through the DR mechanism, and then the users determine their optimal electricity consumption to respond to the prices allocated by the DSM center. In view of our focus on the users' personalized electricity consumption, the uncertainty in the generation of renewable energy is beyond the scope of this paper.

The major contributions of this work are summarized as follows:

(1) We propose a RTP scheme between the DSM center of electricity supplier and multiple residential users in the presence of rapidly updated real-time prices.

(2) A one-leader, N -follower game is formulated to model the strategic interaction between the DSM center and the residential users, with the DSM center being the leader to offer the real-time electricity prices to the residential users. The users purchase the electricity from the DSM center in response to the announced real-time electricity prices.

(3) The dissatisfaction and fairness preference concept are introduced to the residential users considering the satisfaction and irrational behavior of users in the electricity market.

(4) Although users may consume both renewable energy and traditional electricity, they feel guilty about using traditional electricity. The guilt cost of consuming traditional electricity is quantified.

(5) We show the existence and uniqueness of Stackelberg equilibrium, which means that there is an optimal solution for the game. The Stackelberg equilibrium is obtained analytically, which avoids the iterative process as well as alleviates the computational between the DSM center and residential users.

The rest of this paper is organized as follows. Section 2 introduces the system model. Section 3 analyzes the formulation of game model with personalized electricity consumption. Section 4 discusses the number simulation results and sensitivity analysis. Finally, Section 5 draws the conclusion.

2. System Model

In this paper, a regional smart grid market is composed of a centralized electricity supplier and N residential users. The set of users is expressed as $\mathcal{N} \triangleq \{1, 2, \dots, N\}$. One day that is divided into K time slots is regarded as an operating cycle, denoted as $\mathcal{K} \triangleq \{1, 2, \dots, K\}$ where time slot $t \in \mathcal{K}$. The electricity supplier is equipped with a DSM center to manage all users' electricity consumption [31]. The overall market structure and configuration are showed in Figure 1.

2.1 User's Cost Function

The cost of each residential user consists of three parts: the first one is the normal cost, based on the cost of electricity consumption; the second one is the dissatisfaction cost [24], evaluated by DSM center according to electricity consumption; the third one is the guilt cost caused by using traditional electricity (because traditional electricity consumption leads to global warming and pollution problems). The expenses of users

will change when their electricity consumption changes at different time slots. This change information is transmitted to the DSM center through the DR mechanism of different users, and then the DSM center will allocate reasonable electricity prices for each user respectively and inform all users.

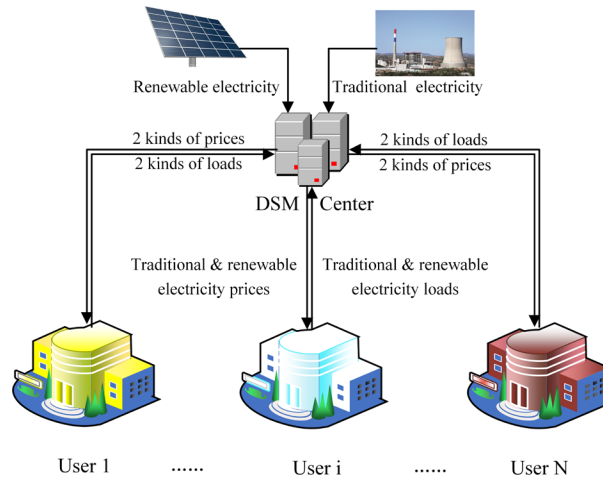


Figure 1. Electricity market structure between the DSM center and the users.

Assuming that at time slot t , user i 's electricity consumption from renewable energy and traditional electricity is x_{ic} and x_{if} , respectively. x_{ic}^{\min} and x_{ic}^{\max} are the minimum and maximum value of x_{ic} , x_{if}^{\min} and x_{if}^{\max} are the minimum and maximum value of x_{if} . Based on the electricity consumption of all users, the DSM center disseminates the price information to all users. The electricity price allocated to user i is denoted as $\mathbf{p}_i = (p_{ic}, p_{if})$, where p_{ic} is price of renewable energy and p_{if} is price of traditional electricity. Then the strategy of DSM center is $\mathbf{p} = [\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_N]$. Obviously, user i 's normal cost based on electricity consumption is $p_{ic}x_{ic} + p_{if}x_{if}$. The dissatisfaction cost is the second cost that evaluates the dissatisfaction level of user i . Let $x_{ic} + x_{if} = d_i$, the function $F_i(d_i)$ is used to express user i 's dissatisfaction. Let m_i be the electricity consumption when the satisfaction of user i is moderate. If user i 's electricity consumption d_i is lower than m_i , it means that user i is not satisfied and $F_i(d_i)$ will drop rapidly with the increasing of d_i . Conversely, when the electricity consumption is greater than m_i , it means user i is satisfied, and $F_i(d_i)$ will be negative and decrease with the increasing of d_i . When $d_i = m_i$, the dissatisfaction function value is zero, which indicates that the user i 's satisfaction is neutral in the median energy demand [24].

The function $F_i(d_i)$ is selected in the following formula based on the above description and theory of Yu and Hong [24].

$$F_i(d_i) = a_i(d_i - d_i^{\max})^2 - a_i(m_i - d_i^{\max})^2, \quad a_i > 0, \quad (1)$$

where $d_i^{\max} = x_{ic}^{\max} + x_{if}^{\max}$, a_i is a non-negative parameter.

The third part cost is the negative utility caused by using traditional electricity which is denoted by as $G_i(x_{if}, b_i)$. Holding the similar idea with Samadi et al. [32], we assume that it should satisfy the following conditions:

- (1) $\frac{\partial G(x_{if}, b_i)}{\partial x_{if}} \geq 0$. That is, users are always interested to consume less traditional electricity if possible.
- (2) $\frac{\partial G(x_{if}, b_i)}{\partial b_i} > 0$, which means when x_{if} is fixed, the larger b_i brings larger guilt.
- (3) $\frac{\partial^2 G(x_{if}, b_i)}{\partial x_{if}^2} \geq 0$. It means that the negative utility for user i cannot get saturated. The more traditional electricity they use, the guiltier they feel.
- (4) $G(0, b_i) = 0$. It shows no traditional electricity consumption brings no guilt.

So the third part cost is depicted by the following function [22]:

$$G_i(x_{if}) = b_i x_{if}^2, b_i > 0. \quad (2)$$

Based on the normal cost of user i , formula (1) and formula (2), the total electricity consumption cost of user i is expressed as

$$u_i(\mathbf{p}, x_{ic}, x_{if}) = p_{ic} x_{ic} + p_{if} x_{if} + \mu_i F_i(d_i) + b_i x_{if}^2, \mu_i > 0, b_i > 0. \quad (3)$$

In formula (3), the parameter μ_i is used to measure user i 's satisfaction at a specified time slot for quantifying the dissatisfaction cost, and different μ_i is selected based on user i 's preference at different time slot. A larger value of μ_i corresponds to the scenario that the user is the less satisfied with the allocated electricity. Thus, the users concerned more with improving their satisfaction, namely, reducing dissatisfaction cost, which can be achieved by increasing electricity demand. Therefore, user i needs to solve the following problem for minimizing his cost.

$$\begin{aligned} & \min u_i(\mathbf{p}, x_{ic}, x_{if}) \\ & \text{s.t.} \begin{cases} x_{ic}^{\min} \leq x_{ic} \leq x_{ic}^{\max}, \\ x_{if}^{\min} \leq x_{if} \leq x_{if}^{\max}, \forall i = 1, 2, \dots, N. \end{cases} \end{aligned} \quad (4)$$

where user i 's strategy set is denoted as

$$\Omega_i = \{\mathbf{x}_i = (x_{ic}, x_{if}) \mid x_{ic}, x_{if} \in R, x_{ic}^{\min} \leq x_{ic} \leq x_{ic}^{\max}, x_{if}^{\min} \leq x_{if} \leq x_{if}^{\max}, \forall i = 1, 2, \dots, N\}.$$

2.2 Revenue of the DSM Center

Since the revenue of electricity supplier with the DSM center is the sum of all users' electricity cost, the revenue function $U_{DSM}(\mathbf{p}, \mathbf{x})$ of electricity supplier can be denoted as follows:

$$U_{DSM}^0(\mathbf{p}, \mathbf{x}) = \sum_{i=1}^N [(p_{ic} - c_c)x_{ic} + (p_{if} - c_f)x_{if}] - \sum_{i=1}^N \mu_i F_i(d_i) - \sum_{i=1}^N b_i x_{if}^2, \mu_i > 0, b_i > 0, \quad (5)$$

where the second part in the right side is negative since users' total dissatisfaction cost must be subtracted from the total revenue of the supplier. $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N) = ((x_{1c}, x_{1f}), (x_{2c}, x_{2f}), \dots, (x_{Nc}, x_{Nf}))$. Besides, c_c and c_f respectively represent the DSM's unit cost of purchasing renewable energy and traditional electricity from electricity market.

Based on all users' electricity consumption, the DSM center needs to maximize its revenue, and then the following problem will be solved.

$$\begin{aligned} \max \quad & U_{DSM}^0(\mathbf{p}, \mathbf{x}) \\ \text{s.t.} \quad & \begin{cases} 0 < p_{ic} < P_{DSM}, \\ 0 < p_{if} < P_{DSM}, \forall i = 1, 2, \dots, N, \end{cases} \end{aligned} \quad (6)$$

where P_{DSM} is the maximum value of real-time electricity price. The strategy set of the DSM center is denoted as

$$\Omega_{DSM} = \{\mathbf{p} = (\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_N) \mid \mathbf{p}_i \in R^2, 0 < p_{ic} < P_{DSM}, 0 < p_{if} < P_{DSM}\}.$$

3. The Leader-follower Game Model for the Personalized Electricity Consumption

3.1 The Model of User Side with Fairness Preference

With the improving of living standard and economy, based on the consumption mentality, each user's electricity cost is affected not only by the user himself, but also by the comparison of electricity costs with other electricity users. If the electricity cost of user i is much higher than that of user j at some time slot, user i will reduce the electricity cost by reducing his own electricity consumption.

