Pricing the Digital Version of a Book: Wholesale vs. Agency Models[1]

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Abstract

We investigate the wholesale and agency pricing models for the digital version of a book made by a publisher, when the print version of the book may exist and one or two retailers sell the book in a market. In the monopoly and duopoly settings, if the revenue-sharing ratio is sufficiently high and the tax rate for the digital version is sufficiently low, then the retail price of the digital version under the agency model is smaller than that under the wholesale model. We also find that the retail price of the digital version under the agency pricing model is more likely to be above that under the wholesale pricing model in the duopoly setting than in the monopoly setting. In the duopoly setting, a sufficiently small degree of substitutability between the two book versions can also make the retail price of the digital version smaller under the agency model than under the wholesale model. Moreover, the existence of the print version is helpful to reducing the retail price of the digital version.

Key words: book; digital version; wholesale pricing; agency pricing; competition.

1 Introduction

The increasing popularity of electronic reading devices—e.g., Amazon Kindle, iPad, Samsung Galaxy Tab, etc.—has been promoting the sales of ebooks, which are usually defined as book-length publications in digital form. Although many ebooks can be viewed as digital versions of existing printed books, a number of ebooks do exist with no printed equivalent. As reported by the Ruediger Wischenbart Content and Consulting Company (2014), the percentage of ebook sales in the world-wide market is significantly small (e.g., 13% in the United States, 11.5% in the United Kingdom, 5% in Germany, etc.). However, the success of ebook sales in the past decade has proved the importance of such a business in today's publishing industry, and ebooks will continue to be popular mainly because of their accessibility, convenience, and environmental protection.

In the publishing industry, a traditional operating model for a publisher and a retailer is called the "wholesale pricing model," under which the publisher determines a wholesale price and the retailer then makes its retail pricing decision. Since 2010, six major publishers (i.e., Simon & Schuster, Penguin, HarperCollins, Hachette, Macmillan, and Random House) have been adopting the "agency pricing model," which is a new operating model championed by Apple Inc. Under the agency pricing model, a publisher makes a retail pricing decision for a retailer who agrees to share a percentage of its sales revenue with the publisher. In reality, the percentage of the sales revenue allotted to the publisher is usually 70%; and the major significance of the agency model is that the retailer cannot discount the retail price if the publisher does not approve any discount. In this paper, we analyze a supply chain under both the wholesale and agency pricing models, and compare our results to derive the conditions under which a model is preferable to the other.

A number of practitioners and academics have noted that many consumers concern high sale prices of ebooks and are thus unwilling to buy ebooks. As McGrellis (2011) reported, a high sale price for an ebook is mainly attributed to (i) the agency pricing model that has been widely adopted by publishers and retailers in practice and (ii) a high tax rate applying to the ebook sales—which is significantly greater than that for printed books. For instance, in Germany (France), the tax rate for ebook consumers is 19% (19.6%), which is significantly greater than that for printed book consumers [7% (5.5%)]. This happens mainly because the tax rate for physical books is a reduced tax rate as these books are treated as "cultural" products, whereas the tax rate for ebooks is a full tax rate as they are viewed as electronic products. Hence, one may naturally question if the status in the European ebook market is ascribed to the high tax rate for ebooks. To address the question, we consider a two-echelon book supply chain involving a publisher and one or two retailers who sell the print and digital versions of a book, and investigate the impact of the tax rate on the supply chain. The International Publisher Association's report (2015) indicates that since 2012, France has reduced the tax rate for ebooks to 5.5%, which is the same as the rate for physical books. The Italy government has also implemented a reduced tax rate for ebooks (i.e., 4%). It thereby behooves us to also examine the book supply chain when the tax rates for the print and digital versions are identical.

In 2013, the U.S. Department of Justice (DOJ) accused Apple and five major publishers of conspiring to increase the price of ebooks, claiming that the agency agreements had played an instrumental role. The DOJ won the case against Apple and reached a settlement with publishers. The settlement prohibited further use of agency agreements. Now, Apple is appealing the judgement. For details, see Gaudin and White's paper (2014). These facts motivate us to explore the impact of the agency pricing models on the retail price of an ebook. In Section 2, we specify two research setting (i.e., a monopoly setting and a duopoly setting), in which we perform our wholesale and agency pricing analyses. For our analyses, we build a demand function, which involves (i) the tax rates that may differ for the digital and the print versions, (ii) the degree of substitutability between the digital and the print versions, and (iii) the degree of substitutability between two retailers.

In Section 3, we investigate a monopoly setting in which only a single retailer serves a market with both the digital and the print versions of a book made by a publisher. While the print version is sold under the wholesale pricing model, the retailer sells the digital version under either the wholesale or the agency pricing model. In Section 4, we examine a duopoly setting in which two retailers compete in the market. We first perform our analysis when only the digital version exists in the market, and then analyze the supply chain when the digital and the print versions coexist.

Next, we review the publications that are concerned with the impact of the wholesale and agency models on the price of an ebook. Dantas, Taboubi, and Zaccour (2014) investigated a manufacturer's pricing decisions of the digital and print versions of a book under the wholesale and agency models in a monopoly (one-retailer) setting. Different from them, we analyze both the monopoly setting and a duopoly (two-retailer) setting in which a manufacturer sells only the digital version or both the digital and print versions of a book to two competing retailers. Moreover, we develop the demand models that involve the tax rate for each book version, the degree of substitutability between the two retailers, and the degree of substitutability between the two book versions, whereas Dantas, Taboubi, and Zaccour's demand model (2014) did not involve the three parameters. We also performed our analysis in the presence of the "Most Favored Nation (MFN)" clauses, which was not considered in Dantas, Taboubi, and Zaccour's paper (2014).

