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Joint Pricing and Contingent Free-Shipping Decisions in B2C Transactions

Mingming Leng
Department of Computing and Decision Sciences, Lingnan University, Tuen Mun, Hong Kong, mmleng@ln.edu.hk

Rafael Becerril-Arreola
Department of Marketing, Anderson School of Management, University of California, Los Angeles, California 90095, USA, rafael.becerril.2013@anderson.ucla.edu

We consider an online retailer's joint pricing and contingent free-shipping (CFS) decisions in both monopoly and duopoly structures, which is an important marketing-operations interface problem. We begin by investigating the impacts of a retailer's decisions on consumers' purchase behaviors, and show that the CFS strategy is useful to acquire the consumers with large order sizes. Then, we compute the probability of repeated purchases, and construct an expected profit function for an online retailer in the monopolistic setting. We find that the fixed shipping fees may have the largest impact on the retailer's profit among all shipping-related parameters, and the retailer can benefit more from homogeneous markets than from heterogeneous ones. Next, we consider the competition between two retailers in the duopoly structure, and analytically show that, if two retailers have identical fixed and variable shipping fees, then their equilibrium decisions are equal. In order to numerically find a Nash equilibrium for two retailers, we develop a simulation approach using Arena and OptQuest. Our simulation-based examples suggest that, as a result of the competition, the two retailers should decrease their profit margins but increase their CFS cutoff levels if they have the same fixed and also the same variable shipping fees.

Key words: marketing-operations interface; profit margin; contingent free shipping; monopoly; duopoly
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1. Introduction

Free shipping (FS) has proved to be significantly effective in improving online retailing operations. In addition to traditional price discounts, many online retailers offer free-shipping promotions to consumers whose purchases exceed a given dollar amount. This paper investigates whether, when, and how price and shipping promotions boost online retailers' profits. The significance of this research stems from the increasing importance of online retailing (see US Census Bureau News 2007) and the prominence of online sales during certain shopping periods (such as holiday shopping seasons). For example, a large number of consumers choose the Internet to make online purchases during holiday shopping seasons. LeClaire (2006) indicated that around 75% of holiday shoppers buy their holiday gifts online, and nearly a third of holiday shoppers do half or more of their holiday shopping on the Internet. Holiday sales are crucial to online retailing. Moreover, as Dilworth (2006) reported, 78% of US small business owners that operate online shopping sites have stated that revenues generated from online shopping during the December holiday season make up a large percentage of their annual revenues. Online retailers are nonetheless disadvantaged as compared with offline merchants. In particular, online stores are currently less efficient than physical stores, since, as Moe and Fader (2004) estimated, their rates of conversion (defined as the percentage of online visitors that are converted into consumers) rarely exceed 5%. Although such inefficiency results largely from the low cost of visiting online stores (Moe and Fader 2004), it is also significantly driven by shipping fees, which may deter consumers. The impact of shipping fees on order incidence is forceful. Market research showed that from 52% (refer to Direct Marketing Association's report 2004) to 60% (refer to Jupiter Communications's report 2001) of online visitors abandon their online shopping carts when presented with shipping and handling fees.

To mitigate the negative impacts of shipping fees on conversion rates, online retailers (hereafter, simply referred to as retailers) implement a variety of shipping policies. The three most common ones are unconditional FS (UFS), contingent FS (CFS), and shipping fees that increase with order size. Under the UFS policy, a retailer absorbs the shipping costs for all orders. When CFS applies, the retailer pays the shipping fees but only for orders with a value equal to or above a predefined cutoff level. Under the third policy, all consumers are responsible for the shipping fees that increase with purchase values. Lewis (2006) showed that, among the three policies, CFS is the most effective in increasing the revenues of the retailers. From the 2007 survey (Advertising.Com 2007) of Advertising.com, we find that around 67% of consumers consider FS as the most enticing promotion offered by online retailers.

The degree of effectiveness of a CFS policy depends on the choice of the cutoff level. On the one hand, a high cutoff level exempts the retailer from a large part of the total shipping expenses and entices consumers to consolidate multiple orders into one, which can reduce the retailer's operational and shipping expenses. On the other hand, this high cutoff level may also hamper the arrival of some potential consumers by pushing them to spend more in order to qualify for FS. As Lewis (2006) and Yang (2006) showed, the number of online consumers decreases when the FS cutoff level increases, and vice versa. Although a low CFS cutoff level favors a large number of orders, it also imposes significant shipping costs on the retailer. For example, Amazon.com introduced everyday CFS in January 2002 and the firm, since then, has gradually lowered the minimum qualifying purchase amount. As a result, the firm's net shipping costs as a percentage of its net consolidated sales have almost steadily increased from 0.61% in 2001 to 1.02% in 2002, 2.58% in 2003, 2.85% in 2004, 2.82% in 2005, and 2.96% in 2006; see Amazon.com's annual reports. The financial burden that online retailers contract by implementing UFS and CFS could be compensated by inflated prices. However, higher prices also have a detrimental effect on order incidence. For recent discussions on pricing decisions in the retailing industry, see Cho et al. (2009), Ketzenberg and Zuidwijk (2009), etc. Thus, when an online retailer adopts the CFS strategy, he should deal with the trade-off between the following two issues: (i) a high CFS threshold deters consumers from placing online orders and (ii) a low CFS cutoff level imposes high operational and shipping expenses on the retailer. How should the retailer take this trade-off into consideration and maximize his expected profit when choosing the price and shipping promotion for a single shopping period? This is a marketing-operations interface problem, because pricing and shipping promotion levels impact operational and shipping expenses and vice versa.

Heuristic experimentation has been used by retailers in practice to search for their optimal cutoff levels (see Aimi 2006, Regan 2002). Two recent academic papers are concerned with the analysis of CFS decision-making problems in two different settings. Leng and Parlar (2005) considered a CFS decision problem in business-to-business (B2B) transactions, where an online seller announces his cutoff level decision and a buyer chooses her purchase amount. Accordingly, this problem was modeled as a leader–follower game in which the seller and the buyer act the roles of the leader and the follower, respectively. The authors solved the game to find the Stackelberg equilibrium. Yang (2006) considered a free shipping and repeat buying problem, in which a rational and cost-minimizing online shopper responds to both the price and the CFS threshold determined by a retailer. However, Yang (2006) considered a single consumer, and investigated only this consumer's purchasing decision problem rather than the retailer's free-shipping decision problem.

In this paper, we develop a two-stage model in a business-to-consumer (B2C) setting to capture the aggregate purchase behaviors of heterogeneous consumers, and characterize the relationship between the retailer's pricing and CFS decisions and his single-period expected profits. More specifically, we first analyze a consumer's net surplus function in a single transaction, which is computed as the consumer-specific utility minus his or her purchase-related expense. To reflect the diverse preferences and incomes of consumers, we introduce a random parameter into the consumer's utility function. Then, we maximize the net surplus function to forecast the consumer's purchase amount, and compute the conversion rate, i.e., the probability that a consumer buys products online.

Using our analytical results for a consumer's purchase amount, we develop the repeat buying model, and then analyze a monopolistic problem in which a single retailer seeks the optimal price and CFS cutoff level that maximize his expected profit for a single period. We perform sensitivity analyses to draw some important managerial insights; for example, we find that an online retailer should set its fixed shipping fee to zero but increase its variable shipping fee, because the impact of the fixed shipping fee on the retailer's profit is the largest compared with other shipping-related parameters. Then, we consider the duopoly structure in which two retailers compete for consumers in a market. We analytically show that, if the two retailers set the same fixed and variable shipping fees, then their equilibrium profit margins and CFS cutoff levels are symmetric; this important managerial insight is illustrated by our numerical examples. We use Arena and OptQuest to simulate several duopoly games and find their Nash equilibria. Note that Arena is a simulation and automation software application commonly used for the simulation of business processes; OptQuest is a software add-in for Arena used to optimize such processes. Our simulation approach is helpful to solve those non-cooperative games that are too complicated to analyze algebraically.

The remainder of our paper is organized as follows: section 2 provides a preliminary discussion about our modeling approach for an online retailer, and section 3 concerns our analysis of a consumer's purchasing decision in a single transaction. We then examine optimal decisions for an online retailer for the monopoly structure in section 4 and the competition between two retailers in section 5. This paper ends with a summary of our major managerial insights in section 6.

2. Preliminaries

In this section, we provide a preliminary discussion about our modeling approach for an online retailer in the monopoly and duopoly structures. For both cases, we consider a joint pricing and free-shipping decision problem of the retailer who sells *multiple* products of a single category and offers a CFS promotion in a market during a single period. We assume that all products for sale at an online retailer belong to a single category. Moreover, as Gupta (1991) discussed, in almost all marketing applications, the covariate values —i.e., prices and other variables that affect purchase rates—remain constant for a time interval (e.g., a week). Accordingly, it is assumed that the single period is short enough for static pricing (as opposed to dynamic pricing) to be the most appropriate strategy.

Since the online retailer sells n products, it has to determine n optimal prices. Note that, as discussed above, these n products that are sold by the online retailer are in a single category. Like a number of scholars in both the marketing and the operations management areas, use the concept of *profit margin* to make the pricing decisions for the online retailer. The profit margin for a product is defined as the ratio of the retailer's per unit profit to its unit acquisition cost. As a result, given the profit margin and the unit acquisition cost for a product, we can easily compute the selling price for the product. More specifically, we denote the retailer's profit margin of product i ($i = 1, 2, \ldots, n$) by m_i and the retailer's unit acquisition cost of the product by c_i. The unit (marginal) profit of the product is $m_i c_i$, and its price p_i can be computed as $p_i = (1+m_i)c_i$. As some marketing scholars (e.g., Anderson et al. 1992, Blattberg and Neslin 1990) have shown, the profit margins of different products in the same category are typically *identical* and setting a uniform margin has been a common pricing rule for retailers. Therefore, it is reasonable to assume that the retailer applies an identical profit margin m to n products belonging to a single category. As a result, making the pricing decisions for these n products is equivalent to determining a single profit margin m. Using this modeling approach, we find that, if the online retailer makes an optimal decision on the profit margin m, the retailer can then easily determine its prices for all of n products in the category. For recent applications of this modeling approach to marketing-operations problems, see Cachon and Kok (2007), Dong et al. (2009), etc.

We notice that, during the single period, each consumer may repeat his or her purchase (see, e.g., Ehrenberg 1988). When repeat-buying occurs, a consumer may make multiple online transactions with possibly correlated purchase amounts. As a result, we cannot assume independence between the transactions of a given customer. Thus, for each consumer, we need to consider the probability of multiple purchases and compute the expected number of purchases. We assume that consumers do not influence each other when they buy products online, such that the purchase amounts of different consumes are independent.

In practice, the cutoff levels for CFS promotions are measured in terms of the value of the purchase. For example, the CFS cutoff levels set by Amazon.com and Barnesandnoble.com are both equal to US$25. Accordingly, in our problem formulation, the retailer announces a CFS cutoff level in US$, and offers the free shipping service to every consumer with a total purchase amount (also measured in US$) no less than the CFS threshold. A consumer's online purchasing process is described as follows: At first, the consumer browses the website of the retailer and collects the prices of the products (in which the consumer is interested) and the shipping-related information. Then, the consumer determines his or her optimal purchase quantity of each product. If the purchase quantity of every product is zero, then the consumer abandons the shopping cart, leaves the retailer, and buys the product(s) from a brick-and-mortar store or another source. Otherwise, the consumer completes an online transaction with the retailer but may or may not qualify for free shipping. In particular, when the consumer's total purchase amount (i.e., the consumer's purchase cost for all products that he or she buys) is greater than or equal to the retailer's CFS cutoff level, the consumer qualifies for FS and is not responsible for the shipping fee. On the other hand, when the purchase amount is less than the CFS threshold, the consumer pays the shipping fee. Note that, when a consumer repeats his or her purchase, the CFS policy applies to the consumer's purchase amount of each transaction rather than to the cumulative amount of all repeated transactions. For example, if the CFS cutoff level is US$100 and a consumer makes two purchases each for an amount of US$60, then the consumer does not qualify for free-shipping service in either transaction. Since the concept of ''shipping fee'' is important to our paper, we define it below.

DEFINITION 1. *Shipping fee is the amount that a consumer, who does not qualify for CFS in a single transaction, pays to the retailer for the delivery of the products sold*.

We assume that, in the market that the online retailer serves, there is a finite consumer base $\mathscr{B}$ consisting of some consumers who may buy the retailer's products. The concept of "consumer base" has been widely used to analyze the impact of marketing strategies on consumer behavior; e.g., Lewis et al. (2006) considered a consumer base that includes 1000 consumers and presented an empirical study regarding the effects of shipping costs on consumer behavior. Whether or not a consumer is likely to complete an online purchase and possibly pay the shipping fee depends on his or her income and willingness to spend in the purchase of this product. Like previous works (e.g., Braden and Oren 1994, Gajanan et al. 2007, Sarvary and Parker 1997, etc.), this research accounts for preferences, income, and other heterogeneous consumer characteristics by introducing a non-negative i.i.d. random parameter θ_i ($i = 1, 2, \ldots, n$) into the consumer-specific net-surplus function. A high value of θ_i implies that the consumer places a high value on product i and that he or she can afford to spend a relatively high amount of money to acquire it. In contrast, a low value of θ_i implies that the consumer is relatively indifferent to product i or that he or she can spend only a relatively low amount of money to buy it.

Next, we examine a consumer's purchasing decision in a single transaction, and then consider a monopoly structure in which a single retailer determines its optimal decisions to maximize its expected profit. We also investigate the competition between two retailers in a duopoly structure.

3. Purchasing Decision of a Consumer in a Single Online Transaction

We now investigate the purchasing decision of a consumer in a single transaction, given the pricing and CFS decisions of an online retailer. We first develop the consumer's net surplus function, and then maximize the net surplus to find the consumer's optimal purchase quantity of each product and optimal purchase amount (that the consumer spends for his or her online purchase). In addition, we analyze the impacts of the retailer's pricing and CFS decisions on the conversion rate (i.e., probability that a consumer buys in an online transaction).

3.1. Net Surplus Function of a Consumer

We propose a net surplus function for a consumer with product-specific parameters θ_i ($i = 1, 2, \ldots, n$) and maximize it to determine the consumer's optimal purchase quantities of n products, given the retailer's profit margin m and CFS cutoff level x. When the consumer buys q_i units of product i from the retailer, the consumer obtains a consumption utility $U(q_1, q_2, \ldots, q_n \mid \theta_1, \theta_2, \ldots, \theta_n)$ (a.k.a. "gross surplus"; see, e.g., Cremer et al. 2001). Since the consumer's product-specific parameter θ_i affects only his or her purchasing decision on product i, we can write the consumer's utility function $U(q_1, q_2, \ldots, q_n \mid \theta_1, \theta_2, \ldots, \theta_n)$ as the sum of the consumer's utilities for all products, i.e.,

$$U(q_1, q_2, \ldots, q_n | \theta_1, \theta_2, \ldots, \theta_n) = \sum_{i=1}^{n} \Lambda_i(q_i|\theta_i), \quad (1)$$

where $\Lambda_i(q_i|\theta_i)$ is the utility of the consumer with specific parameter θ_i who consumes q_i units of product i. Such an additive form has been commonly used to model a consumer's consumption utility (see, e.g., Chung 1994, Coto-Millán 1999, etc.). As commonly assumed, the utility function $\Lambda_i(q_i|\theta_i)$ is positive, increasing and concave in q_i; that is, $\Lambda_i(0|\theta_i) = 0$, $\Lambda_i(q_i|\theta_i) \geq 0$, $\Lambda_i'(q_i|\theta_i) \geq 0$, and $\Lambda_i''(q_i|\theta_i) \leq 0$. (For a detailed discussion on utility functions, see Chung 1994, Coto-Millán 1999.) Since, for a fixed quantity q_i, a consumer with a large value of θ_i should draw a utility higher than that drawn by a consumer with a small value of θ_i, the utility function $\Lambda_i(q_i|\theta_i)$ is increasing in θ_i (see Tirole 1992). In our paper, we assume that the consumer's utility function is linear in the parameter θ_i and in the square root of the purchase quantity q_i, i.e.,

$$\Lambda_i(q_i|\theta_i) = \theta_i\sqrt{q_i}, \quad (2)$$

for $i = 1, 2, \ldots, n$. Note that the *square-root* utility function is widely used in the economics, marketing, and operations management areas (for other applications of the square-root utility function, we refer readers to, e.g., Basu et al. 1985, Leng and Parlar 2005).