Assuming that each user has fairness preference and the electricity supplier is fair and neutral, based on formula (3) in Section 2.1, the user's utility function with fairness preference is described by introducing a reference point. Since the electricity cost is a major factor in the sense of unfairness for each user, the electricity cost of other users is taken as the reference point of his electricity consumption, and the parameter γ is introduced as a fairness preference factor [26, 28], then the cost function of user i with fairness preference is generated as follows:

$$\begin{aligned} U_i(\mathbf{p}, x_{ic}, x_{if}) &= p_{ic}x_{ic} + p_{if}x_{if} + \mu_i F_i(x_{ic} + x_{if}) + \sum_{j=1, j \neq i}^N \gamma [(p_{ic}x_{ic} - p_{jc}x_{jc}) + (p_{ic}x_{ic} - p_{jf}x_{jf})] + b_i x_{if}^2 \\ &= (1 + \gamma N)(p_{ic}x_{ic} + p_{if}x_{if}) + \mu_i a_i (x_{ic} + x_{if} - x_i^{\max})^2 - \mu_i a_i (m_i - d_i^{\max})^2 + b_i x_{if}^2 \\ &\quad - \sum_{j=1}^N \gamma (p_{jc}x_{jc} + p_{jf}x_{jf}), a_i > 0, b_i > 0, \mu_i > 0, \gamma > 0, \end{aligned} \quad (7)$$

where $\gamma[(p_{ic}x_{ic} - p_{jc}x_{jc}) + (p_{ic}x_{ic} - p_{jc}x_{jc})]$ indicates that user i feels unfair if there is a gap between the electricity cost of user i and that of user j .

Similar to formula (4), it is also necessary to find the optimal electricity consumption $x_i(\mathbf{p}) = (x_{ic}(\mathbf{p}), x_{if}(\mathbf{p}))$ to minimize the user's cost, i.e., solving the following optimization problem:

$$\begin{aligned} & \min U_i(\mathbf{p}, x_{ic}, x_{if}) \\ & \text{s.t.} \begin{cases} x_{ic}^{\min} \leq x_{ic} \leq x_{ic}^{\max}, \\ x_{if}^{\min} \leq x_{if} \leq x_{if}^{\max}, \forall i = 1, 2, \dots, N, \end{cases} \end{aligned} \quad (8)$$

where the strategy set of user i is denoted as

$$\Omega_i = \{\mathbf{x}_i = (x_{ic}, x_{if}) \mid x_{ic}, x_{if} \in R, x_{ic}^{\min} \leq x_{ic} \leq x_{ic}^{\max}, x_{if}^{\min} \leq x_{if} \leq x_{if}^{\max}, \forall i = 1, 2, \dots, N\}.$$

3.2 The Model of DSM Center

Considering the user's fairness preference, the revenue function of the DSM center is rewritten as

$$\begin{aligned} U_{DSM}(\mathbf{p}, \mathbf{x}) = & \sum_{i=1}^N [(p_{ic} - c_c)x_{ic}(\mathbf{p}) + (p_{if} - c_f)x_{if}(\mathbf{p})] - \sum_{i=1}^N \mu_i F_i(d_i(\mathbf{p})) - \sum_{i=1}^N b_i x_{if}^2 \\ & - \sum_{i=1}^N \sum_{j=1, j \neq i}^N \gamma[(p_{ic}x_{ic}(\mathbf{p}) - p_{jc}x_{jc}(\mathbf{p})) + (p_{if}x_{if}(\mathbf{p}) - p_{jf}x_{jf}(\mathbf{p}))]. \end{aligned} \quad (9a)$$

However, since

$$\begin{aligned} & \sum_{i=1}^N \sum_{j=1, j \neq i}^N \gamma[(p_{ic}x_{ic}(\mathbf{p}) - p_{jc}x_{jc}(\mathbf{p})) + (p_{if}x_{if}(\mathbf{p}) - p_{jf}x_{jf}(\mathbf{p}))] \\ & = \sum_{i=1}^N \sum_{j=1, j \neq i}^N \gamma[(p_{ic}x_{ic}(\mathbf{p}) - p_{jc}x_{jc}(\mathbf{p}))] - \sum_{i=1}^N \sum_{j=1, j \neq i}^N \gamma[(p_{if}x_{if}(\mathbf{p}) - p_{jf}x_{jf}(\mathbf{p}))] \\ & = \sum_{i=1}^N (N-1)\gamma(p_{ic}x_{ic}(\mathbf{p})) - (N-1)\gamma(p_{ic}x_{ic}(\mathbf{p})) - \sum_{i=1}^N (N-1)\gamma(p_{if}x_{if}(\mathbf{p})) - (N-1)\gamma(p_{if}x_{if}(\mathbf{p})) \\ & = 0, \end{aligned}$$

i.e., the value of fourth term is exactly zero, we can obtain

$$U_{DSM}(\mathbf{p}, \mathbf{x}) = \sum_{i=1}^N [(p_{ic} - c_c)x_{ic}(\mathbf{p}) + (p_{if} - c_f)x_{if}(\mathbf{p})] - \sum_{i=1}^N \mu_i F_i(d_i(\mathbf{p})) - \sum_{i=1}^N b_i x_{if}^2. \quad (9b)$$

Similarly, because the DSM center needs to make optimal response for allocating the electricity price \mathbf{p}_i based on the electricity consumption of all users, we must solve the following optimization problem:

$$\begin{aligned} & \max U_{DSM}(\mathbf{p}, \mathbf{x}) \\ & \text{s.t.} \begin{cases} 0 < p_{ic} < P_{DSM}, \\ 0 < p_{if} < P_{DSM}, \forall i = 1, 2, \dots, N, \end{cases} \end{aligned} \quad (10)$$

where P_{DSM} is the real-time electricity price, and the strategy set of the DSM center is denoted as

$$\Omega_{DSM} = \{\mathbf{p} = (p_1, p_2, \dots, p_N) \mid p_i \in R^2, 0 < p_{ic} < P_{DSM}, 0 < p_{if} < P_{DSM}\}.$$

3.3 Formulation of Leader-Follower Game Model

In this section a leader-follower game is developed to study the strategic interaction between the DSM

center and residential users, and backward induction is used to obtain the Stackelberg equilibrium according to Fudenberg and Tirole [33]. The DSM center maximizes its revenue based on each user's optimal strategy, and then every user reselects his optimal electricity consumption to minimize his cost according to the optimal strategy of the DSM center.

The game is processed as follows:

- (1) The DSM center allocates the electricity price $\mathbf{p} = [\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_N]$ to each user based on the real-time price of smart grid at the previous time slot;
- (2) Each user selects the optimal strategy $\mathbf{x}_i(\mathbf{p})$ from Ω_i according to the electricity price allocated by the DSM center so that formula (4) is established, i.e., $\mathbf{x}_i(\mathbf{p}) = \arg \min_{\mathbf{x}_i \in \Omega_i} U_i(\mathbf{p}, \mathbf{x}_i)$.
- (3) The DSM center reselects the optimal strategy \mathbf{p}^* according to each user's strategy $\mathbf{x}_1(\mathbf{p}), \dots, \mathbf{x}_N(\mathbf{p})$, so that formula (6) is established, i.e., $\mathbf{p}^* = \arg \max_{\mathbf{p} \in \Omega_{DSM}} U_{DSM}(\mathbf{p}, \mathbf{x}_1(\mathbf{p}), \dots, \mathbf{x}_N(\mathbf{p}))$.
- (4) After the optimal strategy is determined, the DSM center reallocates the electricity price to all users, and then all users determine their optimal electricity consumption $(\mathbf{x}_1^*, \mathbf{x}_2^*, \dots, \mathbf{x}_N^*)$. The DSM center and the users will repeat process (2) to (4) until the game equilibrium is reached. At last the Stackelberg equilibrium is obtained.

Definition: Let $\mathbf{x}^* = (\mathbf{x}_1^*, \mathbf{x}_2^*, \dots, \mathbf{x}_N^*)$ be the optimal strategy profile of all users, set $\Omega_j = \Omega_1 \times \Omega_2 \times \dots \times \Omega_N$, $\mathbf{x}^* \in \Omega_j$. The strategy profile $(\mathbf{p}^*, \mathbf{x}^*)$ is the Stackelberg equilibrium of the proposed leader-follower game if and only if the following optimization problem is met.

$$(\mathbf{p}^*, \mathbf{x}^*) = \arg \max_{(\mathbf{p}, \mathbf{x}) \in \Omega_{DSM} \times \Omega_j} U_{DSM}(\mathbf{p}, \mathbf{x}^*) \quad (11)$$

$$\text{s.t. } \mathbf{x}_i^* = \arg \min_{\mathbf{x}_i \in \Omega_i} U_i(\mathbf{p}, \mathbf{x}_i), \quad \forall i = 1, 2, \dots, N. \quad (12)$$

3.4 The Existence and Uniqueness Analysis of Stackelberg Equilibrium

For the leader-follower game, we can use the backward induction to obtain its equilibrium. Firstly, according to problem (12), the best-response strategies of all users are identified in responding to the announced strategy of the DSM center. We then obtain the best strategy of the DSM center based on formula (11) given the identified strategies of all users.

Theorem 1 For the leader-follower game developed between the DSM center and all users, the unique Stackelberg equilibrium exists if and only if the following conditions are satisfied.

- (1) The strategy sets of all game participants are nonempty, convex and compact;
- (2) Given a strategy of the DSM center, each user has a unique best-response strategy;

(3) Given the identified optimal strategies of all users, the DSM center can admit a unique optimal strategy.