In addition, Gans (2012) compared the pricing decisions under the two models and showed that the price is higher under the wholesale model due to the double marginalization. Johnson (2013) compared the wholesale and agency agreements in a model that involves the consumer lock-in, and Johnson (2017) considered the impact of the MFN clauses on the choice of a model for comparing the agency and wholesale agreements. Tan, Carrillo, and Cheng (2016) explored the strategic impact of the agency model in a digital goods supply chain with one supplier and two competing retailers, and found that the agency model can coordinate the competing retailers with a pre-negotiated allocation rule. Tan and Carrillo (2014, 2017) developed a model of vertically differentiated goods to compare the agency model with the wholesale model, and ascertained that both the revenue-sharing structure and the upstream's control over the price can contribute to the benefits of the agency model. Hu and Smith (2016) empirically explored the impact of digital goods channel on traditional goods sales. Different from the above publications, our paper is concerned with the presences of the substitutability between the print and the digital versions of a book, of the competition between two retailers, and of a consumption tax rate, which all influence the pricing decisions in the book supply chain under two pricing models.

This paper ends with a summary of our major insights in Section 5. We relegate the proofs of all propositions, theorems, and corollaries to Appendix A, where the proofs are given in the order that they appear in the main body of our paper.

2 Preliminaries: Research Settings and Demand Function

In practice, a book may only have its digital version or may have both digital and print versions, and a single retailer sells the book or two retailers compete in a market. Accordingly, we consider a book supply chain consisting of a publisher and one or two retailers. The publisher releases the digital version of a book which may also have its print version. Accordingly, in the paper we investigate two research settings:

The Wholesale and Agency Pricing Analysis in a Monopoly Setting. In the setting, the publisher makes both the digital and the print versions of a book, and sells them to a single retailer, who then serves the market with the two versions.

The Wholesale and Agency Pricing Analysis in a Duopoly Setting. In the setting, there are two retailers who compete in the market. For a book, the publisher produces either only the digital version or both the digital and the print versions. That is, there are two scenarios in the setting: (i) the existence of only the digital version and (ii) the coexistence of both the digital and the print versions in the market. We will analyze both scenarios in the duopoly setting.

Next, for scenario (ii) in the duopoly setting, we develop a price-dependent demand function, which, without loss of generality, can be viewed as a general form for the monopoly case and scenario (i) in the duopoly setting. Denoting the publishing version of the book by i, we consider the print version when $i = 1$ and the digital version when $i = 2$. The two versions compete for consumers, in addition to the competition between two retailers (i.e., retailer j, $j = 1, 2$). Referring to the demand functions developed by, e.g., Dobson and Waterson (1997, 2007) and Foros et al. (2014), we characterize the competition between the two versions that are sold by two retailers to the consumers by using the following inverse demand curve:

$$\overline{p}_i^j \equiv p_i^j(1+\alpha_i) = 1 - (q_i^j + \beta q_i^{3-j}) - \gamma(q_{3-i}^j + \beta q_{3-i}^{3-j}), \text{ for } i, j = 1, 2, \tag{1}$$

where p_i^j and q_i^j represent retailer j's retail price and sales for publishing version i of the book, respectively; α_i denotes the tax rate for version i; $\beta \in [0, 1]$ is a parameter regarding the degree of substitutability between the two retailers; and $\gamma \in [0, 1]$ is a parameter denoting the degree of substitutability between the two publishing versions of the book. In (1), $\overline{p}_i^j$ means the tax-inclusive price. Solving (1) for q_i^j gives the demand function for the publishing version i sold by retailer j as

$$q_i^j = \frac{A - (\overline{p}_i^j - \gamma\overline{p}_{3-i}^j) + \beta(\overline{p}_i^{3-j} - \gamma\overline{p}_{3-i}^{3-j})}{J}, \quad \text{for } i, j = 1, 2, \tag{2}$$

where $A \equiv (1-\gamma)(1-\beta)$ and $J \equiv (1-\gamma^2)(1-\beta^2)$.

Because γ and β significantly influence the demand q_i^j in (2), we discuss the two parameters subsequently. We learn from the above that the digital and the print versions of the book substitute for each other at the rate γ. That is, when γ approaches one, i.e., $\gamma = 1$, the two versions perfectly substitutes. When $\gamma = 0$, the two versions are not substitutable, which means that the sales of the digital version are independent of those of the print version. Setting the value of γ in (2) to be zero, we obtain the demand function for the digital version sold by retailer j as

$$q^j = \frac{(1-\beta) - \overline{p}^j + \beta\overline{p}^{3-j}}{(1-\beta^2)}, \quad \text{for } j = 1, 2. \tag{3}$$

The demand function q^j above is a common competition model that was proposed by Singh and Vives (1984). Because the sales of the two versions are independent when $\gamma = 0$, the demand function q^j in (3) will be used to analyze scenario (i) in the duopoly setting.

