Implications for the Role of Retailers in Quality Assurance

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Abstract: We investigate a manufacturer-retailer channel to explore the role of a retailer in assuring the quality of a manufacturer's product as a quality gatekeeper. Such a gatekeeping activity can entail a reduction in the defective rate for consumers, if the retailer charges the manufacturer a penalty for each identified defect that is no smaller than the market penalty for an unidentified defect. As a result of the retailer's gatekeeping, the change in the negotiated wholesale price only depends on the manufacturer's individual benefit, whereas the change in the retailer's optimal retail price is associated with the channel-wide benefit. When the impact of quality relative to retail price on demand is higher, the retailer benefits more from her gatekeeping activity, thus having a greater incentive to take on the quality gatekeeping responsibility. Moreover, the retailer's gatekeeping generates a larger increase in the demand as well as each firm's profit, when the retailer has a stronger relative bargaining power.

Key words: Quality gatekeeping; pricing; game theory; bargaining.

1 Introduction

Quality has been playing an increasingly important role in many industries, and it has been widely treated as one of the key determinants of a firm's business success. In the past two decades, many firms in both the manufacturing and service sectors have paid intensive attention to the improvement in the quality of their products and services. Without a continuous focus on quality improvement, a firm may incur a great loss in profit due to product or service quality problems.

In practice, many manufacturers have spent a considerable effort on quality assurance to prevent defects by adopting the deployment of a quality management system and preventative activities such as failure mode and effects analysis (FMEA). Accordingly, a number of researchers have investigated diverse quality-related problems at the manufacturing level, as indicated by our brief review that is presented later in this section. However, very few extant publications have considered a retailer's effort on quality assurance—which is thought of as the "quality gatekeeping" at the retail level, even though retailers' gatekeeping activities have appeared in reality with a desire to assure product quality and improve channel performance. As Wal-Mart's recent report (2008) indicates, the largest retailer in the world has allocated considerable human resources for its quality assurance program, in which Wal-Mart acts as a quality gatekeeper in its supply chains. The quality gatekeeping effort stems from Wal-Mart's operational strategy that quality is a top priority for the retailer's continuous improvement, as explained by Jeff Macho, the senior vice president and managing director of Wal-Mart Global Procurement. As reported at `Kamcity.com` (2011), Asda Stores Ltd., the second largest supermarket chain in Britain, had invested £27m to significantly increase the frequency of quality control checks in its supply chains and thus enhance quality across its fresh food business. Moreover, Tesco, the largest British supermarket chain, has already constructed a quality assessment center to carry out gatekeeping; see a report by `Telegraph.co.uk` (2008).

Motivated by the above quality-gatekeeping practices, we consider a manufacturer-retailer channel in which a retailer plays the gatekeeping role in quality assurance by identifying defects among a manufacturer's products prior to selling them to consumers. Reeves and Bednar (1994) showed that the most common measurement of the quality of a product is the defective rate, which is defined as the percentage of defects among all products. Since any reduction in the defective rate requires an investment (or, effort) on quality control, the manufacturer may not actually assure zero defective rate, as discussed by Schneiderman (1986). Therefore, it is important to consider the average defective rate as a decision variable in the channel, which is usually negotiated by the manufacturer and the retailer. As observed in practice, the retailer invests in quality assurance as a gatekeeper. Thus, we need to consider the retailer's gatekeeping investment. For similar discussions, see Hwang, Radhakrishnan, and Su (2006) and Zhu, Zhang, and Tsung (2007).

A number of operations (see, e.g., DeFranco 2009) and relevant publications (e.g., Balachandran and Radhakrishnan 2005 and Hwang, Radhakrishnan, and Su 2006) have indicated that, when the retailer identifies a defective product, she immediately returns it to the manufacturer, who then replaces the defect with a good quality product and pays a penalty to the retailer. If the retailer does not recognize a defect but sells it to a consumer, then the consumer will identify the defect and return it to the retailer, who then passes the returned—"unidentified"—defect to the manufacturer. The manufacturer also incurs a penalty cost. For a description of the gatekeeping process, see Section 2, where we discuss a two-stage quality and pricing decision problem. That is, the manufacturer and the retailer bargain over the manufacturer's average defective rate and wholesale price at the first stage, and the retailer decides on her optimal retail price and gatekeeping intensity at the second stage. In Section 3, we analyze the two-stage problem and draw a number of managerial implications. Moreover, in Section 4, we compare the results when the retailer performs quality gatekeeping with those when the retailer does not, in order to explore the impact of the retailer's gatekeeping activity on the channel. Our comparison also exposes some important managerial insights.

In Section 5, we summarize major implications derived from our analysis and conclude that the retailer's quality gatekeeping can help reduce the defective rate for consumers, which becomes more significant when the retailer's bargaining position in her supply chain is stronger. Retailers—especially those with strong powers relative to their suppliers/manufacturers—can benefit more from their quality gatekeeping in supply chains when the impact of quality relative to retail price on demand is higher. Our managerial insights are expected to help practitioners judiciously make their quality and pricing decisions when retailers act as the quality gatekeepers.

Next, we perform a literature review to demonstrate the originality and importance of our research. In the past three decades, several review papers—which mainly include Rao and Monroe (1989), Powell (1995), and Chan and Wu (2002)—have been published to survey publications concerning quality management in specific industries such as food and healthcare. We review major publications that are related to the assurance of product quality, with an emphasis on analytic investigation. We classify our reviewed publications into four categories: (i) quality assurance in a single operations system, (ii) joint pricing and quality assurance decisions in a single system, (iii) quality assurance in a channel with no pricing decision, and (iv) joint pricing and quality assurance decisions in a channel. The publications in Category i include Shapiro (1982), Fine (1986), Schneiderman (1986), Lee and Rose (1987), Ng and Hui (1996), Chan and Spedding (2001), Choo, Linderman, and Schroeder (2007), and Chun (2008), who mainly considered the quality control in a single manufacturing system. The publications in Category ii include Gal-Or (1983), Rao and Monroe (1989), and Wauthy (1996), who analytically or empirically investigated pricing and quality assurance problems for a single firm. In Category iii, the representative publications are Chu and Chu (1994), Zhu, Zhang, and Tsung (2007), Saouma (2008), Chao, Iravani, and Savaskan (2009), and Hsieh and Liu (2010), who did not consider pricing decisions but only focused on quality assurance problems in a channel. In Category iv, a representative publication is Xu (2009), who studied a joint pricing and product quality decision problem in a distribution channel involving a manufacturer and a retailer. Although Xu (2009) also considered a manufacturer-retailer channel, we can note the following significant difference between Xu (2009) and our paper: Xu (2009) investigated a problem in which the manufacturer makes the wholesale pricing and quality decisions for a product, and the retailer determines a retail price. But, we consider a two-stage decision problem in which the manufacturer and the retailer negotiate a defective rate and a wholesale price at the first stage, and the retailer determines her optimal retail price and gatekeeping intensity at the second stage.

The above review shows that our research problem has not been investigated, even though it is important because, in practice, retailers such as Asda, Tesco, and Wal-Mart have been acting as quality gatekeepers in their channels. In online Appendix A, we provide details regarding the retailer's gatekeeping process that is briefly described in Section 2. We relegate the proofs of all lemmas, propositions, theorems, and corollaries to online Appendix B, where the proofs are given in the order that they appear in the main body of our paper. In online Appendix C, we perform numerical experiments with logistic and attraction demand functions to demonstrate the robustness of our major analytical implications that are summarized in Section 5.

2 Preliminaries

The manufacturer sells his products to the retailer at the wholesale price w, and the retailer then serves a market at the retail price p. To assure product quality, the retailer performs her quality gatekeeping activity and reduces the average defective rate faced by consumers. Next, we construct a price- and quality-dependent demand function, and specify a two-stage decision problem.

2.1 Quality Control and the Demand Function

Consumers' purchase decisions are dependent on both the retail price and the quality level that is measured by the average defective rate faced by consumers. Prior to constructing a demand function, we discuss the manufacturer's and the retailer's efforts for quality control. To control product quality, the manufacturer needs to determine an average defective rate $r \in [0, 1]$, which represents his effort on quality control. As discussed by Barros (2013)—the co-founder and former CEO of Contour Cameras, defective units are inevitable for any manufacturer, since it is too costly to identify and remove all defects. Barros also found that, in reality, a product with the defective rate of 2% or less can be deemed to possess a high quality. This is consistent with many practices in which manufacturers with high-quality products usually announce that their *average* defective rates are 2% or less. Barros (2013) observed that some manufacturers may have high defective rates, which could be as high as, e.g., 15%.

The above suggests that the manufacturer is likely to decide a nonzero defective rate. As a result, defects can be sold to the retailer, who may have an incentive to identify some or all defects in order to deliver high-quality products to consumers. The retailer's intention for quality assurance becomes more and more important in today's competitive retail industry. Similar to Asda, Tesco, and Wal-Mart, the retailer adopts the "gatekeeping" approach to inspect the incoming products, which reflects the retailer's effort on quality control. One may find that it would be unrealistic for the retailer to perform her quality inspection for some product categories; for example, the retailer may not open the packages of electronic products for quality inspection. But, the retailer can still inspect a large number of easy-to-check products such as fresh products, apparels, shoes, and some toys, which are usually available for sale at supermarkets like Asda, Tesco, and Wal-Mart.

As argued by Balachandran and Radhakrishnan (2005) and Hwang, Radhakrishnan, and Su (2006), the retailer does not reject any good product, and she may not find all defects in her inspection process but can decrease the defective rate by $1 - y(I)$ from r to $y(I)r$, where $I \in [0, 1]$ denotes the retailer's gatekeeping intensity (i.e., intensity of inspection) for quality improvement (in terms of a reduction in the average defective rate), and $y(I)$ is defined as the percentage of defects unidentified by the retailer such that $0 \leq y(I) \leq 1$. Here, I and $y(I)$ represent the retailer's gatekeeping effort and quality-improvement result, respectively. When the retailer inspects more products, less defects will be unidentified and thus, a smaller percentage of defects will be sold to consumers, i.e., $y'(I) < 0$. In addition, it is naturally more difficult to identify defects when the defective rate is lower, i.e., $y''(I) > 0$. In this paper, to find tractable solutions, we specify the function $y(I)$ as $y(I) = (1 + \mu I)^{-1}$, where $\mu > 0$.

Since the retailer is able to decide on how much effort she will spend on quality assurance, we can consider the gatekeeping intensity I as the retailer's decision variable. For similar concepts such as the quality inspection level, see, e.g., Hwang, Radhakrishnan, and Su (2006).

As a result, the average defective rate for consumers is $y(I)r$. The consumers' aggregate price- and quality-dependent demand function is given as

$$D(p, r, I) = a \exp(-b_1 p - b_2 y(I) r), \tag{1}$$

where $a > 0$; $b_1 > 0$ and $b_2 > 0$ represent the percentage reduction in demand per one-dollar increase in retail price p and that per one-unit increase in average defective rate for consumers $y(I)r$, respectively. Hereafter, we simply call b_1 and b_2 the "pricing effect on demand" and "defective effect on demand." As indicated by a comprehensive review by Huang, Leng, and Parlar (2013), the exponential model structure in (1) has been used in a number of relevant publications (e.g., Jeuland and Shugan 1988, Hanssens and Parsons 1993, Song et al. 2008, and Xu 2009).

2.2 Pricing and Quality Decisions in the Quality Gatekeeping Process

We start by specifying the process for quality gatekeeping at the retail level. As discussed above, even if the retailer performs quality gatekeeping, a defect may not be identified. On average the retailer cannot identify $y(I)rD(p, r, I)$ defects, which are sold to consumers. The consumers find these defects and return them to the retailer, who then passes the defects to the manufacturer. The manufacturer incurs the penalty cost $\delta_1 y(I)rD(p, r, I)$, where $\delta_1 > 0$ denotes the manufacturer's unit penalty cost for each "unidentified" defect. Such a unit penalty cost includes a handling cost, a cost for the loss of his goodwill, and possibly a penalty payment to the consumer who purchased the defect.

In the quality gatekeeping process, the retailer can recognize $(1 - y(I))rD(p, r, I)$ defects (on average) and returns these identified defects to the manufacturer, who then replaces them with good quality products. As the number of defects is usually small, the manufacturer is able to promise such a replacement. Moreover, the manufacturer needs to pay the retailer a penalty fee for each defect identified by the retailer. Since these defects are not sold to any consumer, the manufacturer's unit penalty cost for each identified defect, denoted by δ_2 (where $\delta_2 > 0$), may be different from the unit penalty cost for each unidentified defect δ_1. For details regarding the gatekeeping process, see online Appendix A.

