Competition and Coordination in a Fashion Supply Chain with Wholesale Pricing Schemes

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Accepted AUGUST 2010

Acknowledgement
The first author was supported by the National Natural Science Foundation of China under the Research Project No. 70901036. The second author was supported by the Research and Postgraduate Studies Committee of Lingnan University under the Research Project No. DR08A7.

Abstract
The paper considers a two-echelon supply chain where a supplier determines his production quantity and a retailer chooses her order size and retail price for each period in an infinite horizon. Under a price-discount sharing (PDS) scheme, the supplier's wholesale price linearly depends on the retail price. We develop a stochastic game in which these two supply chain members maximize their discounted profits. We show that a unique Nash equilibrium solution exists for each period, and over the infinite horizon the supplier chooses a stationary base stock policy whereas the retailer's equilibrium solution could be non-stationary. Next, we investigate the problem of whether or not a wholesale pricing scheme can coordinate the supplier and the retailer, and derive the conditions for supply chain coordination. Moreover, we use Nash arbitration scheme to allocate the system-wide profit between the supplier and the retailer.

1 Introduction

During the past decade, many academics and practitioners have paid much attention to supply chain coordination, which is achieved when decisions made by members of a supply chain are identical to globally optimal solutions maximizing total profits or minimizing total costs in the supply chain; see Chopra and Meindl (2004, Ch. 10). Two natural questions regarding the issue arise as follows: What mechanism could be applied to coordinate a supply chain? and how could we implement the mechanism? As Cachon (2003) discussed, a common approach is to design an appropriate contract, and in recent years a number of publications are concerned with supply chain coordination with contracts (e.g., side-payment, buy-back, revenue sharing, profit-discount sharing schemes, etc.).

In practice, many academics and practitioners have identified the importance of improving the performance of an entire supply chain, and focused on the question of how to effectively gain supply chain coordination and integration for the improvement. Next, we briefly review several papers concerning the coordination of fashion supply chains. Kincade et al. (2002) conducted a survey and revealed that the benefits of retailers in the apparel industry are greatly related to the financial promotional support from manufacturers. Motivated by a real case in the apparel industry, Eppen and Iyer (1997) developed a stochastic dynamic programming model to investigate a backup agreement for a fashion supply chain involving a catalog company and a manufacturer. Under the agreement, the catalog company commits to a number of units for a certain fashion season, and the manufacturer holds back a percentage of the committed units and delivers the remaining units before the start of the season. It was shown that the backup agreement can increase both the catalog firm's and the manufacturer's expected profits. Indu and Govind (2008) discussed the practices of three European apparel companies (Zara, H&M, and Benetton) that have successfully integrated their fashion supply chains and increased their profits. Kurata and Yue (2008) examined the scan-back (SB) trade deal—a special type of trade promotion used in fashion supply chains—that monitors a retailer's sales via an IT system. They showed that both the retailer and the manufacturer can benefit from the SB trade deal if the SB deal is accompanied by a buyback contract. In addition, it has been widely recognized that the bullwhip effect that increases the variability of production and order quantity negatively impacts supply chain performance. Cachon and Terwiesch (2006, Ch. 14) concluded that such an effect has been one of two major challenges to supply chain coordination, and discussed several major causes resulting in the undesirable phenomenon: order synchronization, order batch, trade promotion and forward buying, reactive and overreactive ordering and shortage gaming.

Restricting our attention to trade promotion and forward buying, we find a number of evidences about the widespread use of trade promotion and discussions from both academic and practical perspectives. As a survey conducted by MEI Computer Technology Group Inc. in 2010 indicates, trade promotion (e.g., wholesale price reduction) is extensively applied in the consumer packaged goods (CPG) industry, and there is an increased emphasis on improving its effectiveness [i]. According to Investopedia [ii] and TechTarget [iii], the CPG industry is one of the largest in North America, valued at approximately US$2 trillion; some examples of CPGs are food and beverages, footwear and apparel, tobacco, and household products. Ailawadi, Farris, and Shames (1999) stated that trade-promotion expenditures in this industry increased from less than 35 percent in 1983 to nearly 49 percent in 1994, and its budget was more than twice the media advertising budget. The experiences in real business world testify to the conclusions presented in Cachon and Terwiesch (2006, Ch. 14) and Chopra and Meindl (2004, Ch. 10): Trade promotion could result in retail opportunism. When a manufacturer temporarily cuts his wholesale price in order to attract and keep customers, some retailers may use the offer to increase their own margins by purchasing more for future periods rather than sharing the promotion with consumers. This is known as forward buying. Such a response is troubling the manufacturer since he cannot pass the low price to end customers through the retailers and the retailers' opportunistic order behavior leads to the bullwhip effect. On the other hand, without trade promotion, the manufacturer may suffer a significant decline in market share if the competitors continue with trade promotion. More examples and discussions about trade promotion and forward buying can be found in Kumar, Rajiv, and Jeuland (2001).

To encourage the use of trade promotion but avoid forward buying, Ailawadi, Farris, and Shames (1999) suggested that the manufacturers could coordinate their supply chains by using *price-up deal-down* strategies that link the wholesale price to the retail price. Using the strategies, Bernstein and Federgruen (2005) introduced a linear price-discount sharing (PDS) scheme to a game model, and investigated the equilibrium behavior of decentralized supply chains involving N competing retailers in the newsvendor setting. They showed that, when a PDS scheme is reduced to a constant wholesale pricing scheme (where the wholesale price is equal to the supplier's purchase cost), supply chain coordination could be achieved with the constant wholesale pricing scheme and a buy-back rate. Cachon and Lariviere (2005) found that, in the price-setting newsvendor model, a PDS scheme is equivalent to the revenue-sharing contract for supply chain coordination. Based on our above survey, we believe that supply chain coordination has been becoming a prevailing issue in the supply chain management field, and it could be induced through a PDS scheme in the newsvendor setting. However, in reality most supply chain members would like to have a long-term business relationship rather than only for a single period, so it should be more interesting to relax the newsvendor assumption and consider a more general case—supply chain competition and coordination with a pricing scheme in a multi-period context. The major purposes of our paper are to seek an equilibrium decision for each period in a multi-period problem, examine whether the decision is stationary over the periods and investigate whether a wholesale pricing scheme (a PDS scheme or a constant wholesale pricing scheme) can be found to achieve supply chain coordination for each period. Moreover, for each period we utilize a game-theoretic solution concept (e.g., Nash arbitration scheme) to split the chainwide profit generated under supply chain coordination, and compute the side payment transferred between supply chain members to implement the allocation scheme.

In this paper we consider a two-echelon supply chain where a supplier determines his production quantity and a retailer chooses her order size and retail price for each period in an infinite horizon. Using a linear PDS scheme for each period, we base the supplier's wholesale price on the retail price. At the beginning of each period the retailer determines her order quantity for the current period and places an order with the supplier. If customer demand in the period is greater than the retailer's order quantity, the demand is partially filled and the unsatisfied part is lost with a penalty cost charged to the retailer. Otherwise, for the case of overstock, the leftover is carried over to the later period and the retailer incurs a holding cost. The supplier chooses a production quantity and attempts to fill the retailer's order placed at the beginning of each period. In order to immediately fill the order of the retailer, the supplier should make his decision earlier than the arrival of the retailer's order. For simplicity, we assume that the supplier's order can be fully satisfied by his upstream member, and that the lead-time for the supplier to deal with the retailer's order is so short that no more than one order is outstanding at any point in time.

The demand faced by the retailer is an identically-distributed random variable over the infinite horizon, and the demands for different periods are independent of one another. Assuming that the distribution of the demand in a period depends on the retail price in the period, we write the demand function for period t ($t = 1, 2, \ldots, \infty$) in a multiplicative form, i.e.,

$$D_t(p_t, \varepsilon) = D(p_t)\varepsilon, \tag{1}$$

where $D(p_t)$ is a non-negative constant demand term dependent on the retail price p_t (chosen by the retailer for period t). The term ε denotes a non-negative r.v. which is *stationary* over the infinite time horizon with c.d.f. $F(\cdot)$ and p.d.f. $f(\cdot)$. We denote the mean value and variance of ε by μ and σ^2, i.e., $E(\varepsilon) = \mu$ and $Var(\varepsilon) = \sigma^2$. Using Bertrand's model (1883), we write the deterministic demand $D(p_t)$ as a linear, continuous, and strictly decreasing function of the retail price p_t, i.e., $dD(p_t)/dp_t < 0$ and $d^2D(p_t)/dp_t^2 = 0$. For some applications of Bertrand's model, see Palaka, Erlebacher, and Kropp (1998) and Corbett and Karmarkar (2001).

The multiplicative demand function (1) has been extensively used in the management science and operations management field. For example, Petruzzi and Dada (1999) specified the price-sensitive demand function in the multiplicative form in the newsboy context. Chen and Simchi-levi (2004) assumed the demand function is $D_t(p_t, \varepsilon) = \alpha D(p_t) + \beta$, where $\varepsilon = (\alpha, \beta)$ is random perturbations identically distributed and independent across times. Equating β to zero reduces the demand function in Chen and Simchi-levi (2004) to (1).

In this paper we will develop an inventory-related stochastic game (a.k.a. Markov game) to find the equilibrium decisions of supply chain members, and discuss if a PDS scheme can achieve supply chain coordination for each period in an infinite horizon. The theory of stochastic games—that was introduced by Shapley (1953)—describes the time-dependent multi-period game models. For a particular discussion on such games, see Fudenberg and Tirole (1992). As Cachon and Netessine (2004) reviewed, stochastic games have been used to investigate some supply chain-related problems. Cachon and Zipkin (1999) examined a two-echelon supply chain involving a wholesaler and a retailer who determine their inventory decisions. Assuming stationary stochastic demand and fixed transportation times, they considered two stochastic games and designed simple linear side-payment schemes to coordinate the supply chain. Netessine, Rudi, and Wang (2005) analyzed a stochastic multi-period game for a horizontal supply chain where two retailers compete for customers by determining their ordering quantities of a single product with an exogenously given price. Different from Netessine, Rudi, and Wang (2005) where the price is given, the retailer's price in our model is a decision variable and the supplier's wholesale price is determined by a PDS scheme. Moreover, our paper considers a vertical supply chain involving a supplier and a retailer, and also examines supply chain coordination.

Our game analysis is conducted under complete information. To reflect dynamic features in our game model, we consider non-stationary costs. Specifically, the supplier's unit purchase cost and both members' unit holding and shortage penalty costs are non-stationary over the time horizon. (Note that the retailer's unit purchase cost is equal to the supplier's wholesale price that is computed by using a PDS scheme.) The expected profits of the supplier and the retailer are computed as the sum of the discounted expected profits incurred for all periods. We assume that the discount factor β is stationary and the same for both members. Moreover, it is assumed that the unit holding cost of the retailer (h_t, $t = 1, 2, \ldots$) is greater than or equal to that of the supplier (H_t, $t = 1, 2, \ldots$), i.e., $H_t \le h_t$. The assumption does actually make sense due to the following fact: The unit holding cost is commonly computed as a fraction of the unit purchase cost, and the retailer's unit purchase cost is the supplier's wholesale price (w_t, $t = 1, 2, \ldots$) which is no less than the supplier's unit purchase cost (c_t, $t = 1, 2, \ldots$). Using this fact we make two other assumptions that $H_t \le w_t$ and $h_t \le c_t$.

The paper is organized as follows: In Section 2 we develop the discounted profit functions for the supplier and the retailer, and construct a stochastic game model. Section 3 analyzes the game model with a given wholesale pricing (PDS) scheme and obtains the equilibrium decisions for both members. Particularly, in this section we first perform the best-response analysis for each member (player), and then find whether or not a Nash equilibrium exists in each period and the equilibrium solution is unique. If a Nash equilibrium solution exists for any subgame (game for some periods), we use the concept of Markov perfect equilibrium (see, for example, Fudenberg and Tirole 1992) to characterize equilibrium behaviors of the supplier and the retailer. In Section 4 we investigate whether or not supply chain coordination can be achieved by a well-designed PDS scheme or a constant wholesale pricing scheme. Under such a scheme, the supplier and the retailer choose their equilibrium decisions that maximize the supply chain-wide profit and make both members better off than in the non-cooperative situation. As Bernstein and Federgruen (2005) showed, the supplier's wholesale price w_t for supply chain coordination could be equal to his unit purchase cost c_t, which resulting in zero profit at the supplier echelon for period t. For this case, we find that the wholesale price does not depend on the retail price, which implies that the PDS scheme is reduced to a constant wholesale pricing scheme. In order to entice both members to cooperate, we use Nash arbitration scheme to allocate the system-wide profit between the retailer and the supplier, and compute a side payment transferred between them. Under supply chain coordination, both members are better off than in the non-cooperative situation. In Section 5 we summarize our model and major results, and provide some opportunities for the applications of stochastic games.