Denoting the degree of substitutability between retailers 1 and 2, the parameter β in (2) captures how the two retailers differ in their services. Specifically, a higher value of β implies that consumers perceive the retail services as two substitutes at a larger rate. When $\beta \to 1$, retailers 1 and 2 can be regarded as two perfect substitutes. When $\beta = 0$, consumers view the two retailers independently, and the demand function for version i sold by a retailer is derived from (2) to

$$q_i = \frac{(1-\gamma) - \overline{p}_i + \gamma\overline{p}_{3-i}}{(1-\gamma^2)}, \quad \text{for } i = 1, 2. \tag{4}$$

Because for each version of the book, a retailer's sales are not influenced by another retailer when $\beta = 0$, we will use the demand function q_i in (4) to characterize the demand in the monopoly setting.

3 The Wholesale and Agency Pricing Analyses in a Monopoly Setting

In this section, we analyze the pricing models in the monopoly setting in which the publisher sells the digital and print versions of a book to a market through a single retailer. For example, the digital and print versions of the book “Perfect Double” are only available for sales at `Amazon.com` (see `http://www.cindygerard.com/0214_newsletter.html`). In practice, as usual, the publisher first determines his wholesale prices and announces them to the retailer, who then makes its retail pricing decisions. Thus, in the monopoly setting, the decision problem under each pricing model can be considered as a leader-follower game, where the publisher and the retailer act as the “leader” and the “follower,” respectively.

In the market, the print version is always sold under the wholesale pricing model and the digital version is sold under either the wholesale or agency pricing model. Next, under each pricing model for the digital version, we investigate the pricing decisions in Stackelberg equilibrium. Then, we compare our results under the wholesale pricing model with those under the agency pricing model.

3.1 Analysis of the Wholesale Pricing Model for the Digital Version

Under the wholesale pricing model, the retailer purchases the two book versions from the publisher and then sells them to consumers. Denoting the wholesale prices for the print and digital versions by w_1 and w_2, respectively, we can use the demand function q_i in (4) to compute the retailer's and publisher's profits as

$$\pi_{1R} = \sum_{i=1}^{2} (p_i - w_i) \frac{(1-\gamma) - \overline{p}_i + \gamma \overline{p}_{3-i}}{(1-\gamma^2)} \quad \text{and} \quad \pi_{1P} = \sum_{i=1}^{2} (w_i - c_i) \frac{(1-\gamma) - \overline{p}_i + \gamma \overline{p}_{3-i}}{(1-\gamma^2)}, \tag{5}$$

where c_1 and c_2 represent the unit production costs of the print and digital versions of the book, respectively. We assume that $c_1 > c_2$, which is justified as follows: in reality, the cost of a print version includes the physical manufacturing and distribution expenses, which are not considered as the cost components for the digital version. This is demonstrated by Hyatt (2011) who has found that the physical manufacturing and distribution expenses account for about 12% of the retail price of the print version of a book, and the other expenses—such as the copyright fee, editorial development, marketing, overhead cost, etc.—are the same for the print and digital versions.

To find the wholesale and retail prices for the two versions in Stackelberg equilibrium, we need to use the following backward approach. First, we maximize the retailer's profit π_{1R} in (5) to find the retailer's best-response pricing decisions given the wholesale prices w_i ($i = 1, 2$). Secondly, we substitute the retailer's best responses into the publisher's profit function π_{1P} in (5), which is then maximized to obtain the publisher's optimal wholesale prices (viz., the wholesale prices in Stackelberg equilibrium). Thirdly, replacing w_i in the retailer's best responses with the publisher's optimal wholesale prices, we can find the retail prices in Stackelberg equilibrium.

Proposition 1 In the monopoly setting, when the publisher and the retailer adopt the wholesale pricing model, in Stackelberg equilibrium, for publishing version i ($i = 1, 2$) the publisher's wholesale price w_{1i} and the retailer's price p_{1i} are obtained as $w_{1i} = (\eta_i + c_i)/2$, where $\eta_i \equiv 1/(1 + \alpha_i)$; and

$$\begin{aligned} p_{1i} \quad &= \quad \frac{\eta_i}{8\eta_i\eta_{3-i} - 2\gamma^2(\eta_i + \eta_{3-i})^2}\{\gamma(c_{3-i} + \eta_{3-i})(\eta_i - \eta_{3-i}) \\ &\quad +(c_i + \eta_i)[2\eta_{3-i} - \gamma^2(\eta_i + \eta_{3-i})] + 2(1 - \gamma)\eta_{3-i}[2\eta_i + \gamma(\eta_i + \eta_{3-i}]\}. \quad \blacksquare \qquad (6) \end{aligned}$$

Using (6), we can write the tax-inclusive retail price for version i ($i = 1, 2$) as $\overline{p}_{1i} = p_{1i}(1 + \alpha_i) = p_{1i}/\eta_i$, and compute the publisher's and the retailer's profits as

$$\pi_{1P} = \frac{c_2^2\eta_1 + c_1^2\eta_2 - 2(c_1 + c_2)\eta_1\eta_2 + (\eta_1 + \eta_2)\{\eta_2[c_1\gamma + (1 - \gamma)\eta_1] - c_2\gamma(c_1 - \eta_1)\}}{8\eta_1\eta_2 - 2\gamma^2(\eta_1 + \eta_2)^2},$$

and $\pi_{1R} = \pi_{1P}/2$.