In order to get $q_i > 0$ units of product i ($i = 1, 2, \ldots, n$), the consumer pays the purchase amount and the shipping fee (if the consumer does not qualify for free shipping). The consumer's cost of purchasing q_i units of product i is $p_i q_i = (1 + m)c_i q_i$, in which m and c_i denote the retailer's profit margin and unit acquisition cost, respectively. The consumer's total purchase amount (in US$) is then computed as

$$A \equiv (1 + m) \sum_{i=1}^{n} c_i q_i. \quad (3)$$

We next discuss the calculation of the shipping fees that the consumer with parameters θ_i ($i = 1, 2, \ldots, n$) absorbs if he or she does not qualify for free shipping. In practice, there are three common shipping-fee calculation methods: (i) quantity-based shipping rate, (ii) weight-based shipping rate, and (iii) order size-based shipping rate. Using the first method, an online retailer determines the shipping fees as a fixed shipment fee for an online order plus unit fees for each product in the order. In practice, the quantity-based shipping method usually applies to an online retailing system that sells a single product or multiple "similar" products that have similar weights and/or similar prices. Otherwise, for two different products even in the same category, the shipping fees should be different. For example, the shipping fee incurred by a consumer who purchases a USB flash disk should be different from that when the consumer buys a desktop computer. In this paper, we consider an online retailer selling multiple products that belong to a single category but may not have similar weights and prices. Therefore, we do not consider the quantity-based shipping method. An online retailer adopts the second method (i.e., weight-based shipping rate) mostly when the products for sale at the retailer have heterogeneous weights and physical sizes. We do not use this method for our modeling, because we do not consider the products' weights but only investigate the pricing and free-shipping issues that are both associated with a consumer's purchase amount (order size) rather than the weights or physical sizes of the products that the consumer buys.

The third shipping method (i.e., order size-based shipping rate)—which has been the most common one for the online retailers—is used to calculate the shipping fee according to the dollar value of an online order. Specifically, this method is, in practice, implemented by using "By Order Total" shipping method (see, e.g., rmtsupport.com.) with the following formula: the shipping fees for an order equal a fixed shipping base rate plus variable shipping fees that are calculated as a percentage (i.e., shipping charge per unit dollar) times the total dollar value of the order. For example, HayHouse.com, an online bookstore, calculates the shipping fee for an online order as 30% of the total dollar value of the order. HermeticKa.com, a webstore with robes, banners, cloths, etc., charges as shipping fees 10% of the dollar value of each order placed within the United States. As reported by Luening (a staff writer of CNET News) in Luening (2001), a survey of 50 major online retailers found that 54% of them base shipping charges on order size. Accordingly, we, in this paper, assume that the shipping fee of the consumer with purchase amount A is calculated as

$$S(A) = s_0 + sA = s_0 + s(1+m)\sum\nolimits_{i=1}^{n} c_i q_i, \qquad (4)$$

where s_0 and s denote the fixed shipping fee and the variable shipping fee, respectively.

Using the above, we can compute the *net surplus* that the consumer with parameters θ_i $(i = 1, 2, \ldots, n)$ derives from consuming q_i units of product i $(i = 1, 2, \ldots, n)$ as the consumer's gross surplus (utility) minus his or her purchase cost and, possibly, shipping fee. For an application of the concept of net surplus, see Valletti and Cambini (2005). Letting G denote the net consumption surplus of the consumer with parameters θ_i $(i = 1, 2, \ldots, n)$ when the retailer's profit margin is m and its CFS cutoff level is x, we have

$$G = \begin{cases} G_1 \equiv \sum\limits_{i=1}^{n} \theta_i \sqrt{q_i} - (1+m)\sum\limits_{i=1}^{n} c_i q_i, & \text{if } A \geq x, \\ G_2 \equiv \sum\limits_{i=1}^{n} \theta_i \sqrt{q_i} - (1+m)\sum\limits_{i=1}^{n} c_i q_i - S(A), & \text{if } A < x. \end{cases} \qquad (5)$$

3.2. Purchasing Decision of a Consumer

We momentarily ignore the constraints in (5) and find the solution that maximizes G_1 and the solution that maximizes G_2.

THEOREM 1. *The optimal purchase quantities* $\bar{q}_i$ $(i = 1, 2, \ldots, n)$ *that maximize the consumer's net surplus function* G_1 *are*

$$\bar{q}_i = \frac{\theta_i^2}{4(1+m)^2 c_i^2}, \text{ for } i = 1, 2, \ldots, n, \qquad (6)$$

and the resulting purchase amount $\bar{A}$ *and maximum net surplus* G_1^* *are, respectively,*

$$\bar{A} = \frac{\phi}{4(1+m)} \text{ and } G_1^* = \frac{\phi}{4(1+m)}, \qquad (7)$$

where $\phi \equiv \sum_{i=1}^{n} \theta_i^2 / c_i$ *denotes the consumer-specific utility parameter (for all products) in his or her net surplus.*

The optimal purchase quantities $\hat{q}_i$ $(i = 1, 2, \ldots, n)$ that maximize the consumer's net surplus function G_2 are

$$\hat{q}_i = \frac{\theta_i^2}{4(1+s)(1+m)^2 c_i^2}, \text{ for } i = 1, 2, \ldots, n, \qquad (8)$$

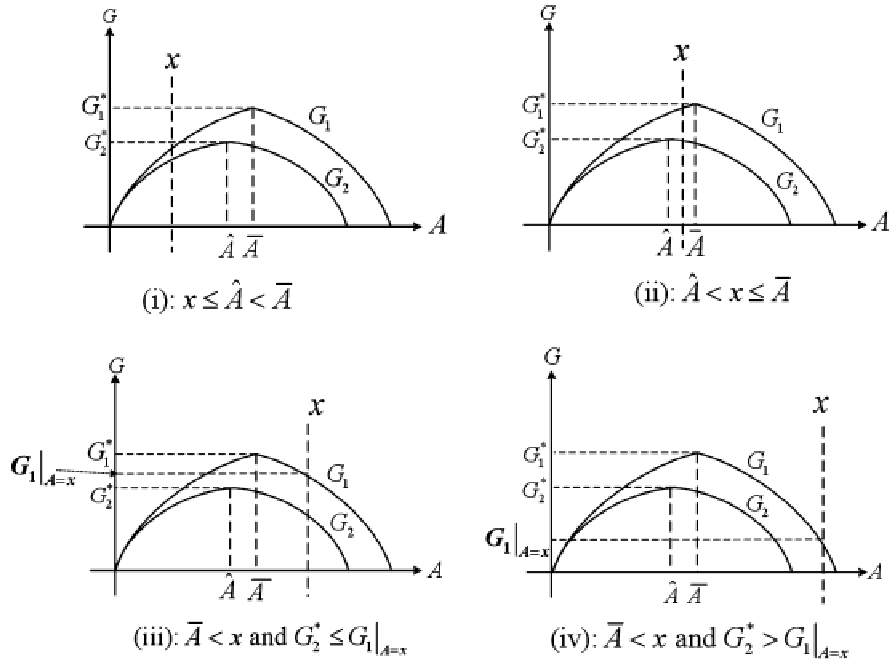
and the resulting purchase amount $\hat{A}$ and maximum net surplus G_2^* are, respectively,

$$\hat{A} = \frac{\phi}{4(1+s)(1+m)} = \frac{\bar{A}}{1+s} \text{ and } \quad G_2^* = \frac{\phi}{4(1+s)(1+m)} - s_0. \qquad (9)$$

PROOF. The proof of this theorem and the proofs of all subsequent theorems in our main paper are given in Appendix S1. □

In a single transaction, whether or not the consumer with the parameter ϕ can obtain the free-shipping service depends on the comparison between the CFS cutoff level x and the consumer's total purchase amount A. This means that we should only pay attention to the amount A, which is a random variable

Figure 1 Four Cases for the Optimal Solution A^* that Maximizes the Net Surplus G of the Consumer with the Parameter ϕ



due to the randomness of the parameter ϕ, as shown in Theorem 1.

THEOREM 2. *The net surplus functions G_1 and G_2 are both concave in the consumer's purchase amount A, and have their maxima at $\bar{A}$ [as given in (7)] and $\hat{A}$ [as given in (9)], respectively.*

Now we maximize the consumer's net surplus G in (5) to find the consumer's optimal purchase quantities q_i^* $(i = 1, 2, \ldots, n)$ and purchase amount A^*. When we consider the constraints in (5), we should perform our analysis according to the position of the CFS cutoff level x. Thus, we need to discuss four cases as shown in Figure 1, and for each case we need to find the optimal purchase amount A^*. For our particular discussion regarding the four cases in Figure 1, see Appendix S2.

Using our analytical results in Appendix S3, we can calculate the optimal purchase amount A^* as shown in the following theorem.

THEOREM 3. *Given a profit margin m and a CFS cutoff level x, the optimal purchase amount A^* that maximizes the net surplus of the consumer with specific parameter ϕ is as follows:*

1. *If $x \geq 4s_0(1+s)$, then*
 (a) *the consumer does not purchase any product online if $\phi < x(1+m)$;*
 (b) *the consumer spends \$x if $x(1+m) \leq \phi \leq 4(1+s)(1+m)[\sqrt{(1+s)x} - \sqrt{sx+s_0}]^2$;*
 (c) *the consumer spends \$$\hat{A}$ if $4(1+s)(1+m)[\sqrt{(1+s)x} - \sqrt{sx+s_0}]^2 < \phi < 4x(1+m)$;*
 (d) *the consumer spends \$$\bar{A}$ if $\phi \geq 4x(1+m)$.*
2. *If $s_0(1+s) < x < 4s_0(1+s)$, then*
 (a) *the consumer does not purchase any product online if $\phi < 4s_0(1+s)(1+m)$;*
 (b) *the consumer spends \$$\hat{A}$ if $4s_0(1+s)(1+m) < \phi < 4x(1+m)$;*
 (c) *the consumer spends \$$\bar{A}$ if $\phi \geq 4x(1+m)$.*
3. *If $0 \leq x < s_0(1+s)$, then*
 (a) *the consumer does not purchase any product online if $\phi < 4x(1+m)$;*
 (b) *the consumer spends \$$\bar{A}$ if $\phi \geq 4x(1+m)$.*

Theorem 3 suggests that a consumer's online purchasing decision depends on the CFS cutoff level x. More specifically, if the CFS cutoff level x is sufficiently high [i.e., when $x \geq 4s_0(1+s)$], then the consumer with the specific parameter ϕ may spend \$$x$, \$$\bar{A}$, \$$\hat{A}$, or may not buy anything online. Note that a consumer with the purchase amount \$$\bar{A}$ qualifies for free shipping; then, the consumer does not need to consider whether or not to change his or her purchasing decision for the free-shipping service. However, if a consumer's purchase amount is \$$\hat{A}$, then the consumer does not qualify for free shipping. Thus, the consumer may increase his or her purchase amount from \$$\hat{A}$ to \$$x$. More specifically, if $\hat{A}$ is close enough to x, then the consumer may increase his or her purchase amount to obtain the free-shipping service; otherwise, if $\hat{A}$ is small, then the consumer is unlikely to increase $\hat{A}$ to x and stays with $\hat{A}$. In addition, the consumers with very small purchase amounts cannot afford the shipping fee and cannot increase their amounts to qualify for free shipping; thus, they are likely to quit their online purchases.

When the CFS threshold x is at the medium level [i.e., $s_0(1+s) < x < 4s_0(1+s)$], the consumer may spend $\$\bar{A}$, $\$\hat{A}$, or leave without any purchase. This implies that a number of consumers with medium or large values of ϕ spend the amount $\$\bar{A}$ and qualify for the free shipping. However, consumers with *small* values of ϕ intend *small* purchase amounts, and may not want to increase their purchase amounts to the CFS cutoff level x to qualify for the free-shipping service. Furthermore, if a consumer has very small value of ϕ and thus his or her purchase amount is very small, then the consumer may abandon his or her online shopping cart because he or she cannot afford the shipping fee and also cannot spend more to qualify for free shipping. Therefore, we conclude that, when a moderate CFS threshold applies, the consumers who do not qualify for the free-shipping service may stay with their small purchase amount $\$\hat{A}$, or may leave without any purchase. When the CFS threshold x is small [i.e., $0 \le x \le s_0(1+s)$], most consumers qualify for free shipping but few other consumers with very small purchase amounts abandon their shopping carts because they do not want to pay for the shipping fee and also do not purchase more to qualify for free shipping.

From the above discussion we draw an important managerial insight, which is presented in the following remark.

REMARK 1. *The CFS strategy should be attractive mostly to consumers with large order sizes. That is, if a consumer's purchase amount is large but it is still smaller than the CFS cutoff level, then the consumer is likely to increase his or her purchase amount to qualify for the free-shipping service. Thus, an online retailer should use the CFS strategy mainly to acquire consumers with large order sizes. Our analytical result is supported by Lewis et al.'s empirical study (Lewis et al. 2006) in which the CFS schedules that involve incentives for large orders are able to successfully induce consumers to shift to larger order sizes.*

3.3. Conversion Rate

We find from Theorem 3 that each consumer may buy or may not buy from the online retailer. One may be interested in the following question: how do the retailer's pricing (profit margin) and CFS decisions affect the conversion rate? The conversion rate is herein defined as the ratio of the number of consumers who buy at least once from the retailer over the total number of consumers who consider buying from the retailer. According to this definition, the conversion rate is the probability that a consumer makes at least one purchase. As in some previous marketing publications (e.g., Lewis et al. 2006), we let all of arriving consumers compose the consumer base $\mathscr{B}$ (e.g., $\mathscr{B} = 1000$ in Lewis et al. 2006). Then we can calculate the number of consumers who buy from the retailer as the size of the consumer base times the conversion rate. That is, the number of consumers who buy from retailer is $\mathscr{B} \times \Pr(A > 0)$, where $\Pr(A > 0)$ is the conversion rate.

$$\Pr(A>0) = \begin{cases} 1 - F[x(1+m)] = \int_{x(1+m)}^{\infty} f(\phi)d\phi, & \text{if } x \ge 4s_0(1+s), \\ 1 - F[4s_0(1+s)(1+m)] = \int_{4s_0(1+s)(1+m)}^{\infty} f(\phi)d\phi, & \text{if } s_0(1+s) < x < 4s_0(1+s), \\ 1 - F[4x(1+m)] = \int_{4x(1+m)}^{\infty} f(\phi)d\phi, & \text{if } 0 \le x \le s_0(1+s), \end{cases}$$

THEOREM 4. *The conversion rate $\Pr(A>0)$ as follows: where $f(\cdot)$ and $F(\cdot)$ are, respectively, the probability density function (p.d.f.) and the cumulative distribution function (c.d.f.) of the random parameter ϕ.*

From Theorem 4, we can find that the conversion rate decreases as the CFS threshold x increases in the range $[0, s_0(1+s)]$. This means that a small CFS threshold deters most consumers with small purchase amounts, because those consumers are unwilling to increase their small amounts and also cannot afford the shipping fee. However, as the CFS cutoff level increases in the range $[s_0(1+s), 4s_0(1+s)]$, the conversion rate is unchanged, because of the following facts: most consumers with small purchase amounts are deterred by the nonzero CFS threshold. The consumers with medium purchase amounts can afford the shipping fee (which may not be large compared with their purchase amounts), but they are unwilling to spend more for the free-shipping service, as shown in Theorem 3. Other consumers have large purchase amounts and qualify for free shipping. Thus, for medium-size orders, the CFS cutoff level does not significantly affect the conversion rate. However, as x increases to a large value, i.e., $x \ge 4s_0(1+s)$, some consumers with medium and large purchase amounts may abandon their shopping cart and the conversion rate thus decreases. In addition, we find from Theorem 4 that the conversion rate decreases as the profit margin m (prices) increases.