2 Stochastic Game Model

We now construct a stochastic game by developing the objective (discounted profit) functions for both supply chain members. In the model the supplier determines his production quantity Q_t and the retailer chooses her order quantity q_t and retail price p_t, $t = 1, 2, \ldots, \infty$. The supplier's wholesale price w_t for period t is computed by using the following linear price-discount sharing (PDS) scheme:

$$w_t = w_t^0 - \alpha_t (p_t^0 - p_t), \qquad (2)$$

where w_t^0 and p_t^0 denote a *base* wholesale price and a *base* retail price in period t, respectively. (For an application of this linear PDS model, see Bernstein and Federgruen [2005].) We assume that $c_t \le w_t^0 \le p_t^0$, in which c_t is the supplier's unit purchase cost. The parameter α_t represents a non-stationary ratio of price changes of the two members, and is assumed to be greater than or equal to zero, i.e., $\alpha_t \ge 0$. Note that when $\alpha_t = 0$, the PDS scheme for period t is reduced to a constant wholesale pricing scheme where $w_t = w_t^0$. The linear PDS model (2) is explained as follows: If the retail price p_t is greater (less) than p_t^0 by one dollar, then the wholesale price w_t is greater (less) than w_t^0 by α_t dollars. To assure that $w_t \ge c_t$, the value of w_t^0 should be properly chosen. Section 3 assumes that w_t^0 is properly given so that w_t is no less than c_t. In Section 4, we find the well-designed value of w_t^0 that satisfies the condition and coordinates the supply chain.

We begin by developing the supplier's discounted profit function. To get it we need to find the expected profit of the supplier for period t, $t = 1, 2, \ldots, \infty$. As assumed in Section 1, the supplier has a short time to process the retailer's order and the supplier's order is fully satisfied by his upstream member. At the beginning of each period the supplier receives the retailer's order with quantity q_t, and attempts to satisfy the order using his products available in stock. If the supplier's inventory level is greater than or equal to the retailer's order size q_t, then the retailer's order is fully satisfied, the leftover is carried over to period $t+1$, and the supplier incurs a holding cost. Otherwise, the retailer's order is partially filled and the unsatisfied part is lost with a shortage penalty cost absorbed by the supplier. Thus, the supplier's expected profit function for period t is formulated as follows:

$$\Pi_t(Q_t; q_t, p_t) = E[w_t \min(Y_t, q_t) - H_t(Y_t - q_t)^+ - K_t(q_t - Y_t)^+ - c_t Q_t], \quad (3)$$

where K_t is the unit shortage costs; Y_t is the supplier's order-up-to level at the beginning of period t, and it is computed as $Y_t = Q_t + X_t$ in which X_t denotes the leftover transferred from period $t-1$. X_t is the *state* at the supplier echelon for period t, and it can be computed as

$$X_t = \begin{cases} X_1, & t = 1, \\ (Y_{t-1} - q_{t-1})^+, & t \geq 2. \end{cases} \quad (4)$$

We assume that, at the beginning of the first period, the starting inventory X_1 is given.

Letting β denote the discount factor per period, we find that the profit Π_t in (3) for period t is equivalent to the value $\beta^{t-1}\Pi_t$ at the beginning of the first period. Thus, the discounted profit of the supplier can be written as follows:

$$\Pi(\mathbf{Q}; \mathbf{q}, \mathbf{p}) = E\left\{\sum_{t=1}^{\infty} \beta^{t-1}[w_t \min(Y_t, q_t) - H_t(Y_t - q_t)^+ - K_t(q_t - Y_t)^+ - c_t Q_t]\right\}, \quad (5)$$

where $\mathbf{Q} \equiv (Q_t,\ t = 1, \ldots, +\infty)$; $\mathbf{q} \equiv (q_t,\ t = 1, \ldots, +\infty)$; and $\mathbf{p} \equiv (p_t,\ t = 1, \ldots, +\infty)$.

Next, we consider the discounted profit function of the retailer. Note that whether or not the retailer's order placed at the beginning of period t can be fully satisfied depends on the supplier's inventory level Y_t. The retailer receives her order quantity q_t or Y_t, whichever is smaller. After receiving the products, the retailer's inventory is increased to the order-up-to level y_t, which can be computed by using the formula $y_t = \min(q_t, Y_t) + x_t$. Here x_t is the *state* of the retailer for period t, and represents the leftover transferred from period $t-1$, which is found as

$$x_t = \begin{cases} x_1, & t = 1, \\ [y_{t-1} - \hat{D}_{t-1}]^+, & t \geq 2, \end{cases} \quad (6)$$

where $\hat{D}_{t-1}$ denotes the actual demand in period $t-1$ and is known to the retailer at the beginning of period t; and x_1 is the given starting inventory level for the first period.

Using y_t, the retailer attempts to satisfy the demand $D_t(p_t, \varepsilon)$ that is determined by (1). If $D_t(p_t, \varepsilon) \geq y_t$, y_t units of demand are satisfied and $[D_t(p_t, \varepsilon) - y_t]$ units are lost with a penalty cost $k_t[D_t(p_t, \varepsilon) - y_t]$ charged to the retailer, where k_t is the unit shortage cost. Otherwise, if $D_t(p_t, \varepsilon) \leq y_t$, the leftover $[y_t - D_t(p_t, \varepsilon)]$ is carried over to the next period (i.e., period $t+1$) and the retailer incurs a holding cost $h_t[y_t - D_t(p_t,\varepsilon)]$, where h_t is the unit holding cost. As a result, we develop the retailer's expected profit function for period t as

$$\begin{aligned}\pi_t(q_t, p_t; Q_t) = E\{&p_t \min[y_t, D_t(p_t,\varepsilon)] - h_t[y_t - D_t(p_t,\varepsilon)]^+ \\ &-k_t[D_t(p_t,\varepsilon) - y_t]^+ - w_t \min(q_t, Y_t)\}.\end{aligned} \tag{7}$$

Thereby, the retailer's objective (discounted profit) function is

$$\begin{aligned}\pi(\mathbf{q},\mathbf{p};\mathbf{Q}) = \sum_{t=1}^{\infty} \beta^{t-1} E\{&p_t \min[y_t, D_t(p_t,\varepsilon)] - h_t[y_t - D_t(p_t,\varepsilon)]^+ \\ &-k_t[D_t(p_t,\varepsilon) - y_t]^+ - w_t \min(q_t, Y_t)\}.\end{aligned}$$

Since $\min[y_t, D_t(p_t,\varepsilon)] = y_t - [y_t - D_t(p_t,\varepsilon)]^+$ and $\min[q_t, Y_t] = q_t - [q_t - Y_t]^+$, we simplify the retailer's objective function to

$$\begin{aligned}\pi(\mathbf{q},\mathbf{p};\mathbf{Q}) = \sum_{t=1}^{\infty} \beta^{t-1} E\{&p_t y_t - w_t Y_t - (p_t + h_t)[y_t - D_t(p_t,\varepsilon)]^+ \\ &-k_t[D_t(p_t,\varepsilon) - y_t]^+ + w_t(Y_t - q_t)^+\}.\end{aligned} \tag{8}$$

We have constructed the two members' objective functions (5) and (8). The supplier and the retailer make their decisions by solving the problems $\max_{Q_t} \Pi(\mathbf{Q};\mathbf{q},\mathbf{p})$ and $\max_{q_t, p_t} \pi(\mathbf{q},\mathbf{p};\mathbf{Q})$, respectively. In the next section, we will analyze the two maximization problems to get the best response functions of the two members, which is then used to obtain the equilibrium solutions.

3 Equilibrium Analysis

We now conduct the best-response analysis for each supply chain member. Specifically, when the retailer's decision is (q_t, p_t), $t = 1,2,\ldots,\infty$, the supplier maximizes his objective function $\Pi(\mathbf{Q};\mathbf{q},\mathbf{p})$ to find the optimal response Q_t^B which is in terms of q_t and p_t. Similarly, the retailer can obtain her best responses q_t^B and p_t^B by maximizing $\pi(\mathbf{q},\mathbf{p};\mathbf{Q})$ given that the supplier's decision is Q_t. The analytical results will be used later to compute the equilibrium solutions.

From (5) and (8), we find that the discounted profit of each supply chain member is the sum of expected profits that are converted to the first period by the factor β. Since our game model assumes a stationary demand distribution that is independent across the periods, we can break down the multi-period game into the multiple *identical* single-period games. As a result, solving (5) and (8) for the best response solutions is equivalent to solving single-period objective functions (3) and (7) for period t, $t = 1,2,\ldots,\infty$. Note that, if we relax the assumption of stationary demand, we cannot break down the multi-period game and our game analysis would be too complicated to be intractable and present the analytical results. Cachon and Netessine (2004) gave a discussion regarding the assumption of stationary demand and the technique of breading down a multi-period stochastic supply-chain game into multiple single-period games. For some similar applications, see Cachon and Zipkin (1999) and Netessine, Rudi, and Wang (2005).

One could find that the stochastic game is quite similar to the single-period static game. We briefly provide our explanations as follows: (i) the stochastic game considers the important and realistic issue that the leftover for a period are carried over to the next period, which is different from a static game; (ii) investigating the stochastic game gives some results (e.g., our discussion on whether or not an equilibrium decision is stationary over the periods) that cannot be envisioned by the static game; (iii) we conduct the game analysis for the case in which the supply chain members have a long-term partnership rather than a single-period temporary business relationship. The long-term partnership no doubt plays an important role in supply chain coordination.

3.1 Supplier's Best-Response Analysis

We analyze the objective function (3) to obtain the supplier's best-response Q_t^B for period t given the retailer's decision (q_t, p_t). For the period, the state of the supplier is the leftover X_t transferred from period $t-1$, which is computed by (4).

Theorem 1 *For period* t, $t = 1, 2, \ldots + \infty$, *the supplier's best response is*

$$Q_t^B = (q_t - X_t)^+, \tag{9}$$

where

$$X_t = \begin{cases} X_1\ (given), & t = 1, \\ [(Q_{t-1}^B + X_{t-1}) - q_{t-1}]^+, & t \geq 2. \end{cases}$$

Proof. The proof of this theorem and the proofs of all subsequent theorems in our main paper are given in Appendix A. ■

Using (9) we compute the supplier's order-up-to level Y_t^B as

$$Y_t^B = Q_t^B + X_t = (q_t - X_t)^+ + X_t = \max(q_t, X_t). \tag{10}$$

Remark 1 Theorem 1 suggests that the supplier adopts the *base stock* inventory policy as his best-response strategy for each period. When the retailer's order quantity q_t is given and known to the supplier, the supplier makes his production decision according to the comparison between X_t and q_t. If $X_t \geq q_t$, the supplier's production quantity for period t is zero. Otherwise, the supplier manufactures $(q_t - X_t)$ units of products to shift his inventory level up to q_t. Furthermore, we conclude that the best response adopted by the supplier is *stationary* over the infinite horizon. ◁

3.2 Retailer's Best-Response Analysis

We analyze the retailer's profit maximization problem (7) for period t to find her best response (q_t^B, p_t^B) as the supplier chooses his production quantity Q_t. Replacing the retailer's order-up-to level y_t with $\min(q_t, Y_t) + x_t$, we re-write the maximization problem for period t to

$$\max_{q_t, p_t} \pi_t(q_t, p_t; Q_t) = E\{p_t \min[\min(q_t, Y_t) + x_t, D_t(p_t, \varepsilon)] - h_t[\min(q_t, Y_t) + x_t - D_t(p_t, \varepsilon)]^+ \\ -k_t[D_t(p_t, \varepsilon) - (\min(q_t, Y_t) + x_t)]^+ - w_t \min(q_t, Y_t)\}, \tag{11}$$

where x_t denotes the state for period t and is computed by using (6).