Corollary 1 When the tax rate for the digital version is the same as that of the print version, i.e., $\alpha \equiv \alpha_1 = \alpha_2$, the retail price for version i in Stackelberg equilibrium is $p_{1i}|_\alpha = (3\eta + c_i)/4$, where $\eta \equiv \eta_1 = \eta_2 = 1/(1 + \alpha)$, and it is decreasing in the tax rate whereas the tax-inclusive retail price is increasing in the tax rate. Moreover, the publisher's profit is

$$\pi_{1P}|_\alpha = \frac{(c_1 - c_2)^2 + 2(1 - \gamma)[\eta^2 - \eta(c_1 + c_2) + c_1c_2]}{8\eta(1 - \gamma^2)}. \quad \blacksquare \qquad (7)$$

3.2 Analysis of the Agency Pricing Model for the Digital Version

Under the agency pricing model, the retail price of the digital version is determined by the publisher rather than the retailer, who does not need to pay for the wholesale price but should allot a percentage of its sales revenue to the publisher. The agency pricing model does not involve the wholesale pricing decision for the publisher. We learn from Tan, Carrillo and Cheng (2016) that some firms (e.g., Apple and Amazon) have adopted the agency pricing model to sell the digital versions of books, while the (traditional) print versions are still sold under the wholesale pricing model. In the monopoly setting where a single retailer sells both versions, the publisher needs to determine the wholesale price of the print version as well as the retail price of the digital version. Then, the retailer decides on the retail price of the print version. After the retailer achieves a sales revenue, the publisher and the retailer allocate the revenue from the sales of the digital version according to a sharing ratio. From the above industry reports, in the ebook industry, the revenue-sharing ratio is commonly specified as 70% for the publisher and 30% for the retailer.

Noting that the retail price of the digital version of the book is p_2, we compute the publisher's and the retailer's gains from selling one copy of the digital version as sp_2 and $(1-s)p_2$, respectively, where $s \in (0,1)$ denotes the revenue-sharing ratio. Thus, under the agency model in the monopoly setting, the retailer's and the publisher's profits are

$$\pi_{2R} = (p_1 - w_1)\frac{(1-\gamma) - \bar{p}_1 + \gamma\bar{p}_2}{(1-\gamma^2)} + (1-s)p_2\frac{(1-\gamma) - \bar{p}_2 + \gamma\bar{p}_1}{(1-\gamma^2)}, \tag{8}$$

and

$$\pi_{2P} = (w_1 - c_1)\frac{(1-\gamma) - \bar{p}_1 + \gamma\bar{p}_2}{(1-\gamma^2)} + (sp_2 - c_2)\frac{(1-\gamma) - \bar{p}_2 + \gamma\bar{p}_1}{(1-\gamma^2)}. \tag{9}$$

In each of π_{2R} and π_{2P}, the first term is the profit generated from selling the print version and the second term is the profit from selling the digital version.

Similar to Section 3.1, we can use a backward approach to find the wholesale and retail pricing decisions in Stackelberg equilibrium, as shown in the following proposition.

Proposition 2 Under the agency pricing model in the monopoly setting, in Stackelberg equilibrium the publisher's wholesale price w_{21} for the print version and the retail price p_{22} for the digital version are obtained as

$$\begin{aligned} w_{21} &= \frac{\{4s\eta_1\eta_2 - \gamma^2[(\eta_1-\eta_2)^2 + s(5\eta_1\eta_2 - \eta_2^2)]\}c_1}{8s\eta_1\eta_2(1-\gamma^2) - \gamma^2(\eta_1-\eta_2)^2} \\ &\quad + \frac{\{\gamma(\eta_1-\eta_2)[(2-\gamma^2)\eta_1 - (1-s)\gamma^2\eta_2]\}c_2 + A_0}{8s\eta_1\eta_2(1-\gamma^2) - \gamma^2(\eta_1-\eta_2)^2}, \end{aligned}$$

where $A_0 = s(1-\gamma)\eta_1\eta_2[(4+2\gamma-\gamma^2)\eta_1 - (2+3\gamma-4s-4s\gamma)\gamma\eta_2]$; and

$$p_{22} = \eta_2\frac{c_1\gamma(\eta_2-\eta_1) + c_2[(4-3\gamma^2)\eta_1 - \gamma^2\eta_2] + (1-\gamma)\eta_1[4s\eta_2(1+\gamma) + \gamma(\eta_1-\eta_2)]}{8s\eta_1\eta_2(1-\gamma^2) - \gamma^2(\eta_1-\eta_2)^2}.$$

Moreover, the retail price for the print version in Stackelberg equilibrium is calculated as

$$p_{21} = \eta_1\frac{B_0 + c_1B_1 + c_2\gamma B_2}{8s\eta_1\eta_2(1-\gamma^2) - \gamma^2(\eta_1-\eta_2)^2},$$

where $B_0 \equiv (1-\gamma^2)(6s\eta_1\eta_2 + 5s\gamma\eta_1\eta_2 + s\gamma\eta_2^2) + (\gamma^2\eta_2 - \gamma^3\eta_1)(\eta_1-\eta_2)$; $B_1 \equiv 2s\eta_2(1-\gamma^2) - \gamma^2(\eta_1-\eta_2)$; $B_2 \equiv (3-2\gamma^3)\eta_1 + (1-2s-2\gamma^2+2s\gamma^2)\eta_2$. ■

Next, we examine the retail price of the digital version when a single tax rate is applied to two versions.