Next, we consider the manufacturer's and the retailer's decision problems. As in practice, the retailer can negotiate with the manufacturer for a "fair" wholesale price. Moreover, according to recent industry reports by KPMG (2010), Peretz (2013), and The Retail Performance Specialists (2013), the two firms usually bargain over the manufacturer's average defective rate, which represents the manufacturer's quality commitment to the retailer and also reflects the retailer's expectation on the quality. The retailer's penalty charge for each identified defect δ_2 should be legitimate and correspond to the defect-related cost incurred by the retailer (Anjoran 2009). As discussed in relevant textbooks by, e.g., Hill (2012), in the process of incoming inspection, if a retailer identifies a defect, then the retailer usually charges its supplier for the time and materials cost of handling the defect. This implies that the value of δ_2 does not result from any negotiation but is mainly based on the cost incurred by the retailer. Such a charge is common in reality. For example, the agreement between Wal-Mart and its suppliers (2003) indicates that the suppliers are responsible for testing charges and the expenses for defect returns. The company iPEK, an international pipeline inspector seller, states in its policy (2012) that its suppliers should reimburse all costs related to defects, which include costs and expenses resulting, directly or indirectly, from defects, as well as the costs for inspections of incoming goods. After the negotiation of the wholesale price and the average defective rate, the retailer determines her optimal gatekeeping intensity and retail price.

According to our above discussion, we can specify a two-stage game problem as described below.

- **Stage 1: Negotiated Wholesale Pricing and Average Defective Rate Decisions in a Two-Player Bargaining Game.** The manufacturer and the retailer negotiate the manufacturer's average defective rate and wholesale price, both of which are characterized by a generalized Nash bargaining (GNB) scheme. The manufacturer and the retailer have relative bargaining powers $\beta \in (0,1)$ and $1-\beta$, respectively.

- **Stage 2: Optimal Retail Pricing and Gatekeeping Intensity Decisions by the Retailer.** The retailer determines an optimal retail price and an optimal gatekeeping intensity that maximize her profit.

3 Analysis of the Two-Stage Decision Problem

We use the backward induction approach to find the manufacturer's and the retailer's decisions in the two-stage game problem that is described in Section 2.2. We begin by developing the retailer's profit function and deriving her optimal retail price and gatekeeping intensity given a wholesale price w and a defective rate r.

3.1 The Optimal Retail Price and Gatekeeping Intensity

The retailer achieves the sale profit $(p-w)D(p,r,I)$, and incurs the cost $\tau I D(p,r,I)$ for her gatekeeping activity, where $\tau > 0$ denotes the unit gatekeeping cost. In addition, she receives the manufacturer's penalty payment $\delta_2(1-y(I))rD(p,r,I)$ for the identified defects. Using the above, we can calculate the retailer's expected profit $\pi_R(p,I)$—when she takes on the gatekeeping responsibility—as

$$\begin{aligned}\pi_R(p,I) &= [p - w - \tau I + \delta_2(1-y(I))r]D(p,r,I) \\ &= a[p - w - \tau I + \delta_2 r(1-y(I))]\exp(-b_1 p - b_2 y(I) r). \qquad (2)\end{aligned}$$

Lemma 1 Given the values of w and r, the retailer's optimal gatekeeping intensity $I^*(r)$ is only dependent on the average defective rate r. It can be uniquely obtained by solving the following equation for I: $(b_1\delta_2 + b_2)y'(I)r = -\tau b_1$. The optimal retail price $p^*(w,r)$ can be uniquely determined as

$$p^*(w,r) = \frac{1 + b_1[w + \tau I^*(r) - \delta_2 r(1 - y(I^*(r)))]}{b_1}. \quad \blacksquare \tag{3}$$

Using $p^*(w,r)$, we find that

$$p^*(w,r) - w = \frac{1}{b_1} + \kappa_R, \tag{4}$$

where $\kappa_R \equiv \tau I^*(r) - \delta_2 r(1 - y(I^*(r)))$ means the retailer's net unit gatekeeping cost. We note that $1/b_1$ is the inverse of the pricing effect on demand, which can be viewed as an increase in price that results in one percentage demand reduction or simply, the price increase for 1% demand reduction. Thus, the retailer's profit margin is equal to the price increase for 1% demand reduction plus the retailer's net unit gatekeeping cost. Substituting $p^*(w,r)$ and $I^*(r)$ into $\pi_R(p,I)$ in (2), we have

$$\begin{aligned} \pi_R(w,r) &\equiv \pi_R(p^*(w,r), I^*(r)) \\ &= \frac{a}{b_1}\exp(-1 - b_1[w + \tau I^*(r) - \delta_2 r(1 - y(I^*(r))] - b_2 y(I^*(r))r). \end{aligned} \tag{5}$$

3.2 The Negotiated Wholesale Price and Average Defective Rate

At the first stage, the manufacturer and the retailer bargain over the wholesale price and the average defective rate. We use the cooperative game concept of GNB scheme (Nash 1950) to characterize the negotiated wholesale price and defective rate. The GNB scheme represents a *unique* bargaining solution that can be obtained by solving the following maximization problem: $\max_{s_1,s_2}(s_1 - s_1^0)^\beta(s_2 - s_2^0)^{1-\beta}$, s.t. $s_1 \geq s_1^0$ and $s_2 \geq s_2^0$, where s_i and s_i^0 correspond to player i's profit and security level (a.k.a. status quo point), respectively, for $i = 1, 2$; β and $(1 - \beta)$ are player 1's and player 2's *relative* bargaining powers.

In our bargaining problem, we, w.l.o.g., assume that the manufacturer and the retailer are player 1 and player 2, respectively. Note that neither the manufacturer nor the retailer can gain any profit if they do not reach an agreement on the wholesale price and the average defective rate. Thus, the status quo point is $(s_1^0, s_2^0) = (0, 0)$. To derive the GNB-based wholesale price and defective rate for the two-player cooperative game, we need to solve the maximization problem that $\max_{w,r} \Omega(w, r) \equiv (\pi_M(w, r))^{\beta}(\pi_R(w, r))^{1-\beta}$, where $\pi_M(w, r)$ denotes the manufacturer's profit.

Next, we construct the profit function for the manufacturer. The manufacturer can achieve the sale profit $(w - c)D(p^*(w, r), r, I^*(r))$, where c is the manufacturer's unit acquisition cost. The manufacturer's penalty cost is $[\delta_1 y(I^*(r))r + \delta_2(1 - y(I^*(r)))r]D(p^*(w, r), r, I^*(r))$. For quality control at the manufacturing level, the manufacturer incurs the unit cost of assuring the average defective rate r, which is denoted by $C(r)$. The manufacturer should exert a larger effort to obtain a smaller defective rate. In addition, the manufacturer's marginal cost $C'(r)$ should be increasing in r, because it is more difficult for the manufacturer to further reduce a smaller defective rate r. The above implies that the cost $C(r)$ should be a decreasing, convex function of the defective rate r, i.e., $C'(r) < 0$ and $C''(r) > 0$. Hence, we can write the manufacturer's profit function as

$$\begin{aligned} \pi_M(w, r) = {} & a\{w - c - [\delta_1 y(I^*(r))r + \delta_2(1 - y(I^*(r)))r] - C(r)\} \\ & \times \exp(-1 - b_1[w + \tau I^*(r) - \delta_2 r(1 - y(I^*(r)))] - b_2 y(I^*(r))r). \end{aligned} \quad (6)$$

Since the GNB objective function $\Omega(w, r) = (\pi_M(w, r))^{\beta}(\pi_R(w, r))^{1-\beta}$ is complicated, we will find the negotiated wholesale price and average defective rate by using three steps. (i) Given a value of r, we maximize $\Omega(w, r)$ to find the negotiated wholesale price $w^*(r)$. (ii) We substitute $w^*(r)$ into $\Omega(w, r)$ and maximize $\Omega(w^*(r), r)$ to find the negotiated average defective rate r^*. (iii) We obtain the negotiated wholesale price $w^* = w^*(r^*)$. Next, we start with the first step.

Lemma 2 Given a value of r, the negotiated wholesale price $w^*(r)$ can be uniquely obtained as

$$w^*(r) = \frac{\beta}{b_1} + c + C(r) + [\delta_1 y(I^*(r))r + \delta_2(1 - y(I^*(r)))r]. \quad \blacksquare \tag{7}$$

In the second step, we obtain the negotiated average defective rate r^* by solving the problem:

$$\max_r \Pi(r) \equiv \Omega(w^*(r), r) = (\pi_M(w^*(r), r))^\beta (\pi_R(w^*(r), r))^{1-\beta} = \frac{\beta^\beta a}{b_1}\Pi_0(r), \tag{8}$$

where $\Pi_0(r) \equiv \exp(-[1 + \beta + b_1(\tau I^*(r) + c + C(r) + \delta_1 y(I^*(r))r)] - b_2 y(I^*(r))r)$.

Lemma 3 If $\bar{C}(r) \equiv C'(r) + rC''(r) > 0$, then $\Pi(r)$ in (8) is strictly concave in r. $\blacksquare$

Next, we investigate the condition $\bar{C}(r) > 0$ in the above lemma. Letting $r(C)$ denote the inverse function of $C(r)$, i.e., $r(C) \equiv C^{-1}(r)$, we have

$$\bar{C}(r) = C'(r) + rC''(r) = \frac{(r'(C))^2 - r(C)r''(C)}{(r'(C))^3}.$$

Since $r(C)$ is the inverse function of $C(r)$ and $C(r)$ is a decreasing, strictly convex function (i.e., $C'(r) < 0$ and $C''(r) > 0$), $r(C)$ is also a decreasing, strictly convex function (i.e., $r'(C) < 0$ and $r''(C) > 0$). It thus follows that $\bar{C}(r) > 0$ if and only if $(r'(C))^2 - r(C)r''(C) < 0$, which must be satisfied when $r(C)$ is a strictly *logarithmically-convex* function because of the following argument. If $r(C)$ is a strictly log-convex function, then

$$\frac{\partial^2(\ln r(C))}{\partial C^2} = \frac{r''(C)r(C) - (r'(C))^2}{(r(C))^2} > 0,$$

or, $\bar{C}(r) = C'(r) + rC''(r) > 0$. That is, the strict log-convexity of $r(C)$ can result in the condition $\bar{C}(r) > 0$. Because $r(C)$ is the manufacturer's average defective rate such that $0 \leq r(C) \leq 1$, $r(C)$ is a decreasing, strictly convex function of C in the range $[0, 1]$. If $r(C)$ possesses the property of strict log-convexity, then $\bar{C}(r)$ must be positive. Next, we examine the log-convexity property of common function forms that are decreasing, strictly convex functions in the range $[0, 1]$.

The following common function forms are decreasing, strictly convex functions in the range $[0, 1]$, and are also strictly log-convex.

1. **Power Functions:** $r(C) = (z_1 + z_2C)^{-\rho}$, where $z_1 \geq 1$, $z_2 > 0$, and $\rho > 0$.
2. **Composite Exponential Functions** (i.e., exponential functions with a functional argument that is a positive, increasing, and strictly concave function): $r(C) = \exp(-\rho x(C))$, where $\rho > 0$, $x(C) > 0$, $x'(C) > 0$, and $x''(C) < 0$. The common mathematical models of $x(C)$ include:
   (a) the logarithm function $x(C) = \ln(z_1 + z_2C)^{\hat{\rho}}$ or $x(C) = \ln(z_1 + z_2C^{\hat{\rho}})$, where $z_1 \geq 1$, $z_2 > 0$, and $\hat{\rho} > 0$;
   (b) the power function $x(C) = \hat{\rho}C^{\nu}$, where $\hat{\rho} > 0$ and $0 < \nu < 1$.
3. **Density Function of Gamma Distribution:**

$$r(C) = \frac{\rho^{-\nu}C^{\nu-1}\exp(-C/\rho)}{\Gamma(\nu)},$$

   where $0 < \nu < 1$, $\rho > 0$, and $\Gamma(\nu)$ is the Gamma function, i.e., $\Gamma(\nu) = \int_0^{+\infty} t^{\nu-1}\exp(-t)dt$.
4. **Density Function of Weibull Distribution:** $r(C) = \nu\rho^{-\nu}C^{\nu-1}\exp(-(C/\rho)^{\nu})$, where $0 < \nu < 1$, and $\rho > 0$.