Using Theorem 4 we can draw some important managerial insights, as shown in the following theorem.

THEOREM 5. *The impacts of profit margin m and CFS threshold x on the conversion rate are described as follows:*

1. *If m is constant and the p.d.f.* $f(\cdot)$ *is unimodal, then a small CFS threshold x [i.e.,* $0 \le x \le s_0(1+s)$*] has greater impact on the conversion rate than a large value of x [i.e.,* $x \ge 4s_0(1+s)$*].*
2. *If x is constant and the p.d.f.* $f(\cdot)$ *is unimodal, then the profit margin m when x is small [i.e.,* $0 \le x \le s_0(1+s)$*]* may *or* may not *have greater impact on the conversion rate than that when x is large [i.e.,* $x \ge 4s_0(1+s)$*], which depends on the p.d.f.* $f(\cdot)$.

According to Theorem 5, we find that the impacts of x and m depend on the shape of the p.d.f. $f(\cdot)$. Since many commonly used probability density functions are unimodal, e.g., Normal, Weibull, Johnson, Lognormal, etc., the results given in Theorem 5 should be applicable to practice. In fact, as the CFS cutoff level increases within a low range, many consumers with small purchase amounts may leave because they cannot afford the shipping fee and are also unwilling to increase their purchase amounts for free shipping. But, as x increases within a high range, most consumers with large purchase amounts are likely to increase their order sizes for free shipping or be willing to pay for shipping fee (which should be small compared with these consumers' purchase amounts). But, the profit margin (pricing decision) affects all consumers, no matter in which range the CFS threshold x is. Summarizing the above gives the following remark.

REMARK 2. *The CFS strategy should be mostly applied to acquire consumers with large order sizes, because, as Theorem 5 indicates, the CFS threshold x's impact on the conversion rate when x is small is greater than that when x is large. This reflects the fact that the CFS strategy does not significantly deter consumers with large purchase amounts from their online transactions. Our result in this remark is similar to that in Remark 1 in which the CFS strategy is also proved to be important to consumers with large purchases but using our analytical results of consumers' purchase amounts rather than the conversion rate.*

4. Optimal Decisions of a Single Retailer in the Monopoly Structure

In this section, we analyze the online retailer's expected profit generated during a single period, and find optimal profit margin and CFS cutoff level for the retailer. Since a consumer may buy nothing, or may buy products once or multiple times, we first consider the repeat buying for a consumer, and compute the probability that the consumer makes r ($r = 0, 1, \ldots, \infty$) purchases during the single period. Next, we construct an expected profit function for the retailer, using our analytical result for repeat buying and that for a consumer's purchasing decision in a single transaction (which is obtained in section 3). We then maximize the expected profit to find optimal decisions for the retailer, and perform sensitivity analysis so as to examine the impacts of some parameters on the retailer's decisions and also draw some important managerial insights.

4.1. Repeat Buying

We now consider the purchasing behavior of a consumer who may not place any order, or may order products online once, or may repeat his or her purchases. We assume that, for a consumer, the number of purchases is a Poisson-distributed random variable, i.e., the probability of r ($r = 0, 1, \ldots, \infty$) purchases during the single period is

$$Pr(T = r) = \frac{exp(-\lambda)\lambda^r}{r!}, \quad (10)$$

where λ denotes the consumer's expected *order frequency* (i.e., expected number of purchases for a period). The Poisson distribution and its compounded extensions with random values of λ, such as the negative binomial distribution, have been widely used to model repeat buying; see Morrison and Schmittlein (1988). Unlike extant work, we do not account for consumer heterogeneity by letting λ be a random variable. We instead incorporate consumer heterogeneity into our model by using our results in Theorem 4, which already account for any distribution $f(\cdot)$ of the parameter ϕ.

THEOREM 6. *For the Poisson distribution* $\Pr(T = r)$ *in (10), a consumer's expected order frequency (i.e., expected number of purchases that a consumer makes during a period) is obtained as follows:*

$$\lambda = \begin{cases} \ln\{1/F[x(1+m)]\}, & \text{if } x \ge 4s_0(1+s), \\ \ln\{1/F[4s_0(1+s)(1+m)]\}, & \text{if } s_0(1+s) < x < 4s_0(1+s), \\ \ln\{1/F[4x(1+m)]\}, & \text{if } 0 \le x \le s_0(1+s). \end{cases} \quad (11)$$

Theorem 6 implies that, when the online retailer increases the CFS cutoff level x and/or the profit margin m, the consumer responds by reducing his or her order frequency.

REMARK 3. *Theorem 6 indicates that a consumer's expected order frequency* λ *is decreasing in the CFS cutoff level x. We recall from Theorem 3 that the CFS cutoff level x affects a consumer's order size (i.e., purchase amount), and learn from Remark 1 that the CFS strategy can induce consumers (whose order sizes are relatively large but smaller than x dollars) to increase their purchase amounts to qualify for free shipping. It thus follows that, as the CFS cutoff level x increases, a consumer's order size increases whereas the consumer's order frequency decreases.*

Using (10) and (11) we can easily compute the probabilities of no purchase, one purchase, and multiple purchases, which are next used for the calculation of the retailer's expected profit during a single period.

4.2. Expected Profit Function of the Retailer

Given the profit margin m and the CFS cutoff level x, an integer-valued number of consumers visit the online retailer during a single period. Like in extant marketing publications (e.g., Lewis et al. 2006), we let all of arriving consumers compose the consumer base $\mathscr{B}$ (e.g., $\mathscr{B} = 1000$ in Lewis et al. 2006). Note that each arriving consumer in the base $\mathscr{B}$ may not buy, may buy once, or may buy multiple times.

THEOREM 7. *The retailer's expected profit is computed as,*

$$\Pi(m,x) = \sum\nolimits_{r=0}^{\infty} \pi(m,x|T=r) \times \Pr(T=r) \times \mathscr{B}, \quad (12)$$

where $\pi(m,x|T=r) = \int_0^{\infty} \pi_i(m,x,\phi|T=r) \times f(\phi)d\phi$ *denotes the retailer's expected profit drawn from a consumer who places his or her orders r times;* $\Pr(T=r)$ *is the probability of r purchases with the expected number* λ *as given in Theorem 6; and* $\Pr(T=r) \times \mathscr{B}$ *is the number of consumers who repeat their purchases r times.*

Next, we compute the retailer's expected revenue and expected cost that are generated when the retailer serves r online orders of a consumer, and then find the expected profit function $\pi(m,x|T=r)$. We notice from Theorem 3 that a consumer with parameter ϕ may abandon his or her shopping cart and leave without any purchase, may spend \$$x$ or \$$\bar{A}$ to qualify for free shipping, or may spend \$$\hat{A}$ and pay the shipping fee. Note that the CFS policy applies to a consumer's single transaction rather than the consumer's aggregate amount for his or her r repeated purchases. Thus, we must calculate $\pi(m,x|T=r)$ as the retailer's expected profit from a single transaction of the consumer times the number of transactions r; that is, $\pi(m,x|T=r) = \pi(m,x) \times r$, in which $\pi(m,x)$ represents the profit that the retailer attains from a single transaction of a consumer. Note that $\pi(m,x)$ is computed as the retailer's revenue minus acquisition and shipping costs. Recalling our discussion in section 2, we can use profit margins to calculate the retailer's revenue minus its acquisition cost. For example, assuming that a consumer's purchase amount is A, the retailer's revenue is A and its acquisition cost is $A/(1+m)$; thus, the retailer's revenue minus its acquisition cost is $mA/(1+m)$. Next, we discuss the calculation of the retailer's shipping cost, which is defined below.

DEFINITION 2. *Shipping cost is the amount that the retailer pays for shipping the products bought by a consumer from stock (e.g., the retailer's warehouse) to the consumer's address.*

Note that "shipping cost" is different from "shipping fee" (which is given by Definition 1). While the shipping fee is always set by the retailer, the shipping cost may be determined by a third-party transportation firm (e.g., UPS, Fedex, etc.) if shipping is outsourced. In fact, regardless of whether a consumer pays the shipping fee or qualifies for CFS, the retailer absorbs the shipping cost.

As Lewis et al. (2006) showed, online retailers may subsidize the shipping fees. This means that, even if a consumer does not qualify for CFS, the shipping fee paid by the consumer may be lower than the shipping cost incurred by the retailer. Actually, the retailer may treat the shipping fee (paid by consumers) as a source of its revenue, and thus choose the shipping fee that is higher than the shipping cost incurred by the retailer. Using a non-negative parameter k, we can compute the shipping cost $K(A)$ for delivering products worth \$$A$ as

$$K(A) = k \times S(A) = k(s_0 + sA), \quad (13)$$

where $S(A)$ is given in (4), and the parameter k may be greater than, equal to, or smaller than 1. If $k > 1$, then the retailer subsidizes the shipping for consumers who spend \$$A$ but do not qualify for CFS. If $k = 1$, then the shipping cost $K(A)$ is equal to the shipping fee $S(A)$; there is no shipping subsidization. Otherwise, if $0 \le k < 1$, then the shipping fee paid by a consumer is higher than the shipping cost incurred by the retailer who thus benefits from shipping products to the consumer. The shipping cost function (13) has been used by some scholars such as Baumol and Vinod (1970).

Using $\pi(m,x)$ we re-write $\Pi(m,x)$ in (12) as

$$\begin{aligned}\Pi(m,x) &= \sum\nolimits_{r=0}^{\infty} \pi(m,x) \times r \times \Pr(T=r) \times \mathscr{B} \\ &= \pi(m,x) \times \lambda(\phi) \times \mathscr{B}, \end{aligned} \quad (14)$$

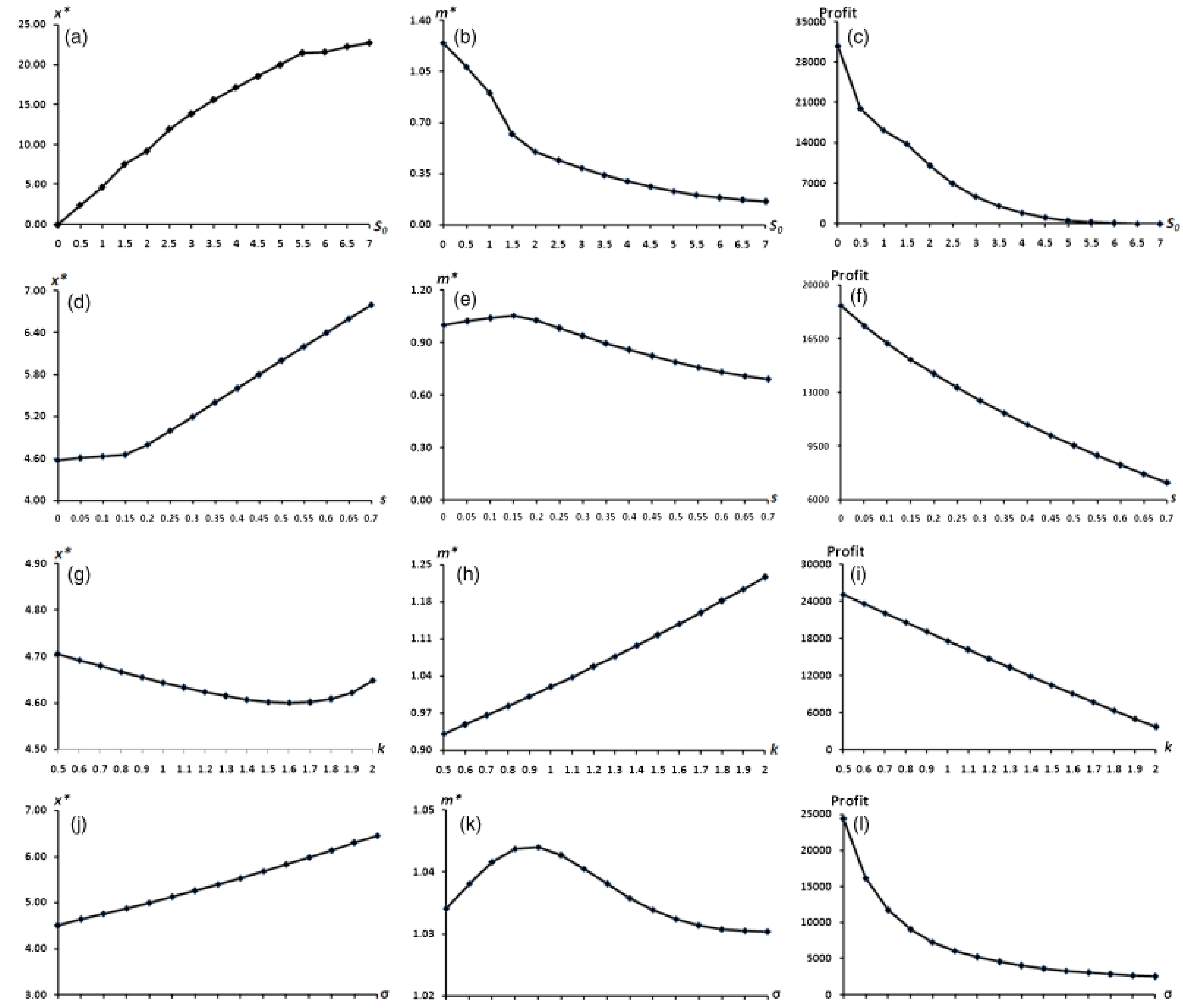
Since a consumer's purchase amount depends on the value of the CFS cutoff level x, as indicated by Theorem 3, we need to calculate $\Pi(m,x)$ for each of three scenarios: $x \ge 4s_0(1+s)$, $s_0(1+s) < x < 4s_0(1+s)$, and $0 \le x \le s_0(1+s)$. For our detailed discussion about these three scenarios, see Appendix S3.

4.3. Numerical Example and Sensitivity Analysis

In order to find optimal decisions for the retailer, we should compare the maximum values of $\Pi(m,x)$ in the three scenarios above. Next, we provide a numerical example to illustrate our analysis.

EXAMPLE 1. *We assume that the fixed shipping fee per shipment is* $s_0 = \$1$*, and the variable shipping fee per dollar value is* $s = \$0.1$*. The consumer base is* $\mathscr{B} = 1000$*, the parameter* $k = 1.1$ *and the consumer parameter* ϕ *is a normally distributed r.v. with mean* $\mu = 30$ *and standard deviation* $\sigma = 5$*. We consider the three scenarios discussed in Appendix S3, and find the optimal profit margin and*

Figure 2 The Impacts of the Fixed Shipping Fee s_0, Variable Shipping Fee s, the Parameter k and the Standard Deviation σ on the Retailer's Optimal CFS Cutoff Level x^*, Optimal Profit Margin m^* and Maximum Profit $\Pi^*(m^*, x^*)$



CFS cutoff level as 1.038 and \$4.63*, respectively, which result in the maximum profit* \$16, 194.34.

Next we will perform sensitivity analyses to examine the impacts of four important parameters—i.e., fixed shipping fee s_0, variable (per dollar) shipping fee s, the parameter k, and the standard deviation σ of the random variable ϕ—on the retailer's optimal decisions and maximum profit. In particular, we investigate how the retailer's optimal decisions and maximum profit change when the parameters (s_0, s, k, σ) vary around their *base* values $(1, 0.1, 1.1, 5)$ used in Example 1. Note that, since the rate k is used to compute the shipping cost $K(A) = k(s_0 + sA)$ according to (13), the parameters s_0, s, and k are all related to the impacts of shipment. We consider the sensitivity analysis of σ in order to examine the impacts of consumer heterogeneity.

Our computational results are presented in Table S1 (which is given in Appendix S4). Using the data in Table S1, we plot twelve graphs (given in Figure 2) to help discuss managerial insights.