Corollary 2 When the tax rate for both the digital and the print versions is α, the retail price of the digital version in Stackelberg equilibrium—i.e., $p_{22}|_\alpha = (c_2 + s\eta)/(2s)$—is decreasing in the tax rate whereas the tax-inclusive retail price (i.e., $\bar{p}_{22}|_\alpha = p_{22}|_\alpha(1+\alpha)$) is increasing in the tax rate. Both $p_{22}|_\alpha$ and $\bar{p}_{22}|_\alpha$ are decreasing in the revenue-sharing ratio s. Moreover, the publisher's and the retailer's profits are computed as

$$\begin{aligned}\pi_{2P}|_\alpha &= \frac{c_2^2[2(1-\gamma^2)+s\gamma^2]-2c_2s[c_1\gamma+(2-\gamma-\gamma^2)\eta]}{8s\eta(1-\gamma^2)}\\ &\quad+\frac{s\{c_1^2-2c_1(1-\gamma)\eta+[(1-\gamma)^2+2s(1-\gamma^2)]\eta^2\}}{8s\eta(1-\gamma^2)},\end{aligned}\tag{10}$$

and

$$\begin{aligned}\pi_{2R}|_\alpha &= \frac{c_1^2-2c_1(1-\gamma)\eta+(1-\gamma)[5+3\gamma-4s(1+\gamma)]\eta^2-2c_2\gamma[c_1-(1-\gamma)\eta]}{16\eta(1-\gamma^2)}\\ &\quad-\frac{c_2^2(4-4s-4\gamma^2+4s\gamma^2-s^2\gamma^2)}{16s^2\eta(1-\gamma^2)}. \quad \blacksquare\end{aligned}\tag{11}$$

3.3 Comparison between the Wholesale and Agency Pricing Analyses in the Monopoly Setting

The analytic retail prices in Propositions 1 and 2 are too complicated to be compared for any meaningful insights. Nevertheless, the results in Corollaries 1 and 2 can be compared to find some meaningful implications. Next, we analytically compare our analytic results under the two pricing models when the government levies an identical tax rate on the digital and the print versions. Then, we perform a numerical study to compare the results when the tax rates for the two versions are different.

3.3.1 Analytic Comparison When the Tax Rates for the Digital and Print Versions Are Identical

When the tax rates for the two versions of the book are equal, we can analytically compare the retail price of the digital version under the wholesale pricing model with that under the agency pricing model, as shown in the following theorem.

Theorem 1 In the monopoly setting, when the tax rates for the digital and the print versions of the book are identical, the retail price of the digital version under the wholesale pricing model (i.e., $p_{12}|_\alpha$) may be greater than that under the agency pricing model (i.e., $p_{22}|_\alpha$), which depends on the value of the revenue-sharing ratio s. Specifically, if $s \geq 2(1+\alpha)c_2/[1+(1+\alpha)c_2]$, then $p_{12}|_\alpha \geq p_{22}|_\alpha$; otherwise, $p_{12}|_\alpha < p_{22}|_\alpha$. ■

The above theorem exposes the important fact that the retail price of an ebook under the agency pricing model may be lower than that under the wholesale pricing model, if the revenue-sharing ratio under the agency model (i.e., s) is sufficiently large and/or the tax rate (i.e., α) is sufficiently small. Specifically, if, given the tax rate α, the revenue-sharing ratio under the agency model is greater than $2(1+\alpha)c_2/[1+(1+\alpha)c_2]$, then the retailer keeps a smaller allocation percentage of the sales revenue and thus induces the publisher to determine a lower retail price under the agency model. If, given the ratio s, the tax rate is increased, then the value of $2(1+\alpha)c_2/[1+(1+\alpha)c_2]$ increases and the inequality $s < 2(1+\alpha)c_2/[1+(1+\alpha)c_2]$ holds with a larger chance, which means that the retail price of the digital version under the agency model is more likely to be greater than that under the wholesale pricing model.

Remark 1 In the monopoly setting, when the tax rates for the digital and the print versions are identical, a sufficient reduction in the tax rate could help make the retail price of the digital version under the agency model smaller than that under the wholesale model. But, if the tax rate cannot be reduced, then a sufficiently high revenue-sharing ratio could also make the retail price smaller under the agency model, because a smaller allocation of the sales revenue to the retailer induces the publisher to reduce the retail price. ■

The above remark indicates that for the case of an identical tax rate in the monopoly setting, both the tax rate and the revenue-sharing ratio significantly influence the comparison between the retail prices of the digital version under the two pricing models. Such an implication may help practitioners reduce the retail price of the digital version of a book. Next, we provide a numerical example to demonstrate the above remark.

Example 1 We suppose that the identical tax rate is $\alpha = 10\%$. In accordance with the practices of most ebook retailers such as Apple and Amazon, we set $s = 70\%$. The normalized costs of the print and the digital versions are 0.5 and 0.4, respectively; that is, $c_1 = 0.5$ and $c_2 = 0.4$. Moreover, $\gamma = 0.5$. We use Corollaries 1 and 2 to find that under the wholesale pricing model, the retail price of the digital version and the publisher's and the retailer's (normalized) profits are \$0.7818, \$0.04, and \$0.02, respectively; and under the agency model, the retail price and the publisher's and the retailer's profits are \$0.7403, \$0.0263, and \$0.0434, respectively.