Proof: (1) According to Section 3.1 and 3.2, we find that the strategy set Ω_i of user i and the strategy set Ω_{DSM} of the DSM center are both sets of convex constraints. Obviously, these sets are nonempty, convex, and compact [34].

(2) We first need to find the best-response strategy of each user according to problem (12), i.e., the user's optimization problem (4). It can be verified that $U_i(\mathbf{p}, x_{ic}, x_{if})$ is continuous and differentiable in Ω_i according to formula (7), which allows us to analyze $U_i(\mathbf{p}, x_{ic}, x_{if})$. Given the strategy \mathbf{p} of the DSM center, in order to obtain each user's best-response function, we calculate the first-order derivative of $U_i(\mathbf{p}, x_{ic}, x_{if})$ with respect to x_{ic} and x_{if} as

$$\frac{\partial U_i(\mathbf{p}, x_{ic}, x_{if})}{\partial x_{ic}} = (1 + \gamma N - \gamma)p_{ic} + 2a_i\mu_i(x_{ic} + x_{if} - d_i^{\max}), \quad (13)$$

$$\frac{\partial U_i(\mathbf{p}, x_{ic}, x_{if})}{\partial x_{if}} = (1 + \gamma N - \gamma)p_{if} + 2b_i x_{if} + 2a_i\mu_i(x_{ic} + x_{if} - d_i^{\max}). \quad (14)$$

Equating (13) and (14) to be zero and solving them yields

$$x_{ic}(\mathbf{p}) = \frac{2a_i\mu_i(d_i^{\max} - x_{if}) - p_{ic}(1 + \gamma N - \gamma)}{2a_i\mu_i}, \quad (15)$$

$$x_{if}(\mathbf{p}) = \frac{2a_i\mu_i(d_i^{\max} - x_{ic}) - p_{if}(1 + \gamma N - \gamma)}{2(b_i + a_i\mu_i)}. \quad (16)$$

Simplifying (15) and (16), we have the best-response function:

$$\begin{aligned} x_{ic}(\mathbf{p}) &= \frac{(1 - \gamma + N\gamma)a_i\mu_i(p_{if} - p_{ic}) - (1 - \gamma + N\gamma)b_i p_{ic} + 2a_i b_i \mu_i d_i^{\max}}{2a_i b_i \mu_i} \\ &= \frac{(1 - \gamma + N\gamma)a_i\mu_i p_{if} - (1 - \gamma + N\gamma)(a_i\mu_i + b_i)p_{ic} + 2a_i b_i \mu_i d_i^{\max}}{2a_i b_i \mu_i}, \end{aligned} \quad (17)$$

$$x_{if}(\mathbf{p}) = \frac{(1 + N\gamma - \gamma)(p_{ic} - p_{if})}{2b_i}. \quad (18)$$

As in real life, $p_{ic} > p_{if}$, $x_{if}(\mathbf{p}) > 0$, $x_{ic}(\mathbf{p}) > 0$, thus,

$$\Omega_{DSM} = \{\mathbf{p} = (\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_N) \mid \mathbf{p}_i \in R^2, 0 < p_{ic} < P_{DSM}, 0 < p_{if} < P_{DSM}, p_{ic} > p_{if}, x_{if}(\mathbf{p}) > 0, x_{ic}(\mathbf{p}) > 0\}.$$

Taking the second-order derivatives of $U_i(\mathbf{p}, x_{ic}, x_{if})$ with respect to x_{ic} and x_{if} yields

$$\frac{\partial^2 U_i(\mathbf{p}, x_{ic}, x_{if})}{\partial x_{ic}^2} = 2a_i\mu_i > 0, \quad \frac{\partial^2 U_i(\mathbf{p}, x_{ic}, x_{if})}{\partial x_{if}^2} = 2b_i + 2a_i\mu_i > 0. \text{ Moreover, user } i \text{ 's cost function is strictly}$$

convex since the strategy set Ω_i of user i is convex in the presence of the user's fairness preference. It is obvious that $\mathbf{x}_i(\mathbf{p})$ is the unique optimal response to \mathbf{p} and the optimal electricity consumption of user i . Therefore, the best-response strategy in terms of (17) and (18) is optimal and unique. Then, condition (2) of Theorem 1 is satisfied.

(3) We next find the DSM center's best strategy by solving the problem in (11), which is built on the users' identified best-response strategies in (17) and (18). By substituting (17) and (18) into the DSM center's revenue function given in (9b), we have

$$\begin{aligned} U_{DSM}(\mathbf{p}, \mathbf{x}) &= \sum_{i=1}^N \left[(p_{ic} - c_c) \frac{(1-\gamma + N\gamma)a_i\mu_i p_{if} - (1-\gamma + N\gamma)(a_i\mu_i + b_i)p_{ic} + 2a_i b_i \mu_i d_i^{\max}}{2a_i b_i \mu_i} + (p_{if} - c_f) \frac{(1+N\gamma-\gamma)(p_{ic} - p_{if})}{2b_i} \right] \\ &\quad - \sum_{i=1}^N \mu F_i \left[\frac{(1-\gamma + N\gamma)a_i\mu_i p_{if} - (1-\gamma + N\gamma)(a_i\mu_i + b_i)p_{ic} + 2a_i b_i \mu_i d_i^{\max}}{2a_i b_i \mu_i} + \frac{(1+N\gamma-\gamma)(p_{ic} - p_{if})}{2b_i} \right] \\ &\quad - \sum_{i=1}^N b_i \left[\frac{(1+N\gamma-\gamma)(p_{ic} - p_{if})}{2b_i} \right]^2. \end{aligned}$$

Since $x_{ic}^{\min} \leq x_{ic} \leq x_{ic}^{\max}$, $x_{if}^{\min} \leq x_{if} \leq x_{if}^{\max}$, $x_{ic}(\mathbf{p})$, $x_{if}(\mathbf{p})$ should satisfy the requirement

$$\begin{aligned} x_{ic}(\mathbf{p}) &= \frac{(1-\gamma + N\gamma)a_i\mu_i p_{if} - (1-\gamma + N\gamma)(a_i\mu_i + b_i)p_{ic} + 2a_i b_i \mu_i d_i^{\max}}{2a_i b_i \mu_i} \in [x_{ic}^{\min}, x_{ic}^{\max}], \\ x_{if}(\mathbf{p}) &= \frac{(1+N\gamma-\gamma)(p_{ic} - p_{if})}{2b_i} \in [x_{if}^{\min}, x_{if}^{\max}]. \end{aligned}$$

We then find

$$\begin{aligned} 2a_i b_i \mu_i x_{ic}^{\min} &\leq (1-\gamma + N\gamma)a_i\mu_i p_{if} - (1-\gamma + N\gamma)(a_i\mu_i + b_i)p_{ic} + 2a_i b_i \mu_i d_i^{\max} \leq 2a_i b_i \mu_i x_{ic}^{\max}, \\ 2b_i x_{if}^{\min} &\leq (1+N\gamma-\gamma)p_{ic} - (1+N\gamma-\gamma)p_{if} \leq 2b_i x_{if}^{\max}, \end{aligned}$$

which means

$$\begin{aligned} (1-\gamma + N\gamma)a_i\mu_i p_{if} - (1-\gamma + N\gamma)(a_i\mu_i + b_i)p_{ic} &\leq 2a_i b_i \mu_i x_{ic}^{\max} - 2a_i b_i \mu_i d_i^{\max}, \\ -(1-\gamma + N\gamma)a_i\mu_i p_{if} + (1-\gamma + N\gamma)(a_i\mu_i + b_i)p_{ic} &\leq 2a_i b_i \mu_i d_i^{\max} - 2a_i b_i \mu_i x_{ic}^{\min}, \\ -(1+N\gamma-\gamma)p_{if} + (1+N\gamma-\gamma)p_{ic} &\leq 2b_i x_{if}^{\max}, \\ (1+N\gamma-\gamma)p_{if} - (1+N\gamma-\gamma)p_{ic} &\leq -2b_i x_{if}^{\min}. \end{aligned}$$

Therefore, the optimal strategy of the DSM center in the presence of the user's fairness preference is:

$$\mathbf{p}^* = \arg \max U_{DSM}(\mathbf{p}, \mathbf{x}) = \sum_{i=1}^N [(p_{ic} - c_c)x_{ic}(\mathbf{p}) + (p_{if} - c_f)x_{if}(\mathbf{p})] - \sum_{i=1}^N \mu_i F_i(d_i(\mathbf{p})) - \sum_{i=1}^N b_i x_{if}^2 \quad (19)$$

where $\mathbf{p}^* \in \Omega_{DSM} = \{\mathbf{p} = (\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_N) \mid \mathbf{p}_i \in \mathbb{R}^2, 0 < p_{ic} < P_{DSM}, 0 < p_{if} < P_{DSM}, (24)\}$. It is straightforward to find

that $U_{DSM}(\mathbf{p}, \mathbf{x})$ is continuous and differentiable in Ω_{DSM} .