The above indicates that common forms of the function $r(C)$ are strictly log-convex. It is thus reasonable to assume in this paper that $r(C)$ is strictly log-convex. Under this assumption, $\bar{C}(r) > 0$. The negotiated average defective rate r^* uniquely satisfies the following equation:

$$[b_1(\delta_1 + \delta_2) + 2b_2]y(I^*(r)) = -2b_1C'(r). \tag{9}$$

Using Lemma 1 and equation (9), we can obtain the negotiated average defective rate r^* and the optimal gatekeeping intensity I^* by solving the following equations for (r, I):

$$\begin{cases} (b_1\delta_2 + b_2)y'(I)r = -\tau b_1, \\ [b_1(\delta_1 + \delta_2) + 2b_2]y(I) = -2b_1 C'(r). \end{cases} \tag{10}$$

The negotiated wholesale price $w^*(r^*)$ and the optimal retail price $p^*(w^*, r^*)$ can be obtained by replacing r and I in (7) and (3) with (r^*, I^*). In addition, substituting w^* and r^* into $\pi_R(w, r)$ in (5) and $\pi_M(w, r)$ in (6) yields the two firms' maximum profits.

Proposition 1 When the retailer and the manufacturer make their decisions as given in Lemmas 1 and 2 and equation (9), the ratio between the retailer's and the manufacturer's unit quality costs and the ratio between their effort elasticities of quality-control effectiveness satisfy the following equation:

$$\frac{1}{2}\left(1 + \frac{b_1\delta_1 + b_2}{b_1\delta_2 + b_2}\right)\frac{\tau I^*}{C(r^*)} = \frac{e_I}{e_r}, \quad \text{where } e_I \equiv \frac{y'(I^*)}{y(I^*)/I^*} \quad \text{and} \quad e_r \equiv 1 \bigg/ \frac{C'(r^*)}{C(r^*)/r^*}\,. \tag{11}$$

In (11), e_I and e_r are two elasticities that measure the impact of the two firms' quality-assurance efforts on the control of defects. ■

The above proposition indicates that the manufacturer's effectiveness in quality assurance influences the retailer's quality gatekeeping activity. If the manufacturer's effectiveness is sufficiently high, then the retailer may only spend a small effort or may not need to take on the quality gatekeeping responsibility. In a recent paper, Yan, Zhao, and Tang (2015) investigated a buyer's choice problem in which the buyer needs to select either quality requirement strategy (QR) or quality promise strategy (QP) for the quality contracts with its suppliers. The buyer's primary motivation for such a choice problem is concerned with who (the buyer or its suppliers) can improve quality and who can improve quality more efficiently when both the buyer and the supplier can make quality efforts. Yan, Zhao, and Tang (2015) found that QP can always induce the suppliers to exert their greatest quality efforts whereas QR limits their efforts. Different from Yan, Zhao, and Tang (2015) who assumed that the suppliers determine their quality efforts by themselves, we consider two firms' quality efforts and effectiveness in quality control when the two firms negotiate both the average defective rate (which represents the manufacturer's quality effort) and the wholesale price. Moreover, the retailer's joint pricing and gatekeeping intensity decisions also distinguish our paper from Yan, Zhao, and Tang (2015).

Corollary 1 The manufacturer gains a profit that is no more than the retailer's profit. Moreover, the ratio of the manufacturer's profit to the retailer's profit is increasing in the manufacturer's relative bargaining power β. ■

According to the above corollary, we conclude that even if the manufacturer has a higher bargaining power than the retailer (i.e., $\beta > 0.5$), the manufacturer still obtains a smaller profit. That is, in the channel, the retailer holds a more "beneficial" position than the manufacturer.

3.3 Impact of the Unit Penalty Costs and the Bargaining Power

We investigate the impact of the unit penalty costs δ_i $(i = 1, 2)$ and the manufacturer's relative bargaining power β on the two firms' quality and pricing decisions, the demand, and their profits.

3.3.1 The Impact of the Unit Penalty Costs δ_i $(i = 1, 2)$

We investigate how the unit penalty costs δ_i $(i = 1, 2)$ incurred by the manufacturer for each defect influence the two firms' decisions, demand, and profits.

The Impact of δ_i $(i = 1, 2)$ on the Two Firms' Quality Decisions We first consider the impact of δ_i on (r^*, I^*).

Proposition 2 As the manufacturer's penalty cost for each unidentified defect δ_1 increases, both the manufacturer's average defective rate r^* and the retailer's optimal gatekeeping intensity I^* will decrease. Moreover, r^* and I^* are decreasing in the manufacturer's penalty payment to the retailer for each identified defect δ_2. The average defective rate for consumers is also decreasing in both δ_1 and δ_2. $\blacksquare$

The above proposition indicates that, as the manufacturer's penalty cost for a defect returned by a consumer is increased, the manufacturer will with spend a greater effort to reduce his average defective rate. Consequently, the retailer will reduce her gatekeeping intensity. In addition, when the retailer charges a higher penalty, the manufacturer will spend more effort on quality assurance. Thus, the retailer's penalty charge can effectively improve the manufacturer's quality control effort. As the defective rate decreases, the retailer reduces her gatekeeping intensity. Moreover, a larger penalty from the market or the retailer for each defect leads to a lower average defective rate for consumers.

The Impact of δ_1 on the Pricing Decisions and the Demand We first investigate the impact of the unit penalty cost for each unidentified defect δ_1 on the negotiated wholesale price $w^*(r^*)$.

Proposition 3 The negotiated wholesale price is increasing in δ_1 if and only if

$$\delta_2 < \eta \equiv \frac{b_2 y(I^*) + b_1(\bar{C}(r^*) + r^* C''(r^*))}{b_1(1 - y(I^*))}, \tag{12}$$

where $\eta > 0$. $\blacksquare$

An interesting observation from the above proposition is that the impact of the unit penalty cost for each unidentified defect δ_1 on the negotiated wholesale price depends on the retailer's charge for each identified defect δ_2. If the retailer's penalty charge δ_2 is larger than the cutoff level η, then the manufacturer and the retailer will negotiate for a lower wholesale price in response to an increase in δ_1; otherwise, they will increase the wholesale price as δ_1 increases. Moreover, even if the manufacturer's penalty to the retailer for each identified defect is higher than his penalty for each unidentified defect (i.e., $\delta_2 > \delta_1$), the negotiated wholesale price may still be increasing in δ_1, which would happen when $\delta_1 < \delta_2 < \eta$.

Next, we investigate the impact of δ_1 on the optimal retail price $p^*(w^*, r^*)$ in (3).

Proposition 4 The optimal retail price is increasing in the unit penalty cost for each unidentified defect δ_1. As a result, the demand is reduced. $\blacksquare$

When the manufacturer experiences a larger penalty due to a defect sold to a consumer, the retailer responds by increasing her retail price (even though the negotiated wholesale price may not be raised, as shown in Proposition 3). Proposition 2 indicates that a higher penalty from the market induces the manufacturer to spend more effort on quality assurance and improve the product quality. This helps mitigate the impact of an increase in the retail price on consumer purchase. Thus, the retailer has an intention to raise her retail price, although the demand is reduced as shown in Proposition 4.

The Impact of δ_2 on the Pricing Decisions and the Demand We consider how the retailer's penalty charge for each identified defect δ_2 influences the negotiated wholesale price and the optimal retail price as well as the demand.

Proposition 5 If $\delta_2 < \eta$, where η is defined as in (12), then the negotiated wholesale price is increasing in the retailer's penalty charge for each identified defect δ_2. Moreover, the optimal retail price is increasing in δ_2, but the demand is independent of δ_2. $\blacksquare$

Propositions 3, 4, and 5 indicate that the market penalty δ_1 and the retailer's penalty charge δ_2 have a similar impact on the negotiated wholesale price and the optimal retail price. However, the retailer's penalty charge does not change the demand, which is different from the impact of the market penalty on the demand. This occurs because, when the penalty δ_2 rises, the retail price increases and the quality also improves. As a result, the negative impact of retail price increase on the demand cancels out the positive impact of quality improvement on the demand.

The Impact of δ_i $(i = 1, 2)$ on the Two Firms' Profits We investigate the effects of δ_i on $\pi_R(w^*, r^*)$ and $\pi_M(w^*, r^*)$.

Proposition 6 Both the manufacturer's and the retailer's profits are decreasing in the unit penalty cost for each unidentified defect δ_1, but are independent of the unit penalty cost for each identified defect δ_2. $\blacksquare$

The above proposition reveals that a higher market penalty for poor quality can further decrease the supply chain profitability. However, the retailer's penalty charge δ_2 does not influence the two firms' profits, because its relevant impact is offset by the effects of the price changes generated by varying the value of δ_2.

3.3.2 The Impact of the Manufacturer's Relative Bargaining Power β

We examine how the manufacturer's relative bargaining power β affects the manufacturer's and the retailer's pricing and quality decisions. As shown in (10), the two firms' quality decisions (r^* and I^*) are independent of β. Such an interesting result happens because the two firms' *relative* bargaining powers mainly influence the wholesale price and thus affect the allocation of the channel-wide profit between the two firms, whereas the two firms' quality decisions mainly influence consumers' purchase behaviors.

The Impact of β on the Negotiated Wholesale Price, the Optimal Retail Price, and the Demand We start with the impact of β on the pricing decisions and the demand.

Proposition 7 When the retailer acts as the quality gatekeeper in the channel, an increase in the manufacturer's power relative to the retailer entails an increase in the negotiated wholesale price and the optimal retail price; but, it leads to a decrease in the demand. $\blacksquare$

When the manufacturer possesses a stronger relative power, he can obtain a larger share of the channel-wide profit through bargaining for a higher wholesale price. As a response, the retailer would set a higher retail price to mitigate her profit loss resulting from the increase in wholesale price. But, this leads fewer consumers to buy from the retailer.

The Impact of β on the Two Firms' Profits We consider the impact of β on $\pi_R(w^*, r^*)$ and $\pi_M(w^*, r^*)$.

Proposition 8 As the manufacturer's relative bargaining power β increases, the manufacturer's profit is increased whereas the retailer's profit is reduced, and the retailer's profit reduction is larger than the manufacturer's profit increase. ■

The above proposition means that the manufacturer benefits from his stronger position in negotiating the wholesale price, but both the retailer and the channel are worse off.

3.3.3 Summary of the Major Results

We have analyzed the impact of δ_1, δ_2, and β on the two firms' pricing and quality decisions, demand, and profits. We summarize the major analytic results in Table 1.

	δ_1	δ_2	β
r^*	$\downarrow$	$\downarrow$	—
I^*	$\downarrow$	$\downarrow$	—
$w^*(r^*)$	$\uparrow$ iff $\delta_2 < \eta$	$\uparrow$ if $\delta_2 < \eta$	$\uparrow$
$p^*(w^*, r^*)$	$\uparrow$	$\uparrow$	$\uparrow$
$D(p^*(w^*, r^*), r^*, I^*)$	$\downarrow$	—	$\downarrow$
π_M	$\downarrow$	—	$\uparrow$
π_R	$\downarrow$	—	$\downarrow$
$\pi_M + \pi_R$	$\downarrow$	—	$\downarrow$

Table 1: The impact of the parameters δ_1, δ_2, and β on the two firms' pricing and quality decisions, demand, and profits. The mark "$\downarrow$"/"$\uparrow$"/"—" indicates that the corresponding decision, demand, or profit is decreasing in/increasing in/independent of the corresponding parameter.

4 The Impact of the Retailer's Quality Gatekeeping Activity

To investigate the impact of the retailer's quality gatekeeping activity on the channel, we first find the two firms' decisions when the retailer does not perform quality gatekeeping, and then compare them with those obtained in Section 3.

4.1 Analysis of the Two-Stage Problem with No Quality Gatekeeping at the Retail Level

Assuming that the retailer does not participate in quality assurance, we analyze a two-stage decision problem where the manufacturer and the retailer first negotiate the average defective rate $\hat{r}^*$ and the wholesale price $\hat{w}^*$; then, the retailer determines her optimal retail price $\hat{p}^*$. Hereafter, we use the hat symbol ($\widehat{\ }$) to indicate the results for the case with no quality gatekeeping, to distinguish them with those in Section 3. We begin by finding the optimal pricing decision for the retailer, who satisfies the demand $\hat{D}(p, r) = a\exp(-b_1 p - b_2 r)$ and obtains the profit $\hat{\pi}_R(p) = \hat{D}(p, r)(p - w)$.

Lemma 4 When the retailer does not act as the quality gatekeeper, the optimal retail price $\hat{p}^*$ can be uniquely obtained as $\hat{p}^*(w, r) = (1 + b_1 w)/b_1$. ■

In the absence of quality gatekeeping at the retail level, the manufacturer's profit is calculated as $\hat{\pi}_M(w, r) = \hat{D}(\hat{p}^*(w, r), r)(w - c - \delta_1 r - C(r))$. We still use the GNB scheme to find the negotiated wholesale price $\hat{w}^*$ and defective rate $\hat{r}^*$.