4.3.1. Impacts of s_0 and s. We begin by examining the effect of the fixed shipping fee s_0 on the retailer's optimal decisions and maximum profit. In this sensitivity analysis, we increase the value of s_0 from 0 to 7.5 in increments of 0.5, and compute optimal solutions and maximum profit for each value of s_0. We find from Figure 2(a) that, as the fixed shipping fee s_0 increases, the retailer should accordingly raise the CFS cutoff level to reduce its shipping-related expenses, which may prevent some consumers from buying online. In order to keep some of those purchases, the retailer has to decrease the profit margin and thus the prices of all products; see Figure 2(b). Nevertheless, as Figure 2(c) indicates, the retailer's profit still decreases; this implies that increasing the fixed shipping fee always inevitably harms the performance of the retailer who cannot change its decisions to eliminate the negative impacts. Especially, when s_0 is very large (e.g., $s_0 \geq 5.5$), the retailer's profit is very close to zero.

In addition, Figures 2(a) to (c) indicate that the retailer's optimal CFS threshold is almost *concave*, increasing in s_0, but its optimal profit margin and maximum profit are almost *convex*, decreasing in s_0. This means that the impacts of s_0 when its value is large are greater than that when its value is small.

The sensitivity analysis of the variable shipping fees s yields some interesting results. We increase the value of s from 0 to 0.7 in steps of 0.05. From Figure 2(d) we find that increasing the value of s forces the retailer to increase its optimal CFS cutoff level. However, as Figure 2(e) shows, the impacts of s on profit margin change as s increases. More specifically, when s is sufficiently small (e.g., $s \leq 0.2$), increasing the value of s leads the retailer to slowly increase its CFS threshold and also slightly increase the profit margin. This happens because of the following fact: When per dollar shipping fee s increases in the range $[0, 0.2]$, the consumers with small purchase amounts may not be willing to pay for the higher shipping fee and thus leave without any purchase. The retailer should not respond by setting a low CFS cutoff level to absorb the shipping fee for those small orders, because a low CFS threshold would otherwise result in more shipping expenses (*only* for small orders) and less profit. Thus, in order to compensate for the loss of consumers with small orders, the retailer has to slightly raise its profit margin, which should not significantly impact the consumers with medium or large purchase amounts because most of these consumers qualify for the free shipping.

When the variable shipping fee s is large (e.g., $s \geq 0.2$), increasing s results in a rise in optimal CFS cutoff level and a reduction in optimal profit margin. Moreover, compared with the changes of optimal solutions when s is small, increasing the large value of s more significantly impacts the retailer's decisions. This can be justified as follows: if a low CFS cutoff level applies, then the large value of per dollar shipping fee s lets the retailer have very high shipping expenses. To reduce these, the retailer should raise its CFS cutoff level, which may deter some consumers with medium or large orders from completing online transactions. The retailer then responds by reducing its profit margin to attract those consumers who may leave because of a higher CFS threshold. Referring to Figure 2(f), we find that increasing s nonetheless deteriorates the retailer's performance no matter how the retailer responds to a higher value of s.

4.3.2. Impacts of *k*. We now investigate how changing the rate k impacts the retailer's optimal decisions and its maximum profit. For the sensitivity analysis, the value of k is increased from 0.5 to 2.0 in increments of 0.1. Note that, as discussed in section 1, the value of the rate k depends on whether the retailer subsidizes the shipment for consumers or treats the shipping fee as a source of its operating revenue. More specifically, if the retailer hopes to increase its revenue from shipping, then $k < 1$; if the shipping cost incurred by the retailer is the same as the shipping fee paid by consumers, then $k = 1$; otherwise, if the retailer shares a part of shipping cost with consumers who do not qualify for the free shipping, then $k > 1$.

From Figure 2(g) we find that, as k increases, the retailer's CFS cutoff level first decreases but then increases. This interesting result reflects the following fact: When k is smaller than 1, the retailer earns revenue from shipping when consumers pay for the shipment; but, some consumers may abandon their shopping carts because of the high shipping fee. Thus, in order to entice the consumers to place online orders, the retailer has to reduce its CFS cutoff level. However, increasing the value of k reduces the retailer's revenue; so, the retailer increases its profit margin, as shown in Figure 2(h).

When $k > 1$ but is not very high (i.e., $k \leq 1.6$), the retailer shares shipping cost with consumers who do not qualify for the free shipping. Thus, as k increases in the range $[1, 1.6]$, the retailer experiences a smaller difference between the retailer's shipping payment when a consumer qualifies for the free shipping and that when the consumer does not qualify for the free shipping. This implies that reducing the CFS cutoff level does not bring significantly high shipping expense to the retailer. As a result, the retailer is willing to decrease the CFS threshold to attract more consumers. To ensure its profitability, the retailer still increases its profit margin, as indicated by Figure 2(h).

When k is very high (i.e., $k > 1.6$), we find that the retailer increases its CFS cutoff level, which differs from the retailer's free-shipping decision when $k \leq 1.6$. This occurs because of the following reason. Note that the retailer's cost savings generated when a consumer with purchase amount $A \leq x$ does not qualify for free shipping is actually the shipping fee $S(A) = K(A)/k$ according to (13). This means that, when k is very large, the retailer's benefit from the CFS strategy is reduced. As a consequence, the retailer is willing to increase its CFS cutoff level. But, the retailer still shares considerable shipping costs with the consumers with very large order sizes, thus increasing its profit margin as shown in Figure 2(h).

From Figure 2(i) we find that, as in the case of s_0 and s, the retailer's profit decreases in k. This happens because increasing the value of k makes the retailer incur more shipping expenses no matter how the retailer behaves. Observing Figure 2(a)–(i), we find that, among the three shipping-related parameters, the impacts of s_0 are the largest whereas the impacts of k are the smallest. This suggests that reducing s_0 could be more useful to increase the retailer's operating profit, which is observed in the practice of many online retailers (e.g., HayHouse.com, HermeticKa.com, etc.) that do not charge a fixed shipping fee.

4.3.3. Impacts of σ. For this sensitivity analysis, the value of σ is increased from 4 to 18 in increments of 1. As shown in Figure 2(j), the numerical results suggest that the heterogeneity of consumers (measured by σ) affects the retailer's free-shipping decision; more specifically, as the value of σ increases, the retailer should increase its CFS threshold. This means that, in a relatively homogeneous market (i.e., σ is small), the retailer should set a low cutoff level. In relatively heterogeneous markets (i.e., σ is large), the retailer should choose a high cutoff level. Figure 2(k) indicates that the retailer's profit margin decision is unimodal in σ. That is, when σ is small (e.g., $\sigma \leq 7$) and the market is still relatively homogeneous, increasing σ results in a few more consumers with small order sizes and also a few more consumers with large order sizes. Those consumers with small order sizes may be deterred by the CFS threshold from their online purchases. In order to ensure its profitability, the retailer has to increase its profit margin. However, when σ is large (e.g., $\sigma > 7$) and the market is relatively heterogeneous, the retailer raises its CFS cutoff level as a result of increasing σ, which may deter a number of consumers from their online transactions. To reduce this impact, the retailer needs to decrease its profit margin to keep consumers. Our study also indicates (see Figure 2(l)) that the retailer can benefit more from homogeneous markets than from heterogeneous ones.

5. Competition between Two Retailers in the Duopoly Structure

In this section, we consider a duopoly problem in which two online retailers (i.e., Retailers 1 and 2) sell identical products to consumers in a common market. These two retailers choose their profit margins and CFS cutoff levels to compete for consumers. For online retailing operations, we notice the following two facts: (i) consumers' search costs are lower in online (virtual) markets than in traditional (physical) ones (see, e.g., Bakos 1997), (ii) the majority of online consumers are more likely to go to a comparison shopping site rather than directly to a web-based store (see, e.g., LeClaire 2006). As a result, the competition among online retailers should be higher than that among the brick-and-mortar stores.

Because of the above two facts, consumers can cheaply and conveniently visit two retailers before they make their purchasing decisions. Accordingly, we can assume that both retailers' pricing and free-shipping information is known to each consumer, who maximizes his or her net surplus (defined in section 1) to determine an optimal purchase amount and make a single or multiple online transactions with either Retailer 1 or Retailer 2. Recall from section 4 that, when a consumer with parameter ϕ visits a single retailer in the monopoly structure, the consumer's optimal purchase amount is calculated as shown in Theorem 3. Now, we consider the consumer's optimal purchasing decision in the duopoly structure, and compute the equilibrium profit margins and CFS thresholds for two retailers.

In the duopoly structure, Retailer i $(i = 1, 2)$ announces its profit margin m_i and CFS threshold x_i to consumers in a market. The fixed and variable shipping fees of Retailer i $(i = 1, 2)$ are denoted by $s_0^{(i)}$ and $s^{(i)}$; thus, the shipping fee of a consumer with purchase amount A at Retailer i is $S^{(i)}(A) = s_0^{(i)} + s^{(i)}A$. Moreover, we denote by $k^{(i)}$ $(i = 1, 2)$ the parameter in Retailer i's shipping cost function (13). For the duopoly case, the consumer with specific parameter ϕ needs to compare his or her maximum net surpluses generated from purchasing at Retailers 1 and 2. Note that we can compute the consumer's maximum net surplus from a single retailer (i.e., Retailer i, $i = 1, 2$) by using the consumer's optimal purchase amount A^* given in Theorem 3. After comparing maximum net surpluses at the two retailers, the consumer decides to make online transaction(s) with the retailer at which his or her net surplus is higher. If the consumer's net surpluses from buying at two retailers are equal, then we assume that the consumer chooses Retailer 1 with the probability γ $(0 \leq \gamma \leq 1)$ and Retailer 2 with the probability $1 - \gamma$. The probability of choosing a retailer can be determined based on the preference of the consumer for each online retailer, which depends on the consumer's subjective perception of each retailer's brand name (Smith and Brynjolfsson 2001), web-site usability (Montgomery et al. 2004, Venkatesh and Agarwal 2006), his or her shopping habit (Reibstein 2002), and experience (Johnson et al. 2004), etc.

According to the above discussion, we should use Nash equilibrium to characterize optimal decisions of Retailers 1 and 2 for the duopoly problem. We denote Retailer i's equilibrium profit margin and CFS threshold, respectively, by m_i^N and x_i^N, for $i = 1, 2$. Note that our game is similar to Bertrand game (Bertrand 1883) in which two retailers make their pricing decisions to compete for consumers; however, in our game, we also consider the CFS decisions.

THEOREM 8. *If* $s_0^{(1)} = s_0^{(2)}$ *and* $s^{(1)} = s^{(2)}$*, then two retailers' equilibrium profit margins and CFS cutoff levels are identical, i.e.,* $m_1^N = m_2^N$ *and* $x_1^N = x_2^N$.

We learn from Theorem 8 that the two retailers should choose similar profit margins and CFS cutoff levels when they charge consumers the same shipping fees. This result may explain the actual strategies of Amazon.com and Barnesandnoble.com—which charge very similar shipping fees (see, e.g., Dinlersoz and Li 2006)—set same CFS cutoff levels (currently, US\$25), and also determine very similar profit margins (see, e.g., Chevalier and Goolsbee 2003).

One may notice from the proof of Theorem 8 that our analysis for the duopoly structure is very complicated because we should consider the consumer's decision for nine cases each corresponding to one of three ranges of x_1 and one of three ranges of x_2. Moreover, in each case, there is a large number of possibilities for the position of ϕ. Thus, it is unrealistic to analytically solve our game; instead, we have to develop a simulation approach to find the equilibrium solutions.

In this section, we construct a simulation model by using "Arena," which is a primary simulation software in industry, and then use "OptQuest"—an optimization add-in for Arena—to find the optimal (equilibrium) decisions for the two retailers. This software has been previously used in academic research by, for example, Aras et al. (2006) and Askin and Chen (2006). For more information regarding "Arena" and "OptQuest" (see, e.g., Kelton et al. 2007). We present our simulation framework in detail in Appendix S5.

Next, we provide two numerical examples to illustrate our simulation approach. As Theorem 8 indicates, the two retailers' equilibrium decisions are identical when their fixed and variable shipping fees are identical, i.e., $s_0^{(1)} = s_0^{(2)}$ and $s^{(1)} = s^{(2)}$. To illustrate this result, we first consider an example in which two retailers have equal fixed and variable shipping fees, and use simulation to find Nash equilibrium. Then, we consider another example in which two retailers have different fixed and variable shipping fees.

EXAMPLE 2. *We now consider a duopoly structure in which Retailers 1 and 2 determine their profit margins and CFS thresholds to compete for consumers. We assume that the two retailers set identical fixed and variable shipping fees as* $s_0 = \$1$ *and* $s = \$0.1$. *As in Example 1, we assume that the consumer base is* $\mathscr{B} = 1000$*, and the consumer parameter* ϕ *is a normally distributed r.v. with mean* $\mu = 30$ *and standard deviation* $\sigma = 5$. *Moreover, we assume that, in the shipping cost functions (13) for Retailers 1 and 2,* $k_1 = 1.1$ *and* $k_2 = 0.5$. *When a consumer can draw the same net surpluses from buying at either retailer, we assume that the consumer buys from Retailer 1 with the probability* $\gamma = 0.6$*, and buys from Retailer 2 with the probability* $1 - \gamma = 0.4$. *From Example 1 we learn that the Retailer 1's optimal monopolistic profit margin and CFS cutoff level are 1.038 and \$4.63, respectively. We also find that the Retailer 2's optimal monopolistic profit margin and CFS cutoff level are 0.9311 and \$4.704, respectively.*

Using our simulation approach (presented in Appendix S5) we perform 11 simulations (for detailed results, see Appendix S6), and find that the two retailers' equilibrium decisions are as follows: $m_1^N = m_2^N = 0.8750$ and $x_1^N = x_2^N = 4.846$. The resulting profits are \$8,749.31 for Retailer 1 and \$9,333.49 for Retailer 2.

The above example indicates that, when the two retailers' fixed and variable shipping fees are the same, their equilibrium decisions should be equal, as shown in Theorem 8. In fact, we use our simulation approach in Example 2 to solve a large number of games with different parameter values. We find that two retailers always choose identical equilibrium decisions, in agreement with our analytical results in Theorem 8. Since these results hold even when $k_1 \neq k_2$, it follows that the retailers may have different profits regardless of implementing identical shipping and pricing policies. A retailer may be more profitable by using shipping fees as a source of revenue than by subsidizing shipping even when its shipping costs are high. In addition, when we compare Examples 1 and 2, we find that the competition in the duopoly structure "forces" the two retailers to decrease their profit margins (i.e., prices of their products) but increase their CFS cutoff levels. These important managerial insights also hold for all of other games that we simulated.

Next, we provide another example to show that the two retailers' equilibrium solutions may not be identical when their fixed and variable shipping fees are unequal.

EXAMPLE 3. *We again consider Example 2 but change only Retailer 2's fixed and variable shipping fees to* $s_0 = \$2$ *and* $s = \$0.2$. *We then calculate Retailer 2's optimal decisions for the monopoly structure as* $m_2^* = 0.2417$ *and* $x_2^* = \$9.6$. *We perform 13 simulations (see Appendix S6 for our simulation results) to find the equilibrium solutions as* $m_1^N = 0.6591$, $x_1^N = 10.114$*; and* $m_2^N = 0.6898$, $x_2^N = 16.139$. *The resulting profits are* \$7119.38 *for Retailer 1 and* \$16,997.46 *for Retailer 2.*

Our result in Example 3 suggests that, when the two retailers' shipping fees are different, they should make asymmetric equilibrium decisions. Moreover, when we compare Examples 2 and 3, we find that the retailer with higher shipping fees (i.e., Retailer 2 in our Example 3) may increase both its profit margin and CFS cutoff level. If we assume that profit margins and prices are positively correlated, then our results would agree with those of Dinlersoz and Li (2006) who empirically found that shipping fees and prices are correlated, and explained this correlation as a result of imperfect consumer information. Our model and results suggest that this positive correlation may also arise because of competitive effects even under perfect consumer information.

Example 3 also suggests that, in the duopoly structure, a retailer should achieve more profits from insensitive consumers than from highly sensitive consumers. This insight is drawn from the following result in Example 3: Retailer 2 with higher shipping fees sets a higher cutoff level but its profit margin is not significantly different from Retailer 1's profit margin. As a consequence, Retailer 2 obtains a significantly higher profit than Retailer 1, even though the two retailers' shipping policies in terms of k_1 and k_2 are the same. It thus follows that we can attribute the result (about the profits) to the difference between the shipping fees of the two retailers. Because of the fact that higher shipping fees and cutoff levels deter shipping-sensitive consumers, we can conclude that, in Example 3, most consumers who buy from Retailer 2 should be insensitive to shipping fees; and thus, Retailer 2 could profit more from serving fewer shipping fee-sensitive consumers.