In order to determine that (19) is the solution for maximizing the DSM center's revenue, we need to calculate the Hessian matrix of $U_{DSM}(\mathbf{p}, \mathbf{x})$. The second derivatives of $U_{DSM}(\mathbf{p}, \mathbf{x})$ with respect to $p_{ic}, p_{jc}, p_{if}, p_{jf}$ are computed as

$$\frac{\partial^2 U_{DSM}(\mathbf{p}, \mathbf{x})}{\partial p_{ic} \partial p_{jc}} = \begin{cases} -\frac{(1+\gamma N-\gamma)^2}{2b_i} - 2a_i \mu_i \left(\frac{1+\gamma N-\gamma}{2b_i} - \frac{(1+\gamma N-\gamma)(a_i \mu_i + b_i)}{2a_i b_i \mu_i} \right)^2 - \frac{(1+\gamma N-\gamma)(a_i \mu_i + b_i)}{a_i b_i \mu_i}, & j=i, \\ 0, & j \neq i. \end{cases}$$

$$\frac{\partial^2 U_{DSM}(\mathbf{p}, \mathbf{x})}{\partial p_{if} \partial p_{jf}} = \begin{cases} -\frac{(1+\gamma N-\gamma)^2}{2b_i} - \frac{1+\gamma N-\gamma}{b_i}, & j=i, \\ 0, & j \neq i. \end{cases}$$

$$\frac{\partial^2 U_{DSM}(\mathbf{p}, \mathbf{x})}{\partial p_{ic} \partial p_{jf}} = \begin{cases} \frac{1+\gamma N-\gamma}{b_i} + \frac{(1+\gamma N-\gamma)^2}{2b_i}, & j=i, \\ 0, & j \neq i. \end{cases}$$

$$\frac{\partial^2 U_{DSM}(\mathbf{p}, \mathbf{x})}{\partial p_{if} \partial p_{jc}} = \begin{cases} \frac{1+\gamma N-\gamma}{b_i} + \frac{(1+\gamma N-\gamma)^2}{2b_i}, & j=i, \\ 0, & j \neq i. \end{cases}$$

Denote the Hessian matrix of $U_{DSM}(\mathbf{p}, \mathbf{x})$ by $\mathbf{A} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$, where

$$A_{11} = (a_{ij}^{11}), a_{ij}^{11} = \begin{cases} -\frac{(1+\gamma N-\gamma)^2}{2b_i} - \frac{1+\gamma N-\gamma}{b_i}, & j=i, \\ 0, & j \neq i. \end{cases}$$

$$A_{12} = (a_{ij}^{12}), a_{ij}^{12} = \begin{cases} \frac{1+\gamma N-\gamma}{b_i} + \frac{(1+\gamma N-\gamma)^2}{2b_i}, & j=i, \\ 0, & j \neq i. \end{cases}$$

$$A_{21} = (a_{ij}^{21}), a_{ij}^{21} = \begin{cases} \frac{1+\gamma N-\gamma}{b_i} + \frac{(1+\gamma N-\gamma)^2}{2b_i}, & j=i, \\ 0, & j \neq i. \end{cases}$$

$$A_{22} = (a_{ij}^{22}), a_{ij}^{22} = \begin{cases} -\frac{(1+\gamma N-\gamma)^2}{2b_i} - 2a_i \mu_i \left(\frac{1+\gamma N-\gamma}{2b_i} - \frac{(1+\gamma N-\gamma)(a_i \mu_i + b_i)}{2a_i b_i \mu_i} \right)^2 - \frac{(1+\gamma N-\gamma)(a_i \mu_i + b_i)}{a_i b_i \mu_i}, & j=i, \\ 0, & j \neq i. \end{cases}$$

The congruent transformation matrix of matrix \mathbf{A} is denoted as $\mathbf{B} = \text{diag}(b_1, \dots, b_{2N})$, where

$$b_1 = b_2 = \dots = b_N = a_{ii}^{11},$$

$$\begin{aligned}
b_{N+1} = b_{N+2} = \dots = b_{2N} = & -\frac{a_{ij}^{12} a_{ji}^{21} + a_{ii}^{22}}{a_{ij}^{11}} = -\frac{(1+\gamma N - \gamma)^2}{2b_i} - 2a_i \mu_i \left(\frac{1+\gamma N - \gamma}{2b_i} - \frac{(1+\gamma N - \gamma)(a_i \mu_i + b_i)}{2a_i b_i \mu_i} \right)^2 \\
& - \frac{(1+\gamma N - \gamma)(a_i \mu_i + b_i)}{a_i b_i \mu_i} + \frac{1+\gamma N - \gamma}{b_i} + \frac{(1+\gamma N - \gamma)^2}{2b_i} = -2a_i \mu_i \left(\frac{1+\gamma N - \gamma}{2b_i} - \frac{(1+\gamma N - \gamma)(a_i \mu_i + b_i)}{2a_i b_i \mu_i} \right)^2 \\
& - \frac{(1+\gamma N - \gamma)(a_i \mu_i + b_i)}{a_i b_i \mu_i} + \frac{1+\gamma N - \gamma}{b_i}.
\end{aligned}$$

Considering

$$-\frac{(1+\gamma N - \gamma)(a_i \mu_i + b_i)}{a_i b_i \mu_i} + \frac{1+\gamma N - \gamma}{b_i} < -\frac{(1+\gamma N - \gamma)a_i \mu_i}{a_i b_i \mu_i} + \frac{1+\gamma N - \gamma}{b_i} = -\frac{1+\gamma N - \gamma}{b_i} + \frac{1+\gamma N - \gamma}{b_i} = 0,$$

we then obtain $b_{N+1} = b_{N+2} = \dots = b_{2N} < 0$. Therefore, matrix \mathbf{B} is strictly negative definite, implying that the Hessian matrix \mathbf{A} of $U_{DSM}(\mathbf{p}, \mathbf{x})$ is also strictly negative definite in view of the congruent relationship of \mathbf{A} and \mathbf{B} . Hence, $U_{DSM}(\mathbf{p}, \mathbf{x})$ is a strictly concave function in Ω_{DSM} , i.e., the problem in (19) is a convex optimization problem that can be solved using convex programming methods [34]. Meanwhile, the best strategy \mathbf{p}^* for the problem in (19) is determined to be unique based on the strictly negative definite Hessian matrix \mathbf{A} . As a result, condition (3) of Theorem 1 is satisfied.

Once the DSM center's unique optimal strategy \mathbf{p}^* is determined, the optimal strategies of all users can be subsequently identified according to (17) and (18). Finally, the strategy profile $(\mathbf{p}^*, \mathbf{x}^*)$ constitutes the unique Stackelberg equilibrium of the proposed leader-follower game. It is then straightforward to show that Theorem 1 holds when conditions (1)-(3) are satisfied. The proof of Theorem 1 is then completed.

Since the analytical solutions exist for both the DSM center and the users according to the proof of Theorem 1, the Stackelberg equilibrium can be directly obtained using the backward induction. Moreover, the Stackelberg equilibrium is unique and optimal.

4. Numerical Simulations

4.1 Simulation Scenario and Parameter Setting

This section reveals the numerical results for our RTP scheme, which help evaluate its performance. A regional electricity market of smart grid with three users is considered for our simulation. Each user procures both renewable energy and traditional electricity from the same DSM center of an electricity supplier. The set of users is denoted as $\mathcal{I} = \{1, 2, 3\}$. Note that our findings can apply to any electricity market that serves more than three users; and, we consider three users in our simulation only for a clear presentation of simulation results.

The values of relevant parameters are listed in Tables 1 and 2. In addition, $m_1 \in [2.1, 2.2]$, $m_2, m_3 \in [2, 2.1]$, $a_1, a_3 \in [0.3, 0.31]$, $a_2 \in [0.31, 0.32]$, and $b_1, b_2, b_3 \in [0.08, 0.09]$. The parameter values are particular to this simulation, which may vary according to different local electricity markets or different types of users.

However, the change does not distort our analytic results. Next, we examine various aspects to evaluate the performance of our RTP scheme.

Table 1. Values of relevant parameters.

Parameter	Value
x_{if}^{\min}	0.5
x_{ic}^{\min}	0.5
x_{if}^{\max}	4.2
x_{ic}^{\max}	4
c_c	0.5
c_f	0.3
γ	0.3
P_{DSM}	4

Table 2. Values of μ_i at different time slots.

Time	0:00-9:00; 23:00-24:00	9:00-13:00; 19:00-23:00	13:00-19:00
μ_i	0.18	0.17	0.16

4.2 Simulation Result

Based on the above scenarios and parameters, we first obtain the prices in Stackelberg equilibrium for each time slot (one hour), as is given in Figure 2. We can observe that the peaks of electricity prices are consistent with those in reality, which validates the RTP scheme. The renewable electricity prices and traditional electricity prices have little fluctuation (the difference between peak and valley electricity price is less than 0.04 cents), which indicates that our RTP scheme is in line with practices and ensures the stability of electricity prices. We note that the cost of renewable energy from electricity market is higher than that of traditional electricity, which is due to the uncertainty of renewable energy. Then, the price of renewable electricity is higher, moreover, the users' dissatisfaction decrease faster ($\frac{\partial F_i}{\partial d_i} = 2a_i(d_i - d_i^{\max}) \leq 0$, greater a_i makes F_i drop faster). The DSM center charges user i a higher price for a greater revenue, because a larger value of a_i means that it is easier for user i to feel satisfied with his electricity consumption.

Since the DSM center hopes to not only secure its own revenue but also meet the users' electricity demand, which are both involved in its revenue function, the revenue of the DSM center decreases if the dissatisfaction of users increases. Figure 2 demonstrates that the prices of traditional and renewable energies are both less than their costs, which entices the DSM center to care more about the users' dissatisfaction than its own revenue.

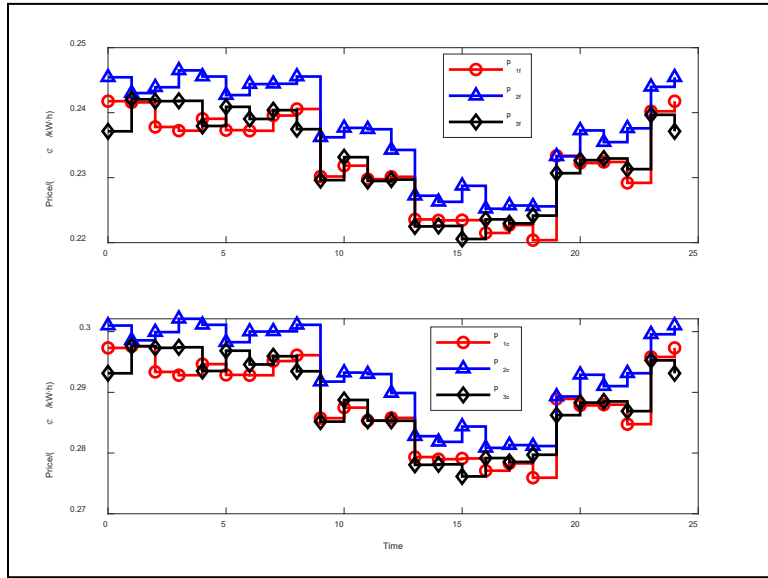


Figure 2. The users' optimal prices at the equilibrium allocated by the DSM center.