In order to investigate the impact of s on the retail price as well as the publisher's and the retailer's profits, we change the value of s from 0.5 to 0.9 with a step equal to 0.1. For each value of s, we compute the corresponding retail price, as shown in Figure 1(a), where we find that if the value of s is sufficiently small, then the retail price of the digital version under the agency model is higher than that under the wholesale model; otherwise, the retail price is higher under the wholesale model. This is consistent with Remark 1. In addition, we note from Figure 1(b) that when the value of the tax rate α is increased from 2% to 20% in steps of 2%, the retail price under the wholesale model is always higher than that under the agency model; but, the difference between the two retail prices is decreasing in α, which is also in agreement with Remark 1. From the above, we can conclude that, for an reduction in the retail price of the digital version under the agency model, increasing the revenue-sharing ratio may be more effective than decreasing the tax rate. ■

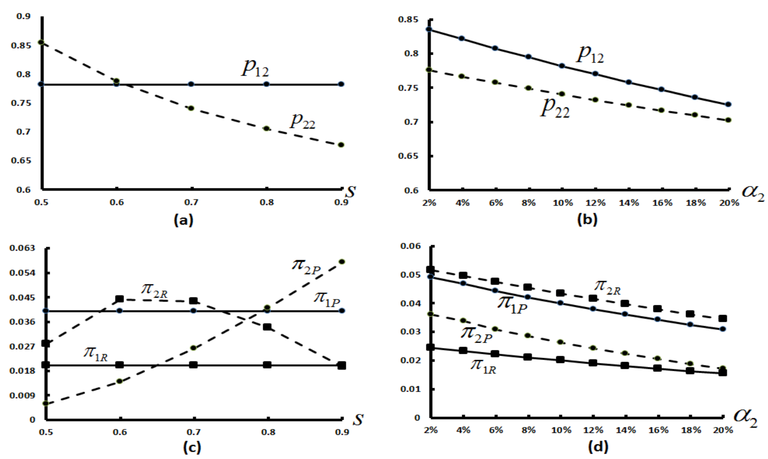

Figure 1: The impact of the revenue-sharing ratio s and the identical tax rate α on the retail price of the digital version as well as the publisher's and the retailer's profits in the monopoly setting.

Next, we compare the publisher's profits under two pricing models.

Theorem 2 When the tax rates for the digital and the print versions are identical in the monopoly setting, we find that if $s(c_2 + \eta)^2 \geq 2s^2\eta^2 + 2c_2^2$, then the publisher's profit under the wholesale pricing model is greater than that under the agency pricing model (i.e., $\pi_{1P}|_\alpha \geq \pi_{2P}|_\alpha$); otherwise, $\pi_{1P}|_\alpha < \pi_{2P}|_\alpha$. ■

We learn from the above theorem that the publisher may or may not benefit from the agency pricing model, which depends on the values of the revenue-sharing ratio s, the tax rate α, and the unit production cost of the digital version c_2. Figure 1(c) indicates that the publisher enjoys a higher profit as the revenue allocation to him is increased; moreover, if the value of s is sufficiently large, then the profit under the agency model is higher than that under the wholesale model. We also learn from Figure 1(d) that the tax rate does not significantly influence the difference between the publisher's profits under two pricing models. The above implies that, to entice the publisher to adopt the agency model, the retailer needs to suggest a large revenue-sharing ratio. Moreover, we learn from Figures 1(c) and (d) that the retailer enjoys a higher profit under the agency model than under the wholesale model, and the retailer's profit reaches its maximum when the value of s is moderate under the agency model.

3.3.2 Numerical Comparison When the Tax Rates for the Digital and Print Versions Are Different

To further investigate the impact of different tax rates for the two versions on two pricing models, we perform a numerical study to conduct the comparison. We use the parameter values specified in Example 1 but assume that $\alpha_1 = 10\%$ and $\alpha_2 = 20\%$, which are consistent with the current tax rates for electronic and physical books in Germany (France)—viz., 19% (19.6%) for electronic books and 7% (5.5%) for physical books. We use Propositions 1 and 2 to find that the retail price of the digital version and the (normalized) profit of the publisher under the wholesale pricing model are \$0.7263 and \$0.0343, and those under the agency model are \$0.7043 and \$0.0208.

Next, *ceteris paribus* we change the value of s from 0.5 to 0.9 with a step equal to 0.1, and compute the retail price of the digital version as well as the publisher's and the retailer's profits for each value of s. Our results are plotted in Figures 2(a) and (b), where we observe that when the value of s is sufficiently high, the retail price of the digital version (the publisher's profit) under the agency model is smaller (greater) than that under the wholesale model. Such results are consistent with those when the tax rates for two versions of the book are identical as shown in Remark 1. In addition, we find from Figures 2(c) and (d) that when an increase in the tax rate α_2 results in a reduction in both the retail price of the digital version and the profit of the publisher, which is similar to the corresponding results for the case of an identical tax rate for two book versions. Figures 2(e) and (f) indicate the impact of the parameter γ on the retail price of the digital version and the publisher's profit. From Figure 2(e), we learn that the degree of substitutability between the two book versions has a small impact on the retail price. But, when the two book versions substitute with a certain degree, the retail price of the digital version arrives to its highest value. Figure 2(f) shows that as the degree increases, the publisher's profit is reduced. Moreover, Figures 2(b), (d), and (f) indicate that the retailer's profit under the agency model is very likely to be higher than that under the wholesale model.