Lemma 5 When the retailer does not act as the quality gatekeeper, the negotiated average defective rate $\hat{r}^*$ can be uniquely obtained by solving the equation $b_1(\delta_1 + C'(r)) + b_2 = 0$; the negotiated wholesale price $\hat{w}^*$ can be uniquely determined as

$$\hat{w}^* = \frac{\beta}{b_1} + c + \delta_1 \hat{r}^* + C(\hat{r}^*). \quad ■ \tag{13}$$

The optimal retail price can be re-written as

$$\hat{p}^*(\hat{r}^*) \equiv \hat{p}^*(\hat{w}^*, \hat{r}^*) = \frac{1 + \beta}{b_1} + c + \delta_1 \hat{r}^* + C(\hat{r}^*).$$

We also find that $\hat{p}^*(\hat{r}^*) - \hat{w}^* = 1/b_1$, which means that, in the absence of the retailer's quality gatekeeping, the retailer's profit margin is equal to the price increase for 1% demand reduction, similar to our argument on equation (4). A comparison between (4) and the above equation shows that the retailer's profit margin for the gatekeeping case is her profit margin for the case of no gatekeeping plus her net unit gatekeeping cost.

Using $\hat{r}^*$ in Lemma 5, we compute the demand as $\hat{D}(\hat{p}^*(\hat{r}^*), \hat{r}^*) = a \exp(-1 - \beta - b_1(c + \delta_1 \hat{r}^* + C(\hat{r}^*)) - b_2 \hat{r}^*)$.

Proposition 9 Regardless of whether the retailer takes on the gatekeeping responsibility, the retailer's stronger relative bargaining power can always increase the demand, the retailer's profit, and the channel-wide profit but decrease the manufacturer's profit. ■

4.2 The Quality and Pricing Decisions with vs. without the Retailer's Quality Gatekeeping

To examine the impact of the retailer's quality gatekeeping on the channel, we compare the manufacturer's and the retailer's quality and pricing decisions when the retailer takes on the gatekeeping responsibility with those when the retailer does not.

4.2.1 The Impact of the Retailer's Gatekeeping on the Manufacturer's Average Defective Rate and the Average Defective Rate for Consumers

Since the retailer implements the quality gatekeeping strategy to reduce the average defective rate for consumers, the gatekeeping activity is effective *if and only if* $y(I^*)r^* < \hat{r}^*$.

Theorem 1 The retailer's quality gatekeeping activity may or may not reduce the manufacturer's average defective rate and the average defective rate for consumers, which depends on the retailer's penalty charge for each identified defect δ_2.

1. If $\delta_2 \geq \omega_1$, where

$$\omega_1 \equiv \frac{(b_1\delta_1 + 2b_2)(1 - y(I^*)) + b_1\delta_1}{b_1 y(I^*)}, \tag{14}$$

then as a result of the retailer's gatekeeping, both the manufacturer's average defective rate and the average defective rate for consumers are reduced, i.e., $r^* \leq \hat{r}^*$ and $y(I^*)r^* < \hat{r}^*$.

2. If $\omega_2 \leq \delta_2 < \omega_1$, where

$$\omega_2 \equiv \frac{\delta_1 \hat{r}^* - 2(C'(r^*)r^* - C'(\hat{r}^*)\hat{r}^*)}{y(I^*)r^*}, \tag{15}$$

then $r^* > \hat{r}^*$ and $y(I^*)r^* \leq \hat{r}^*$.

3. If $\delta_2 < \omega_2$, then $r^* > \hat{r}^*$ and $y(I^*)r^* > \hat{r}^*$. ■

The above theorem shows a somewhat surprising result that in response to the retailer's gatekeeping activity, the manufacturer and the retailer *may* not agree to reduce the manufacturer's average defective rate but decide to increase it. This happens *only* when the retailer charges the manufacturer a sufficiently small penalty for each identified defect. This result implies that the retailer's high penalty charge for each identified defect δ_2 can help induce a reduction in the manufacturer's average defective rate. Moreover, whether the average defective rate for consumers is reduced or not also depends on the penalty charge δ_2. We note from Theorem 1 that the range of the value of δ_2 in which $y(I^*)r^* \leq \hat{r}^*$ is wider than the range of δ_2 in which $r^* \leq \hat{r}^*$. That is, when $\omega_2 \leq \delta_2 < \omega_1$, consumers can always enjoy a lower defective rate, although the retailer's quality gatekeeping leads to an increase in the manufacturer's average defective rate. This demonstrates the importance of the retailer's penalty charge in effectively improving the quality performance in the supply chain.

The conditions in Theorem 1 (viz., the comparison between δ_2 and ω_i, $i = 1, 2$) can also be interpreted in terms of δ_1 and δ_2. Noting that $\omega_1 > \delta_1$ and $\omega_2 < \delta_1$, we conclude that the retailer's quality gatekeeping activity leads to a lower defective rate for consumers (i.e., $y(I^*)r^* \leq \hat{r}^*$), if the manufacturer's penalty payment for each identified defect δ_2 is higher than or equal to the penalty for each unidentified defect δ_1. That is, the positive impact of the retailer's gatekeeping on quality improvement can result from her penalty charge that is no smaller than the market penalty. In practice, the retailer's unit penalty charge is likely to be no smaller than δ_1, mainly because the retailer's gatekeeping should not lead the retailer to share the market penalty with the manufacturer but should instead motivate the manufacturer to improve his quality. Otherwise, the manufacturer "benefits" from the gatekeeping by absorbing a penalty smaller than the market penalty, thus lowering his incentive for quality improvement. In conclusion, the retailer's gatekeeping could be viewed as an effective activity for reducing the defective rate for consumers.

4.2.2 The Impact of the Retailer's Gatekeeping on the Negotiated Wholesale Price, the Optimal Retail Price, the Demand, and the Profits

We compare the negotiated wholesale prices, the optimal retail prices, the demands, and the profits with and without the retailer's gatekeeping.

Theorem 2 The retailer enjoys a higher profit from the gatekeeping activity and is thus willing to exert the gatekeeping effort, if

$$b_2(\hat{r}^* - y(I^*)r^*) > b_1(\kappa_M + \kappa_R). \tag{16}$$

The above condition is independent of the retailer's relative bargaining power. In (16), $\kappa_M \equiv C(r^*) + \delta_1 y(I^*)r^* + \delta_2(1 - y(I^*))r^* - \delta_1\hat{r}^* - C(\hat{r}^*)$ and $\kappa_R = \tau I^*(r) - \delta_2 r(1 - y(I^*(r)))$ are the manufacturer's and the retailer's net unit quality-related costs generated by the retailer's gatekeeping activity. Under the condition in (16), the demand and the manufacturer's profit are also increased, and the retailer's stronger relative bargaining power can enhance the increases in the demand and the two firms' profits. If the condition in (16) does not hold, then the retailer will have no incentive to take on the quality gatekeeping responsibility.

Moreover, when the retailer implements quality gatekeeping, the negotiated wholesale price is increased (i.e., $w^* > \hat{w}^*$) if $\kappa_M > 0$, and the optimal retail price is increased (i.e., $p^* > \hat{p}^*$) if $\kappa_M + \kappa_R > 0$. ■

In (16), b_1 and b_2 reflect the impact of retail price and that of quality on demand, respectively. Thus, the first observation from the above theorem is that the retailer's incentive to take on the quality gatekeeping responsibility depends on the impact of quality relative to retail price on demand, and it increases with the retailer's relative bargaining power. Theorem 2 also shows that when the retailer decides to assure the quality as a gatekeeper, the change in the negotiated wholesale price depends on the change in the manufacturer's quality-related cost, whereas the change in the optimal retail price is dependent on the change in the channel-wide quality-related cost. This means that, for the pricing decisions, the negotiated wholesale price is dependent on the manufacturer's "individual" benefit but the optimal retail price is related to the channel-wide benefit. Moreover, the retailer's gatekeeping activity may or may not increase the demand and each channel member's profit, which depends on the impact of quality relative to retail price on demand. Nevertheless, a higher relative bargaining power of the retailer can help enhance the increase or mitigate the decrease in demand and each firm's profit.

The positive impact of the retailer's bargaining power on the demand and the two firms' profits may result from the retailer's position in the supply chain. That is, the retailer is closer to consumers than the manufacturer, thereby greatly influencing consumers' purchases. Moreover, as indicated by Theorem 2, the retailer's benefit from quality gatekeeping increases with the retailer's relative bargaining power, which may be consistent with the practice that powerful retailers such as Asda, Tesco, and Wal-Mart have decided to act as quality gatekeepers in their supply chains.

5 Summary and Concluding Remarks

We investigated a quality-assurance problem where a manufacturer supplies products with a defective rate to a retailer who then inspects the products as a quality gatekeeper. We constructed and analyzed a two-stage decision model, in which (i) the manufacturer and the retailer negotiate an average defective rate and also bargain over the wholesale price, and (ii) the retailer decides on her optimal retail price and gatekeeping intensity. We then examined the effects of unit penalty costs and the two firms' relative bargaining powers on their quality and pricing decisions, demand, and profits. In addition, we analyzed the two-stage problem without the retailer's gatekeeping. A comparison between our analytical results for the problems with and without the retailer's gatekeeping activity generated a number of insights regarding the impact of the retailer's quality gatekeeping on the channel.

This paper is focused on the impact of quality gatekeeping at the retail level, which can be viewed as the theme of our study. Next, we summarize our most important analytic insights, which also contribute to answering three questions that *naturally* arise under the theme.

1. **How can a retailer perform an effective quality gatekeeping activity?** We learn from our analysis that the retailer's quality gatekeeping can effectively reduce the channel-wide defective rate, if her penalty charge for each identified defect is larger than or equal to the market penalty for each unidentified defect. Such a penalty charge is reasonable as the retailer should not share the market penalty with the manufacturer through her gatekeeping activity but should "force" the manufacturer to improve his quality.

2. **What is the impact of a retailer's quality gatekeeping on pricing decisions in the supply chain?** Our analytic results indicate that when the retailer implements quality gatekeeping, the change in the negotiated wholesale price *only* depends on the manufacturer's "individual" benefit, and the change in the retailer's optimal retail price is *only* related to the channel-wide benefit. This shows that the quality gatekeeping activity at the retail level exposes the two firms' different concerns in their pricing decisions, which may be attributed to the firms' different positions in the supply chain. More specifically, the retailer is close to consumers and thus concerns the channel-wide performance, whereas the manufacturer directly serves the retailer with a focus on his own benefit.

3. **When/why does a retailer have an incentive to take on the quality gate-**

keeping responsibility? We find that the impact of quality relative to retail price on demand plays an important role in increasing the demand and assuring the retailer's incentive to implement the quality gatekeeping strategy. A higher impact of quality relative to retail price on demand can lead the retailer's gatekeeping activity to generate a higher demand and greater profits for the retailer and the supply chain. Moreover, if the retailer possesses a stronger power in bargaining with the manufacturer, then she and the wholesale supply chain can profit more from her gatekeeping activity, which happens mainly because her stronger position in the supply chain can enhance the effectiveness of her gatekeeping in improving quality and increasing demand. It thus follows that the retailer is willing to take on the quality gatekeeping responsibility, when/because the impact of quality relative to retail price on demand is high and/or the retailer has a strong bargaining power.

From the above, we conclude that the retailer's quality gatekeeping can help reduce the defective rate for consumers, which becomes more significant when the retailer's bargaining position in her supply chain is stronger. Retailers—especially, those with strong powers relative to their suppliers/manufacturers—can benefit more from their quality gatekeeping in supply chains when the impact of quality relative to retail price on demand is higher.

We used the exponential demand function in (1) for our analysis. In order to examine the robustness of the above major implications, we consider two commonly-used demand functions in online Appendix C: the logistic demand function (see, e.g., Phillips 2005) and the attraction demand function (see, e.g., Benjaafar, Elahi, and Donohue 2007), which are written as follows:

$$\begin{cases} \text{Logistic:} \quad D(p,r,I) = \dfrac{a}{1+\exp(a' + b_1 p + b_2 y(I) r)}, \text{ where } a, a', b_1, b_2 > 0; \\ \text{Attraction:} \quad D(p,r,I) = \dfrac{a(b_1 p + b_2 y(I) r)}{(b_1 p + b_2 y(I) r) - a'}, \text{ where } a, a', b_1, b_2 > 0. \end{cases}$$

For details regarding the two demand function forms, see a recent review by Huang, Leng, and Parlar (2013). Since our two-stage analysis with the logistic and the attraction demand functions are *intractable*, we perform numerical experiments in online Appendix C. The numerical results demonstrate that our major managerial insights summarized in Section 5 hold for both the logistic and the attraction demand functions. Thus, the analytical implications based on the demand function in (1) should be robust.