Figures 3 and 4 show the hourly optimal renewable/traditional electricity demand and total energy demand of each user in Stackelberg equilibrium under the RTP scheme, respectively. From Figure 3, we learn that the difference among traditional electricity demands of three users is not conspicuous, whereas the difference among renewable electricity demands is larger. As shown in Figure 4, the user with a greater value of a_i consumes more electricity. This occurs because a greater value of a_i makes the value of F_i drop faster, which makes the user more satisfied and thus incentivizes them to buy more electricity. Moreover, the peak and valley distributions of electricity demands are influenced by μ_i . The electricity demands at time slots with a smaller value of μ_i are higher than those with a greater value of μ_i , which is consistent with our assumption in Section 2.1 (i.e., a smaller value of μ_i causes a higher satisfaction for the user, thus achieving more electricity demand).

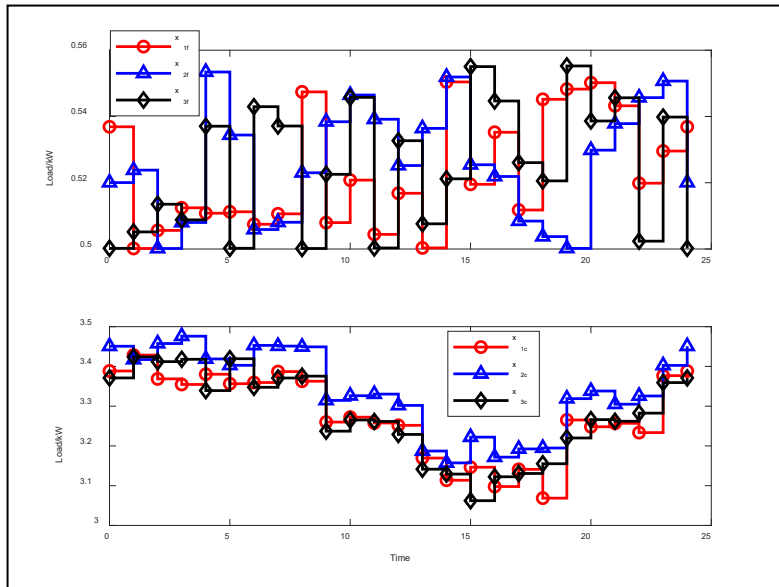


Figure 3. Optimal renewable/traditional electricity demands of the users at the equilibrium.

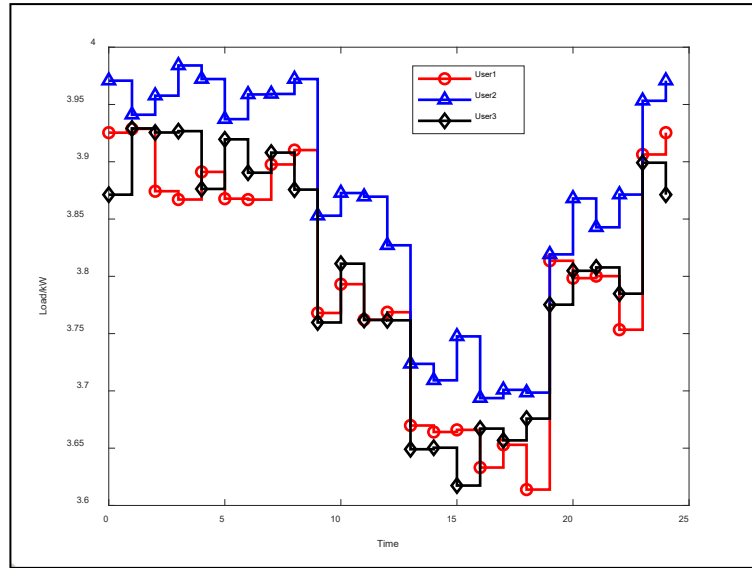


Figure 4. Optimal total electricity demands of the users at the equilibrium.

Figure 5 and Figure 6 show all users' costs and the DSM center's revenue at the equilibrium, respectively. As can be seen from Figure 5, the user with a larger value of m_i incurs a higher cost of consuming electricity. That is, a greater value of m_i means that *ceteris paribus*, using the same amount of electricity generates a higher dissatisfaction and a greater cost. We also note from Figure 6 that the segment change of the DSM center's revenue is obvious, and the peak and valley distributions are synchronized with the optimal electricity prices, electricity demands, and the values of μ_i in Stackelberg equilibrium.

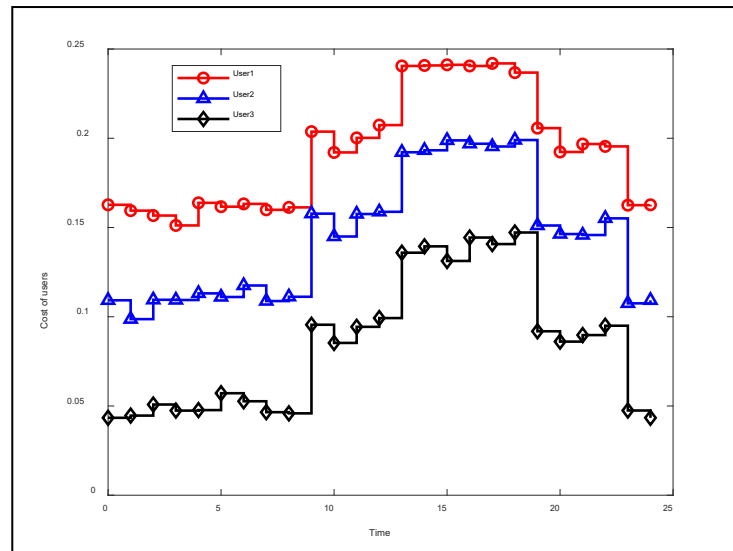


Figure 5. Costs of the users at the equilibrium.

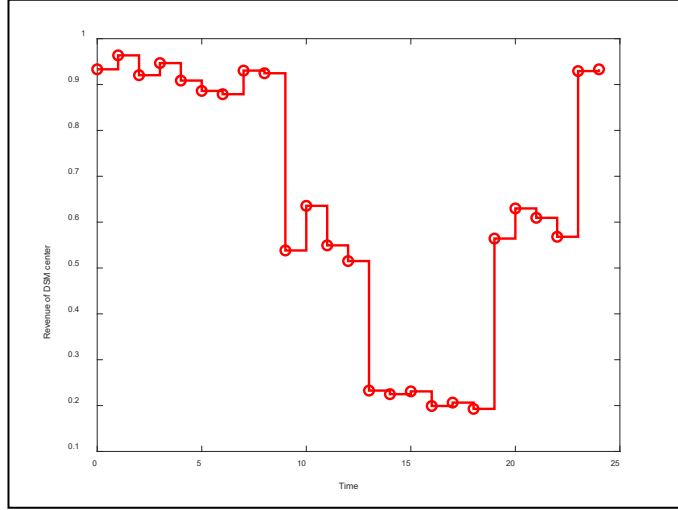


Figure 6. Revenue of the DSM center at the equilibrium.

4.3 Sensitivity Analysis

4.3.1 Sensitivity Analysis about Dissatisfaction Parameter μ_i

We use the parameter values of Table 1 and scenarios in Section 4.1, and set other values of relevant parameters as $m_i = 2$, $b_i = 0.08$, $a_1 = 0.03, a_2 = 0.35, a_3 = 0.25$, $\mu_1 = \mu_2 = 0.16, \mu_3 \in [0, 0.22, 1]$. Next, we investigate the influence of μ_3 on the optimal electricity prices, electricity demands, and the users' costs, as well as the DSM center's revenue.

Figure 7 presents how μ_3 affects the price of renewable/traditional electricity in Stackelberg equilibrium, and Figures 8 and 9 depict the influence of μ_3 on the resulting renewable/traditional electricity demands and total electricity demands in equilibrium. We find that both renewable energy price and traditional electricity price of user 3 increase with μ_3 , whereas the prices for users 1 and 2 do not change. This reflects the fact that user 3 is less satisfied. It thus follows that μ_i has a positive impact on the electricity price. From Figures 8 and 9, we find that the renewable/traditional electricity demand and total electricity demand of user 3 are also increasing with μ_3 . We conclude that when user 3 is more dissatisfied, he is more willing to consume electricity, which may lead to an increase in price. Note that the increase of the optimal price and the resulting electricity demand resulting from the influence of μ_3 do not mean that an increase in electricity demand is caused by an increase in electricity price.

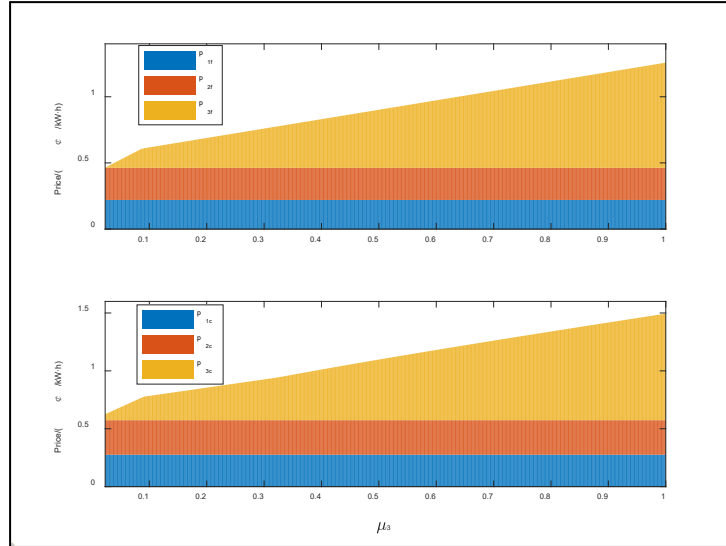


Figure 7. Influence of μ_3 on optimal prices at the equilibrium.