6. Summary and Concluding Remarks

This paper contributes to the body of literature on shipping promotions by exploring the managerial implications and optimality of joint pricing (profit margin) and CFS decisions during a single period. We investigated this problem by considering both monopoly and duopoly structures. After analyzing a consumer's purchase decision given an online retailer's pricing and CFS decisions, we then considered the monopoly structure and found the optimal profit margin and CFS cutoff level that maximize a retailer's single-period expected profit. We performed sensitivity analysis to examine the impacts of shipping- and consumer heterogeneity-related parameters on the retailer's decisions and profit. Then, we considered the duopoly structure, and used Arena and OptQuest to find Nash equilibria for several duopolistic games.

Next, we summarize major managerial insights that we have drawn analytically and numerically.

1. The main managerial insights based on our *analytical* results include:

(a) An online retailer's CFS strategy is useful to acquire consumers with large order sizes; this implies that the retailer should mainly consider the CFS strategy to entice the consumers whose purchase amounts are large. This result is supported by Lewis et al.'s empirical study (Lewis et al. 2006).

(b) An online retailer's CFS strategy when the CFS cutoff level is small has greater impact on the conversion rate than that when the CFS cutoff level is large.

This result also demonstrates our result (a), because of the following facts: the CFS strategy when the CFS cutoff level is small largely deters the consumers with small order sizes from their online purchases whereas that when the CFS cutoff level is large does not significantly impact the consumers with large order sizes. Thus, our result regarding the impact of CFS strategy on the conversion rate also implies that the CFS strategy should be mainly used to acquire consumers with large order sizes.

(c) An online retailer's profit margin (pricing decision) when the CFS threshold is small *may* or *may not* have greater impact on the conversion rate than that when the CFS cutoff level is large, which depends on the preference and incomes of consumers [i.e., the p.d.f. $f(A)$].

(d) In the duopoly structure, if the two retailers' fixed shipping fees are equal and their variable shipping fees are also equal, then the retailers should determine identical equilibrium decisions. This result is exemplified by the practice of Amazon.com and Barnesandnoble.com which set very similar shipping fees (see, e.g., Dinlersoz and Li 2006), the same CFS cutoff level \$25, and very similar profit margins (Chevalier and Goolsbee 2003).

2. The main managerial insights based on our *numerical* results (i.e., sensitivity analysis and computer simulation with Arena and OptQuest) include:

(a) Among three shipping-related parameters (i.e., s_0, s and k), the fixed shipping fee s_0 has the largest impacts on the retailer's profit. This suggests that reducing s_0 should be most useful to increase the retailer's operating profit, which is justified by the practice that many online retailers (e.g., HayHouse.com, HermeticKa.com, etc.) set their fixed shipping fee to zero. For more discussion, see section 4.3.1.

(b) If an online retailer's variable shipping fee is significantly small, then increasing such a fee should lead the retailer to slightly raise both CFS cutoff level and profit margin. However, if the variable shipping fee is significantly large, then increasing the fee should result in raising the CFS cutoff level but decreasing the profit margin. For our justification, see section 4.3.1.

(c) If an online retailer treats the shipping fee as a source of its operating revenue, i.e., $k<1$, then we find that, when k increases (that is, the retailer's revenue from shipping decreases), the retailer should reduce its CFS cutoff level but raise its profit margin. However, if the retailer is willing to subsidize the shipment for consumers, i.e., $k>1$, then increasing k results in a higher CFS cutoff level and a higher profit margin. For our justification, see section 4.3.2.
(d) An online retailer should set a low cutoff level in a relatively homogeneous market and a high cutoff level in a relatively heterogeneous market. Moreover, the retailer can benefit more from homogeneous markets than from heterogeneous ones. For more discussion, see section 4.3.3.
(e) In the duopoly structure, if the two retailers have equal fixed and variable shipping fees, then they should choose identical equilibrium decisions. This supports our *analytical* result (d). As a result of the competition, both retailers decrease their profit margins but increase their CFS cutoff levels. However, if two retailers' fixed and variable shipping fees are unequal, then they *may not* determine identical equilibrium decisions. For a more detailed discussion, see section 5.

In the future, we may extend this work to consider some supply chain-related problems. For instance, we could incorporate inventory costs into our pricing and CFS decision model and investigate how online retailers make their ordering, pricing, and shipping promotion decisions. In addition, considering the competition between online retailers and brick-and-mortar stores may be of particular interest because the presence of physical stores significantly influences the purchasing decisions of customers. In reality, a consumer may choose between picking up the products from a local store and increasing the size of the online order to qualify for free shipping. Teltzrow et al. (2003) showed that 75% of customers prefer the option of picking up their goods at a local brick-and-mortar store. Furthermore, the retailers may choose to list their products at some third-party web sites such as shopping.com or yahoo.com. The costs of employing third-party web sites and/or third-party physical stores differ from the costs of using proprietary web sites, as found by Chen et al. (2007).

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Supporting Information

Additional supporting information may be found in the online version of this article:

Appendix S1: Proofs of Theorems.
Appendix S2: A Discussion of Four Cases in Figure 1.
Appendix S3: An Online Retailer's Expected Profit Function.
Appendix S4: Numerical Results for the Sensitivity Analysis in Section 4.3.
Appendix S5: A Simulation Approach for the Game Analysis in Section 5.
Appendix S6: Simulation Results for two Examples in Section 5.

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Appendix A Proofs of Theorems

Proof of Theorem 1. We first find optimal purchase quantities $\bar{q}_i$ $(i = 1, 2, \ldots, n)$ that maximize the net surplus function G_1 of a consumer with parameters θ_i $(i = 1, 2, \ldots, n)$ who qualifies for CFS. Taking the first- and second-order derivatives of G_1 w.r.t. q_i $(i = 1, 2, \ldots, n)$ gives

$$\frac{\partial G_1}{\partial q_i} = \frac{\theta_i}{2\sqrt{q_i}} - (1+m)c_i \qquad \text{and} \qquad \frac{\partial G_1^2}{\partial^2 q_i} = -\frac{\theta_i}{4q_i\sqrt{q_i}} < 0, \tag{15}$$

which implies that G_1 is concave in the purchase quantity q_i. To find the optimal quantity $\bar{q}_i$ that maximizes G_1, we set $\partial G_1/\partial q_i$ to zero, solve the resulting equation, and find that

$$\bar{q}_i = \frac{\theta_i^2}{4(1+m)^2 c_i^2};$$

and the consumer's total purchase amount (in terms of $\bar{q}_i$, $i = 1, 2, \ldots, n$) for a single online transaction, according to (3), is

$$\bar{A} \equiv (1+m)\sum\nolimits_{i=1}^{n} c_i \bar{q}_i = \frac{1}{4(1+m)}\phi,$$

where $\phi \equiv \sum_{i=1}^{n} \theta_i^2/c_i$ denotes the consumer's random consumer-specific utility parameter (for all products) in his or her net surplus. Note that the consumer-specific parameter ϕ is a random variable, since θ_i $(i = 1, 2, \ldots, n)$ are random. If the probability distribution functions (p.d.f.) of θ_i $(i = 1, 2, \ldots, n)$ are given, we can then find the p.d.f. of the r.v. ϕ. To compute the maximum net surplus $\bar{G}_1$, we substitute $\bar{q}_i$ into the function $\bar{G}_1$ and find

$$G_1^* = \sum\nolimits_{i=1}^{n} \frac{\theta_i^2}{4(1+m)c_i} = \frac{1}{4(1+m)}\phi,$$

which is equal to $\bar{A}$.

Next, we maximize the function G_2 to find optimal purchase quantities $\hat{q}_i$ (for $i = 1, 2, \ldots, n$) and purchase amount $\hat{A}$, when the consumer doesn't qualify for the free shipping and thus has to absorb the shipping fee $S(\hat{A})$ in which $\hat{A} \equiv (1+m)\sum_{i=1}^{n} c_i \hat{q}_i$. We compute the first- and second-order derivatives of G_2 w.r.t. q_i $(i = 1, 2, \ldots, n)$ as,

$$\frac{\partial G_2}{\partial q_i} = \frac{\theta_i}{2\sqrt{q_i}} - (1+s)(1+m)c_i \qquad \text{and} \qquad \frac{\partial G_2^2}{\partial^2 q_i} = -\frac{\theta_i}{4q_i\sqrt{q_i}} < 0,$$

which implies that G_2 is concave in the purchase quantity q_i. To find the optimal quantity $\hat{q}_i$ that maximizes G_2, we set $\partial G_2/\partial q_i$ to zero, solve the resulting equation, and find that

$$\hat{q}_i = \frac{\theta_i^2}{4(1+s)(1+m)^2 c_i^2},$$

and corresponding optimal purchase amount as

$$\hat{A} \equiv (1+m)\sum\nolimits_{i=1}^{n} c_i \hat{q}_i = \frac{\phi}{4(1+s)(1+m)} = \frac{\bar{A}}{1+s}.$$

Substituting $\hat{q}_i$ into the function G_2 gives

$$G_2^* = \frac{\phi}{4(1+s)(1+m)} - s_0.$$

■

Proof of Theorem 2. We first consider the properties of G_1 w.r.t. A. Using (3) we take the second-order derivative of G_1 w.r.t. the purchase amount A and find

$$\frac{\partial G_1^2}{\partial^2 A} = -\frac{1}{4(1+m)^2}\sum\nolimits_{i=1}^{n} \frac{\theta_i}{c_i^2 q_i \sqrt{q_i}} < 0,$$

which means that G_1 is concave in the purchase amount A. As indicated by Theorem 1, the optimal purchase amount $\bar{A}$ maximizing G_1 is given by (7).

Similarly, taking the second-order derivative of G_2 w.r.t. the purchase amount A gives

$$\frac{\partial G_2^2}{\partial^2 A} = -\frac{1}{4(1+m)^2}\sum\nolimits_{i=1}^{n} \frac{\theta_i}{c_i^2 q_i \sqrt{q_i}} < 0,$$

which is the same as $\partial G_1^2/\partial^2 A$. This means that G_2 is also concave in the purchase amount A. As indicated by Theorem 1, the optimal purchase amount $\hat{A}$ maximizing G_2 is given by (9). ■

Proof of Theorem 3. We perform our analysis according to the four cases indicated by Figure 1.

1. In Cases (i) and (ii), $x \le \bar{A}$ and the consumer's optimal purchase amount is $A^* = \bar{A}$. Therefore, we can easily find that, if $\phi \ge 4x(1+m)$, then $A^* = \bar{A} = \phi/[4(1+m)]$.
2. In Case (iii), $x > \bar{A}$ and $G_2^* \le G_1|_{A=x}$. For this case, if $G_1|_{A=x} \ge 0$, then the consumer makes a purchase; but, if $G_1|_{A=x} < 0$, then the consumer leaves the retailer without any purchase. Hence, when $x > \bar{A}$, $G_1|_{A=x} \ge 0$ and $G_2^* \le G_1|_{A=x}$, the consumer's optimal purchase amount is $A^* = x$. Next, we specify these three conditions. The condition that $x > \bar{A}$ can be re-written as

$$\phi < 4x(1+m). \tag{16}$$

From (21) we find that $G_1|_{A=x} \ge 0$ when

$$\phi \ge x(1+m). \tag{17}$$

According to our above analysis for Case (iii), we find that $G_2^* \le G_1|_{A=x}$ depends on the comparison between x and s_0. Specifically, if $x < s_0$, then $G_2^* \le G_1|_{A=x}$ when

$$\phi \ge 4(1+m)(1+s)[\sqrt{(1+s)x} + \sqrt{sx+s_0}]^2; \tag{18}$$

otherwise, if $x \geq s_0$, then $G_2^* \leq G_1|_{A=x}$ when

$$\phi \geq 4(1+m)(1+s)[\sqrt{(1+s)x}+\sqrt{sx+s_0}]^2, \text{ or, } \phi \leq 4(1+m)(1+s)[\sqrt{(1+s)x}-\sqrt{sx+s_0}]^2. \quad (19)$$

Hence, we consider the following issues: when $x < s_0$, we find that, if ϕ satisfies (18), then ϕ cannot satisfy (16). This means that, if $x < s_0$, then Case (iii) doesn't happen.
When $x \geq s_0$, we find that Case (iii) applies if and only if

$$x(1+m) \leq 4(1+m)(1+s)[\sqrt{(1+s)x} - \sqrt{sx+s_0}]^2,$$

or simply,

$$0 \leq sx + (1+s)(3x+8sx+4s_0) - 8(1+s)\sqrt{(1+s)x}\sqrt{sx+s_0},$$

which requires that

$$x \geq 4s_0(1+s).$$

This means that, if $x \geq 4s_0(1+s)$, then Case (iii) happens, and the consumer spends \$$x$. But, from (16) and (19), we find that we need to compare x and $(1+s)[\sqrt{(1+s)x} - \sqrt{sx+s_0}]^2$. Our comparison shows that, for Case (iii),

$$x > (1+s)[\sqrt{(1+s)x} - \sqrt{sx+s_0}]^2,$$

which implies that, if $x \geq 4s_0(1+s)$, then the consumer spends \$$x$ only when

$$x(1+m) \leq \phi \leq 4(1+m)(1+s)[\sqrt{(1+s)x} - \sqrt{sx+s_0}]^2;$$

otherwise, if $x < 4s_0(1+s)$, then the consumer abandons his or her shopping cart.

3. In Case (iv), $x > \bar{A}$ and $G_2^* > G_1|_{A=x}$. For this case, if $G_2^* \geq 0$, then the consumer makes a purchase; but, if $G_2^* < 0$, then the consumer leaves without any online purchase. Hence, when $x > \bar{A}$, $G_2^* \geq 0$ and $G_2^* > G_1|_{A=x}$, the consumer's optimal purchase amount is $A^* = \hat{A}$. Next, we specify these three conditions. The condition that $x > \bar{A}$ can be re-written as (16).
From (9) we find that $G_2^* \geq 0$ when

$$\phi \geq 4(1+m)(1+s)s_0. \quad (20)$$

Comparing (16) and (20) suggests that $x > (1+s)s_0$.
Similar to our above analysis, we find that $G_2^* > G_1|_{A=x}$ depends on the comparison between x and s_0. Specifically, if $x < s_0$, then $G_2^* > G_1|_{A=x}$ when

$$0 \leq \phi < 4(1+m)(1+s)[\sqrt{(1+s)x} + \sqrt{sx+s_0}]^2;$$

otherwise, if $x \geq s_0$, then $G_2^* > G_1|_{A=x}$ when

$$4(1+m)(1+s)[\sqrt{(1+s)x} - \sqrt{sx+s_0}]^2 < \phi < 4(1+m)(1+s)[\sqrt{(1+s)x} + \sqrt{sx+s_0}]^2.$$

Because $x > (1+s)s_0$, we don't need to consider the case that $x < s_0$. When $x > (1+s)s_0$, we find that, if $x \geq 4s_0(1+s)$, then $x > \bar{A}$, $G_2^* \geq 0$ and $G_2^* > G_1|_{A=x}$ when

$$4(1+m)(1+s)[\sqrt{(1+s)x} - \sqrt{sx+s_0}]^2 < \phi < 4x(1+m),$$

otherwise, if $(1+s)s_0 < x < 4s_0(1+s)$, then $x > \bar{A}$, $G_2^* \geq 0$ and $G_2^* > G_1|_{A=x}$ when

$$4(1+m)(1+s)s_0 < \phi < 4x(1+m).$$

In conclusion, we reach the result as shown in this theorem. ■

Proof of Theorem 4. Given that the r.v. ϕ is distributed with p.d.f. $f(\cdot)$ and c.d.f. $F(\cdot)$, we can easily find from Theorem 3 that the probability that the consumer with the specific parameter ϕ doesn't make any purchase is

$$\Pr(A=0) = \begin{cases} \int_0^{x(1+m)} f(\phi)d\phi, & \text{if } x \geq 4s_0(1+s), \\ \int_0^{4s_0(1+s)(1+m)} f(\phi)d\phi, & \text{if } s_0(1+s) < x < 4s_0(1+s), \\ \int_0^{4x(1+m)} f(\phi)d\phi, & \text{if } 0 \leq x \leq s_0(1+s). \end{cases}$$

Thus, we can compute the probability $\Pr(A>0) = 1 - \Pr(A=0)$, as shown in this theorem. ■

Proof of Theorem 5. The first-order derivative of the probability $\Pr(A>0)$ w.r.t. x is

$$\begin{aligned} &\frac{\partial \Pr(A>0)}{\partial x} \\ = &\begin{cases} -(1+m)f[x(1+m)], & \text{if } x \geq 4s_0(1+s), \\ 0, & \text{if } s_0(1+s) < x < 4s_0(1+s), \\ -4(1+m)f[4x(1+m)], & \text{if } 0 \leq x \leq s_0(1+s), \end{cases} \end{aligned}$$

which implies that the probability $\Pr(A>0)$ is non-increasing in the CFS cutoff level x. We notice that, when $x \geq 4s_0(1+s)$, the first-order derivative is $-(1+m)f[x(1+m)]$; but when $0 \leq x \leq s_0(1+s)$, the first-order derivative is $-4(1+m)f[4x(1+m)]$. Moreover, we find that the impacts of x when $x = s_0(1+s)$ and that when $x = 4s_0(1+s)$ are the same. In order to compare the impact of x when $x > 4s_0(1+s)$ and that when $0 < x < s_0(1+s)$, we can arbitrarily consider the CFS cutoff level $x_1 \in (4s_0(1+s), \infty)$. If we assume that the p.d.f. of x is unimodal with two tails, then we can always find a corresponding value $x_2 \in (0, s_0(1+s))$ such that $f[4x_2(1+m)] = f[x_1(1+m)]$. Thus, $-4(1+m)f[4x_2(1+m)] < -(1+m)f[x_1(1+m)]$; this means that the impact of x when $x = x_2$ is greater than that when $x = x_1$. Therefore, we have the conclusion as in the first item of the theorem.