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Online Appendices

"Implications for the Role of Retailers in Quality Assurance"

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Appendix A Details Regarding the Retailer's Gatekeeping Process

One may note that in reality the retailer could adopt a different gatekeeping process by setting an acceptance threshold to determine whether or not all products should be accepted or rejected. That is, if the percentage of identified defects is greater than or equal to the threshold, then the retailer rejects all products (sold by the manufacturer to the retailer); otherwise, the retailer accepts the products. We do not consider the retailer's gatekeeping process with a threshold because of the following facts: By consulting with a number of quality practitioners in the retail industry, we identify that the retailer may determine such an acceptance threshold if she performs her quality inspection at the manufacturer's site before the shipment of the products. If the retailer's sampling test indicates that the percentage of defects exceeds the retailer's threshold, then the manufacturer is required to inspect all products, identify all defects, and replace them with good products. This means that the gatekeeping process with an acceptance threshold takes place at the manufacturer's site rather than at the retailer's, which differs from our study in which the gatekeeping process occurs at the retailer's site, as observed in the practices of Asda, Tesco, and Wal-Mart. However, the gatekeeping process with an acceptance threshold could be viewed as if the retailer acts as a gatekeeper at her "own store." Observing that the shipment may be delayed due to quality problems, the retailer usually prepares a "safety lead time" for ordering products with the manufacturer, in order to guarantee her service level.

Moreover, if a retailer inspects incoming products at her own store—which is common in the retail industry, then the retailer is unlikely to implement a gatekeeping process involving an acceptance threshold but instead uses the process that is considered in our paper, because, otherwise, she may experience a long shipping time, a high shipping cost for the "round trip" of the products, and a large stockout probability. Actually, extant relevant publications (including, e.g., Balachandran and Radhakrishnan 2005 and Hwang, Radhakrishnan, and Su 2006) investigated only the quality inspection process similar to that in our paper.

Appendix B Proofs

Proof of Lemma 1. Partially differentiating $\pi_R(p, I)$ in (2) once and twice w.r.t. p, we have

$$\frac{\partial \pi_R(p,I)}{\partial p} = a\{1 - b_1[p - w - \tau I + \delta_2 r(1 - y(I))]\} \exp(-b_1 p - b_2 y(I) r),$$
$$\frac{\partial^2 \pi_R(p,I)}{\partial p^2} = -ab_1\{2 - b_1[p - w - \tau I + \delta_2 r(1 - y(I))]\} \exp(-b_1 p - b_2 y(I) r),$$

which is negative at the point that $\partial \pi_R(p, I)/\partial p = 0$. Thus, solving the first order condition yields that given w, I, and r, the optimal retail price can be uniquely found as

$$p^*(w, r, I) = \frac{1 + b_1[w + \tau I - \delta_2 r(1 - y(I))]}{b_1}.$$

Substituting $p^*(w, r, I)$ into $\pi_R(p, I)$ in (2) gives

$$\pi_R(p^*(w, r, I), I) = \frac{a}{b_1} \exp(-1 - b_1[w + \tau I - \delta_2 r(1 - y(I))] - b_2 y(I) r).$$

Partially differentiating $\pi_R(p^*(w, r, I), I)$ once and twice w.r.t. I, we have

$$\begin{aligned}\frac{\partial \pi_R(p^*(w,r,I),I)}{\partial I} &= -\frac{a}{b_1} \exp(-1 - b_1[w + \tau I - \delta_2 r(1 - y(I))] - b_2 y(I) r) \\ &\quad \times [b_1(\tau + \delta_2 r y'(I)) + b_2 y'(I) r], \\ \frac{\partial^2 \pi_R(p^*(w,r,I),I)}{\partial I^2} &= \frac{a}{b_1} \exp(-1 - b_1[w + \tau I - \delta_2 r(1 - y(I))] - b_2 y(I) r) \\ &\quad \times \{[b_1(\tau + \delta_2 r y'(I)) + b_2 y'(I) r]^2 - (b_1\delta_2 + b_2) r y''(I)\}.\end{aligned}$$

At the point that $\partial \pi_R(p^*(w, r, I), I)/\partial I = 0$, $b_1(\tau+\delta_2 r y'(I))+b_2 y'(I) r = 0$ and $\partial^2 \pi_R(p^*(w, r, I), I)/\partial I^2 < 0$. Therefore, we prove this lemma. ■

Proof of Lemma 2. To find the GNB-characterized wholesale price, we need to solve the maximization problem:

$$\max_{w,r} \Omega(w, r) = (\pi_M(w, r))^\beta (\pi_R(w, r))^{1-\beta}.$$

Using (5) and (6), we re-write $\Omega(w, r)$ as,

$$\Omega(w, r) = \frac{a\Lambda(w, r)}{b_1^{1-\beta}},$$

where

$$\Lambda(w,r) \equiv \{w - c - [\delta_1 y(I^*(r))r + \delta_2(1 - y(I^*(r)))r] - C(r)\}^{\beta} \\ \times \exp(-\{1 + b_1[w + \tau I^*(r) - \delta_2 r(1 - y(I^*(r))]\} - b_2 y(I^*(r))r).$$

Taking the logarithm of $\Lambda(w,r)$ we obtain $\ln(\Lambda(w,r)) = \beta\xi_1 + \xi_2$, where

$$\xi_1 \equiv \ln\{w - c - [\delta_1 y(I^*(r))r + \delta_2(1 - y(I^*(r)))r] - C(r)\},$$
$$\xi_2 \equiv -\{1 + b_1[w + \tau I^*(r) - \delta_2 r(1 - y(I^*(r))]\} - b_2 y(I^*(r))r.$$

The first- and second-order derivatives of ξ_1 w.r.t. w are computed as,

$$\frac{\partial \xi_1}{\partial w} = \frac{1}{w - c - [\delta_1 y(I^*(r))r + \delta_2(1 - y(I^*(r)))r] - C(r)},$$
$$\frac{\partial^2 \xi_1}{\partial w^2} = -\frac{1}{[w - c - [\delta_1 y(I^*(r))r + \delta_2(1 - y(I^*(r)))r] - C(r)]^2} < 0.$$

The first– and second-order derivatives of ξ_2 w.r.t. w are

$$\frac{\partial \xi_2}{\partial w} = -b_1 \text{ and } \frac{\partial^2 \xi_2}{\partial w^2} = 0.$$

Therefore, given r, $\ln(\Lambda(w,r))$ is a strictly concave function of w, which means that $\Lambda(w,r)$ and $\Omega(w,r)$ are log-concave functions of w. There must exist a unique GNB solution $w^*(r)$, which satisfies the first order condition $\partial(\ln(\Lambda(w,r)))/\partial w = 0$. That is,

$$\frac{\partial(\ln(\Lambda(w)))}{\partial w} = \frac{\beta}{w - c - [\delta_1 y(I^*(r))r + \delta_2(1 - y(I^*(r)))r] - C(r)} - b_1 = 0.$$

We can then find the negotiated wholesale price $w^*(r)$ as in (7). ■

Proof of Lemma 3. The first-order partial derivative of $\Pi(r)$ with respect to r is computed as follows**:**

$$\frac{\partial \Pi(r)}{\partial r} = -\Pi(r)\left[b_1\left(\tau\frac{\partial I^*(r)}{\partial r} + C'(r) + \delta_1 y'(I^*(r))r\frac{\partial I^*(r)}{\partial r} + \delta_1 y(I^*(r))\right)\right. \\ \left. + b_2 y'(I^*(r))r\frac{\partial I^*(r)}{\partial r} + b_2 y(I^*(r))\right].$$

Lemma 1 indicates that $(b_1\delta_2 + b_2)y'(I^*(r))r = -\tau b_1$. The first-order derivative $\partial\Pi(r)/\partial r$ can be simplified as

$$\frac{\partial \Pi(r)}{\partial r} = -\Pi(r)\left[(\delta_1 - \delta_2)b_1 y'(I^*(r))r\frac{\partial I^*(r)}{\partial r} + b_1 C'(r) + (b_1\delta_1 + b_2)y(I^*(r))\right].$$

We differentiate both sides of $(b_1\delta_2 + b_2)y'(I^*(r))r = -\tau b_1$ w.r.t. r, and find

$$\frac{\partial I^*(r)}{\partial r} = -\frac{y'(I^*(r))}{y''(I^*(r))r} = \frac{1+\mu I^*(r)}{2\mu r}.$$

We re-write $\partial\Pi(r)/\partial r$ as

$$\frac{\partial \Pi(r)}{\partial r} = -\Pi(r)\left[\frac{1}{2}b_1(\delta_1+\delta_2)y(I^*(r)) + b_1 C'(r) + b_2 y(I^*(r))\right].$$

Then, we compute the second-order derivative of $\Pi(r)$ with respect to r and find

$$\begin{aligned}\frac{\partial^2 \Pi(r)}{\partial r^2} &= \Pi(r)\left[\frac{1}{2}b_1(\delta_1+\delta_2)y(I^*(r)) + b_1 C'(r) + b_2 y(I^*(r))\right]^2 \\ &- \Pi(r)\left[\frac{1}{2}b_1(\delta_1+\delta_2)y'(I^*(r))\frac{\partial I^*(r)}{\partial r} + b_1 C''(r) + b_2 y'(I^*(r))\frac{\partial I^*(r)}{\partial r}\right] \\ &= \Pi(r)\left[\frac{1}{2}b_1(\delta_1+\delta_2)y(I^*(r)) + b_1 C'(r) + b_2 y(I^*(r))\right]^2 \\ &- \Pi(r)\left[-\frac{1}{4r}b_1(\delta_1+\delta_2)y(I^*(r)) + b_1 C''(r) - \frac{1}{2r}b_2 y(I^*(r))\right].\end{aligned}$$

At the point satisfying the first-order condition that $\partial\Pi(r)/\partial r = 0$, we have the following equation:

$$b_1(\delta_1+\delta_2)y(I^*(r)) + 2b_1 C'(r) + 2b_2 y(I^*(r)) = 0.$$

Using this equation, we have

$$\frac{\partial^2 \Pi(r)}{\partial r^2} = -\Pi(r)\frac{b_1}{r}\left[\frac{1}{2}C'(r) + rC''(r)\right] < -\Pi(r)\frac{b_1}{r}\bar{C}(r),$$

where $\bar{C}(r) \equiv (C'(r) + rC''(r))$. If $\bar{C}(r) > 0$, then $\partial^2\Pi(r)/\partial r^2 < 0$ and $\Pi(r)$ is strictly concave in r. This lemma is thus proved. ■

Proof of Proposition 1. Comparing the two equations in (10), we reach the equation in (11), where on the left-hand side, $\tau I^*/C(r^*)$ is the ratio of the retailer's per product gatekeeping cost to the manufacturer's per product quality-control cost. The right-hand side is the ratio of e_I to e_r. The term e_I is the elasticity of the percentage of unidentified defects with respect to the retailer's gatekeeping intensity. Since the denominator in the term e_r (i.e., $C'(r^*)/(C(r^*)/r^*)$) is the elasticity of the manufacturer's unit quality-control cost with respect to his average defective rate, the term e_r can be explained as the elasticity of the average defective rate with respect to the manufacturer's unit quality-control cost. Simply put, $e_I < 0$ and $e_r < 0$ denote the retailer's and the manufacturer's effectiveness in quality assurance, respectively. A smaller negative value of e_I (e_r)—or a larger absolute value of e_I (e_r)—indicates the retailer's (the manufacturer's) higher effectiveness. It thus follows from the above that e_I/e_r is the ratio between the retailer's and the manufacturer's effort elasticities of quality-control effectiveness. ■

Proof of Corollary 1. From the two firms' profit functions, we can compute the ratio

$$\frac{\pi_R(w^*, r^*)}{\pi_M(w^*, r^*)} = \frac{1}{\beta},$$

which results in this corollary. ■

Proof of Proposition 2. Differentiating both sides of the equation in (9) w.r.t. δ_1 yields

$$b_1 y(I^*(r^*)) + [b_1(\delta_1 + \delta_2) + 2b_2] y'(I^*(r^*)) \frac{\partial I^*(r^*)}{\partial r} \frac{\partial r^*}{\partial \delta_1} = -2b_1 C''(r^*) \frac{\partial r^*}{\partial \delta_1}. \tag{17}$$

From the proof of Lemma 3, we have

$$\frac{\partial I^*(r^*)}{\partial r} = \frac{1 + \mu I^*(r^*)}{2\mu r^*}.$$

Noting that

$$y'(I^*(r^*)) = -\mu(1 + \mu I^*(r^*))^{-2},$$

and using (9), we re-write (17) as

$$y(I^*(r^*)) = -\frac{1}{r^*}(\bar{C}(r^*) + r^* C''(r^*)) \frac{\partial r^*}{\partial \delta_1},$$

which implies that $\partial r^*/\partial \delta_1 < 0$. Moreover, $\partial I^*(r^*)/\partial \delta_1 = (\partial I^*(r)/\partial r)|_{r=r^*} \times (\partial r^*/\partial \delta_1) < 0$, which follows as $\partial I^*(r)/\partial r > 0$.