For the best-response analysis, we use the following three steps: (i) fix the retail price p_t and find the best-response order quantity q_t^B; (ii) fix q_t and find the best-response price p_t^B; (iii) find the retailer's best-response (q_t^B, p_t^B). In the first step, we compare q_t and Y_t and have

1. If $q_t \le Y_t$, the expected profit function (11) is reduced to

$$\pi_{t1}(q_t, p_t; Q_t) = E\{p_t \min[q_t + x_t, D_t(p_t, \varepsilon)] - h_t[q_t + x_t - D_t(p_t, \varepsilon)]^+ \\ -k_t[D_t(p_t, \varepsilon) - (q_t + x_t)]^+ - w_t q_t\}. \tag{12}$$

2. If $q_t \ge Y_t$, the expected profit function (11) is written as

$$\pi_{t2}(q_t, p_t; Q_t) = E\{p_t \min[Y_t + x_t, D_t(p_t, \varepsilon)] - h_t[Y_t + x_t - D_t(p_t, \varepsilon)]^+ \\ -k_t[D_t(p_t, \varepsilon) - (Y_t + x_t)]^+ - w_t Y_t\}, \tag{13}$$

which is independent of the decision variable q_t.

Lemma 1 *For a given value of* p_t*, the function* $\pi_{t1}(q_t, p_t; Q_t)$ *is strictly concave in order quantity* q_t.

Proof. The proof of this lemma and the proofs of all subsequent lemmas in our main paper are given in Appendix B. ■

Solving (27) we can find the optimal order quantity for a given p_t.

Theorem 2 *For a retail price* p_t*, the retailer's optimal order quantity (*$\hat{q}_t$*) is*

$$\hat{q}_t = \begin{cases} q_t^0(p_t), & q_t^0(p_t) \le Y_t, \\ Y_t + \theta_t, & q_t^0(p_t) \ge Y_t, \end{cases}$$

where θ_t *is an arbitrary value in the range* $[0, +\infty)$*, and*

$$q_t^0(p_t) = \left(D(p_t) F^{-1}\left[\frac{p_t + k_t - w_t}{p_t + h_t + k_t} \right] - x_t \right)^+. \quad \square \tag{14}$$

Next we find the optimal retail price p_t for a given order quantity q_t.

Lemma 2 *For any given* q_t*, the objective function* $\pi_t(q_t, p_t; Q_t)$ *for period* t *is strictly concave in the retail price* p_t. □

Using Theorem 2 and Lemma 2, we can find the best-response retail price p_t^B and order quantity q_t^B which maximize the objective (profit) function $\pi_t(q_t, p_t; Q_t)$ in (11).

Theorem 3 *The retailer's best response is obtained as follows:*

$$(q_t^B, p_t^B) = \begin{cases} (q_t^0(\tilde{p}_t), \tilde{p}_t), & \text{if } \pi_t(q_t^0(\tilde{p}_t), \tilde{p}_t; Q_t) \ge \pi_t(Y_t + \theta_t, \hat{p}_t; Q_t), \\ (Y_t + \theta_t, \hat{p}_t), & \text{if } \pi_t(q_t^0(\tilde{p}_t), \tilde{p}_t; Q_t) \le \pi_t(Y_t + \theta_t, \hat{p}_t; Q_t), \end{cases} \tag{15}$$

where $\tilde{p}_t \equiv \arg\max_{q_t^0(p_t) \le Y_t} \pi_t(q_t^0(p_t), p_t; Q_t)$ *and* $\hat{p}_t \equiv \arg\max_{q_t^0(p_t) \ge Y_t} \pi_t(Y_t + \theta_t, p_t; Q_t)$. □

3.3 Equilibrium Solution

We use the best-response results given by Theorems 1 and 3 to find the Nash equilibrium for period t.

Theorem 4 *The Nash equilibrium for period* t *is obtained as*

$$(Q_t^N, q_t^N, p_t^N) = ((q_t^0(\bar{p}_t) - X_t)^+, q_t^0(\bar{p}_t), \bar{p}_t), \tag{16}$$

where $\bar{p}_t$ *is the optimal solution maximizing*

$$\max_{p_t} \pi_t(q_t^0(p_t), p_t; Q_t) = E\{(p_t - w_t)q_t^0(p_t) + p_t x_t - (p_t + h_t)[q_t^0(p_t) + x_t - D_t(p_t, \varepsilon)]^+ \\ - k_t[D_t(p_t, \varepsilon) - (q_t^0(p_t) + x_t)]^+\}.$$

□

The Nash equilibrium in Theorem 4 suggests that the supplier should adopt a *base stock* policy in each time period. We notice that, by using the base-stock strategy, the supplier's production quantity is always equal to the retailer's order quantity after a period. Particularly, if, in a period, the supplier's stock is reduced to a level that is lower than the retailer's order quantity, the supplier schedules his production and increases his inventory level to the size of the retailer's order. As a result, at the end of the period, the supplier's inventory level is decreased to zero after the retailer's order is satisfied; and for any following periods, the state (starting inventory) for each period is zero and the supplier's equilibrium production lot size equals the retailer's order quantity. We plot Figure 1 to illustrate the equilibrium solutions in the infinite horizon.

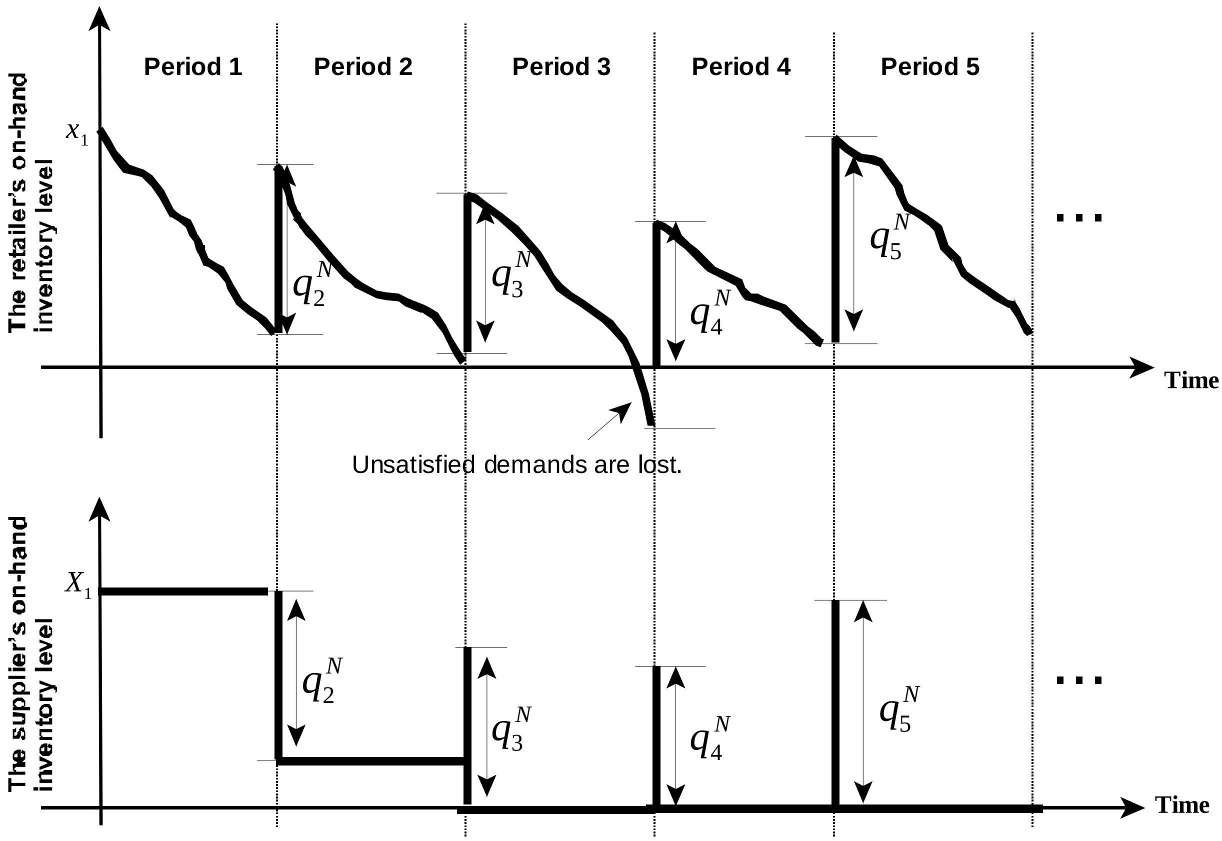


Figure 1. The equilibrium production quantity of the supplier and order size of the retailer in the infinite horizon.

Remark 2 According to Figure 1, the Nash equilibrium solutions over the time horizon are *stationary* after some specific period when the inventory level is reduced to zero. However, prior to the period, the equilibrium could be *non-stationary*. For example, we assume that the starting inventory level X_t for period t is positive. If $q_t^0(\bar{p}_t) \le X_t$, Theorem 4 indicates that the Nash equilibrium for this period is $(Q_t^N, q_t^N, p_t^N) = (0, q_t^0(\bar{p}_t), \bar{p}_t)$, which implies that the supplier does not schedule his production for period t. In period $t+1$, if $q_{t+1}^0(\bar{p}_{t+1}) > X_{t+1}$, then the Nash equilibrium for the period is $(Q_{t+1}^N, q_{t+1}^N, p_{t+1}^N) = (q_{t+1}^0(\bar{p}_{t+1}) - X_{t+1}, q_{t+1}^0(\bar{p}_{t+1}), \bar{p}_{t+1})$, which is different from the schedule for period t. For this example, the equilibria for all periods after period $t+1$ are stationary. $\triangleleft$

In the theory of stochastic games, we use the concept of Markov perfect equilibrium (MPE) to characterize the equilibrium behaviors of the supplier and the retailer for all periods. The MPE is defined as a profile of Markov strategies that yields a Nash equilibrium in every proper subgame; see Fudenberg and Tirole (1992, Ch. 13). Theorem 4 indicates that a unique Nash equilibrium for each time period always exists if the PDS scheme is designed such that the wholesale price w_t is positive.

Theorem 5 *If the base wholesale price* w_t^0 *is properly designed to ensure positive wholesale price* w_t*, then Markov perfect equilibrium (MPE) always exists in the two-echelon supply chain in an infinite horizon.* □

Example 1 In the numerical example we compute the Nash equilibrium solutions for 10 periods. For the demand function (1), we assume that $D(p_t) = -2p_t + 100$ and ε is a normally-distributed random variable with mean value 5 and variance 1, i.e., $\varepsilon \sim N(5,1)$. In order to assure the positive value of ε, we truncate the normal distribution function at zero and assume that the probability of negative values are added to that of zero. (For an application of the assumption, see Netessine, Rudi, and Wang [2005].) Moreover, for simplicity we set the values of PDS parameters in (2) to the following: $\alpha_t = 0.2$, $t = 1,\ldots,10$, and the discount factor $\beta = 0.6$. The values of the other parameters and actual demands for ten periods (i.e., $\hat{D}_t$, $t = 1,\ldots,10$) are given in Table 1.

$\hat{D}_1$	$\hat{D}_2$	$\hat{D}_3$	$\hat{D}_4$	$\hat{D}_5$	$\hat{D}_6$	$\hat{D}_7$	$\hat{D}_8$	$\hat{D}_9$	$\hat{D}_{10}$
160	130	140	150	120	150	110	100	120	120
H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8	H_9	H_{10}
2	3	3	2.5	3.5	2	3.5	3	3.5	3
K_1	K_2	K_3	K_4	K_5	K_6	K_7	K_8	K_9	K_{10}
5	7	8	6.5	8	6	8.5	7.5	9	8.5
c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}
15	16	18	17	16	16	17	17	18	19
w_1^0	w_2^0	w_3^0	w_4^0	w_5^0	w_6^0	w_7^0	w_8^0	w_9^0	w_{10}^0
20	22	25	23	24	22	25	24	26	26
h_1	h_2	h_3	h_4	h_5	h_6	h_7	h_8	h_9	h_{10}
5	5.5	6.5	6	5.5	4.5	4	4.5	4	4
k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}
11	12	12	12	13	12	10	11	12	11
p_1^0	p_2^0	p_3^0	p_4^0	p_5^0	p_6^0	p_7^0	p_8^0	p_9^0	p_{10}^0
30	32	36	35	36	33	35	35	36	36

Table 1: The parameter values in Example 1.