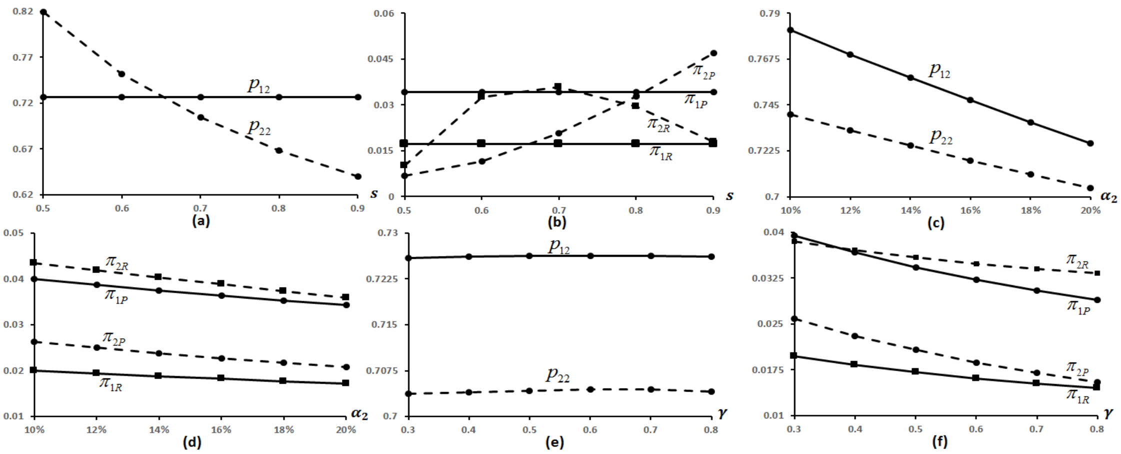

Figure 2: The impact of the revenue-sharing ratio s, the tax rates α_2, and the degree of substitutability between two book versions γ on the retail price of the digital version as well as the publisher's and the retailer's profits in the monopoly setting.

Remark 2 In the monopoly setting, a tax system with different tax rates for the digital and print versions does not significantly influence our results regarding (i) the difference between the retail prices of the digital version under two pricing models and (ii) the publisher's incentive to adopt the agency pricing model. That is, the managerial insights in Remark 1, which are drawn for the case of an identical tax rate for two book versions, still hold for the case of different tax rates for the two versions. ■

4 The Wholesale and Agency Pricing Analyses in a Duopoly Setting

In this section, we consider a duopoly setting in which the publisher sells his books through two competing retailers (i.e., retailers 1 and 2) in a market. In practice, for some books (e.g., "Five Waves to Financial Freedom: Learn Elliott Wave Analysis;" see `http://wavetimes.net/books_more`), only their digital versions are available. For many of other books (e.g., "War Brides;" see `http://www.amazon.com/`), both the digital and the print versions are available. Accordingly, we investigate the following two scenarios: first, the two retailers compete when only the digital version exists in the market; and secondly, the two retailers sell both the digital and the print versions in the market.

4.1 The Pricing Analyses When Only the Digital Version Exists

We start our analysis when there is no print version and only the digital version exists. First, we analyze (i) the wholesale pricing model under which Retailers 1 and 2 buy the digital version from the publisher and then sell them to consumers, and (ii) the agency pricing model under which the two retailers sell the digital version to consumers as agents and the retail price of the digital version is determined by the publisher. Then, we compare the retail price of the digital version and the profit of the publisher under the wholesale model with those under the agency model, in order to examine the impact of the pricing models for the digital version on the consumer purchase, and also investigate the publisher's incentive to implement the wholesale or agency pricing model. Lastly, we investigate the impact of the Most-Favored-Nation clause on the pricing decisions for the digital version.

4.1.1 Analysis of the Wholesale Pricing Model

Under the wholesale pricing model in the duopoly setting, we construct and solve a two-stage problem to find the pricing decisions for the publisher and two retailers. The two stages are specified as follows. In the first stage, the publisher determines the wholesale price of the digital version and announces it to the retailers. In the second stage, the two retailers compete in a "simultaneous-move" game in which each retailer maximizes its own profit to determine the optimal retail price of the digital version. To solve the two-stage problem, we use a backward approach as in Section 3.

Using (2), we compute the publisher's and retailer j's ($j = 1, 2$) profits (denoted by π_{3P} and π^j_{3R}, respectively) as

$$\pi_{3P} = \sum_{j=1}^{2}(w - c_2)\frac{(1-\beta) - \overline{p}^j_2 + \beta\overline{p}^{3-j}_2}{(1-\beta^2)} \quad \text{and} \quad \pi^j_{3R} = (p^j_2 - w)\frac{(1-\beta) - \overline{p}^j_2 + \beta\overline{p}^{3-j}_2}{(1-\beta^2)}, \tag{12}$$

where p^j_2 is the retail price of the digital version decided by retailer j and $\overline{p}^j_2 = (1 + \alpha_2)p^j_2$ is the corresponding tax-inclusive retail price.