In addition, for the average defective rate for consumers, we have

$$\begin{aligned} \frac{\partial\left(r^* y(I^*(r^*))\right)}{\partial \delta_1} &= \frac{\partial r^*}{\partial \delta_1} y(I^*(r^*)) + r^* y'(I^*(r^*)) \frac{\partial I^*(r^*)}{\partial r} \frac{\partial r^*}{\partial \delta_1} \\ &= \frac{y(I^*(r^*))}{2} \frac{\partial r^*}{\partial \delta_1} < 0. \end{aligned}$$

Similar to the above, we find that $\partial r^*/\partial \delta_2 < 0$, $\partial I^*(r^*)/\partial \delta_2 < 0$, and $\partial\left(r^* y(I^*(r^*))\right)/\partial \delta_2 < 0$. This proposition is thus proved. ■

Proof of Proposition 3. We replace r in $w^*(r)$ in (7) with r^* that is obtained by solving the equation in (9), and differentiate $w^*(r^*)$ once w.r.t. δ_1 as

$$\frac{\partial w^*(r^*)}{\partial \delta_1} = C'(r^*)\frac{\partial r^*}{\partial \delta_1} + y(I^*)r^* + \delta_1 y'(I^*)r^*\frac{\partial I^*}{\partial \delta_1} + \delta_1 y(I^*)\frac{\partial r^*}{\partial \delta_1} - \delta_2 y'(I^*)r^*\frac{\partial I^*}{\partial \delta_1} + \delta_2(1-y(I^*))\frac{\partial r^*}{\partial \delta_1}.$$

From the proof of Proposition 2, we have

$$y(I^*(r^*)) = -\frac{1}{r^*}(\bar{C}(r^*) + r^*C''(r^*))\frac{\partial r^*}{\partial \delta_1},$$

and

$$y'(I^*)r^*\frac{\partial I^*}{\partial \delta_1} = y'(I^*)r^*\left.\frac{\partial I^*}{\partial r}\right|_{r=r^*}\frac{\partial r^*}{\partial \delta_1} = -\frac{y(I^*)}{2}\frac{\partial r^*}{\partial \delta_1}. \tag{18}$$

We re-write $\partial w^*(r^*)/\partial \delta_1$ as

$$\frac{\partial w^*(r^*)}{\partial \delta_1} = \left[\delta_2 - 2r^*C''(r^*) + \frac{1}{2}(\delta_1 - \delta_2)y(I^*)\right]\frac{\partial r^*}{\partial \delta_1},$$

where $\partial r^*/\partial \delta_1 < 0$. It thus follows that $\partial w^*(r^*)/\partial \delta_1 > 0$ if and only if

$$\frac{1}{2}y(I^*)\delta_1 + \left(1 - \frac{1}{2}y(I^*)\right)\delta_2 < 2r^*C''(r^*).$$

Using (9), we have

$$-\left[(\delta_1 + \delta_2) + \frac{2b_2}{b_1}\right]y(I^*) = 2C'(r^*), \tag{19}$$

and obtain the condition specified in this proposition. The proposition is thus proved. ∎

Proof of Proposition 4. Using (3) and the proof of Proposition 3, we compute the first-order derivative of $p^*(w^*, r^*)$ w.r.t. δ_1 as

$$\begin{aligned}\frac{\partial p^*(w^*, r^*)}{\partial \delta_1} &= \frac{\partial w^*(r^*)}{\partial \delta_1} + \tau\frac{\partial I^*}{\partial \delta_1} - \delta_2\frac{\partial r^*}{\partial \delta_1}(1-y(I^*)) + \delta_2 r^* y'(I^*)\frac{\partial I^*}{\partial \delta_1} \\ &= \left[-2r^*C''(r^*) + \frac{\delta_1}{2}y(I^*)\right]\frac{\partial r^*}{\partial \delta_1} + \tau\frac{\partial I^*}{\partial r}\frac{\partial r^*}{\partial \delta_1} \\ &= \left[-2r^*C''(r^*) + \frac{\delta_1}{2}y(I^*)\right]\frac{\partial r^*}{\partial \delta_1} + \tau\frac{1+\mu I^*}{2\mu r^*}\frac{\partial r^*}{\partial \delta_1}.\end{aligned}$$

As Lemma 1 indicates, $(b_1\delta_2 + b_2)y'(I^*)r^* = -\tau b_1$. We thus have

$$\tau = -\left(\delta_2 + \frac{b_2}{b_1}\right)y'(I^*)r^* = \left(\delta_2 + \frac{b_2}{b_1}\right)\mu r^*(1+\mu I^*)^{-2}, \tag{20}$$

and we can re-write $\partial p^*(w^*, r^*)/\partial \delta_1$ as

$$\frac{\partial p^*(w^*, r^*)}{\partial \delta_1} = \left[-2r^*C''(r^*) + \frac{\delta_1}{2}y(I^*)\right]\frac{\partial r^*}{\partial \delta_1} + \left(\frac{\delta_2}{2} + \frac{b_2}{2b_1}\right)y(I^*)\frac{\partial r^*}{\partial \delta_1}.$$

Recalling from (9) that $[b_1(\delta_1 + \delta_2) + 2b_2]y(I^*) = -2b_1C'(r^*)$, we have $\delta_1 y(I^*) = -2C'(r^*) - \delta_2 y(I^*) - 2b_2 y(I^*)/b_1$ and re-write $\partial p^*(w^*, r^*)/\partial \delta_1$ as

$$\begin{aligned}\frac{\partial p^*(w^*, r^*)}{\partial \delta_1} &= -\left[r^*C''(r^*) + \bar{C}(r^*) + \left(\frac{\delta_2}{2} + \frac{b_2}{b_1}\right)y(I^*)\right]\frac{\partial r^*}{\partial \delta_1} + \left(\frac{\delta_2}{2} + \frac{b_2}{2b_1}\right)y(I^*)\frac{\partial r^*}{\partial \delta_1}\\ &= -\left[r^*C''(r^*) + \bar{C}(r^*) + \frac{b_2}{2b_1}y(I^*)\right]\frac{\partial r^*}{\partial \delta_1}\\ &> 0,\end{aligned}$$

which means that the retail price is increasing in the unit penalty cost for each unidentified defect.

Next, we examine the impact of δ_1 on the demand $D(p^*(w^*, r^*), r^*, I^*)$. Differentiating $D(p^*(w^*, r^*), r^*, I^*)$ w.r.t. δ_1 and using (18), we obtain

$$\begin{aligned}\frac{\partial D(p^*(w^*, r^*), r^*, I^*)}{\partial \delta_1} &= -a\left[b_1\frac{\partial p^*(w^*, r^*)}{\partial \delta_1} + \frac{b_2 y(I^*)}{2}\frac{\partial r^*}{\partial \delta_1}\right]\exp(-b_1p^*(w^*, r^*) - b_2y(I^*)r^*)\\ &= ab_1\left[r^*C''(r^*) + \bar{C}(r^*)\right]\frac{\partial r^*}{\partial \delta_1}\exp(-b_1p^*(w^*, r^*) - b_2y(I^*)r^*)\\ &< 0.\end{aligned}$$

The proposition is proved. ■

Proof of Proposition 5. We compute the first-order derivative of $w^*(r^*)$ w.r.t. δ_2 as

$$\begin{aligned}\frac{\partial w^*(r^*)}{\partial \delta_2} &= C'(r^*)\frac{\partial r^*}{\partial \delta_2} + \delta_1 y'(I^*)r^*\frac{\partial I^*}{\partial \delta_2} + \delta_1 y(I^*)\frac{\partial r^*}{\partial \delta_2}\\ &\quad + (1 - y(I^*(r)))r - \delta_2 y'(I^*)r^*\frac{\partial I^*}{\partial \delta_2} + \delta_2(1 - y(I^*))\frac{\partial r^*}{\partial \delta_2}.\end{aligned}$$

Similar to the proof of Proposition 3, we can simplify $\partial w^*(r^*)/\partial \delta_2$ as

$$\frac{\partial w^*(r^*)}{\partial \delta_2} = r^* + \left[\delta_2 + 2\bar{C}(r^*) + \frac{1}{2}y(I^*)(\delta_1 - \delta_2)\right]\frac{\partial r^*}{\partial \delta_2}.$$

Using (19), we re-write $\partial w^*(r^*)/\partial \delta_2$ as

$$\frac{\partial w^*(r^*)}{\partial \delta_2} = r^* + \left[\delta_2(1 - y(I^*)) + \bar{C}(r^*) + r^*C''(r^*) - \frac{b_2}{b_1}y(I^*)\right]\frac{\partial r^*}{\partial \delta_2},$$

which must be positive if $\delta_2 < \eta$ where η is defined as in (12).

We also calculate the first-order derivative of $p^*(w^*, r^*)$ w.r.t. δ_2 as

$$\begin{aligned}\frac{\partial p^*(w^*, r^*)}{\partial \delta_2} &= \frac{\partial w^*(r^*)}{\partial \delta_2} + \tau \frac{\partial I^*}{\partial \delta_2} - r^*(1 - y(I^*)) - \delta_2(1 - y(I^*))\frac{\partial r^*}{\partial \delta_2} + \delta_2 r^* y'(I^*)\frac{\partial I^*}{\partial \delta_2} \\ &= \left[\bar{C}(r^*) + r^* C''(r^*) - \frac{b_2}{b_1} y(I^*)\right]\frac{\partial r^*}{\partial \delta_2} + r^* y(I^*) + [\tau + \delta_2 r^* y'(I^*)]\frac{\partial I^*}{\partial \delta_2} \\ &= \left[\bar{C}(r^*) + r^* C''(r^*) - \frac{b_2}{b_1} y(I^*)\right]\frac{\partial r^*}{\partial \delta_2} + r^* y(I^*) + \frac{b_2}{2b_1} y(I^*)\frac{\partial r^*}{\partial \delta_2},\end{aligned}$$

where the last equality follows from (20).

Similar to the proof of Proposition 2, we can show that

$$y(I^*(r^*)) = -\frac{1}{r^*}(\bar{C}(r^*) + r^* C''(r^*))\frac{\partial r^*}{\partial \delta_2}.$$

It follows that

$$\frac{\partial p^*(w^*, r^*)}{\partial \delta_2} = -\frac{b_2}{2b_1} y(I^*)\frac{\partial r^*}{\partial \delta_2} > 0,$$

and

$$\begin{aligned}\frac{\partial D(p^*(w^*, r^*), r^*, I^*)}{\partial \delta_2} &= -a\left[b_1 \frac{\partial p^*(w^*, r^*)}{\partial \delta_2} + \frac{b_2 y(I^*)}{2}\frac{\partial r^*}{\partial \delta_2}\right]\exp(-b_1 p^*(w^*, r^*) - b_2 y(I^*) r^*) \\ &= 0.\end{aligned}$$

Therefore, we prove this proposition. ■

Proof of Proposition 6. Using (5), (6), and (7), we write $\pi_R(w^*, r^*)$ and $\pi_M(w^*, r^*)$ as

$$\pi_R(w^*, r^*) = \frac{a}{b_1}\Omega(r^*, I^*) \text{ and } \pi_M(w^*, r^*) = \frac{\beta a}{b_1}\Omega(r^*, I^*), \tag{21}$$

where

$$\Omega(r^*, I^*) \equiv \exp(-[1 + \beta + b_1(\tau I^* + c + C(r^*) + \delta_1 y(I^*) r^*)] - b_2 y(I^*) r^*). \tag{22}$$

Thus, we only need to analyze the impact of δ_i $(i = 1, 2)$ on $\Omega(r^*, I^*)$ in (22). Differentiating