Next, we specify our computation for period 1, and directly present our results for the other periods in a table. At the beginning of period 1, the available stocks for the supplier and the retailer are assumed to be 200 units and 180 units, respectively. According to Theorem 4 we maximize $\pi_1(q_1^0(p_1), p_1; Q_1)$ and find Nash-equilibrium price $p_1^N = \$33.1$. Using the linear PDS scheme (2) we have the wholesale price $w_1 = w_1^0 - \alpha_t(p_1^0 - p_1^N) = \$20.62$, and then compute the retailer's order quantity $q_1^0(p_1^N) = 0$ units, which means that the retailer does not order new products and her inventory level is 180 units, i.e., $y_1 = q_1^0(p_1^N) + x_1 = 180$. Since $q_1^0(p_1^N)$ is zero, the supplier has no sale revenue. However, the supplier incurs a holding cost, and his profit for period 1 is $\Pi_1 = -2 \times 200 = -\$400$. The expected profit of the retailer for period 1 is computed as \$5112, and the actual profit for the retailer, denoted by $\hat{\pi}_1$, is computed as

$$\begin{aligned}\hat{\pi}_1 = {} & p_1^N \min[q_1^0(p_1^N) + x_1, \hat{D}_1] - h_1[q_1^0(p_1^N) + x_1 - \hat{D}_1]^+ \\ & -k_1[\hat{D}_1 - (q_1^0(p_1^N) + x_1)]^+ - w_1 q_t^0(p_t^N) \\ & = 33.1 \times 160 - 5 \times 20 - 11 \times 0 - 20.62 \times 0 = \$5196.\end{aligned}$$

The unsold products totaling 20 [obtained by computing $(y_1 - \hat{D}_1)$] units are carried to period 2. At the beginning of the second period, the available inventory levels at the supplier and the retailer are 200 and 20, i.e., $X_2 = 200$ and $x_2 = 20$. We find the optimal decisions for the other nine periods and provide the results in Table 2.

Period		1	2	3	4	5	6	7	8	9	10
Supplier	X_t	200	200	92	0	0	0	0	0	0	0
	Q_t^N	0	0	21	126	121	132	114	117	84	108
	Y_t	200	200	113	126	121	132	114	117	84	108
	w_t	20.62	23.02	25.46	23.44	24.34	22.72	25.64	24.52	26.64	25.56
	Π_t	-400	2199	2501	811	1010	888	985	881	726	818
Retailer	x_t	180	20	0	0	0	1	0	4	21	0
	p_t^N	33.1	37.1	38.3	37.2	37.7	36.6	38.2	37.6	39.2	38.8
	q_t^N	0	108	113	126	121	132	114	117	84	108
	y_t	180	128	113	126	121	133	114	121	105	108
	π_t	5112	1716	979	1199	1093	1315	998	1198	1445	897
	$\hat{\pi}_t$	5196	2227	1135	1442	1571	1669	1264	794	1698	1193

Table 2: The Results (i.e., optimal decisions and corresponding profits) obtained for the first ten periods.

Using the profits above, we compute ten-period discounted profits for the supplier and the retailer as follows: the supplier's discounted profit is

$$\Pi(\mathbf{Q};\mathbf{q},\mathbf{p})=\sum_{t=1}^{10}\beta^{(t-1)}\Pi_t(Q_t;q_t,p_t)=\$2285.94;$$

and the retailer's expected and actual discounted profits are respectively

$$\pi(\mathbf{q},\mathbf{p};\mathbf{Q})=\sum_{t=1}^{10}\beta^{(t-1)}\pi_t(q_t,p_t;Q_t)=\$7110.34,$$

$$\hat{\pi}(\mathbf{q},\mathbf{p};\mathbf{Q})=\sum_{t=1}^{10}\beta^{(t-1)}\hat{\pi}_R^t(q_t,p_t;Q_t)=\$7707.4. \lhd$$

Now, using the results in Table 2, we discuss the equilibrium decisions and expected profits of the two supply chain members for ten periods. From Figure 2, we find that,

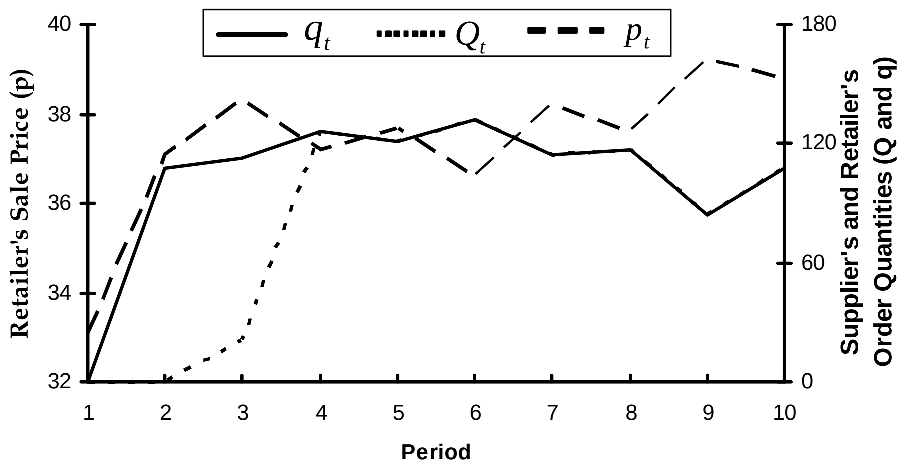


Figure 2. The supplier's equilibrium production quantity (Q_t^N) and the retailer's equilibrium price (p_t^N) and order size (q_t^N) over the first ten periods in an infinite horizon.

except for periods 1 to 3, the retailer's equilibrium prices (p_t^N, $t=3,\ldots,10$) and order quantities (q_t^N, $t=3,\ldots,10$) are negatively correlated since they change in a reverse direction across the periods. For each period the retailer's ordering decision is based on her demand forecast. Note that demand is decreasing in the retailer's price p_t since $dD(p_t)/dp_t<0$. If the retailer increases her price, the demand decreases and the retailer correspondingly reduces her order quantity. Otherwise, if the retailer decreases the price, her order quantity should be increased. For the first period, the retailer has a sufficient starting inventory to satisfy the end demand, so that the retailer needn't place an order with the supplier and a few units of products are carried to period 2. To meet the customer demand and reduce the inventory cost for the second period, the retailer increases the price and orders some units from the supplier. For period 3, the retailer continues increasing her price but orders less products.

The supplier's production quantity is equal to the retailer's order size after period 4. For period t, $t \geq 4$, the supplier's starting inventory is zero; and, according to Theorem 4, the supplier adopts the base stock policy so that the inventory level (after production) equals the retailer's order quantity.

We plot Figure 3 to show the changes of profits at each supply chain member over ten periods in an infinite horizon. We find that the retailer's expected and actual profits

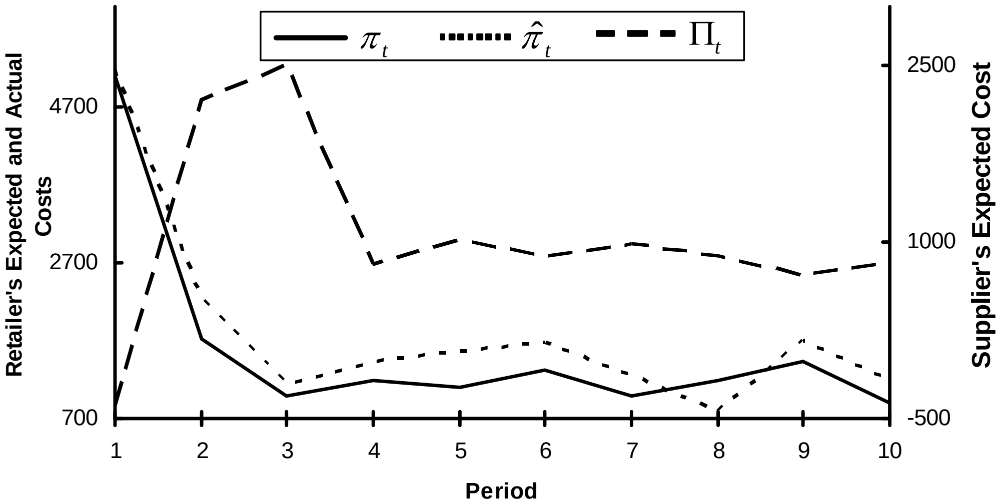


Figure 3. The supplier's profits (Π_t, $t = 1, \ldots, 10$) and the retailer's expected and actual profits (π_t and $\hat{\pi}_t$, $t = 1, \ldots, 10$) over the first ten periods in an infinite horizon.

are not significantly different for each period. Moreover, we find that the supplier's profit (Π_t) and the retailer's profits ($\hat{\pi}_t$ and π_t) change in a reverse direction. Particularly, the supplier's profit increases (decreases) when the retailer's profit decreases (increases), and vice versa. During period 1, the retailer has sufficient inventory to satisfy the demand and does not need to order the new products from the supplier, thus resulting in zero revenue at the supplier echelon. In period 2, the supplier sells most of products in stock to the retailer and carries a few units to period 3 so that the supplier incurs a higher profit, whereas the retailer has to pay for the shortage cost and her profit decreases. After period 2 the supplier chooses production quantity to make his inventory level equal to the retailer's order quantity. In addition, the supplier's wholesale price is determined based on the retailer's price, as indicated in (2). As a result, the supplier's profit depends on not only the retailer's purchase quantity but also her price. If the retailer reduces purchase quantity and sale price, then the supplier's sale quantity and wholesale price are both decreasing and the supplier's profit is reduced. However, the retailer's profit would increase as her purchase cost is reduced.

4 Supply Chain Coordination

In this section we examine whether or not a wholesale pricing scheme (a PDS scheme or a constant wholesale pricing scheme) can be found to achieve supply chain coordination for each period. Under a wholesale pricing scheme, the equilibrium solutions chosen by both supplier and retailer are identical to the globally optimal solutions maximizing the supply chain-wide discounted profit. In addition, the two members are better off than in the non-cooperative situation. Thus, in order to find a proper wholesale pricing scheme, we need to find the globally optimal solutions and compare the solutions with the Nash equilibrium solutions obtained in Section 3. By equating the two solutions, we can derive the conditions under which the parameters α_t, p_0, and w_0 make the supply chain coordinated. To entice both members to cooperate for supply chain coordination, we use the game-theoretic solution concept—Nash arbitration scheme—to divide the chainwide profit generated by cooperation between the two supply chain members, and compute the side-payment transfer.

4.1 Maximization of Supply Chain-Wide Discounted Profit

Now we find the globally optimal solution that maximizes the system-wide discounted profit. We compute the system-wide discounted profit function for period t, denoted by $G_t(Q_t, q_t, p_t)$, as the sum of the discounted profits of the supplier and the retailer, i.e., $G_t(Q_t, q_t, p_t) = \Pi_t(Q_t; q_t, p_t) + \pi_t(q_t, p_t; Q_t)$. Since $Y_t = Q_t + X_t$, we can simplify the objective function for the following two cases:

1. If $q_t \le Y_t = Q_t + X_t$, then the discounted profit function is

$$\begin{aligned}G_t(Q_t, q_t, p_t \mid q_t \le X_t) &= \Pi_t(Q_t; q_t, p_t) + \pi_t(q_t, p_t; Q_t)\\ &= E\{q_t(p_t + H_t) - (c_t + H_t)(Q_t + X_t) + c_t X_t + p_t x_t \\ &\quad -(p_t + h_t)[q_t + x_t - D_t(p_t, \varepsilon)]^+ - k_t[D_t(p_t, \varepsilon) - (q_t + x_t)]^+\}.\end{aligned} \tag{17}$$

2. If $q_t \ge Y_t = Q_t + X_t$, then the discounted profit function is

$$\begin{aligned}G_t(Q_t, q_t, p_t \mid q_t \ge X_t) &= \Pi_t(Q_t; q_t, p_t) + \pi_t(q_t, p_t; Q_t)\\ &= E\{(p_t - c_t)(Q_t + X_t) + p_t x_t - K_t[q_t - (Q_t + X_t)] \\ &\quad -(p_t + h_t)[Y_t + x_t - D_t(p_t, \varepsilon)]^+ - k_t[D_t(p_t, \varepsilon) - (Y_t + x_t)]^+\}.\end{aligned} \tag{18}$$

Theorem 6 *Given the retailer's decisions (q_t, p_t), the supplier's globally optimal production quantity is obtained as* $Q_t^*(q_t, p_t) = (q_t - X_t)^+$, $t = 1, 2, \ldots, +\infty$. □

Theorem 6 indicates that, in order to maximize the supply chain-wide profit, the supplier chooses the production quantity by comparing X_t and q_t for each period. Next, we find the globally optimal solutions (q_t^*, p_t^*) that maximize $G_t((q_t - X_t)^+, q_t, p_t)$.