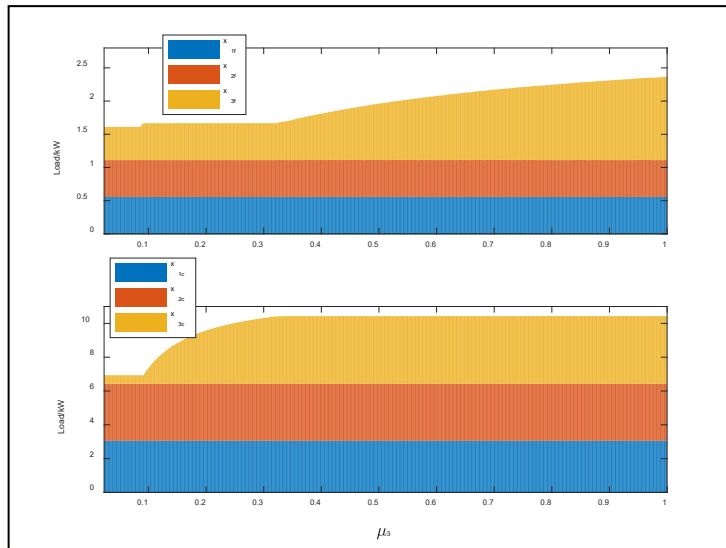


Figure 8. Influence of μ_3 on the users' traditional/renewable electricity demands at the equilibrium.

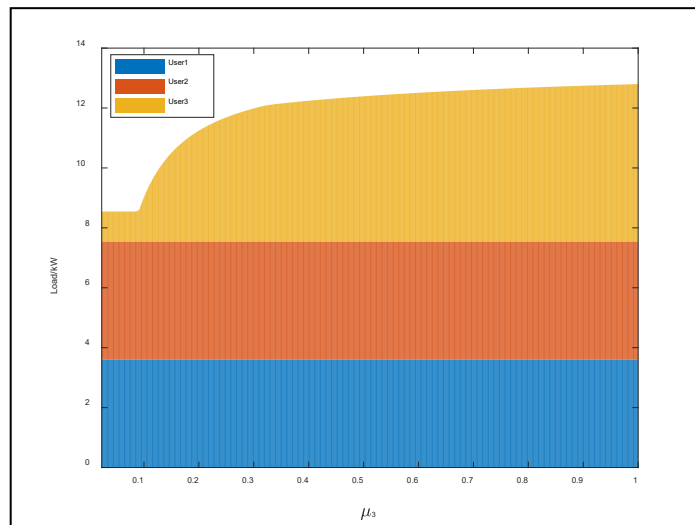


Figure 9. Influence of μ_3 on the users' total electricity demands at the equilibrium.

From Figure 10, we find that the revenue of the DSM center begins with slightly diminishing and then

increases when the value of μ_3 rises. Figure 11 indicates that the cost of user reaches its maximum value first, and then decreases until it reaches its minimum value. Obviously, the dissatisfaction of users has a great influence on the DSM center's revenue. Moreover, since the cost of user is divided into four parts: normal cost, dissatisfaction cost, guilt cost, and unfairness cost, μ_3 mainly affects the second part. A greater value of μ_3 results in a larger value of x_3 (see Figure 9), which makes the user's dissatisfaction higher. When the value of x_3 is sufficiently large, F_3 is negative and the revenue of the DSM center rises firstly. Figure 11 also shows that parameter μ_3 influences all users' costs. The change of μ_3 mainly affects electricity demand of user 3, which also has a direct effect on unfairness costs of users 1 and 2.

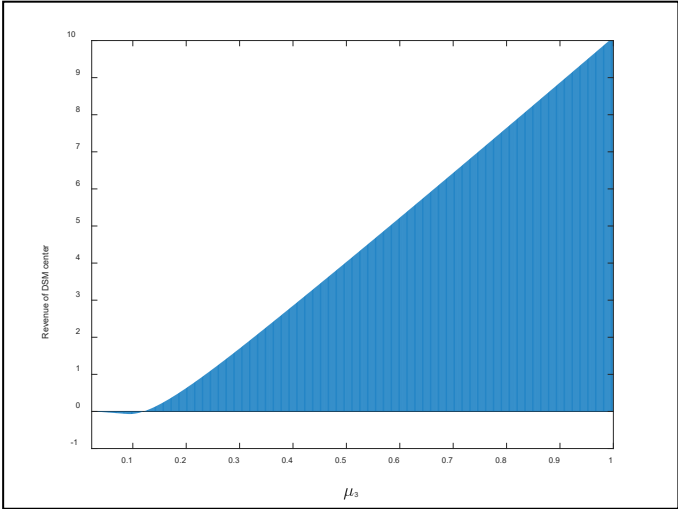


Figure 10. Influence of μ_3 on DSM center's revenue at the equilibrium.

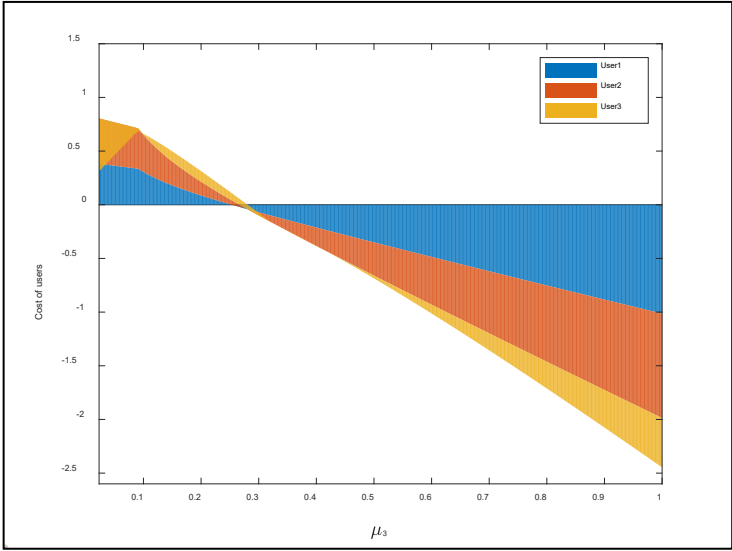


Figure 11. Influence of μ_3 on costs of the users at the equilibrium.

4.3.2 Sensitivity Analysis about Unfair Parameter γ

To investigate the influences of γ on the optimal electricity prices, electricity demands, and costs of the users, as well as the DSM center's revenue, we change the values of parameters μ_i and γ in Section 4.3.1

as $\mu_t = 0.16$, $\gamma \in [0, 1]$.

Figure 12 shows how γ affects the optimal renewable/traditional electricity prices. As Figure 12 exposes, both renewable electricity price and traditional electricity price decrease when the value of γ increases. This reflects the negative effect of γ , which occurs because a larger value of γ brings about a higher unfairness cost if the user consumes the same amount of traditional electricity, and the DSM center thus has to set a higher price to decrease the electricity demand and thus reduce the unfairness cost of the user. As Figures 13 and 14 show, the renewable electricity demand of the user decreases whereas the traditional electricity load increases with γ . The increase of traditional electricity demand is more than the decrease of renewable electricity demand. Due to the concern about fairness, the users feel more unfair with the increase of γ , thus consuming less electricity.

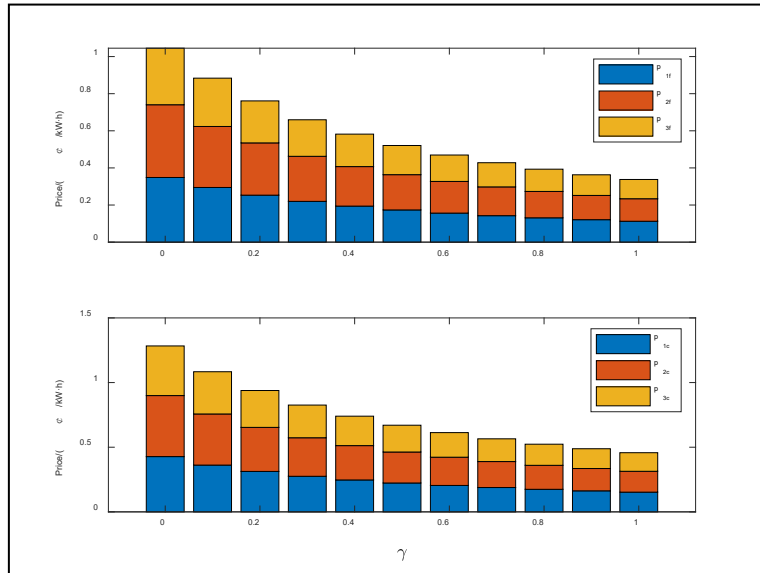


Figure 12. Influence of γ on optimal electricity prices at the equilibrium.