$\Omega(r^*, I^*)$ once w.r.t. δ_1, we have

$$
\begin{aligned}
\frac{\partial \Omega(r^*, I^*)}{\partial \delta_1} &= \Omega(r^*, I^*) \left[-b_1 \left(\tau \frac{\partial I^*}{\partial \delta_1} + C'(r^*) \frac{\partial r^*}{\partial \delta_1} + y(I^*) r^* + \delta_1 y'(I^*) r^* \frac{\partial I^*}{\partial \delta_1} + \delta_1 y(I^*) \frac{\partial r^*}{\partial \delta_1} \right) \right. \\
&\quad \left. - b_2 \left(y'(I^*) r^* \frac{\partial I^*}{\partial \delta_1} + y(I^*) \frac{\partial r^*}{\partial \delta_1} \right) \right] \\
&= -\Omega(r^*, I^*) \left[b_1 y(I^*) r^* + \left(b_1 \tau + b_1 \delta_1 y'(I^*) r^* + b_2 y'(I^*) r^* \right) \frac{\partial I^*}{\partial \delta_1} \right. \\
&\quad \left. + \left(b_1 C'(r^*) + b_1 \delta_1 y(I^*) + b_2 y(I^*) \right) \frac{\partial r^*}{\partial \delta_1} \right].
\end{aligned}
$$

Using (18) and (20), we can re-write the above as

$$
\begin{aligned}
\frac{\partial \Omega(r^*, I^*)}{\partial \delta_1} &= -\Omega(r^*, I^*) \left[b_1 y(I^*) r^* + b_1 \left(\delta_1 - \delta_2 \right) y'(I^*) r^* \frac{\partial I^*}{\partial \delta_1} + \left(b_1 C'(r^*) + b_1 \delta_1 y(I^*) + b_2 y(I^*) \right) \frac{\partial r^*}{\partial \delta_1} \right] \\
&= -\Omega(r^*, I^*) \left[b_1 y(I^*) r^* - b_1 \left(\delta_1 - \delta_2 \right) \frac{y(I^*)}{2} \frac{\partial r^*}{\partial \delta_1} + \left(b_1 C'(r^*) + b_1 \delta_1 y(I^*) + b_2 y(I^*) \right) \frac{\partial r^*}{\partial \delta_1} \right] \\
&= -b_1 \Omega(r^*, I^*) \left\{ y(I^*) r^* + \left(C'(r^*) + \left[(\delta_1 + \delta_2) + \frac{2b_2}{b_1} \right] \frac{y(I^*)}{2} \right) \frac{\partial r^*}{\partial \delta_1} \right\}.
\end{aligned}
$$

It follows from (19) that

$$
\frac{\partial \Omega(r^*, I^*)}{\partial \delta_1} = -b_1 \Omega(r^*, I^*) y(I^*) r^* < 0.
$$

Similarly, we can show that

$$
\frac{\partial \Omega(r^*, I^*)}{\partial \delta_2} = 0.
$$

The proposition thus follows. ■

Proof of Proposition 7. We differentiate $w^*(r^*)$ once w.r.t. β, and find $\partial w^*(r^*)/\partial \beta = 1/b_1 > 0$. The first-order derivative of $p^*(w^*, r^*)$ w.r.t. β is computed as $\partial p^*(w^*, r^*)/\partial \beta = 1/b_1$. We also find

$$
\frac{\partial D(p^*(w^*, r^*), r^*, I^*)}{\partial \beta} = -a \exp(-b_1 p^*(w^*, r^*) - b_2 y(I^*) r^*) < 0.
$$

We thus obtain this proposition. ■

Proof of Proposition 8. According to (21) and (22), we compute

$$
\frac{\partial \pi_R(w^*, r^*)}{\partial \beta} = \frac{a}{b_1} \frac{\partial \Omega(r^*, I^*)}{\partial \beta} = -\frac{a}{b_1} \Omega(r^*, I^*) < 0,
$$

and

$$\frac{\partial \pi_M(w^*, r^*)}{\partial \beta} = \frac{(1-\beta)a}{b_1}\Omega(r^*, I^*) > 0.$$

Moreover,

$$\frac{\partial(\pi_R(w^*, r^*) + \pi_M(w^*, r^*))}{\partial \beta} = -\frac{\beta a}{b_1}\Omega(r^*, I^*) < 0.$$

In addition,

$$\frac{|\partial \pi_R(w^*, r^*)/\partial \beta|}{\partial \pi_M(w^*, r^*)/\partial \beta} = \frac{1}{1-\beta} > 1.$$

We thus prove this proposition. ■

Proof of Lemma 4. The first- and second-order derivatives of $\hat{\pi}_R(p)$ w.r.t. p are

$$\begin{aligned}\frac{\partial \hat{\pi}_R(p)}{\partial p} &= a\exp(-b_1 p - b_2 r)[1 - b_1(p-w)],\\ \frac{\partial^2 \hat{\pi}_R(p)}{\partial p^2} &= -b_1 a\exp(-b_1 p - b_2 r)[2 - b_1(p-w)],\end{aligned}$$

which is negative at the point that $\partial\hat{\pi}_R(p)/\partial p = 0$. The optimal retail price is thus found as shown in this lemma. ■

Proof of Lemma 5. We use $\hat{p}^*(w,r)$ in Lemma 4 to specify $\hat{\pi}_M(w,r)$ and $\hat{\pi}_R(\hat{p}^*(w,r))$ as

$$\begin{aligned}\hat{\pi}_M(w,r) &= a\exp(-(1+b_1 w + b_2 r))(w - c - \delta_1 r - C(r));\\ \hat{\pi}_R(\hat{p}^*(w,r)) &= a\exp(-(1+b_1 w + b_2 r))/b_1.\end{aligned}$$

The GNB solutions $\hat{w}^*$ and $\hat{r}^*$ can be found by solving

$$\max_{w,r}(\hat{\pi}_M(w,r))^{\beta} \times (\hat{\pi}_R(\hat{p}^*(w,r))^{1-\beta} = \frac{a}{b_1^{1-\beta}}\hat{\Lambda}(w,r),$$

where

$$\hat{\Lambda}(w,r) \equiv \exp(-(1+b_1 w + b_2 r))(w - c - \delta_1 r - C(r))^{\beta}.$$

Taking the logarithm of both sides of $\hat{\Lambda}(w,r)$ yields,

$$\hat{L}(w,r) \equiv \ln\hat{\Lambda}(w,r) = \hat{\xi}_1 + \beta\hat{\xi}_2,$$

where

$$\hat{\xi}_1 \equiv -(1 + b_1 w + b_2 r) \text{ and } \hat{\xi}_2 \equiv \ln(w - c - \delta_1 r - C(r)).$$

Next, we find $\hat{w}^*$ and $\hat{r}^*$ by using the following steps. First, given a value of r, we compute the GNB solution $\hat{w}^*(r)$. Then, we substitute $\hat{w}^*(r)$ into $\hat{\pi}_M(w,r)$ and $\hat{\pi}_R(\hat{p}^*(w,r))$, and calculate the GNB solution $\hat{r}^*$, which can be used to find $\hat{w}^* = \hat{w}^*(r^*)$. In the first step, we compute the first– and second-order derivatives of $\hat{\xi}_1$ w.r.t. w as

$$\frac{\partial \hat{\xi}_1}{\partial w} = -b_1 \qquad \text{and} \qquad \frac{\partial^2 \hat{\xi}_1}{\partial w^2} = 0.$$

The first- and second-order derivatives of $\hat{\xi}_2$ w.r.t. w are

$$\frac{\partial \hat{\xi}_2}{\partial w} = \frac{1}{w - c - \delta_1 r - C(r)} \qquad \text{and} \qquad \frac{\partial^2 \hat{\xi}_2}{\partial w^2} = -\frac{1}{(w - c - \delta_1 r - C(r))^2} < 0.$$

Therefore, $(\hat{\pi}_M(w,r))^\beta \times (\hat{\pi}_R(\hat{p}^*(w,r))^{1-\beta}$ is a log-concave function of w. The GNB solution $\hat{w}^*(r)$ satisfies the first-order condition $\partial \hat{L}(w,r)/\partial w = 0$, where

$$\frac{\partial \hat{L}(w,r)}{\partial w} = \frac{\beta}{w - c - \delta_1 r - C(r)} - b_1.$$

The negotiated wholesale price $\hat{w}^*(r)$ is thus found as

$$\hat{w}^*(r) = \frac{\beta}{b_1} + c + \delta_1 r + C(r).$$

In the second step, to compute the GNB-characterized average defective rate $\hat{r}^*$, we need to solve the maximization problem: $\max_{\hat{r}} \hat{\Pi}(r) \equiv (\hat{\pi}_M(\hat{w}^*(r), r))^\beta \times (\hat{\pi}_R(\hat{p}^*(\hat{w}^*(r), r))^{1-\beta}$, where

$$\begin{aligned}
\hat{\pi}_M(\hat{w}^*(r), r) &= \frac{a\beta}{b_1} \exp(-[1 + \beta + b_1(c + \delta_1 r + C(r)) + b_2 r]), \\
\hat{\pi}_R(\hat{p}^*(\hat{w}^*(r), r)) &= \frac{a}{b_1} \exp(-[1 + \beta + b_1(c + \delta_1 r + C(r)) + b_2 r]).
\end{aligned}$$

The first- and second-order derivatives of $\hat{\Pi}(r)$ w.r.t. r are

$$\begin{aligned}
\frac{\partial \hat{\Pi}(r)}{\partial r} &= -\frac{a\beta^\beta}{b_1}[b_1(\delta_1 + C'(r)) + b_2] \exp(-[1 + \beta + b_1(c + \delta_1 r + C(r)) + b_2 r]), \\
\frac{\partial^2 \hat{\Pi}(r)}{\partial r^2} &= -\frac{a\beta^\beta}{b_1} b_1 C''(r) \exp(-[1 + \beta + b_1(c + \delta_1 r + C(r)) + b_2 r]) \\
&\quad + \frac{a\beta^\beta}{b_1}[b_1(\delta_1 + C'(r)) + b_2]^2 \exp(-[1 + \beta + b_1(c + \delta_1 r + C(r)) + b_2 r]).
\end{aligned}$$

Setting $\partial \hat{\Pi}(r)/\partial r$ to zero and solving it for r, we find that the optimal solution must satisfy the equation $b_1(\delta_1 + C'(r)) + b_2 = 0$. At the point that satisfies the equation $b_1(\delta_1 + C'(r)) + b_2 = 0$,

$$\frac{\partial^2 \hat{\Pi}(r)}{\partial r^2} = -\frac{a\beta^\beta}{b_1} b_1 C''(r) \exp(-[1 + \beta + b_1(c + \delta_1 r + C(r)) + b_2 r]) < 0,$$

and $\hat{\Pi}(r)$ is strictly concave. This lemma is then proved. ■

Proof of Proposition 9. The demand function $\hat{D}(\hat{p}^*(\hat{r}^*), \hat{r}^*) = a\exp(-1-\beta-b_1(c+\delta_1\hat{r}^*+C(\hat{r}^*))-b_2\hat{r}^*)$ implies that the retailer's stronger relative bargaining power can help increase the demand. Such a result also holds when the retailer takes on the gatekeeping responsibility, as shown in Proposition 7.

Next, we prove the impact of β on the two firms' profits. The first-order derivatives of $\hat{\pi}_M(\hat{w}^*(\hat{r}^*), \hat{r}^*)$ and $\hat{\pi}_R(\hat{p}^*(\hat{w}^*(\hat{r}^*), \hat{r}^*))$ w.r.t. β are

$$\frac{\partial \hat{\pi}_M(\hat{w}^*(\hat{r}^*), \hat{r}^*)}{\partial \beta} = \frac{a(1-\beta)}{b_1}\exp(-1-\beta-b_1(c+\delta_1\hat{r}^*+C(\hat{r}^*))-b_2\hat{r}^*) > 0,$$

and

$$\frac{\partial \hat{\pi}_R(\hat{p}^*(\hat{w}^*(\hat{r}^*), \hat{r}^*))}{\partial \beta} = -\frac{a}{b_1}\exp(-1-\beta-b_1(c+\delta_1\hat{r}^*+C(\hat{r}^*))-b_2\hat{r}^*) < 0.$$

It then follows that

$$\frac{\partial(\hat{\pi}_M(\hat{w}^*(\hat{r}^*), \hat{r}^*)+\hat{\pi}_R(\hat{p}^*(\hat{w}^*(\hat{r}^*), \hat{r}^*)))}{\partial \beta} = -\frac{a\beta}{b_1}\exp(-1-\beta-b_1(c+\delta_1\hat{r}^*+C(\hat{r}^*))-b_2\hat{r}^*) < 0.$$

Comparing the above result with Proposition 8, we prove this proposition. ■

Proof of Theorem 1. Using (9) and Lemma 5, we find

$$2b_1(C'(r^*)-C'(\hat{r}^*)) = (b_1\delta_1+2b_2)(1-y(I^*))+b_1(\delta_1-\delta_2 y(I^*)) \le 0,$$

when $\delta_2 \ge \omega_1$ with ω_1 defined as in (14). This implies that if $\delta_2 \ge \omega_1$, then $r^* \le \hat{r}^*$; otherwise, $r^* > \hat{r}^*$.