Theorem 7 *For a given retail price* p_t*, the retailer's globally optimal order quantity is*

$$q_t^*(p_t) = \begin{cases} \zeta_t, & \text{if } \zeta_t \le X_t, \\ \eta_t, & \text{if } \eta_t \ge X_t, \\ X_t, & \text{otherwise,} \end{cases} \tag{19}$$

where

$$\zeta_t = \left[D(p_t) F^{-1}\left(\frac{p_t + H_t + k_t}{p_t + h_t + k_t} \right) - x_t \right]^+, \eta_t = \left[D(p_t) F^{-1}\left(\frac{p_t + k_t - c_t}{p_t + h_t + k_t} \right) - x_t \right]^+. \ \square$$

In (19) ζ_t and η_t can be explained as the retailer's *desired* order quantities under different conditions for period t. As Theorem 7 suggests, at the beginning of each period, the retailer compares the supplier's starting inventory level with her desired order quantities to determine whether or not to place an order with the supplier. Particularly, if the supplier's stock available at the beginning of a period is less than the two desired order quantities, then the retailer should place an order for the smaller desired quantity η_t. (Note that $\zeta_t \ge \eta_t$ according to (24).) If the starting inventory at the supplier is greater than the two desired order quantities, then the retailer should order ζ_t (the larger desired) units of products. Otherwise, the retailer's order quantity equals the supplier's starting inventory.

Remark 3 Our above analysis implies that the optimal order quantity chosen by the retailer must be close to the supplier's starting inventory level for each period in an infinite horizon. This follows the fact that, in order to maximize the system-wide profit, the two supply chain members should jointly determine their inventory decisions in the supply chain. According to Theorem 6, the supplier adopts the *base stock* policy. In particular, if, at the beginning of a period, the retailer's order quantity is greater than the starting inventory of the supplier, the supplier produces new products and makes his order-up-to level equal to the retailer's order quantity. Otherwise, the supplier does not schedule his production and uses the stock (carried from the last period) to satisfy the retailer's order. Therefore, we base the retailer's ordering decision on the supplier's starting inventory and make the retailer's order quantity as close to the supplier's starting inventory as possible. Otherwise, both members could experience the losses in their profits. For example, if the supplier's starting inventory is greater than the two desired order quantities and the retailer chooses the smaller one rather than the larger desired one, then the supplier incurs a greater holding cost and the retailer's shortage cost is increased. As another example, we assume that, when the supplier's starting inventory is less than the retailer's two desired order quantities, the retailer chooses the larger one rather than the smaller one. This leads to a higher purchasing and holding costs for the retailer. Thus, the optimal solution maximizing the system-wide profit is given as shown in Theorem 7. ◁

We now analyze the profit function G_t to find the retail price p_t for any given order quantities (Q_t, q_t).

Theorem 8 For period t the system-wide profit function is strictly concave in the retail price p_t. □

From Theorems 7 and 8, we develop the following procedure for finding the globally optimal solution.

Step 1. We solve the maximization problem $\max_{p_t} G_t(0, \zeta_t, p_t)$, subject to $\zeta_t \le X_t$. If the optimal solution exists, we denote it by $p_t^{'}$ and compute the corresponding objective value as $G_t(0, \zeta_t(p_t^{'}), p_t^{'})$.

Step 2. We solve the maximization problem $\max_{p_t} G_t(\eta_t - X_t, \eta_t, p_t)$ subject to $X_t \le \eta_t$. If the optimal solution exists, we denote it by $p_t^{''}$ and compute the corresponding objective value as $G_t(\eta_t(p_t^{''}) - X_t, \eta_t(p_t^{''}), p_t^{''})$.

Step 3. We solve the maximization problem $\max_{p_t} G_t(0, X_t, p_t)$ subject to $\eta_t \le X_t \le \zeta_t$. If the optimal solution exists, we denote it by $p_t^{'''}$ and compute the corresponding objective value as $G_t(0, X_t, p_t^{'''})$.

Step 4. Letting $\bar{\pi}^t$ denote the maximum of the objective values computed in the above steps, we obtain the globally optimal solution (Q_t^*, q_t^*, p_t^*) $(t = 1, 2, \ldots, +\infty)$ as follows:

$$(Q_t^*, q_t^*, p_t^*) = \begin{cases} (0, \zeta_t(p_t^{'}), p_t^{'}), & \text{if } p_t^{'} \text{ exists and } G_t(0, \zeta_t(p_t^{'}), p_t^{'}) = \bar{\pi}^t, \\ (\eta_t(p_t^{''}) - X_t, \eta_t(p_t^{''}), p_t^{''}), & \text{if } p_t^{''} \text{ exists and } G_t(\eta_t(p_t^{''}) - X_t, \eta_t(p_t^{''}), p_t^{''}) = \bar{\pi}^t, \\ (0, X_t, p_t^{'''}) & \text{if } p_t^{'''} \text{ exists and } G_t(0, X_t, p_t^{'''}) = \bar{\pi}^t. \end{cases} \quad (20)$$

4.2 Design of a Wholesale Pricing Scheme for Supply Chain Coordination

In order to properly design a wholesale pricing scheme to coordinate the two-echelon supply chain, we find the appropriate values of the PDS parameters (w_t^0, p_t^0, and α_t) that make the Nash equilibrium solutions (16) identical to the optimal solutions (20) maximizing the discounted system-wide profit.

Theorem 9 *If the globally optimal solution for period* t *is* $(\eta_t(p_t^{''}) - X_t, \eta_t(p_t^{''}), p_t^{''})$ *and* $p_t^{''}$ *is also the optimal solution for the unconstrained maximization problem* $\max_{p_t} G_t(\eta_t(p_t) - X_t, \eta_t(p_t), p_t)$*, then the supply chain can be coordinated in period* t *with a wholesale pricing scheme where* $\alpha_t = 0$ *and* $w_t^0 = c_t$. □

Remark 4 From Theorem 9 we obtain two important conclusions below.

1. A condition for supply chain coordination is $\alpha_t = 0$, which implies that, when the supply chain is coordinated, the supplier's wholesale price w_t (paid by the retailer to the supplier) is not associated with the retailer's price. This means that, when the PDS scheme is reduced to a constant wholesale pricing scheme, the supply chain could be coordinated. Since the supplier adopts the base stock inventory policy where the inventory level after production equals the retailer's order quantity, the inventory risk in the supply chain is absorbed by the retailer. Moreover, the retailer's price affects the demand. Hence, the retailer's purchasing and pricing decisions play the important roles in improving supply chain performance.
2. Under a properly-designed wholesale pricing scheme, w_t is equal to the supplier's unit purchasing cost c_t, so resulting in zero profit for the supplier. Although the supplier is worse off, the system-wide performance is still improved. ◁

According to our discussion in Remark 4, we find that the supplier is worse off while the supply chain is coordinated by a proper wholesale pricing scheme (which is a special form of the PDS scheme). If no other scheme is involved to allocate the chainwide profit between the supplier and the retailer, the supplier would lose an incentive to cooperate with the retailer to achieve the supply chain coordination. In order to entice both supply chain members, we should consider the problem of "fairly" allocating the system-wide profit generated by supply chain coordination. Here, under a "fair" allocation scheme, two supply chain members are better off than in the non-cooperative situation. Moreover, Theorem 9 indicates that this supply chain couldn't be coordinated by using a wholesale pricing scheme. For this case, we should also find an allocation scheme that "fairly" divides the chainwide profit between two members so that both of them have incentives to cooperate for supply chain improvement.

4.3 Allocation of System-Wide Profit under Supply Chain Coordination

We apply Nash arbitration scheme—a solution concept in the theory of cooperative games—to derive the formula of allocating the system-wide profit $G_t(Q_t^*, q_t^*, p_t^*)$, which is incurred by both members under supply chain coordination for period t. Nash arbitration scheme (a.k.a. Nash bargaining scheme) was developed by Nash (1950). For a two-player game with the *status quo* (φ_0, ϕ_0), the scheme suggests a unique solution (φ, ϕ) by solving the following constrained nonlinear problem $\max\,(\varphi - \varphi_0)(\phi - \phi_0)$, *s.t.* $\varphi \geq \varphi_0$ and $\phi \geq \phi_0$. Let φ and ϕ denote the allocations to the supplier and the retailer, respectively. The solution (φ, ϕ) is an undominated Pareto optimal solution, so that any other solutions cannot make both players better. In our game, for period t we determine φ_t and ϕ_t such that $\varphi_t + \phi_t = G_t(Q_t^*, q_t^*, p_t^*)$. Thus, any point satisfying the equality is an undominated Pareto optimal. The status quo point corresponds to the minimum profits that two players could achieve if they do not cooperate, thereby representing the "security" levels guaranteed to the two players. For our game, the status quo is $(\Pi_t(Q_t^N; q_t^N, p_t^N), \pi_t(q_t^N, p_t^N; Q_t^N))$.

Theorem 10 *When we use Nash arbitration scheme to allocate the system-wide profit* $G_t(Q_t^*, q_t^*, p_t^*)$ *for period* t*, the supplier and the retailer respectively obtain*

$$\varphi_t = \frac{G_t(Q_t^*, q_t^*, p_t^*) + \Pi_t(Q_t^N; q_t^N, p_t^N) - \pi_t(q_t^N, p_t^N; Q_t^N)}{2},$$

$$\phi_t = \frac{G_t(Q_t^*, q_t^*, p_t^*) - \Pi_t(Q_t^N; q_t^N, p_t^N) + \pi_t(q_t^N, p_t^N; Q_t^N)}{2}. \quad \square$$

After finding the allocation approach suggested by Nash arbitration scheme, we consider how to implement the allocation scheme. In particular, when the supplier and the retailer cooperate for supply chain coordination, the system-wide profit is $G_t(Q_t^*, q_t^*, p_t^*) = \Pi_t(Q_t^*; q_t^*, p_t^*) + \pi_t(q_t^*, p_t^*; Q_t^*)$ where the supplier and the retailer have their "local" profits $\Pi_t(Q_t^*; q_t^*, p_t^*)$ and $\pi_t(q_t^*, p_t^*; Q_t^*)$, respectively. In order to let the two supply chain members obtain φ_t and ϕ_t, we need to compute the side payment transferred from one member to the other member. For example, as Theorem 9 indicates, the profit incurred by the supplier is zero under supply chain coordination with a wholesale pricing (reduced PDS) scheme. For this case, we compute the side-payment transfer from the retailer to the supplier so that the allocation to the supplier is φ_t. Otherwise, the supplier would have no incentive to stay with the retailer for supply chain coordination.

Theorem 11 *For period* t*, the side payment transferred between the supplier and the retailer is* $[\max(S,R) - \min(S,R)]/2$, *where* $S \equiv \Pi_t(Q_t^*; q_t^*, p_t^*) - \Pi_t(Q_t^N; q_t^N, p_t^N)$ *and* $R \equiv \pi_t(q_t^*, p_t^*; Q_t^*) - \pi_t(q_t^N, p_t^N; Q_t^N)$. □

As shown by Theorem 9, the supplier's local profit $\Pi_t(Q_t^*; q_t^*, p_t^*)$ would be zero when the supply chain is coordinated by a constant wholesale pricing scheme where the wholesale price is equal to the supplier's purchase cost, i.e., $w_t = c_t$. For this case, the retailer transfers the side payment $(R - S)/2$ to the supplier.

To illustrate supply chain coordination with the constant wholesale pricing scheme (i.e., $w_t = c_t$) and the allocation approach in terms of Nash arbitration scheme, we consider the following example with the parameter values given in Example 1.