The first-order derivative of the probability $\Pr(A > 0)$ w.r.t. m is

$$\frac{\partial \Pr(A > 0)}{\partial m} = \begin{cases} -xf[x(1+m)], & \text{if } x \geq 4s_0(1+s), \\ -4s_0(1+s)f[4s_0(1+s)(1+m)], & \text{if } s_0(1+s) < x < 4s_0(1+s), \\ -4xf[4x(1+m)], & \text{if } 0 \leq x \leq s_0(1+s), \end{cases}$$

which implies the decreasing property of the probability $\Pr(A > 0)$. We find that, as x decreases, the profit margin m's impact on the conversion rate when x is small [i.e., $0 \leq x \leq s_0(1+s)$] *may* be larger. This differs from our above analysis, because, for any large value m_1, we cannot ensure to find a corresponding small value m_2 such that $f[4x(1+m_2)] = f[x(1+m_1)]$. [Note that the minimum value of m is zero and the corresponding the p.d.f. is $f(4x)$.] ■

Proof of Theorem 6. From Theorem 4 we can find the probability $\Pr(A > 0)$, which is the probability that the consumer buys online. For the Poisson distribution $\Pr(T = r)$ in (10), we can compute the probability of no purchase (i.e., $r = 0$) as

$$\Pr(T = 0) = \exp[-\lambda(\phi)].$$

Equating $\exp[-\lambda(\phi)]$ to the probability $\Pr(A = 0) = 1 - \Pr(A > 0)$ where $\Pr(A > 0)$ is obtained from Theorem 4, we can compute the consumer-specific parameter $\lambda(\phi)$, as shown in this theorem. ■

Proof of Theorem 7. We denote by ϕ_i and r_i the parameters for the purchase amount and number of purchases of consumer i. Let $\pi_i(m, x|\phi_i, r_i)$ be the profit that the retailer draws from consumer i for given values of the random parameters ϕ_i and r_i.

We then compute $\Pi(m, x)$ as,

$$\Pi(m, x) = E\left[\sum_{i=1}^{\mathcal{B}} \pi_i(m, x|\phi_i, r_i)\right] = \sum_{i=1}^{\mathcal{B}} E[\pi_i(m, x|\phi_i, r_i)].$$

Using the law of iterated expectations, we have

$$\begin{aligned} \Pi(m, x) &= \sum_{i=1}^{\mathcal{B}} E\left[E\left[\pi_i(m, x|\phi_i, r_i)|\phi_i\right]\right] \\ &= \sum_{i=1}^{\mathcal{B}} E\left[\sum\nolimits_{r_i=0}^{\infty} \pi_i(m, x|\phi_i, r_i) \times \Pr(r_i)\right]. \end{aligned}$$

Then, using the law of conditional probability, we find that

$$\begin{aligned}
\Pi(m,x) &= \sum_{i=1}^{\mathcal{B}} \int \sum\nolimits_{r_i=0}^{\infty} \pi_i(m,x,\phi_i|r_i) \times f(\phi_i) \times \Pr(r_i) d\phi_i, \\
&= \mathcal{B} \int \sum\nolimits_{r_i=0}^{\infty} \pi_i(m,x,\phi_i|r_i) \times \Pr(r_i) \times f(\phi_i) d\phi_i \\
&= \int \sum\nolimits_{r_i=0}^{\infty} \pi_i(m,x,\phi_i|r_i) \times f(\phi_i) d\phi_i \times \Pr(r_i) \times \mathcal{B} \\
&= \sum\nolimits_{r_i=0}^{\infty} \int \pi_i(m,x,\phi_i|r_i) \times f(\phi_i) d\phi_i \times \Pr(r_i) \times \mathcal{B} \\
&= \sum\nolimits_{r=0}^{\infty} \pi(m,x|T=r) \times \Pr(T=r) \times \mathcal{B}.
\end{aligned}$$

This proves the theorem. ■

Proof of Theorem 8. We now analyze the two retailers' optimal decisions when the parameters in their shipping fee functions are equal. For notational simplicity, we set $s_0 \equiv s_0^{(1)} = s_0^{(2)}$ and $s \equiv s^{(1)} = s^{(2)}$. For the duopoly case, a consumer makes his or her purchasing decision when two retailers (i.e., Retailer i, $i = 1, 2$) sell identical products online. Thus, we need to compare the consumer's maximum net surpluses from buying at either of the two retailers, and find the optimal purchase amount A^* that maximizes the consumer's overall net surplus. Theorem 3 indicates that, for a monopoly scenario, the consumer's purchasing decision depends on the value of x and we thus need to consider three cases for the monopoly structure. Now, in the duopoly structure, two retailers' CFS cutoff levels x_1 and x_2 may be different; as a result, we should analyze the consumer's decision for nine cases each corresponding to one of three ranges of x_1 and one of three ranges of x_2.

We first analyze the two retailers' profit margin decisions for any given pair of x_1 and x_2, and show that, to compete for consumers, the two retailers always set identical profit margins for any values of x_1 and x_2. As mentioned above, we need to consider nine cases in which x_1 and x_2 fall in some specific ranges, and for each case examine how the two retailers choose their profit margins to compete for consumer with specific parameter ϕ.

Case 1: $x_1 \geq 4s_0(1+s)$ **and** $x_2 \geq 4s_0(1+s)$**.** For this case, we find that, if $\phi < \min[x_1(1+m_1), x_2(1+m_2)]$, then the consumer doesn't purchase any product from either retailer, and thus his or her optimal purchase amount is $A^* = 0$. But, if $\min[x_1(1+m_1), x_2(1+m_2)] < \phi < \max[x_1(1+m_1), x_2(1+m_2)]$, then the consumer decides to buy products from the retailer with $\min[x_1(1+m_1), x_2(1+m_2)]$. In order to compete for the consumer, the retailer with $\max[x_1(1+m_1), x_2(1+m_2)]$ should decrease its profit margin so that it has the minimum value $\min[x_1(1+m_1), x_2(1+m_2)]$.

Letting

$$\begin{aligned}
\underline{\vartheta} &\equiv \min\left\{4(1+s)(1+m_j)\left[\sqrt{(1+s)x_j} - \sqrt{sx_j + s_0}\right]^2, j = 1,2\right\}, \\
\bar{\vartheta} &\equiv \max\left\{4(1+s)(1+m_j)\left[\sqrt{(1+s)x_j} - \sqrt{sx_j + s_0}\right]^2, j = 1,2\right\},
\end{aligned}$$

we find that, if $\max[x_1(1+m_1), x_2(1+m_2)] \leq \underline{\vartheta}$, then the consumer with parameter ϕ such that $\max[x_1(1+m_1), x_2(1+m_2)] \leq \phi \leq \underline{\vartheta}$ can achieve the net surpluses $\sqrt{x_j\phi/[1+m_j]} - x_j$ from Retailer j, $j = 1, 2$. In order to win the consumer, the two retailers should reduce their profit margins to increase the consumer's net surplus. Otherwise, if $\max[x_1(1+m_1), x_2(1+m_2)] \geq \underline{\vartheta}$, then we, w.l.o.g., assume that the retailer with $\bar{\vartheta}$ is Retailer i ($i = 1, 2$) and the retailer with $\underline{\vartheta}$ is Retailer j ($j = 1, 2$, $j \neq i$), and consider the following two possibilities:

1. If the consumer with parameter ϕ such that $\max[x_1(1+m_1), x_2(1+m_2)] \leq \phi \leq \bar{\vartheta}$ determines the purchase amount x_i from Retailer i, then the consumer can achieve the net surplus $\sqrt{x_i\phi/[1+m_i]} - x_i$;
2. If the consumer with parameter ϕ such that $\max[x_1(1+m_1), x_2(1+m_2)] \leq \phi \leq \bar{\vartheta}$ determines the purchase amount $\hat{A}_j$ or $\bar{A}_j$ from Retailer j ($j = 1, 2$, $j \neq i$), then the consumer can achieve the net surplus $\phi/[4(1+s)(1+m_j)] - s_0$ or $\phi/[4(1+m_j)]$.

In order to compete for the consumer, each retailer reduces its profit margin m_i or m_j to increase the consumer's net surplus. Otherwise, the retailer with a high value of profit margin may lose the consumer.

Next, we find that, if $\phi \geq \bar{\vartheta}$, then the consumer spends $\$\hat{A}_i$ or $\$\bar{A}_i$ from Retailer i, or the consumer spends $\$\hat{A}_j$ or $\$\bar{A}_j$ from Retailer j. As a result, the net surplus achieved from Retailer i is computed as $\phi/[4(1+s)(1+m_i)] - s_0$ or $\phi/[4(1+m_i)]$, and that from Retailer j is calculated as $\phi/[4(1+s)(1+m_j)] - s_0$ or $\phi/[4(1+m_j)]$. Like in our above discussion, each retailer should have an incentive to reduce its profit margin so as to win the consumer.

In addition, we notice that, as Retailer i ($i = 1, 2$) reduces its profit margin, the range for no purchase is smaller and the range for the purchase amount $\bar{A}_i$ is greater. That is, as a result of the competition, each retailer reduces its profit margins for this case.

Case 2: $x_1 \geq 4s_0(1+s)$ and $s_0(1+s) < x_2 < 4s_0(1+s)$. For this case, we find that, if $\phi < \min[x_1(1+m_1), 4s_0(1+s)(1+m_2)]$, then the consumer doesn't purchase any product from either retailer, and thus his or her optimal purchase amount is $A^* = 0$. But, if $\min[x_1(1+m_1), 4s_0(1+s)(1+m_2)] < \phi < \max[x_1(1+m_1), 4s_0(1+s)(1+m_2)]$, then the consumer decides to buy products from the retailer with $\min[x_1(1+m_1), 4s_0(1+s)(1+m_2)]$. To compete for the consumer, the retailer with $\max[x_1(1+m_1), 4s_0(1+s)(1+m_2)]$ should decrease its profit margin so that it has the minimum value $\min[x_1(1+m_1), 4s_0(1+s)(1+m_2)]$.

When $\phi > \max[x_1(1+m_1), 4s_0(1+s)(1+m_2)]$, the consumer's purchase amount from Retailer 1 is x_1, $\hat{A}_1$ or $\bar{A}_1$; and the consumer's purchase amount from Retailer 2 is $\hat{A}_2$ or $\bar{A}_2$. Similar to our analysis for Case 1, the two retailers should reduce their profit margins to compete for the consumer. Therefore, for this case, the two retailers have incentives to decrease their profit margins as a consequence of competition.

Case 3: $x_1 \geq 4s_0(1+s)$ and $0 \leq x_2 < s_0(1+s)$. For this case, we find that, if $\phi < \min[x_1(1+m_1), 4x_2(1+m_2)]$, then the consumer doesn't purchase any product from both retailers, and thus his or her optimal purchase amount is $A^* = 0$. But, if $\min[x_1(1+m_1), 4x_2(1+m_2)] < \phi < \max[x_1(1+m_1), 4x_2(1+m_2)]$, then the consumer decides to buy products from the retailer with $\min[x_1(1+m_1), 4x_2(1+m_2)]$. To compete for the consumer, the retailer with

$\max[x_1(1+m_1), 4x_2(1+m_2)]$ should decrease its profit margin so that it has the minimum value $\min[x_1(1+m_1), 4x_2(1+m_2)]$.

When $\phi > \max[x_1(1+m_1), 4x_2(1+m_2)]$, the consumer's purchase amount from Retailer 1 is x_1, $\hat{A}_1$ or $\bar{A}_1$; and the consumer's purchase amount from Retailer 2 is $\bar{A}_2$. Therefore, for this case, the two retailers should decrease their profit margins as a consequence of competition.

Case 4: $s_0(1+s) < x_1 < 4s_0(1+s)$ and $x_2 \geq 4s_0(1+s)$. The analysis for this case is similar to that for Case 2.

Case 5: $s_0(1+s) < x_1 < 4s_0(1+s)$ and $s_0(1+s) < x_2 < 4s_0(1+s)$. For this case, if $\phi < \min[4s_0(1+s)(1+m_1), 4s_0(1+s)(1+m_2)]$, then the consumer doesn't purchase any product from either retailer, and thus his or her optimal purchase amount is $A^* = 0$. But, if $\min[4s_0(1+s)(1+m_1), 4s_0(1+s)(1+m_2)] < \phi < \max[4s_0(1+s)(1+m_1), 4s_0(1+s)(1+m_2)]$, then the consumer decides to buy products from the retailer with $\min[4s_0(1+s)(1+m_1), 4s_0(1+s)(1+m_2)]$ (i.e., the retailer with the smaller profit margin). To compete for the consumer, the retailer with $\max[4s_0(1+s)(1+m_1), 4s_0(1+s)(1+m_2)]$ (i.e., the retailer with the larger profit margin) should decrease its profit margin so that it has the minimum value $\min[4s_0(1+s)(1+m_1), 4s_0(1+s)(1+m_2)]$.

If $\phi > \max[4s_0(1+s)(1+m_1), 4s_0(1+s)(1+m_2)]$, then the consumer's purchase amount from Retailer 1 is $\hat{A}_1$ or $\bar{A}_1$; and the consumer's purchase amount from Retailer 2 is $\hat{A}_2$ or $\bar{A}_2$. As discussed for Case 1, two retailers are likely to reduce their profit margins in order to compete for consumers.

Case 6: $s_0(1+s) < x_1 < 4s_0(1+s)$ and $0 \leq x_2 < s_0(1+s)$. For this case, if $\phi < \min[4s_0(1+s)(1+m_1), 4x_2(1+m_2)]$, then the consumer doesn't purchase any product from both retailers, and thus his or her optimal purchase amount $A^* = 0$. But, if $\min[4s_0(1+s)(1+m_1), 4x_2(1+m_2)] < \phi < \max[4s_0(1+s)(1+m_1), 4x_2(1+m_2)]$, then the consumer decides to buy products from the retailer with $\min[4s_0(1+s)(1+m_1), 4x_2(1+m_2)]$. To compete for the consumer, the retailer with $\max[4s_0(1+s)(1+m_1), 4x_2(1+m_2)]$ should decrease its profit margin so that it has the minimum value $\min[4s_0(1+s)(1+m_1), 4x_2(1+m_2)]$.