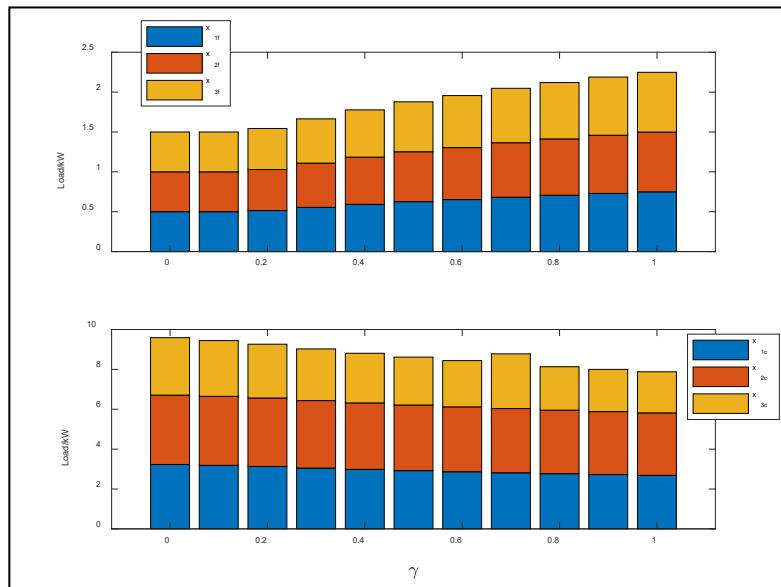


Figure 13. Influence of γ on traditional/renewable electricity demands at the equilibrium.

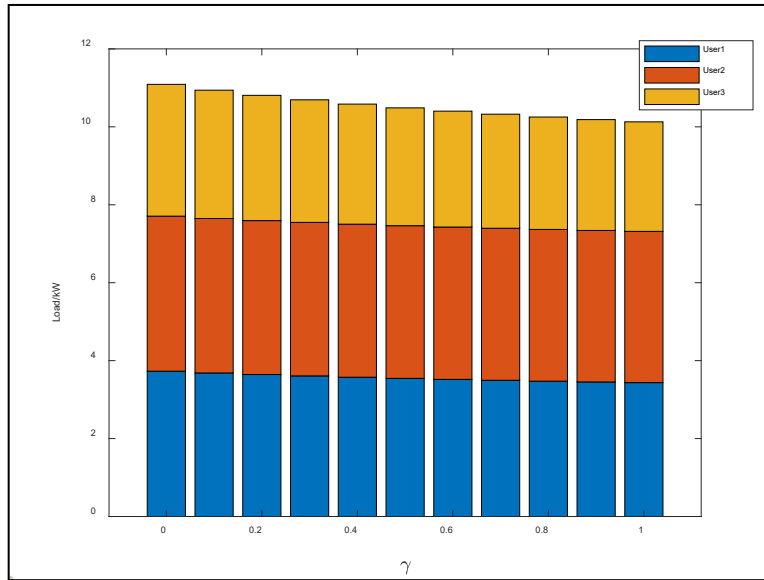


Figure 14. Influence of γ on the users' electricity demands at the equilibrium.

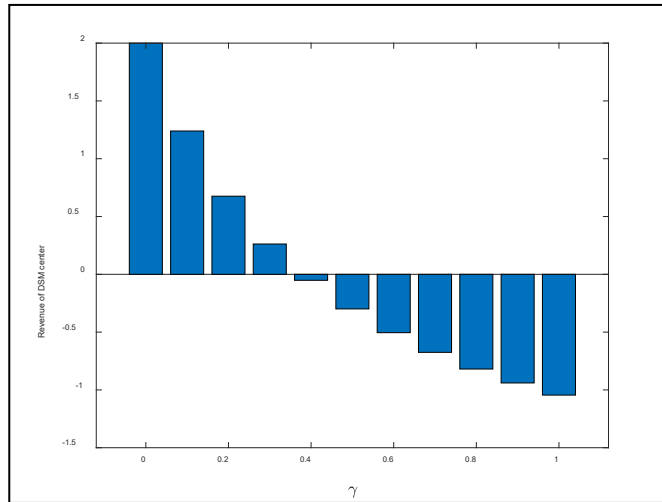


Figure 15. Influence of γ on DSM center's revenue at the equilibrium.

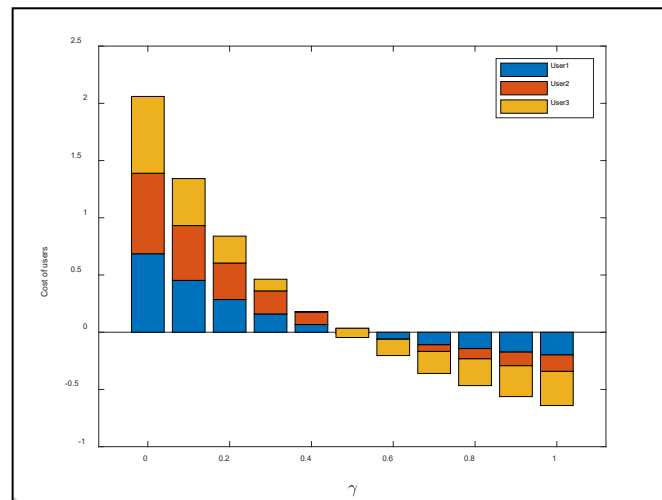


Figure 16. Influence of γ on costs of the users at the equilibrium.

We can learn from Figures 15 and 16 that both the DSM center's revenue and the costs of users decrease as γ increases. In fact, as Figure 14 indicates, the electricity demands of users decrease due to the increase of

γ , which lowers the normal costs of users and the revenue of the DSM center.

4.3.3 Sensitivity Analysis about Guilt Parameter b_i

In this section, we investigate the influence of b_i on the optimal electricity prices, electricity demands, and user costs, as well as the DSM center's revenue. The values of parameter b_i and μ_i in Section 4.3.1 are changed to $\mu_i = 0.16$, $b_1 = b_2 = 0.08$, $b_3 \in [0, 0.55]$.

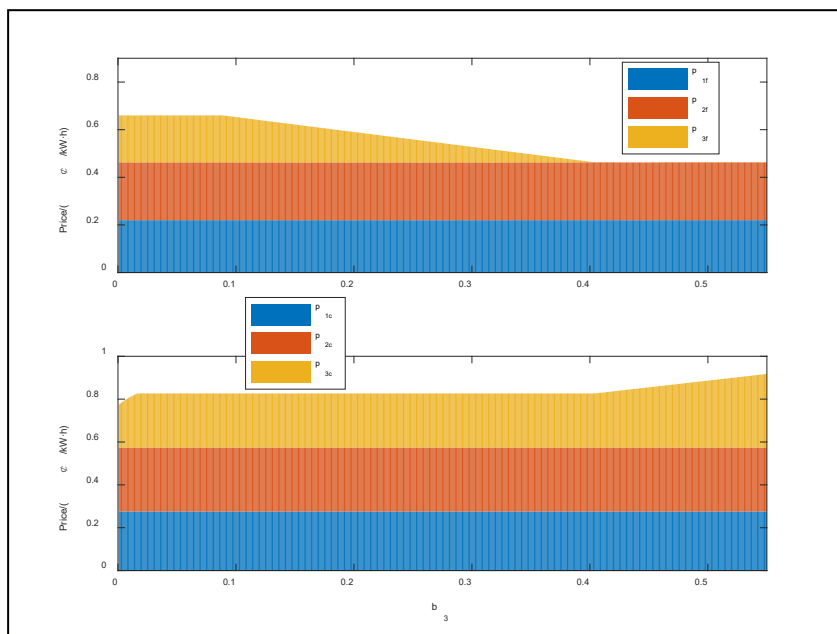


Figure 17. Influence of b_3 on optimal electricity prices at the equilibrium.

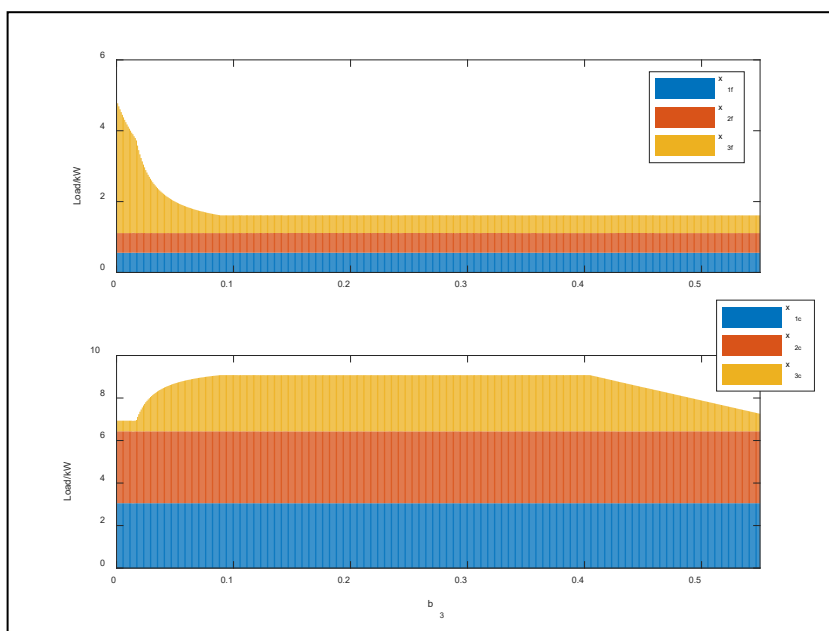


Figure 18. Influence of b_3 on traditional/renewable electricity demands at the equilibrium.

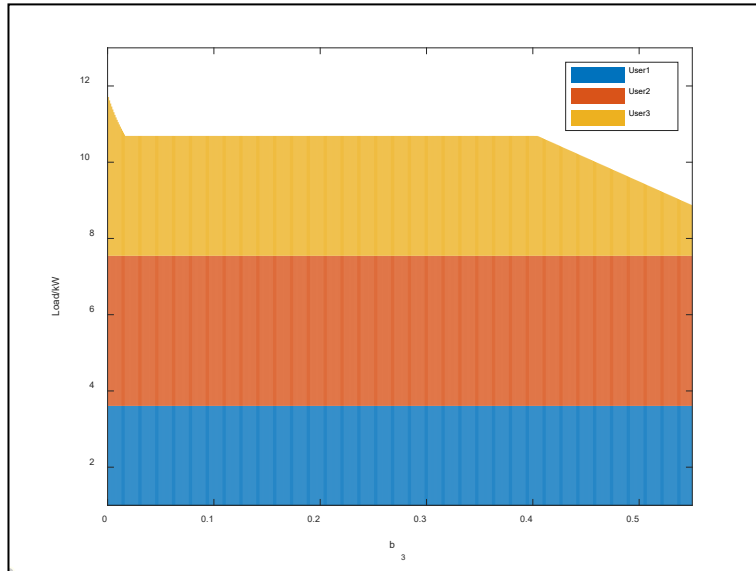


Figure 19. Influence of b_3 on users' total electricity demands at the equilibrium.