Example 2 Using the parameter values in Example 1, we numerically investigate whether or not a constant wholesale pricing scheme can coordinate the supply chain, and compute the allocations of the system-wide profit to the two supply chain members. Similar to Example 1, we present a particular computation for period 1 and directly give the results for the other periods. For the first period we begin by obtaining the globally optimal solutions (Q_t^*, q_1^*, p_1^*). We find that $p_1^{''}$ does not exist, and we get two possible optimal solutions as: $(0, \zeta_1(p_1^{'}), p_1^{'}) = (0,149,24.5)$ and $(0, X_1, p_1^{'''}) = (0,200,15.93)$. The corresponding system-wide profits are $G_t(0, \zeta_t(p_t^{'}), p_t^{'}) = \$5708$ and $G_t(0, X_t, p_t^{'''})$ =\$4851. Thus, the optimal solution is $(Q_t^*, q_1^*, p_1^*) = (0, \zeta_1(p_1^{'}), p_1^{'}) = (0,149,24.5)$ and the resulting system-wide profit is \$5708, which is greater than the sum of two members' profits given in Table 2. However, for the period, we cannot achieve supply chain coordination with a constant wholesale pricing scheme, as indicated in Theorem 9. The supplier's wholesale price is attained as $w_1 = w_1^0 - \alpha_t(p_1^0 - p_1^*) = \$18.9$. The supplier's expected profit is computed as $\Pi_t = \$2118$ and the retailer's expected profit is $\pi_t = \$3590$. We notice that the supplier is better off than in the non-cooperative situation but the retailer is worse off. In order to entice the retailer to cooperate with the supplier in the first period, we use Theorem 11 to compute the side payment $(S - R)/2$ transferred from the supplier to the retailer. Since $S = \$2518$ and $R = -1522$, the side payment is computed as \$2020. According to the side-payment scheme (SPS), the supplier transfers \$2020 to the retailer, and the supplier and the retailer have the profits as \$98(\$2118 – \$2020) and \$5610(\$3590 + \$2020), respectively. With the allocation both supply chain members are better off.

Next, we find the globally optimal solutions and compute the corresponding system-wide profit for periods 2 through 10. If in a period the retailer's optimal price is $p_t^{''}$, the supply chain can be coordinated with the constant wholesale pricing scheme where $w_t = c_t$. Moreover, for each period, we find the side payment scheme so that both members have incentives to cooperate for supply chain coordination. Our result is given in Table 3.

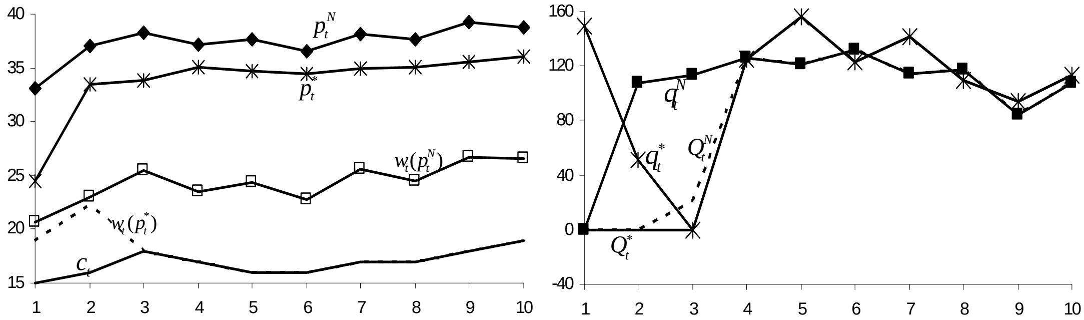
Period		1	2	3	4	5	6	7	8	9	10
Supplier	X_t	200	51	0	0	0	0	0	0	0	0
	Q_t^*	0	0	0	125	156	123	142	110	94	113

	Y_t	200	51	0	125	156	123	142	110	94	113
	w_t	18.9	22.3	18	17	16	16	17	17	18	19
Retailer	x_t	180	169	169	29	4	40	13	45	55	29
	p_t^*	24.5	33.5	33.9	35.1	34.7	34.5	35	35.1	35.6	36.1
	q_t^*	149	51	0	125	156	123	142	110	94	113
	y_t	329	220	169	154	160	163	155	155	149	142
Pricing Scheme?		No	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
SPS?		Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Before SPS	Π_t	2118	1137	0	0	0	0	0	0	0	0
	π_t	3590	4061	4922	2567	2294	2896	2344	2871	2942	2366
After SPS	Π_t	98	2841	3222	1090	1106	1235	1166	1277	1112	1144
	π_t	5610	2357	1700	1477	1188	1661	1178	1594	1830	1222

Table 3: The globally optimal solutions and side-payment schemes (SPS) for supply chain coordination over the first ten periods.

From Table 3 we find that, from period 1 to period 2, the supply chain cannot be coordinated by a wholesale pricing scheme. For the two periods, we use the side-payment schemes given by Theorem 11 to fairly allocate the chainwide profit so that both members have incentives to cooperate for supply chain coordination. The supplier's inventory level is reduced to zero at the beginning of period 3 and afterward the supply chain can be coordinated with the constant wholesale pricing schemes ($w_t = c_t,\ t = 3, 4, \ldots, +\infty$), which leads to zero profits for the supplier. In order to entice the supplier to cooperate with the retailer for supply chain coordination, we use the side-payment schemes to allocate the system-wide profits between the two supply chain members. It is noticed that both supply chain members are better off after profit allocation with side-payment schemes. In addition, we compute the discounted profits for ten periods as follows: (i) before the wholesale pricing and side-payment schemes the supplier and the retailer have discounted profits \$2800 and \$9138, respectively; (ii) after the side-payment schemes they have the discounted profits \$3108 and \$8831. Thus, under supply chain coordination, both supply chain members are better off than in the non-cooperative case. (Note that, as Example 1 indicates, for the non-cooperative case, the supplier's and the retailer's expected discounted profits are \$2285.94 and \$7110.34, respectively.) $\triangleleft$

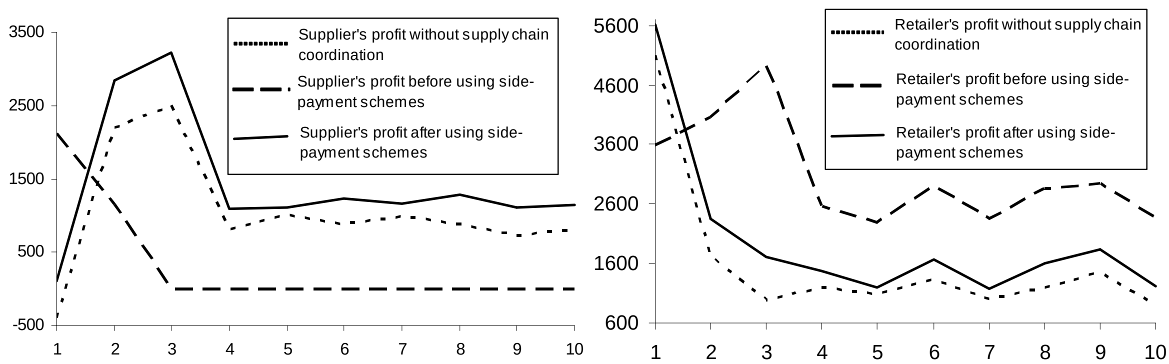
Next we examine the variations in the wholesale prices (w_t) and sale prices (p_t) after coordinating the supply chain. From Figure 4 (a), we find that the prices (i.e., $w_t(p_t^*)$ and p_t^*) chosen by the supplier and the retailer under the constant wholesale pricing



(a) **(b)**

Figure 4. The supplier's and the retailer's prices and production/order quantities before and after supply chain coordination.

and side-payment schemes are lower than those (i.e., $w_t(p_t^N)$ and p_t^N) in Example 1. In this example, the two supply chain members make their pricing and purchasing decisions to maximize the system-wide profit. Since the demand is a decreasing function of the retail price, they could choose a lower sale price to attract more demand and thus increase the overall profit. As Figure 4 (b) indicates, under supply chain coordination, the retailer's purchase quantities q_t^* for periods 1 through 3 are significantly different from those (q_t^N, $t = 1,2,3$) in the non-cooperative situation. For example, without supply chain coordination, the retailer does not place an order with the supplier for the first period since he has sufficient stock (i.e., 180 units). This results in a negative profit for the supplier and a lower overall profit. In order to increase the system-wide profit, the retailer's ordering decision for the first period is raised to q_t^1, as shown in Figure 4 (b). The supplier's production quantities are based on the retailer's ordering decisions, since the supplier adopts the base stock policy.



(a) **(b)**

Figure 5. The expected profits of the supplier and the retailer under supply chain coordination.

From Figure 5, we can find that, when the supply chain is coordinated with the constant wholesale pricing and SPS schemes, both supply chain members are better off. Moreover, when the two members cooperate for supply chain coordination, the supplier incurs a lower profit locally and the retailer accomplishes a higher profit except for period 1. In order to entice the supplier to join the coalition we use constant side-payment schemes to transfer some amounts from the retailer to the supplier so that they are both better off.

5 Summary and Concluding Remarks

In this paper, we consider two-echelon supply chains with wholesale pricing schemes in an infinite horizon. Assuming that the supplier's wholesale price is determined by a linear PDS function, we develop a stochastic game where the supplier chooses his production quantity and the retailer determines her order size and retail price. We first solve the stochastic game to find the best response functions for both members. According to the best response analysis, the supplier uses a *base stock* policy, whereas the retailer chooses her optimal solutions based on some specific conditions. By using the best response analysis, we show that for each period a unique equilibrium—which could be *non-stationary* over the periods—always exists for the supply chain.

We next compute the optimal solution that maximizes the supply chain-wide profit for each period. In order to examine whether or not the supply chain can be coordinated, we analyze the globally optimal solution and Nash equilibrium for each time period, and find the conditions under which supply chain coordination is achieved. We show that, if the supply chain is coordinated, the supplier's wholesale price w_t must equal the unit purchasing cost c_t so that the PDS scheme is reduced to a constant wholesale pricing scheme. However, the supplier is worse off under the constant PDS scheme since the supplier's profit is reduced to zero due to $w_t = c_t$. In order to entice the supplier to cooperate with the retailer for supply chain coordination, we use the Nash arbitration scheme to allocate the system-wide profit between the two supply chain members, and compute the side-payment scheme to implement the allocation approach suggested by Nash arbitration scheme.

Our paper assumes the stationary demand across the periods in the infinite horizon. It would be more interesting to investigate the supply chain in future by relaxing the assumption. Moreover, we could relax the assumption to examine the game models in Cachon and Zipkin (1999) and Netessine, Rudi, and Wang (2005).

Appendix A Proofs of Theorems

Proof of Theorem 1. In order to simplify our analysis, we compare Y_t and q_t to re-write the supplier's objective function (3). Recalling that $Y_t = Q_t + X_t$, we have

$$\Pi_t(Q_t;q_t,p_t) = \begin{cases} \Pi_{t1}, & \text{if } Q_t + X_t \geq q_t, \\ \Pi_{t2}, & \text{if } Q_t + X_t \leq q_t, \end{cases} \tag{21}$$

where $\Pi_{t1} \equiv (w_t + H_t)q_t - H_t(Q_t + X_t) - c_tQ_t$ and $\Pi_{t2} \equiv (w_t + K_t)(Q_t + X_t) - K_tq_t - c_tQ_t$. Note that $\Pi_{t1} = \Pi_{t2}$ when $Q_t + X_t = q_t$. To find the supplier's best response, we consider the following two cases:

1. $q_t \geq X_t$. We find from (21) that the supplier's profit function is Π_{t2} when $Q_t = 0$. As Π_{t2} is an increasing function of Q_t due to $w_t \geq c_t$, we increase the value of Q_t to raise the supplier's profit Π_{t2}. Noticing that $\Pi_{t1} = \Pi_{t2}$ when $Q_t = q_t - X_t$, we find that, if Q_t is greater than $q_t - X_t$, the profit Π_{t1} is reduced since Π_{t1} is a decreasing function of Q_t. Hence, for this case, the optimal solution maximizing $\Pi_t(Q_t;q_t,p_t)$ is obtained as $Q_t = q_t - X_t$.
2. $q_t \leq X_t$. In this case, the objective function is the decreasing function Π_{t1}, so the optimal production quantity is zero, i.e., $Q_t = 0$.