If $\phi > \max[4s_0(1+s)(1+m_1), 4x_2(1+m_2)]$, then the consumer's purchase amount from Retailer 1 is $\hat{A}_1$ or $\bar{A}_1$; and the consumer's purchase amount from Retailer 2 is $\bar{A}_2$. Comparing them we find that each retailer has an incentive to reduce its profit margin.

Case 7: $0 \leq x_1 < s_0(1+s)$ and $x_2 \geq 4s_0(1+s)$. The analysis for this case is similar to that for Case 3.

Case 8: $0 \leq x_1 < s_0(1+s)$ and $s_0(1+s) < x_2 < 4s_0(1+s)$. The analysis for this case is similar to that for Case 6.

Case 9: $0 \leq x_1 < s_0(1+s)$ and $0 \leq x_2 < s_0(1+s)$. For this case, if $\phi < \min[4x_1(1+m_1), 4x_2(1+m_2)]$, then the consumer doesn't purchase any product from either retailer, and thus his or her optimal purchase amount $A^* = 0$. But, if $\min[4x_1(1+m_1), 4x_2(1+m_2)] < \phi < \max[4x_1(1+m_1), 4x_2(1+m_2)]$, then the consumer decides to buy products from the retailer with $\min[4x_1(1+m_1), 4x_2(1+m_2)]$. To compete for the consumer, the retailer with $\max[4x_1(1+m_1), 4x_2(1+m_2)]$ should decrease its profit margin so that it has the minimum value $\min[4x_1(1+m_1), 4x_2(1+m_2)]$.

If $\phi > \max[4x_1(1+m_1), 4x_2(1+m_2)]$, then the consumer's purchase amount from Retailer 1 is $\bar{A}_1$;

and the consumer's purchase amount from Retailer 2 is $\bar{A}_2$. Each retailer thus has an incentive to reduce its profit margin.

From the above discussion of the nine cases, we conclude that each retailer can reduce its profit margin to win consumers. Then, the two retailers would reduce their profit margins to zero; this means that their prices are set equal to their acquisition costs, as shown by Bertrand [9]. However, we find from Section 4.2 that, if a retailer reduces its profit margin to zero, then the retailer's profit would be negative because it should pay shipping fee to consumers who qualify for free shipping. Thus, in order to compete for consumers but ensure their profitability, the two retailers should choose identical, nonzero profit margins.

When the two retailers set identical profit margins, we let $m \equiv m_1 = m_2$. We next investigate how two retailers change their CFS cutoff levels to compete for consumers. Like in our above discussion, we consider the following nine cases:

Case 1: $x_1 \geq 4s_0(1+s)$ **and** $x_2 \geq 4s_0(1+s)$**.** For this case, we find that consumers don't make any online transactions with the retailer with the greater CFS cutoff level, because the consumer with $\phi < x_i(1+m_i)$ doesn't buy from Retailer i ($i = 1, 2$). Thus, the two retailers have incentives to finally set identical CFS cutoff levels. If $\phi > \max[x_1(1+m_1), x_2(1+m_2)]$, then the consumer's purchase amount from Retailer i ($i = 1, 2$) is x_i, $\hat{A}_i$ or $\bar{A}_i$. We learn from Theorem 1 that $\hat{A}_1 = \hat{A}_2$ and $\bar{A}_1 = \bar{A}_2$. So, we should compare the consumer's net surpluses when the consumer chooses x_i at Retailer i. From (21) we find that when $\phi > \max[x_1(1+m_1), x_2(1+m_2)]$, the consumer's net surplus is higher when he or she chooses the retailer with the smaller CFS cutoff level. Therefore, to compete, two retailers should finally set identical CFS cutoff level.

Case 2: $x_1 \geq 4s_0(1+s)$ **and** $s_0(1+s) < x_2 < 4s_0(1+s)$**.** For this case, we find that, similar to Case 1, consumers don't make any online transactions with the retailer with the greater CFS cutoff level. In addition, we find that when $\phi > \max[x_1(1+m_1), 4s_0(1+s)(1+m_2)]$, the consumer's purchase amount from Retailer 1 is x_1, $\hat{A}_1$ or $\bar{A}_1$, and the consumer's purchase amount from Retailer 2 is $\hat{A}_2$ or $\bar{A}_2$. Since $\hat{A}_1 = \hat{A}_2$ and $\bar{A}_1 = \bar{A}_2$, we need to compare the consumer's net surplus in terms of x_1 and that in terms of $\hat{A}_2$. Note that for this case $x_2 < x_1$. Because

$$G^{(2)}|_{A=\hat{A}_2} > G^{(2)}|_{A=x_2} = \sqrt{\frac{x_2\phi}{(1+m)}} - x_2 > \sqrt{\frac{x_1\phi}{(1+m)}} - x_1 = G^{(1)}|_{A=x_1},$$

which means that Retailer 1 should change its CFS threshold x_1 to x_2 for this case.

Case 3: $x_1 \geq 4s_0(1+s)$ **and** $0 \leq x_2 < s_0(1+s)$**.** Similarly, we find that consumers don't make any online transactions with the retailer with the greater CFS cutoff level. In addition, we find that when $\phi > \max[x_1(1+m_1), 4x_2(1+m_2)]$, the consumer's purchase amount from Retailer 1 is x_1, $\hat{A}_1$ or $\bar{A}_1$, and the consumer's purchase amount from Retailer 2 is $\bar{A}_2$. Since $\bar{A}_1 = \bar{A}_2$, we need to compare the consumer's net surplus in terms of x_1, that in terms of $\hat{A}_1$ and that in terms of $\bar{A}_2$. We find that

$$G^{(2)}|_{A=\bar{A}_2} > G^{(2)}|_{A=\hat{A}_2} = \frac{\phi}{4(1+s)(1+m)} - s_0 = G^{(1)}|_{A=\hat{A}_1},$$

and

$$G^{(2)}|_{A=\bar{A}_2} > G^{(2)}|_{A=\hat{A}_2} > G^{(2)}|_{A=x_2} > G^{(1)}|_{A=x_1};$$

thus, the consumer's net surplus in terms of $\bar{A}_2$ is higher. As a result, Retailer 1 should change its CFS threshold x_1 to x_2 for this case.

Case 4: $s_0(1+s) < x_1 < 4s_0(1+s)$ and $x_2 \geq 4s_0(1+s)$. Similar to Case 2, Retailer 2 should change its CFS threshold x_2 to x_1 for this case.

Case 5: $s_0(1+s) < x_1 < 4s_0(1+s)$ and $s_0(1+s) < x_2 < 4s_0(1+s)$. For this case, we find from Theorem 3 that the consumer with specific parameter ϕ has the same purchase amounts at the two retailers. However, the range for no purchase at the retailer with the larger CFS threshold is wider; this means that the probability of no purchase at the retailer with the larger CFS threshold is larger. Similarly, we find that, if a retailer's CFS threshold is larger, then the probability for the larger purchase amount $\bar{A}$ at the retailer is smaller. Thus, we conclude that, to compete for consumers, the two retailers should change their CFS cutoff levels until they have identical CFS thresholds.

Case 6: $s_0(1+s) < x_1 < 4s_0(1+s)$ and $0 \leq x_2 < s_0(1+s)$. For this case, we find that $x_1 > x_2$. Similar to Case 3, Retailer 1 should change its CFS threshold x_1 to x_2 for this case.

Case 7: $0 \leq x_1 < s_0(1+s)$ and $x_2 \geq 4s_0(1+s)$. Similar to Case 3, Retailer 2 should change its CFS threshold x_2 to x_1 for this case.

Case 8: $0 \leq x_1 < s_0(1+s)$ and $s_0(1+s) < x_2 < 4s_0(1+s)$. Similar to Case 6, Retailer 2 should change its CFS threshold x_2 to x_1 for this case.

Case 9: $0 \leq x_1 < s_0(1+s)$ and $0 \leq x_2 < s_0(1+s)$. For this case, in order to compete for consumers, the two retailers have incentives to eventually set identical CFS cutoff levels.

From the above analysis, we conclude that the two retailers' equilibrium CFS thresholds should be identical. The theorem is thus proved. ■

Appendix B A Discussion of Four Cases in Figure 1

We discuss four cases in Figure 1: [Note that, as implies by Theorem 1, $\bar{A} > \hat{A}$ and $G_1^* > G_2^*$.]

(i): $x \leq \hat{A} < \bar{A}$. According to Theorem 1, we find that $G_1^* = \phi/[4(1+m)] \geq 0$. Therefore, for this case, the consumer qualifies for CFS and his or her optimal purchase quantities are $q_i^* = \bar{q}_i$ $(i = 1, 2, \ldots, n)$, which are given by (6); and his or her optimal purchase amount is $A^* = \bar{A}$, which is given by (7). For this case, see Figure 1(i).

(ii): $\hat{A} < x \leq \bar{A}$. The analytical results for this case are the same as those for Case (i). For this case, see Figure 1(ii).

(iii): $\bar{A} < x$ and $G_2^* \leq G_1|_{A=x}$. Using Figure 1(iii), we find that, if $G_1|_{A=x} \geq 0$, then the optimal purchase amount is $A^* = x$ and the consumer qualifies for CFS; otherwise, the net surplus is negative, and the consumer abandons his or her shopping cart. It, however, follows from (5) that we must determine the consumer's purchase quantities q_i^* $(i = 1, 2, \ldots, n)$ when the consumer's purchase amount $A = x$. Otherwise, if we don't have any information about purchase quantities but only have purchase amount $A = x$, we cannot compute the consumer's net surplus $G_1|_{A=x}$.

To find the purchase quantities, we should maximize G_1 subject to $A = x$; that is,

$$\max_{q_i} \sum_{i=1}^{n} \theta_i \sqrt{q_i} - (1+m) \sum_{i=1}^{n} c_i q_i, \text{ s.t. } (1+m) \sum_{i=1}^{n} c_i q_i = x.$$

Solving the problem gives

$$q_i^* = \frac{\theta_i^2}{c_i^2} \frac{x}{(1+m)\phi},$$

and the resulting maximum net surplus

$$G_1|_{A=x} = \sqrt{\frac{x\phi}{(1+m)}} - x. \tag{21}$$

Hence, when $\phi \geq x(1+m)$, $G_1|_{A=x} \geq 0$ and the consumer makes his or her purchase; when $\phi < x(1+m)$, $G_1|_{A=x} < 0$ and the consumer abandons his or her online shopping cart. Moreover, the condition that $G_2^* \leq G_1|_{A=x}$ can be written as

$$\frac{\phi}{4(1+s)(1+m)} - s_0 \leq \sqrt{\frac{x\phi}{(1+m)}} - x.$$

Solving the inequality we find that, if $x < s_0$, then $G_2^* \leq G_1|_{A=x}$ when

$$\phi \geq 4(1+m)(1+s)[\sqrt{(1+s)x} + \sqrt{sx+s_0}]^2;$$

otherwise, if $x \geq s_0$, then $G_2^* \leq G_1|_{A=x}$ when

$$\phi \geq 4(1+m)(1+s)[\sqrt{(1+s)x} + \sqrt{sx+s_0}]^2, \text{ or, } \phi \leq 4(1+m)(1+s)[\sqrt{(1+s)x} - \sqrt{sx+s_0}]^2.$$

(iv): $\bar{A} < x$ **and** $G_2^* > G_1|_{A=x}$**.** Using Figure 1(iv), we find that, if $G_2^* \geq 0$, then $q_i^* = \hat{q}_i$ ($i = 1, 2, \ldots, n$), which are given by (8); and the optimal purchase amount is $A^* = \hat{A}$, which is given by (9). For this case the consumer buys but doesn't qualify for free shipping. Otherwise, if $G_2^* < 0$, the net surplus is then negative, and the consumer abandons his or her shopping cart. According to (9), we find that, if $\phi \geq 4s_0(1+s)(1+m)$, then $G_2^* \geq 0$; otherwise, if $\phi < 4s_0(1+s)(1+m)$, then $G_2^* < 0$. In addition, if $x < s_0$, then $G_2^* > G_1|_{A=x}$ only when

$$0 \leq \phi < 4(1+m)(1+s)[\sqrt{(1+s)x} + \sqrt{sx+s_0}]^2;$$

otherwise, if $x \geq s_0$, then $G_2^* > G_1|_{A=x}$ when

$$4(1+m)(1+s)[\sqrt{(1+s)x} - \sqrt{sx+s_0}]^2 < \phi < 4(1+m)(1+s)[\sqrt{(1+s)x} + \sqrt{sx+s_0}]^2.$$

Appendix C An Online Retailer's Expected Profit Function

We now specify an online retailer's expected profit function (14) for the following three scenarios.

C.1 Scenario 1: $x \geq 4s_0(1+s)$

In this scenario, we find from Theorem 3 that, if $\phi < x(1+m)$, the consumer with parameter ϕ doesn't purchase any product online and the retailer's profit is thus zero. If $x(1+m) \leq \phi \leq 4(1+s)(1+m)[\sqrt{(1+s)x} - \sqrt{sx+s_0}]^2$, the consumer then spends \$$x$ in a single transaction, and qualifies for free shipping. To satisfy the consumer, the online retailer needs to absorb the shipping fee. When $x(1+m) \leq \phi \leq 4(1+s)(1+m)[\sqrt{(1+s)x} - \sqrt{sx+s_0}]^2$, $\pi(m,x)$ should be computed as revenue minus acquisition and shipping costs. Because the consumer spends \$$x$ for each of r transactions, the sale revenue per transaction is \$$x$. Since the profit margin is m, the retailer's acquisition cost is computed as $x/(1+m)$.

Therefore, when $x(1+m) \leq \phi \leq 4(1+s)(1+m)[\sqrt{(1+s)x} - \sqrt{sx+s_0}]^2$, $\pi(m,x)$ can be computed as

$$\pi(m,x) = x - \frac{x}{1+m} - K(x) = \frac{mx}{1+m} - k(s_0 + sx) = \left[\frac{m-(1+m)ks}{1+m}\right]x - ks_0,$$

where $K(x)$ is defined in (13)

Next, when $4(1+s)(1+m)[\sqrt{(1+s)x} - \sqrt{sx+s_0}]^2 < \phi < 4x(1+m)$, the consumer spends \$$\hat{A}$ and pays for the shipping fee $S(\hat{A})$; as a result, the retailer's per transaction profit $\pi(m,x)$ is

$$\pi(m,x) = \frac{m\hat{A}}{1+m} - (k-1)(s_0 + s\hat{A}) = \frac{[m-(k-1)(1+m)s]\phi}{4(1+s)(1+m)^2} - (k-1)s_0,$$

where if $k < 1$, then $(k-1)(s_0 + sx) < 0$ and the retailer's profit increases.

When $\phi \geq 4x(1+m)$, the consumer spends \$$\bar{A}$ and obtains the free-shipping service. Hence, the retailer's profit $\pi(m,x)$ is found as

$$\pi(m,x) = \left[\frac{m-(1+m)ks}{1+m}\right]\bar{A} - ks_0 = \frac{[m-(1+m)ks]\phi}{4(1+m)^2} - ks_0.$$

In conclusion, for the first scenario [i.e., $x \geq 4s_0(1+s)$], we can compute

$$\begin{aligned}\pi(m,x) \quad = \quad & \int_{x(1+m)}^{4(1+s)(1+m)[\sqrt{(1+s)x}-\sqrt{sx+s_0}]^2} \left\{\left[\frac{m-(1+m)ks}{1+m}\right]x - ks_0\right\} f(\phi)d\phi \\ & + \int_{4(1+s)(1+m)[\sqrt{(1+s)x}-\sqrt{sx+s_0}]^2}^{4x(1+m)} \left\{\frac{[m-(k-1)(1+m)s]\phi}{4(1+s)(1+m)^2} - (k-1)s_0\right\} f(\phi)d\phi \\ & + \int_{4x(1+m)}^{\infty} \left\{\frac{[m-(1+m)ks]\phi}{4(1+m)^2} - ks_0\right\} f(\phi)d\phi.\end{aligned}$$

and use (14) to find

$$\Pi(m,x) = \pi(m,x) \times \lambda(\phi) \times \mathcal{B} = \pi(m,x) \times \ln\{1/F[x(1+m)]\} \times \mathcal{B},$$

where $\lambda(\phi) = \ln\{1/F[x(1+m)]\}$ for the scenario, as shown in Theorem 6.