Next, we investigate the change in the average defective rate for consumers that results from the quality gatekeeping at the retail level. When the retailer implements her quality gatekeeping strategy, the average defective rate for consumers is $y(I^*)r^*$. When the retailer does not inspect the quality, the average defective rate for consumers is the same as the manufacturer's average defective rate $\hat{r}^*$. According to (9) and Lemma 5, we obtain

$$(b_1\delta_1+2b_2)(y(I^*)r^*-\hat{r}^*) = -2b_1(C'(r^*)r^*-C'(\hat{r}^*)\hat{r}^*)-b_1(\delta_2 y(I^*)r^*-\delta_1\hat{r}^*) \le 0,$$

when $\delta_2 \ge \omega_2$ with ω_2 defined as in (15). That is, if $\delta_2 \ge \omega_2$, then $y(I^*)r^* \le \hat{r}^*$; otherwise, $y(I^*)r^* > \hat{r}^*$.

We find from (15) that

$$\omega_2 = \frac{\delta_1\hat{r}^*-2(C'(r^*)r^*-C'(\hat{r}^*)\hat{r}^*)}{y(I^*)r^*} = \frac{\delta_1\hat{r}^*}{y(I^*)r^*}-\frac{2(C'(r^*)r^*-C'(\hat{r}^*)\hat{r}^*)}{y(I^*)r^*}.$$

When $\delta_2 < \omega_1$, because $r^* > \hat{r}^*$ and $C'(r^*)r^* > C'(\hat{r}^*)\hat{r}^*$, we have

$$\omega_2 < \frac{\delta_1 \hat{r}^*}{y(I^*)r^*} < \frac{\delta_1}{y(I^*)} < \omega_1.$$

The theorem is thus proved. ■

Proof of Theorem 2. We calculate

$$w^* - \hat{w}^* = \kappa_M \equiv C(r^*) + \delta_1 y(I^*)r^* + \delta_2(1 - y(I^*))r^* - \delta_1 \hat{r}^* - C(\hat{r}^*), \tag{23}$$

which represents the increase in the manufacturer's unit quality-related cost that results from the retailer's quality gatekeeping. Note that if $\kappa_M < 0$, then $-\kappa_M$ is the manufacturer's quality cost saving generated by the retailer's gatekeeping.

We then compute the difference between the optimal retail prices $p^*(w^*, r^*)$ and $\hat{p}^*(\hat{r}^*)$ as

$$p^*(w^*, r^*) - \hat{p}^*(\hat{r}^*) = \kappa_M + \kappa_R, \tag{24}$$

where $\kappa_R = \tau I^* - \delta_2 r^*(1 - y(I^*))$ denotes the retailer's *net* unit gatekeeping cost, as defined in Section 3.1. If $\kappa_R < 0$, then $-\kappa_R$ is viewed as the retailer's unit cost saving generated by her gatekeeping activity. When there is a channel-wide cost saving (i.e., $\kappa_M + \kappa_R < 0$), the retailer is willing to reduce her optimal retail price. As indicated by (23) and (24), the signs of $(w^* - \hat{w}^*)$ and $(p^*(w^*, r^*) - \hat{p}^*(\hat{r}^*))$ depend on κ_M and κ_R.

We compute

$$\begin{aligned}
&D(p^*(w^*, r^*), r^*, I^*) - \hat{D}(\hat{p}^*(\hat{r}^*), \hat{r}^*) \\
&= a\exp(-b_1 p^*(r^*, I^*) - b_2 y(I^*)r^*) - a\exp(-b_1 \hat{p}^*(\hat{r}^*) - b_2 \hat{r}^*) \\
&= a\exp(-1 - \beta - b_1(c + C(r^*) + \delta_1 y(I^*)r^* + \tau I^*) - b_2 y(I^*)r^*) \\
&- a\exp(-1 - \beta - b_1(c + \delta_1 \hat{r}^* + C(\hat{r}^*)) - b_2 \hat{r}^*).
\end{aligned} \tag{25}$$

We find that

$$\begin{aligned}
&[-1 - \beta - b_1(c + C(r^*) + \delta_1 y(I^*)r^* + \tau I^*) - b_2 y(I^*)r^*] \\
&- [-1 - \beta - b_1(c + \delta_1 \hat{r}^* + C(\hat{r}^*)) - b_2 \hat{r}^*] \\
&= -b_1(\kappa_M + \kappa_R) - b_2(y(I^*)r^* - \hat{r}^*).
\end{aligned}$$

Moreover, we find from (25) that if the inequality in (16) is satisfied, then $D(p^*(w^*, r^*), r^*, I^*) >$

$\hat{D}(\hat{p}^*(\hat{r}^*), \hat{r}^*)$, and

$$\frac{\partial\left(D(p^*(w^*,r^*),r^*,I^*)-\hat{D}(\hat{p}^*(\hat{r}^*),\hat{r}^*)\right)}{\partial\beta} = -\left(D(p^*(w^*,r^*),r^*,I^*)-\hat{D}(\hat{p}^*(\hat{r}^*),\hat{r}^*)\right) < 0,$$

which means that the difference between $D(p^*(w^*, r^*), r^*, I^*)$ and $\hat{D}(\hat{p}^*(\hat{r}^*), \hat{r}^*)$ is decreasing in β.

Next, we examine the impact of β on the channel-wide profit. We compute

$$\begin{aligned}&(\pi_R(w^*,r^*)+\pi_M(w^*,r^*))-(\hat{\pi}_M(\hat{w},\hat{r}^*)+\hat{\pi}_R(\hat{w}^*,\hat{r}^*))\\&=\frac{(1+\beta)}{b_1}[D(p^*(w^*,r^*),r^*,I^*)-\hat{D}(\hat{p}^*(\hat{r}^*),\hat{r}^*)],\end{aligned}$$

which implies that the impact of β on the channel-wide profit is the same as that on the demand.

Moreover, we compute

$$\begin{aligned}&\pi_R(w^*,r^*)-\hat{\pi}_R(\hat{w}^*,\hat{r}^*)\\&=\frac{1}{b_1}[D(p^*(w^*,r^*),r^*,I^*)-\hat{D}(\hat{p}^*(\hat{r}^*),\hat{r}^*)],\end{aligned}$$

and

$$\begin{aligned}&\pi_M(w^*,r^*)-\hat{\pi}_M(\hat{w}^*,\hat{r}^*)\\&=\frac{\beta}{b_1}[D(p^*(w^*,r^*),r^*,I^*)-\hat{D}(\hat{p}^*(\hat{r}^*),\hat{r}^*)],\end{aligned}$$

which means that both the manufacturer's and the retailer's profit changes are the same as the change in demand. We can thus prove this theorem. ■

Appendix C Robustness of Major Implications for Different Demand Functions

We now examine whether our major managerial insights that are summarized in Section 5 still hold if we consider different demand functions. We consider two commonly-used nonlinear demand functions: the logistic demand function (see, e.g., Phillips 2005) and the attraction demand function (see, e.g., Benjaafar, Elahi, and Donohue 2007). For details regarding the two demand function forms, see a recent review by Huang, Leng, and Parlar (2013).

C.1 Robustness of Major Implications for the Logistic Demand Function

We consider the following logistic demand function (see, e.g., Phillips 2005):

$$D(p, r, I) = \frac{a}{1 + \exp(a' + b_1 p + b_2 y(I) r)}, \tag{26}$$

where a represents the market size and the parameters $a', b_1, b_2 > 0$. Since it is intractable to analyze the two-stage problem with the demand function in (26), we subsequently perform numerical experiments with the parameter values as $a = 1000$, $a' = 0.03$, $b_1 = 0.01$, and $b_2 = 0.03$. In addition, the percentage of unidentified defects $y(I)$ is a power function $y(I) = (1 + I)^{-1}$. We also assume that the retailer's unit gatekeeping cost $\tau = 3$, the manufacturer's unit acquisition cost $c = 40$, the manufacturer's relative bargaining power $\beta = 0.4$, the unit penalty costs $\delta_1 = 1$ and $\delta_2 = 1.2$, and the manufacturer's unit quality-control cost function $C(r) = 10/\sqrt{1 + 5r}$.

We use the software of Maple 2015 to solve the two-stage problem, and obtain the negotiated average defective rate and the retailer's optimal gatekeeping intensity as $r^* = 9.03\%$ and $I^* = 67\%$, respectively; the negotiated wholesale price is $w^* = \$59.01$, and the optimal retail price is $p^* = \$95.66$. The demand is 272; the retailer's and the manufacturer's profits are $\$9,406.39$ and $\$2,877.73$, respectively; and the supply chain profit is $\$12,284.12$.

Without the retailer's gatekeeping, the negotiated average defective rate is $\hat{r}^* = 9.24\%$, which is greater than $r^* y(I^*) = 5.41\%$. But, if we reduce the value of δ_2 from 1.2 to 0.5, then $r^* y(I^*) = 9.35\% > \hat{r}^*$. This means that our **first** implication in Section 5 holds. In addition, for the case of no gatekeeping, we obtain the negotiated wholesale price as $\hat{w}^* = \$59.78$ and the optimal retail price as $\hat{p}^* = \$91.32$. We note that $\hat{w}^* > w^* = \$59.01$ and $\kappa_M = -0.01$, which implies that as a result of the gatekeeping activity at the retail level, the manufacturer achieves the cost saving $0.01 and thus, the negotiated wholesale price is reduced from $59.78 to $59.01. We also find that $\hat{p}^* < p^* = \$95.66$ and $\kappa_M + \kappa_R = 0.09$. It then follows that the retailer's gatekeeping activity makes the channel incur a higher quality-related cost and thus the optimal retail price is increased. The above result is consistent with the **second** implication given in Section 5.

When the retailer does not perform gatekeeping, the retailer's profit is $\$8,555.65$, which is smaller than that when the retailer has the gatekeeping activity. If we increase the value of b_1 from 0.01 to 0.02, then the retailer's gatekeeping reduces the retailer's profit from $\$4,338.66$ to $\$3,946.25$. But, if the value of β is reduced from 0.4 to 0.2, then the retailer's profit is increased to $\$5,305.71$. This shows that the **third** implication listed in Section 5 holds.

C.2 Robustness of Major Implications for the Attraction Demand Function

We consider the following attraction demand function (see, e.g., Benjaafar, Elahi, and Donohue 2007):

$$D(p, r, I) = \frac{a(b_1 p + b_2 y(I) r)}{(b_1 p + b_2 y(I) r) - a'}, \tag{27}$$

where $a, a', b_1, b_2 > 0$. It is also intractable to analyze the two-stage problem with the demand function in (27). We thus have to conduct numerical experiments with the parameter values in (27) as $a = 500$, $a' = 0.1$, $b_1 = 0.01$, and $b_2 = 0.03$. Other parameter values and functions are the same as those used in Appendix C.1.

We use the software of Maple 2015 to solve the two-stage problem, and obtain the negotiated average defective rate and the retailer's optimal gatekeeping intensity as $r^* = 8.76\%$ and $I^* = 56\%$, respectively; the negotiated wholesale price is $w^* = \$72.91$; and the optimal retail price is $p^* = \$101.33$. The demand is 555; the retailer's and the manufacturer's profits are $\$14,848.67$ and $\$13,573.31$, respectively; and the supply chain profit is $\$28,421.98$.

Without the retailer's gatekeeping, the negotiated average defective rate is $\hat{r}^* = 9.23\%$, which is greater than $r^* y(I^*) = 5.62\%$. But, if we reduce the value of δ_2 from 1.2 to 0.5, then $r^* y(I^*) = 9.41\% > \hat{r}^*$. This means that our **first** implication in Section 5 holds. In addition, we obtain the negotiated wholesale price as $\hat{w}^* = \$74.12$ and the optimal retail price as $\hat{p}^* = \$99.34$. We note that $\hat{w}^* > w^*$and $\kappa_M = -0.02$, which implies that as a result of the gatekeeping activity at the retail level, the manufacturer achieves the cost saving \$0.02 and thus, the negotiated wholesale price is reduced. We also find that $\hat{p}^* < p^*$ and $\kappa_M + \kappa_R = 0.17$. It then follows that the retailer's gatekeeping activity makes the channel incur a higher cost and thus the optimal retail price is increased. The above result is consistent with the **second** implication given in Section 5.

When the retailer does not perform gatekeeping, the retailer's profit is $\$14,017.10$, which is smaller than that when the retailer has the gatekeeping activity. If we increase the value of b_1 from 0.01 to 0.02, then the retailer's gatekeeping reduces the retailer's profit to $\$13,212.53$. But, if the value of β is reduced from 0.4 to 0.2, then the retailer's profit is increased to $\$14,848.67$. This shows that the **third** implication listed in Section 5 holds.