According to Figures 17-19, b_3 has a small effect on the optimal electricity prices and demands of other users. However, b_3 has a positive effect on the optimal renewable electricity price of user 3 whereas it has a negative effect on his optimal traditional electricity price and electricity demand. As shown by (2), a consumption of the same amount of electricity can bring about a greater guilt cost if the value of b_i increases. Thus, when the value of b_3 increases, user 3 has to reduce his traditional electricity demand to lower the guilt cost. The DSM center responds by charging user 3 a lower traditional electricity price to entice him to consume more electricity. Meanwhile, user 3 increases his renewable electricity procurement to meet his electricity demand. The DSM center then sets a higher renewable electricity price for a greater revenue. However, the higher price finally leads to a decrease of total electricity demand, which results in the decrease of the DSM center's revenue, as shown in Figure 20. Especially, the DSM center's revenue drops faster when b_3 is larger than 0.4 because the electricity demand of user 3 drops faster at that time. The reason is that when user 3 is more concerned about guilt caused by using traditional electricity, he incurs a larger guilt cost. As Figure 21 reveals, the increase of b_3 can affect not only user 3's cost but also other users' costs. This is due to the fact that the change of b_3 affects the traditional electricity demand x_{3f} of user 3, and then has a direct impact on the unfairness costs of users 1 and 2.

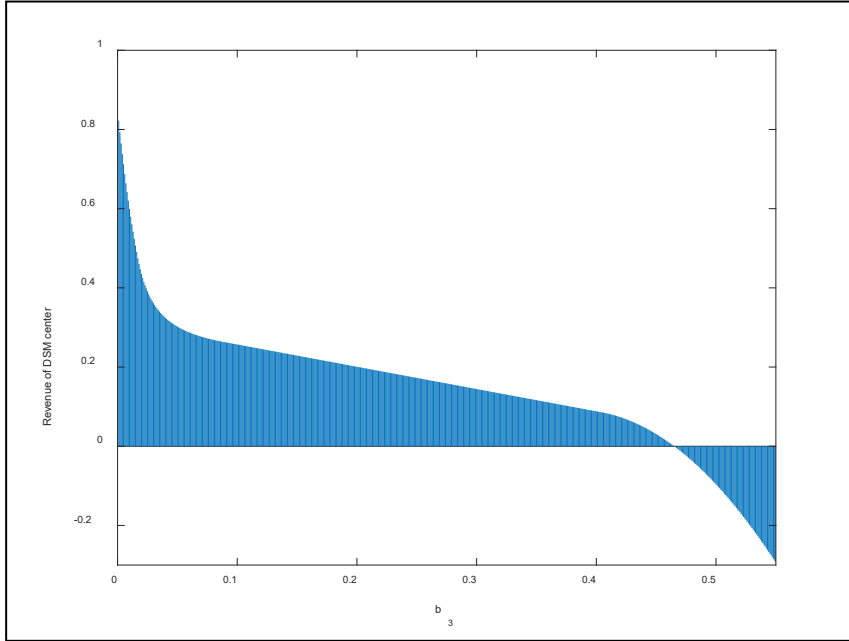


Figure 20. Influence of b_3 on DSM center's revenue at the equilibrium.

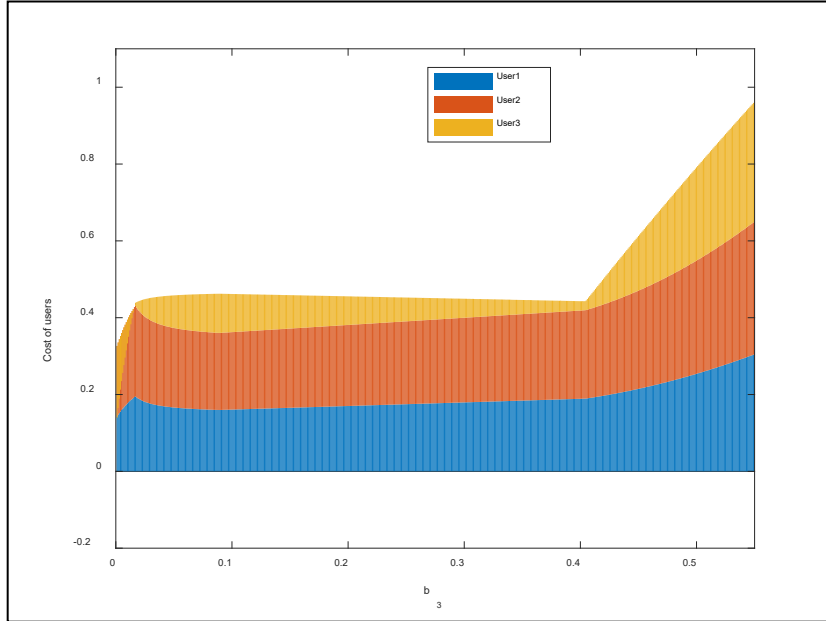


Figure 21. Influence of b_3 on costs of the users at the equilibrium.

4.4 Comparative Analysis

In this section, we evaluate our RTP scheme. From the perspectives of peak load reduction and price stabilization, the CPP scheme relevant to renewable energy can reduce the peak-to-average ratio by up to 22%, while the RTP scheme relevant to renewable energy can reduce the peak-to-average ratio by 26.55% [35]. Moreover, as mentioned in [36], the CPP scheme can considerably reduce the operation cost of microgrids and control the peak-to-valley load difference to 18.29%. The PLP scheme [37] can also obviously cut peaks and fill valleys. However, the above CPP and PLP schemes make it impossible to recover the cost of generating renewable resources. If subsidy incentives are provided, then the optimal results can be changed by the

incentive schemes. Differently, the RTP scheme proposed in this paper can control the peak-to-valley price difference for electricity loads to less than 0.4 cents, which shows that the RTP scheme can ensure the stability of electricity price, so as to maintain the revenues of all market partners.

Similar to this paper, Dai et al. [38] developed a leader-follower game for studying the RTP scheme, and designed two iterative algorithms to solve their game model. They found that the RTP scheme may ease the valley to peak power loads, and the fluctuation of electricity price is within 2.5. However, Dai et al. [38] did not consider the user consumption behavior nor do they conduct research under renewable energy. Observing this, we introduce the dissatisfaction and fairness preference to the utility function of user, and quantify the guilt of user consuming traditional energy. Both renewable and traditional energies are supplied for sustainable development and stable electricity prices. This makes our research results more applicable to real operations. In addition, we obtain analytical Stackelberg equilibrium.

Today, China is staying in a transitional period of energy conversion, and many users are choosing renewable energy in addition to traditional energy. This is consistent with our analysis that involves both renewable energy and traditional energy. Moreover, noting the lack of researches about behavior analysis in the existing RTP scheme, we introduce one more complex personalized electricity consumption behaviors of users to our game model. This breaks through the limitations of existing researches regarding the user behavior [22], which better stabilizes electricity prices and balances supply and demand.

5. Conclusion

This paper proposes a RTP scheme to improve the management of personalized electricity consumptions in a smart grid market where the users consume both traditional and renewable energies. We develop a leader-follower game to study the strategic interaction between the DSM center and residential users, and use the backward induction to obtain the Stackelberg equilibrium. The DSM center maximizes its revenue based on each user's optimal strategy, and each user then reselects his optimal electricity consumption to minimize his cost as a response to the optimal strategy of the DSM center. Finally, we set the scenarios and parameters for our numerical simulations and sensitivity analysis. According to our findings, we summarize the conclusions below.

(1) The dissatisfaction in the users' personalized electricity consumption has a positive effect on the electricity price and a negative effect on the electricity demand. When the dissatisfaction increases faster, then the price of renewable electricity is higher but the revenue of the DSM center changes with a concave appearance.

(2) The fairness preference factor has a negative effect on the DSM center's revenue and two kinds of different electricity prices. Moreover, the more users concern about fairness, the less electricity they consume. When the fairness preference factor increases, traditional electricity load increases whereas renewable energy load decreases. Nonetheless, the increased amount of traditional electricity load is no more than the decreased amount of renewable energy load.

(3) The change of user's guilt factor using traditional electricity has a small effect on the optimal prices and loads of other users, but has a positive effect on his own optimal renewable electricity price and negative effect on his own optimal traditional electricity price and demand. In addition, the increase of guilt factor can affect all users' costs.

The RTP scheme in this paper provides a new research direction for the personalized pricing in an electricity market with renewable energy. First, this model takes into account the dissatisfaction of electricity consumption on the user side, which personally measures the satisfaction of each user on electricity consumption and is involved in the user cost function. Secondly, the users feel guilty when using traditional electricity, which is more in line with the current concept of low-carbon life and sustainable development. Finally, the fairness preference is also considered in our model. Thus, the user's cost function can be analyzed more accurately since one user's electricity cost is affected by the electricity consumption of other users who have personalized electricity consumptions.

This paper involves some factors along with personalized electricity consumptions in the RTP study with traditional and renewable energies. However, it does not consider the instability of renewable energy generation. If the users are more inclined to renewable energy due to the guilt of using traditional electricity, then the large amount of electricity demand may lead to a shortage of supply. How to solve this dilemma and focus on the intermittent and volatile effects of renewable energy generation may be an important subject in our research calendar.

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