C.2 Scenario 2: $s_0(1+s) < x < 4s_0(1+s)$

For this scenario, as Theorem 3 indicates, the consumer may not buy, may spend $\$\hat{A}$ but pay for shipping cost $K(\hat{A})$, or may spend $\$\bar{A}$ without shipping payment. Similar to Section C.1, we can compute the retailer's expected profit $\Pi(m, x)$ as follows:

$$\Pi(m, x) = \pi(m, x) \times \ln\{1/F[4s_0(1+s)(1+m)]\} \times \mathcal{B},$$

where

$$\begin{aligned}\pi(m, x) &= \int_{4s_0(1+s)(1+m)}^{4x(1+m)} \left\{ \frac{[m-(k-1)(1+m)s]\phi}{4(1+s)(1+m)^2} - (k-1)s_0 \right\} f(\phi)d\phi \\ &\quad + \int_{4x(1+m)}^{\infty} \left\{ \frac{[m-(1+m)ks]\phi}{4(1+m)^2} - ks_0 \right\} f(\phi)d\phi.\end{aligned}$$

C.3 Scenario 3: $0 \le x < s_0(1+s)$

For this scenario the consumer may not buy or may buy products worth $\$\bar{A}$. We can similarly compute the retailer's expected profit $\Pi(m, x)$ as follows:

$$\Pi(m, x) = \pi(m, x) \times \ln\{1/F[4x(1+m)]\} \times \mathcal{B},$$

where

$$\pi(m, x) = \int_{4x(1+m)}^{\infty} \left\{ \frac{[m-(1+m)ks]\phi}{4(1+m)^2} - ks_0 \right\} f(\phi)d\phi.$$

Appendix D Numerical Results for the Sensitivity Analysis in Section 4.3

s_0	x^*	m^*	$\Pi(m^*, x^*)$	s	x^*	m^*	$\Pi(m^*, x^*)$	k	x^*	m^*	$\Pi(m^*, x^*)$	σ	x^*	m^*	$\Pi(m^*, x^*)$
0.0	0.00	1.2472	30,797.99	0.00	4.58	1.0000	18,652.68	0.5	4.704	0.9311	25,086.14	4	4.50	1.0341	24,418.77
0.5	2.47	1.0841	17,216.38	0.05	4.61	1.0204	17,355.15	0.6	4.692	0.9481	23,577.52	5	4.63	1.0381	16,194.34
1.0	4.63	1.0381	16,194.34	0.10	4.63	1.0381	16,194.34	0.7	4.679	0.9653	22,079.37	6	4.76	1.0416	11,738.93
1.5	7.57	0.6230	13,779.05	0.15	4.66	1.0539	15,150.90	0.8	4.667	0.9830	20,591.81	7	4.88	1.0438	9,047.25
2.0	9.20	0.5017	9,966.09	0.20	4.80	1.0268	14,200.88	0.9	4.655	1.0010	19,114.98	8	5.00	1.0440	7,289.86
2.5	11.95	0.4401	6,970.17	0.25	5.00	0.9809	13,309.02	1.0	4.644	1.0194	17,649.08	9	5.12	1.0427	6,074.47
3.0	13.87	0.3877	4,721.78	0.30	5.20	0.9372	12,467.98	1.1	4.633	1.0381	16,194.34	10	5.26	1.0405	5,196.34
3.5	15.61	0.3410	3,060.23	0.35	5.40	0.8960	11,672.90	1.2	4.623	1.0573	14,751.04	11	5.39	1.0381	4,539.96
4.0	17.17	0.2986	1,862.66	0.40	5.60	0.8574	10,919.56	1.3	4.615	1.0769	13,319.53	12	5.54	1.0358	4,035.95
4.5	18.56	0.2612	1,038.53	0.45	5.80	0.8215	10,204.45	1.4	4.608	1.0969	11,900.24	13	5.68	1.0339	3,640.37
5.0	20.00	0.2297	514.37	0.50	6.00	0.7884	9,524.75	1.5	4.603	1.1173	10,493.71	14	5.83	1.0324	3,324.25
5.5	21.50	0.2046	218.35	0.55	6.20	0.7584	8,878.32	1.6	4.600	1.1382	9,100.61	15	5.98	1.0314	3,067.76
6.0	21.62	0.1852	76.32	0.60	6.40	0.7318	8,263.67	1.7	4.601	1.1596	7,721.84	16	6.14	1.0308	2,856.94
6.5	22.28	0.1706	21.04	0.65	6.60	0.7088	7,679.94	1.8	4.608	1.1814	6,358.59	17	6.29	1.0306	2,681.74
7.0	22.76	0.1598	4.37	0.70	6.80	0.6901	7,126.86	1.9	4.622	1.2037	5,012.51	18	6.45	1.0304	2,534.73

Table 1: The impacts of the parameters s_0, s, k, σ on the retailer's optimal decisions and maximum profits.

Appendix E A Simulation Approach for the Game Analysis in Section 5

In our simulation model, consumers in the base $\mathcal{B}$ arrive to the two retailers and make their purchasing decisions. We develop a flow chart in Figure 3 to show the simulation of a consumer's behavior in a single transaction. Given the p.d.f. of the parameter ϕ, we randomly generate a specific value for an arriving consumer, and use Theorem 3 to compute the consumer's optimal purchase amount $A^{(i)}$ and maximum net surplus $G^{(i)}$ if the consumer buys from Retailer i, $i = 1, 2$. Then, we compare $G^{(1)}$ and $G^{(2)}$ to find at which retailer the consumer should buy. If $G^{(i)} = G^{(j)}$, then the consumer spends $A^{(1)}$ to buy products from Retailer 1 with the probability γ, and spends $A^{(2)}$ to buy products from Retailer 2 with the probability $(1 - \gamma)$. If $G^{(i)} > G^{(j)}$ $(i, j = 1, 2;\ i \neq j)$, then the consumer gains more from purchasing at Retailer i, and decides to buy from the retailer. Otherwise, if $G^{(i)} < G^{(j)}$ $(i, j = 1, 2;\ i \neq j)$, then the consumer buys from Retailer j. Next, we compare the purchase amount of the consumer and the CFS cutoff level of the retailer from which the consumer buys, in order to find whether the consumer qualifies for the free shipping. We can then compute both retailers' profits from the consumer, as shown in Figure 3. Note that, if the consumer chooses Retailer i, then Retailer j's profit from the consumer is zero; otherwise, if the consumer chooses Retailer j, then Retailer i's profit from the consumer is zero.

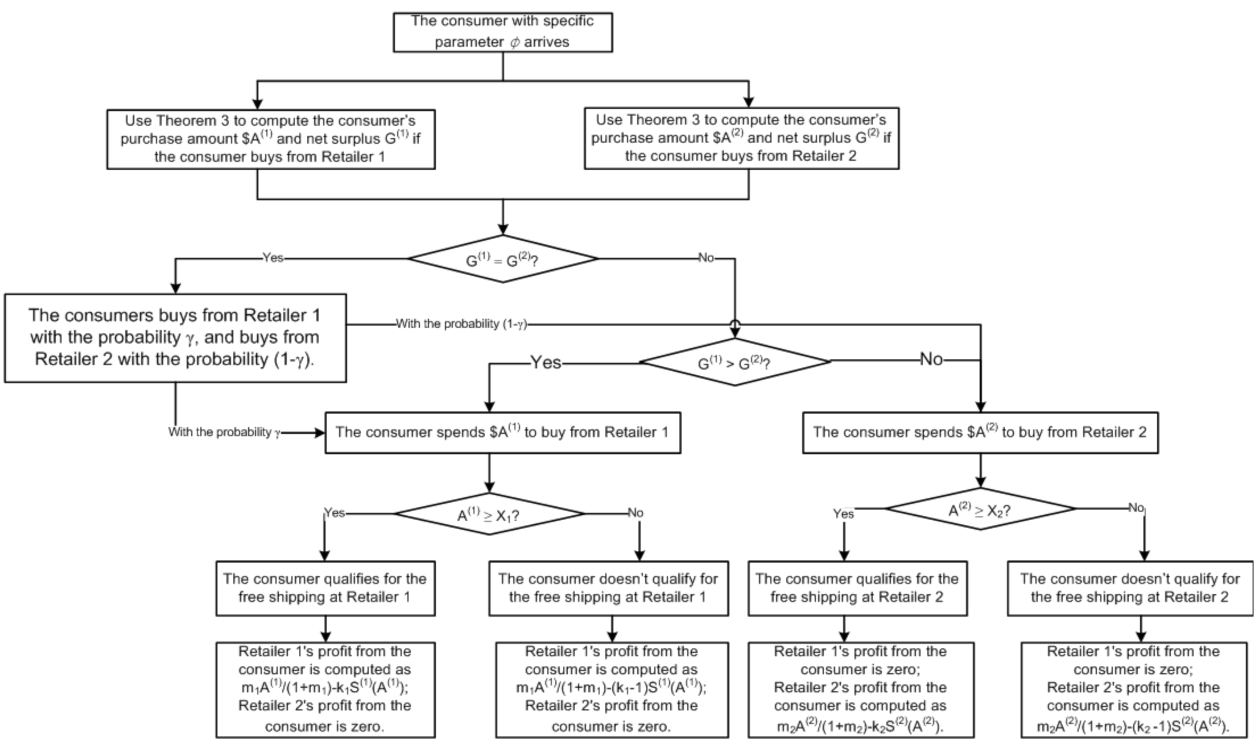

Figure 3: The flow chart of a consumer's online purchase in our simulation model.

From the simulation for the consumer with ϕ in a single transaction, we can find at which online retailer the consumer buys. If the consumer chooses Retailer i $(i = 1, 2)$, then we can use Theorem

6 and the retailer's decisions x_i and m_i to compute $\lambda(\phi)$, i.e., the consumer's expected number of repeated purchases. Thus, the total profit that Retailer i earns from the consumer's purchases is calculated as the profit in a single transaction times $\lambda(\phi)$.

We use "Arena"—a primary simulation software in industry—to develop a simulation model. Arena provides a variety of modules for simulation. For example, we can use the module "Create" to generate a new consumer in Figure 3. In our simulation we generate 10,000 consumers, as in Lewis *et al.* [33]. Moreover, we can use the module "Decide" to determine at which retailer a consumer buys. For more information regarding how to use Arena for simulation, see, e.g., Kelton *et al.* [28]. After all consumers leave, we can compute the total profit that each retailer achieves during the single period. Denote Retailer 1's and Retailer 2's total profits by $\Pi_1(m_1, x_1; m_2, x_2)$ and $\Pi_2(m_2, x_2; m_1, x_1)$, respectively. Given the values of m_i and x_i $(i = 1, 2)$, we can use the above simulation approach to find $\Pi_1(m_1, x_1; m_2, x_2)$ and $\Pi_2(m_2, x_2; m_1, x_1)$.

In order to find a Nash equilibrium, we should compute a retailer's best-response function in terms of the other retailer's decisions. However, due to the complexity of our game, we have to use simulation to find a retailer's best response when the other retailer's decisions are given. We develop the following procedure to search for Nash equilibrium.

1. We first use our method to compute Retailer i's optimal profit margin m_i^* and CFS cutoff level x_i^*, for $i = 1, 2$.
2. Given that Retailer 2's decisions are m_2^* and x_2^*, we maximize Retailer 1's profit $\Pi_1(m_1, x_1; m_2^*, x_2^*)$ to find its best-response solutions $m_1^B(m_2^*, x_2^*)$ and $x_1^B(m_2^*, x_2^*)$. To maximize $\Pi_1(m_1, x_1; m_2^*, x_2^*)$ in our Arena simulation model, we use "OptQuest"—which is an optimization add-in for Arena—to search for $m_1^B(m_2^*, x_2^*)$ and $x_1^B(m_2^*, x_2^*)$.
3. Given that Retailer 1's best-response decisions are $m_1^B(m_2^*, x_2^*)$ and $x_1^B(m_2^*, x_2^*)$, we use "OptQuest" to maximize Retailer 2's profit $\Pi_2(m_2, x_2; m_1^B(m_2^*, x_2^*), x_1^B(m_2^*, x_2^*))$ and find its best-response solutions $m_2^B(m_1^B(m_2^*, x_2^*), x_1^B(m_2^*, x_2^*))$ and $x_2^B(m_1^B(m_2^*, x_2^*), x_1^B(m_2^*, x_2^*))$.
4. If

$$\begin{aligned} m_1^* &= m_1^B(m_2^*, x_2^*),\ x_1^* = x_1^B(m_2^*, x_2^*); \\ m_2^* &= m_2^B(m_1^B(m_2^*, x_2^*), x_1^B(m_2^*, x_2^*)),\ x_2^* = x_2^B(m_1^B(m_2^*, x_2^*), x_1^B(m_2^*, x_2^*)), \end{aligned}$$

then we arrive to Nash equilibrium $(m_i^N, x_i^N) = (m_i^*, x_i^*)$, for $i = 1, 2$. Otherwise, we let

$$m_2^* \equiv m_2^B(m_1^B(m_2^*, x_2^*), x_1^B(m_2^*, x_2^*)) \text{ and } x_2^* \equiv x_2^B(m_1^B(m_2^*, x_2^*), x_1^B(m_2^*, x_2^*)),$$

and then go to Step 2 to continue with our search.

Appendix F Simulation Results for two Examples in Section 5

F.1 Simulation Results for Example 2

Simulations	Retailer 1		Retailer 2	
	m_1^*	x_1^*	m_2^*	x_2^*
Starting Point	—	—	0.9311	4.704
Simulation 1 (Retailer 1's best response)	0.9308	4.705	—	—
Simulation 2 (Retailer 2's best response)	—	—	0.9187	4.737
Simulation 3 (Retailer 1's best response)	0.9182	4.739	—	—
Simulation 4 (Retailer 2's best response)	—	—	0.8907	4.807
Simulation 5 (Retailer 1's best response)	0.8894	4.810	—	—
Simulation 6 (Retailer 2's best response)	—	—	0.8761	4.844
Simulation 7 (Retailer 1's best response)	0.8759	4.845	—	—
Simulation 8 (Retailer 2's best response)	—	—	0.8751	4.846
Simulation 9 (Retailer 1's best response)	0.8750	4.846	—	—
Simulation 10 (Retailer 2's best response)	—	—	0.8750	4.846
Simulation 11 (Retailer 1's best response)	0.8750	4.846	—	—

F.2 Simulation Results for Example 3

Simulations	Retailer 1		Retailer 2	
	m_1^*	x_1^*	m_2^*	x_2^*
Starting Point	—	—	0.2417	9.600
Simulation 1 (Retailer 1's best response)	0.5685	10.202	—	—
Simulation 2 (Retailer 2's best response)	—	—	0.1331	10.839
Simulation 3 (Retailer 1's best response)	0.4179	9.517	—	—
Simulation 4 (Retailer 2's best response)	—	—	0.0679	10.958
Simulation 5 (Retailer 1's best response)	0.3314	7.811	—	—
Simulation 6 (Retailer 2's best response)	—	—	0.0200	12.101
Simulation 7 (Retailer 1's best response)	0.2639	8.653	—	—
Simulation 8 (Retailer 2's best response)	—	—	0.6737	16.307
Simulation 9 (Retailer 1's best response)	0.6938	13.971	—	—
Simulation 10 (Retailer 2's best response)	—	—	0.6898	16.139
Simulation 11 (Retailer 1's best response)	0.6591	10.114	—	—
Simulation 12 (Retailer 2's best response)	—	—	0.6898	16.139
Simulation 13 (Retailer 1's best response)	0.6591	10.114	—	—