In conclusion, we reach the best response function (9). ■

Proof of Theorem 2. According to Lemma 1, we can find the optimal solution $q_t^0(p_t)$ by equating (27) to zero and solving the resulting equation. If $q_t^0(p_t) \leq Y_t$, then the retailer's objective function is (12) and the optimal order quantity is $q_t^0(p_t)$. Otherwise, her objective function is (13) which is independent of q_t. That is, when the supplier's order quantity Q_t and the retail price p_t are not changed, the retailer's profit for period t is a constant. Hence, when $q_t^0(p_t) \geq Y_t$, the retailer's optimal order quantity is an arbitrary number greater than or equal to Y_t. ■

Proof of Theorem 3. According to Theorem 2, we find an optimal order quantity, considering some specific conditions. In order to get the best response for the retailer, we should compute the optimal retail price for each possible order quantity and compare the resulting objective values (profits). If the condition $q_t^0(p_t) \leq Y_t$ is satisfied, Theorem 2 suggests that $q_t = q_t^0(p_t)$. For this case we can find the optimal price by maximizing the objective function $\pi_t(q_t^0(p_t), p_t;Q_t)$ subject to $q_t^0(p_t) \leq Y_t$. The solution is denoted by $\tilde{p}_t$. Lemma 2 shows that $\tilde{p}_t$ exists. The corresponding profit is $\pi_t(q_t^0(\tilde{p}_t), \tilde{p}_t;Q_t)$.

If $q_t^0(p_t) \geq Y_t$, Theorem 2 indicates that the optimal order quantity is $Y_t + \theta_t$ where $\theta_t \in [0,+\infty)$. We maximize $\pi_t(Y_t + \theta_t, p_t;Q_t)$ subject to $q_t^0(p_t) \geq Y_t$, and denote the solution by $\hat{p}_t$. Substituting $\hat{p}_t$ into (13) gives the profit $\pi_t(Y_t + \theta_t, \hat{p}_t;Q_t)$.

Comparing $\pi_R^t(q_t^0(\tilde{p}_t), \tilde{p}_t; Q_t)$ and $\pi_R^t(Y_t + \theta_t, \hat{p}_t; Q_t)$, we obtain the best-response solution (q_t^B, p_t^B) for the retailer as (15). ■

Proof of Theorem 4. Theorem 3 indicates that the retailer's best response is $(q_t^0(\tilde{p}_t), \tilde{p}_t)$ or $(Y_t + \theta_t, \hat{p}_t)$, where $q_t^0(\tilde{p}_t) \le Y_t$. From (9) and (10) we find the supplier's best-response quantity $Q_t^B = (q_t - X_t)^+$ and order-up-to inventory level $Y_t^B = \max(q_t, X_t)$. We analyze the following two cases:

Case 1. If the retailer's order quantity q_t^B is $q_t^0(\tilde{p}_t)$ which is less than or equal to Y_t, then $Y_t^B = X_t \ge q_t^0(\tilde{p}_t)$ and $Q_t^B = (q_t^B - X_t)^+ = 0$. In order to find $\tilde{p}_t$, we replace q_t in (12) with $q_t^0(p_t)$ and maximize the resulting function $\pi_t(q_t^0(p_t), p_t; Q_t)$ subject to $q_t^0(p_t) \le Y_t = X_t$.

Case 2. If the retailer's order quantity q_t^B is $Y_t + \theta_t$, we find from (10) that $\theta_t = 0$ and $q_t^B = Y_t^B \ge X_t$, so that $Q_t^B = (q_t^B - X_t)^+ = Y_t^B - X_t$. In order to find $\hat{p}_t$, we maximize the function (13) subject to $q_t^0(p_t) \ge Y_t$. We notice that the function (13) is identical to (12) when the supplier manufactures $(Y_t^B - X_t)$ units of products to increase his order-up-to level Y_t to the retailer's order quantity q_t. Hence, if the retailer's order quantity q_t is greater than or equal to X_t, the supplier's order-up-to level Y_t equals q_t, as indicated by (10). For this case, the retailer chooses her optimal order quantity q_t and price p_t by maximizing (12) subject to $q_t \ge X_t$. Since $q_t^0(p_t)$ is the optimal solution of maximizing (12) without any constraint, we can find $\hat{p}_t$ by replacing q_t with $q_t^0(p_t)$ and maximizing $\pi_t(q_t^0(p_t), p_t; Q_t)$ subject to $q_t^0(p_t) \ge X_t$.

Note that the retailer has the identical objective function $\pi_t(q_t^0(p_t), p_t; Q_t)$ for both cases, but has different constraints ($q_t^0(p_t) \le X_t$ and $q_t^0(p_t) \ge X_t$ for the cases 1 and 2, respectively). We can find an optimal price for the retailer by simply maximizing the unconstrained function $\pi_t(q_t^0(p_t), p_t; Q_t)$ below.

$$\begin{aligned}
\pi_t(q_t^0(p_t), p_t; Q_t) &= E\{p_t \min[q_t^0(p_t) + x_t, D_t(p_t, \varepsilon)] - h_t[q_t^0(p_t) + x_t - D_t(p_t, \varepsilon)]^+ \\
&\quad - k_t[D_t(p_t, \varepsilon) - (q_t^0(p_t) + x_t)]^+ - w_t q_t^0(p_t)\} \\
&= E\{(p_t - w_t)q_t^0(p_t) + p_t x_t - (p_t + h_t)[q_t^0(p_t) + x_t - D_t(p_t, \varepsilon)]^+ \\
&\quad - k_t[D_t(p_t, \varepsilon) - (q_t^0(p_t) + x_t)]^+\}.
\end{aligned}$$

Let $\bar{p}_t$ denote the optimal price. If the retailer's order quantity $q_t^0(\bar{p}_t)$ is less than or equal to the supplier's starting inventory level X_t at the beginning of period t, i.e., $q_t^0(\bar{p}_t) \le X_t$, then the retailer's order is fully satisfied and the supplier does not schedule his production for the period. Otherwise, if the supplier's beginning inventory X_t is less than the retailer's order $q_t^0(\bar{p}_t)$, then the supplier should produce new products to increase his inventory level to $q_t^0(\bar{p}_t)$. Thus, the supplier's equilibrium solution Q_t^N is $(q_t^0(\bar{p}_t) - X_t)^+$, and the retailer's equilibrium quantity q_t^N is $q_t^0(\bar{p}_t)$ and price p_t^N is $\bar{p}_t$. ■

Proof of Theorem 5. The result is obtained according to our discussion presented immediately before this theorem. ■

Proof of Theorem 6. For period t, the supplier determines his optimal production quantity Q_t based on his state (the starting inventory X_t). When $X_t \geq q_t$, the objective function (17) applies. Since $c_t \geq 0$ and $H_t \geq 0$, we reduce the supplier's order quantity to zero in order to maximize $G_t(Q_t, q_t, p_t)$. Thus, $Q_t^*(q_t, p_t) = 0$ when $X_t \geq q_t$.

When $X_t \leq q_t$, the supplier produces new products to satisfy the retailer's order since the starting inventory is insufficient. When the retailer's order quantity equals the supplier's order-up-to level (i.e., $q_t = Y_t = Q_t + X_t$), the functions (17) and (18) are both reduced to

$$\begin{aligned} G_t(Q_t, q_t, p_t) = E\{&(p_t - c_t)q_t + p_t x_t \\ &-(p_t + h_t)[q_t + x_t - D_t(p_t, \varepsilon)]^+ - k_t[D_t(p_t, \varepsilon) - (q_t + x_t)]^+\}, \end{aligned} \tag{22}$$

which does not include the holding and shortage costs of the supplier. On the other hand, when $q_t \neq Y_t$, the two costs are involved and reduce the supply chain-wide profit. Thus, in order to maximize the supply chain profit for period t, the supplier's order-up-to level is equal to the retailer's order quantity, i.e., $Y_t^* = Q_t^* + X_t = q_t$. In conclusion, we obtain the result. ■

Proof of Theorem 7. Similar to the proof of Theorem 6, we consider the problem for each of two case: $X_t \geq q_t$ and $X_t \leq q_t$.

1. When $q_t \leq X_t$, Theorem 6 suggests that the supplier's order quantity is zero. This results in the system-wide profit function

$$\begin{aligned} G_t(Q_t^*, q_t, p_t \mid q_t \leq X_t) = E\{&q_t(p_t + H_t) - H_t X_t + p_t x_t \\ &-(p_t + h_t)[q_t + x_t - D_t(p_t, \varepsilon)]^+ - k_t[D_t(p_t, \varepsilon) - (q_t + x_t)]^+\}. \end{aligned}$$

The first- and second-order partial derivatives of $G_t(Q_t^*, q_t, p_t \mid q_t \leq X_t)$ w.r.t. q_t are

$$\frac{\partial G_t(Q_t^*, q_t, p_t \mid q_t \le X_t)}{\partial q_t} = p_t + H_t + k_t - (p_t + h_t + k_t)F\left[\frac{q_t + x_t}{D(p_t)}\right],$$

$$\frac{\partial^2 G_t(Q_t^*, q_t, p_t \mid q_t \le X_t)}{\partial q_t^2} = -\frac{p_t + h_t + k_t}{D(p_t)} f\left[\frac{q_t + x_t}{D(p_t)}\right] < 0,$$

which implies that the function $G_t(Q_t^*, q_t, p_t \mid q_t \le X_t)$ is a strictly concave function in q_t. Equating the first-order derivative to zero and solving it yields

$$q_t = \left[D(p_t)F^{-1}\left(\frac{p_t + H_t + k_t}{p_t + h_t + k_t}\right) - x_t\right].$$

Recalling that $H_t \le h_t$, the ratio $(p_t + H_t + k_t)/(p_t + h_t + k_t)$ is in the range $(0,1]$. Since $0 \le q_t \le X_t$, we write the retailer's optimal order quantity as

$$\hat{q}_t(p_t) = \min\left\{\left[D(p_t)F^{-1}\left(\frac{p_t + H_t + k_t}{p_t + h_t + k_t}\right) - x_t\right]^+, X_t\right\},$$

2. When $q_t \ge Y_t$, we find from Theorem 6 that the supplier's order-up-to level equals the retailer's order size (i.e., $Q_t^* + X_t = q_t$). Taking the first- and second-order partial derivatives of (22) w.r.t. q_t, we have

$$\begin{aligned} \frac{\partial G_t(Q_t^*, q_t, p_t \middle| q_t \ge X_t)}{\partial q_t} &= p_t + k_t - c_t - (p_t + h_t + k_t)F\left[\frac{q_t + x_t}{D(p_t)}\right], \\ \frac{\partial^2 G_t(Q_t^*, q_t, p_t \middle| q_t \ge X_t)}{\partial q_t^2} &= -\frac{p_t + h_t + k_t}{D(p_t)} f\left[\frac{y_t}{D(p_t)}\right] < 0. \end{aligned} \tag{23}$$

As a result, the profit function $G_t(Q_t^*, q_t, p_t \mid q_t \ge X_t)$ for period t is strictly concave in q_t. Equating (23) to zero and solving it for q_t gives the solution as

$$q_t = \left[D(p_t)F^{-1}\left(\frac{p_t + k_t - c_t}{p_t + h_t + k_t}\right) - x_t\right].$$

Similarly, considering the constraint $q_t \ge X_t$, we obtain

$$\overline{q}_t(p_t) = \max\left\{\left[D(p_t)F^{-1}\left(\frac{p_t + k_t - c_t}{p_t + h_t + k_t}\right) - x_t\right]^+, X_t\right\}.$$

To determine the globally optimal solution $q_t^*(p_t)$, we need to compare $\hat{q}_t(p_t)$ and $\overline{q}_t(p_t)$. Since $(p_t + H_t + k_t) > (p_t + k_t - c_t)$, we find that $F^{-1}[(p_t + H_t + k_t)/(p_t + h_t + k_t)] > F^{-1}[(p_t + k_t - c_t)/(p_t + h_t + k_t)]$ and

$$\left[D(p_t)F^{-1}\left(\frac{p_t + H_t + k_t}{p_t + h_t + k_t}\right) - x_t\right]^+ \ge \left[D(p_t)F^{-1}\left(\frac{p_t + k_t - c_t}{p_t + h_t + k_t}\right) - x_t\right]^+. \tag{24}$$

Let ζ_t and η_t respectively denote the LHS and RHS of (24). Comparing them with X_t, we analyze four cases as follows:

1. If $\zeta_t \le X_t$, we find from (24) that $\eta_t \le X_t$. Consequently, we have $\hat{q}_t(p_t) = \zeta_t$ and $\bar{q}_t(p_t) = X_t$, so that the maximum conditional profits are $G_t(0, \zeta_t, p_t \mid q_t \le X_t)$ and $G_t(0, X_t, p_t \mid q_t \ge X_t)$. As $G_t(0, X_t, p_t \mid q_t \ge X_t) = G_t(0, X_t, p_t \mid q_t \le X_t)$, the globally optimal order quantity of the retailer for the case is ζ_t.
2. If $\eta_t \ge X_t$, then $\zeta_t \ge X_t$. As a result, we have that $\hat{q}_t(p_t) = X_t$ and $\bar{q}_t(p_t) = \eta_t$, so the maximum conditional profits are $G_t(0, X_t, p_t \mid q_t \le X_t)$ and $G_t(\eta_t - X_t, \eta_t, p_t \mid q_t \ge X_t)$. Similarly, we obtain the globally optimal order quantity of the retailer as η_t.
3. If $\zeta_t \ge X_t$ and $\eta_t \le X_t$, then $\hat{q}_t(p_t) = X_t$ and $\bar{q}_t(p_t) = X_t$, so the globally optimal solution is $q_t^*(p_t \mid \zeta_t \le X_t, \eta_t \le X_t) = X_t$.
4. If $\zeta_t \le X_t$ and $\eta_t \ge X_t$, we find that $\zeta_t \le \eta_t$ which is contrary to (24).

Summarizing the above results, we have (19). ■

Proof of Theorem 8. Using (2) we replace w_t with $(w^0 - \alpha_t(p^0 - p_t))$ for the function $G_t(Q_t, q_t, p_t)$. Partially taking the first-order derivative of $G_t(Q_t, q_t, p_t)$ w.r.t. p_t, we have

$$\begin{aligned}\frac{\partial G_t(Q_t, q_t, p_t)}{\partial p_t} &= \frac{\partial \Pi_t(Q_t; q_t, p_t)}{\partial p_t} + \frac{\partial \pi_t(q_t, p_t; Q_t)}{\partial p_t} \\ &= (q_t + x_t) - k_t D'(p_t)\mu - E[q_t + x_t - D_t(p_t, \varepsilon)]^+ \\ &\quad + (p_t + h_t + k_t) D'(p_t) \int_0^{\frac{q_t + x_t}{D(p_t)}} x f(x) dx.\end{aligned}$$

We find the second-order partial derivative as

$$\frac{\partial^2 G_t(Q_t, q_t, p_t)}{\partial p_t^2} = \frac{\partial^2 \Pi_t(Q_t; q_t, p_t)}{\partial p_t^2} + \frac{\partial^2 \pi_t(q_t, p_t; Q_t)}{\partial p_t^2} = \frac{\partial^2 \pi_t(q_t, p_t; Q_t)}{\partial p_t^2}.$$

Using Lemma 2, we have that $\partial^2 G_t(Q_t, q_t, p_t) / \partial p_t^2 < 0$, and reach the concavity of the objective function $\Pi_t(Q_t; q_t, p_t)$. ■

Proof of Theorem 9. For supply chain coordination we choose the values of PDS parameters to make (Q_t^N, q_t^N, p_t^N) identical to (Q_t^*, q_t^*, p_t^*), i.e., (Q_t^N, q_t^N, p_t^N) $= (Q_t^*, q_t^*, p_t^*)$ when the parameter values are properly designed. From Theorems 4 and 7 we find that $q_t^N = q_t^*$ if and only if

$$q_t^* = \eta_t = \left[D(p_t) F^{-1}\left(\frac{p_t + k_t - c_t}{p_t + h_t + k_t} \right) - x_t \right]^+ \text{ and } w_t = c_t.$$

Moreover, we notice from Theorems 4 and 7 that p_t^N and p_t^* are obtained by respectively solving

$$\begin{aligned} \max_{p_t} \pi_t(q_t^N, p_t; (q_t^N - X_t)^+) = E\{(p_t - w_t)q_t^N + p_t x_t - (p_t + h_t)[q_t^N + x_t - D_t(p_t, \varepsilon)]^+ \\ -k_t[D_t(p_t, \varepsilon) - (q_t^N + x_t)]^+\}, \end{aligned} \tag{25}$$

and

$$\begin{aligned} \max_{p_t} G_t(q_t^* - X_t, q_t^*, p_t \mid q_t^* \geq X_t) = E\{(p_t - c_t)q_t^* + p_t x_t - (p_t + h_t)[q_t^* + x_t - D_t(p_t, \varepsilon)]^+ \\ -k_t[D_t(p_t, \varepsilon) - (q_t^* + x_t)]^+\}. \end{aligned} \tag{26}$$

The problem (26) is the same as (25) if we add the constraint $q_t^* \geq X_t$ to (25) and replace w_t with c_t. Hence, in order to coordinate the supply chain with a wholesale pricing scheme, the globally optimal solution for period t must be $(\eta_t(p_t^{''}) - X_t, \eta_t(p_t^{''}), p_t^{''})$, and $p_t^{''}$ is the optimal solution of the unconstrained maximization problem $\max_{p_t} G_t(\eta_t(p_t) - X_t, \eta_t(p_t), p_t)$. Note that w_t in (25) is a variable of p_t but c_t in (26) is a given constant. In order to assure that $p_t^N = p_t^*$, we set the value of α_t to 0, so that $w_t = w_t^0$, which is a constant. ■

Proof of Theorem 10. To allocate the profit $G_t(Q_t^*, q_t^*, p_t^*)$ between the supplier and the retailer, we solve the following maximization problem

$$\max\,[\varphi_t - \Pi_t(Q_t^N; q_t^N, p_t^N)][\phi_t - \pi_t(q_t^N, p_t^N; Q_t^N)]$$

$$\text{s.t. } \varphi_t \geq \Pi_t(Q_t^N; q_t^N, p_t^N), \phi_t \geq \pi_t(q_t^N, p_t^N; Q_t^N), \text{and } \varphi_t + \phi_t = G_t(Q_t^*, q_t^*, p_t^*).$$

Replacing ϕ_t with $G_t(Q_t^*, q_t^*, p_t^*) - \varphi_t$ in the objective function, we write our maximization problem to the following:

$\max[\varphi_t - \Pi_t(Q_t^N; q_t^N, p_t^N)][-\varphi_t + G_t(Q_t^*, q_t^*, p_t^*) - \pi_t(q_t^N, p_t^N; Q_t^N)]$. Differentiating the function w.r.t. φ_t, equating it to zero and solving the resulting equation, we find

$$\varphi_t = \frac{G_t(Q_t^*, q_t^*, p_t^*) + \Pi_t(Q_t^N; q_t^N, p_t^N) - \pi_t(q_t^N, p_t^N; Q_t^N)}{2}.$$

Since $\phi_t = G_t(Q_t^*, q_t^*, p_t^*) - \varphi_t$, we have

$$\phi_t = \frac{G_t(Q_t^*, q_t^*, p_t^*) - \Pi_t(Q_t^N; q_t^N, p_t^N) + \pi_t(q_t^N, p_t^N; Q_t^N)}{2}.$$

Noting that $G_t(Q_t^*, q_t^*, p_t^*) \geq G_t(Q_t^N, q_t^N, p_t^N) = \Pi_t(Q_t^N; q_t^N, p_t^N) + \pi_t(q_t^N, p_t^N; Q_t^N)$, we find that $\varphi_t \geq \Pi_t(Q_t^N; q_t^N, p_t^N)$ and $\phi_t \geq \pi_t(q_t^N, p_t^N; Q_t^N)$. Thus, we arrive to the result. ■

Proof of Theorem 11. The differences between local profits incurred by two members and the allocations of $G_t(Q_t^*, q_t^*, p_t^*)$ to them are

$$\Pi_t(Q_t^*; q_t^*, p_t^*) - \varphi_t = \frac{[\Pi_t(Q_t^*; q_t^*, p_t^*) - \Pi_t(Q_t^N; q_t^N, p_t^N)] - [\pi_t(q_t^*, p_t^*; Q_t^*) - \pi_t(q_t^N, p_t^N; Q_t^N)]}{2},$$

$$\pi_t(q_t^*, p_t^*; Q_t^*) - \phi_t = \frac{[\pi_t(q_t^*, p_t^*; Q_t^*) - \pi_t(q_t^N, p_t^N; Q_t^N)] - [\Pi_t(Q_t^*; q_t^*, p_t^*) - \Pi_t(Q_t^N; q_t^N, p_t^N)]}{2}.$$

Defining $S \equiv \Pi_t(Q_t^*; q_t^*, p_t^*) - \Pi_t(Q_t^N; q_t^N, p_t^N)$ and $R \equiv \pi_t(q_t^*, p_t^*; Q_t^*) - \pi_t(q_t^N, p_t^N; Q_t^N)$, we consider two cases as follows:

1. If $S \ge R$, we find that $\Pi_t(Q_t^*; q_t^*, p_t^*) \ge \varphi_t$ and $\pi_t(q_t^*, p_t^*; Q_t^*) \le \phi_t$. For this case, the supplier incurs a higher profit locally than the allocation to him, whereas the retailer's local profit is less than the allocation to her. Hence, the supplier transfers a side payment [amounting to $\Pi_t(Q_t^*; q_t^*, p_t^*) - \varphi_t$] to the retailer. Using S and R, we write the side-payment transfer as $(S-R)/2$.
2. If $S \le R$, we find that $\Pi_t(Q_t^*; q_t^*, p_t^*) \le \varphi_t$ and $\pi_t(q_t^*, p_t^*; Q_t^*) \ge \phi_t$. For this case, the side-payment transfer from the retailer to the supplier is amount of $[\pi_t(q_t^*, p_t^*; Q_t^*) - \phi_t]$ which is computed as $(R-S)/2$.

In conclusion, we find the side-payment transfer between the two members for period t. ■

Appendix B Proofs of Lemmas

Proof of Lemma 1. The first- and second-order partial derivatives of function (12) with respect to q_t are

$$\frac{\partial \pi_{t1}(q_t, p_t; Q_t)}{\partial q_t} = p_t + k_t - w_t - (p_t + h_t + k_t) F\left[\frac{q_t + x_t}{D(p_t)}\right], \tag{27}$$

$$\frac{\partial^2 \pi_{t1}(q_t, p_t; Q_t)}{\partial q_t^2} = -\frac{p_t + h_t + k_t}{D(p_t)} f\left[\frac{q_t + x_t}{D(p_t)}\right] < 0,$$

which leads to the result. ■

Proof of Lemma 2. Replacing w_t in (11) with RHS of (2) and partially differentiating the function $\pi_t(q_t, p_t; Q_t)$ w.r.t. p_t, we have

$$\begin{aligned}\frac{\partial \pi_t(q_t, p_t; Q_t)}{\partial p_t} &= y_t - \alpha_t \min(q_t, Y_t) - k_t D'(p_t)\mu - E[y_t - D_t(p_t, \varepsilon)]^+ \\ &\quad + (p_t + h_t + k_t) D'(p_t) \int_0^{y_t / D(p_t)} x f(x) dx.\end{aligned} \tag{28}$$

The second-order partial derivative is

$$\frac{\partial^2 \pi_t(q_t, p_t; Q_t)}{\partial p_t^2} = -kD''(p_t)\mu + 2D'(p_t)\int_0^{y_t/D(p_t)} xf(x)dx + (p_t + h_t + k_t)D''(p_t)\int_0^{y_t/D(p_t)} xf(x)dx$$
$$-(p_t + h_t + k_t)[D'(p_t)]^2 \frac{y_t^2}{[D(p_t)]^3} f\left(\frac{y_t}{D(p_t)}\right).$$

Since $dD(p_t)/dp_t < 0$ and $d^2D(p_t)/dp_t^2 = 0$, we simplify the above and have $\frac{\partial^2 \pi_t(q_t, p_t; Q_t)}{\partial p_t^2} = 2D'(p_t)\int_0^{y_t/D(p_t)} xf(x)dx - (p_t + h_t + k_t)[D'(p_t)]^2 \frac{y_1^2}{[D(p_t)]^3} f\left(\frac{y_t}{D(p_t)}\right) < 0,$ there by reaching the concavity of the function $\pi_R^t(q_t, p_t; Y_t)$ in p_t. ■

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Keywords: fashion supply chain, coordination, stochastic game, Markov perfect equilibrium, price-discount sharing scheme, Nash arbitration scheme, side payments.

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