Proposition 3 In the duopoly setting, when the wholesale pricing model is adopted, the wholesale and retail prices in equilibrium—denoted by w_{32} and p^j_{32} ($j = 1, 2$), respectively—are obtained as

$$w_{32} = \frac{1 + (1 + \alpha_2)c_2}{2(1 + \alpha_2)} \quad \text{and} \quad p_{32} \equiv p^1_{32} = p^2_{32} = \frac{2(1-\beta) + 1 + (1 + \alpha_2)c_2}{2(2-\beta)(1 + \alpha_2)}. \quad \blacksquare \tag{13}$$

The above proposition indicates that in the duopoly setting, under the wholesale model, the two retailers should choose an identical retail price. We can also calculate the resulting sales of the two retailers as well as the publisher's and the retailer's profits as

$$q_{32} \equiv q^1_{32} = q^2_{32} = \frac{1 - (1 + \alpha_2)c_2}{2(2-\beta)(1+\beta)},$$

and

$$\pi_{3P} = \frac{[1-(1+\alpha_2)c_2]^2}{2(1+\alpha_2)(1+\beta)(2-\beta)} \quad \text{and} \quad \pi_{3R} \equiv \pi_{3R}^1 = \pi_{3R}^2 = \frac{1-\beta}{2(2-\beta)}\pi_{3P}. \tag{14}$$

Corollary 3 The retail price p_{32} and sales q_{32} for each retailer are both decreasing in the tax rate α_2 whereas the tax-inclusive retail price $\bar{p}_{32} = (1+\alpha_2)p_{32}$ is increasing in α_2. ■

The above corollary implies that under the wholesale pricing model for the digital version, if the tax rate is reduced, then more consumers are willing to purchase the digital version, although the retail price is increased.

4.1.2 Analysis of the Agency Pricing Model

Under the agency model, $s_j \in (0,1)$ is the revenue-sharing ratio that is adopted by the publisher and retailer j $(j = 1,2)$. When retailer j achieves a revenue generated from selling the digital version, the publisher and the retailer enjoy s_j and $1 - s_j$ of the sales revenue, respectively. Therefore, we can calculate the publisher's and the retailer j's profits (denoted by π_{4P} and π_{4R}^j, respectively) as

$$\pi_{4P} = \sum_{j=1}^{2}[s_j p_2^j - c_2]\frac{(1-\beta) - \bar{p}_2^j + \beta\bar{p}_2^{3-j}}{(1-\beta^2)} \quad \text{and} \quad \pi_{4R}^j = (1-s_j)p_2^j\frac{(1-\beta) - \bar{p}_2^j + \beta\bar{p}_2^{(3-j)}}{(1-\beta^2)}.$$

Proposition 4 Under the agency model in the duopoly setting, when only the digital version exists for the book, the optimal retail price of the digital version for retailer j $(j = 1,2)$ is

$$p_{42}^j = \eta_2(1-\beta)\frac{c_2(1+\alpha_2)[\beta s_j + (2+\beta)s_{3-j}] + s_{3-j}[(2+\beta)s_j + \beta s_{3-j}]}{4s_j s_{3-j} - \beta^2(s_j + s_{3-j})^2}, \tag{15}$$

which is decreasing in the tax rate α_2, whereas the tax-inclusive retail price $\bar{p}_{42}^j = p_{42}^j(1+\alpha_2)$ is increasing in α_2. ■

Next, we focus on the case of $s \equiv s_1 = s_2$, in which two retailers use an identical revenue-sharing ratio. Such a case is common in reality; for example, most ebook retailers (e.g., Amazon and Apple) are using the ratio $s = 70\%$ to share their revenues with publishers, as reported by Gaudin and White (2014) and Owen (2014), etc. This phenomenon is mainly attributed to the competition between ebook retailers, because, if a retailer allocates a smaller share to the publisher, then the publisher may quit the cooperation with the retailer and thus abandon the sales channel.

Corollary 4 When two retailers implement the same revenue-sharing ratio, they charge an identical optimal retail price as $p_{42}|_s \equiv p_{42}^1|_s = p_{42}^2|_s = (c_2 + s\eta_2)/(2s)$, which is decreasing in s. We also calculate the publisher's and the retailer's profits as $\pi_{4P}|_s = [s-(1+\alpha_2)c_2]^2/[2s(1+\alpha_2)(1+\beta)]$ and $\pi_{4R}|_s \equiv \pi^1_{4R}|_s = \pi^2_{4R}|_s = (1-s)[s^2-(1+\alpha_2)^2c_2^2]/[2s^2(1+\alpha_2)(1+\beta)]$, which are decreasing in β. ■

The above corollary indicates that, for the case of an identical revenue-sharing ratio, the retail price of the digital version is independent of the competition between two retailers. This happens mainly because of the following fact: as the retailers keep the same share of their revenues from selling the digital version, they act as if they were an entity, from the perspective of the publisher. Thus, the value of β (which measures the competition between the two retailers) does not influence the retail pricing decision of the publisher.

4.1.3 Comparison between Two Pricing Analyses When Only the Digital Version Exists

For our comparison, we first focus on the case of an identical revenue-sharing ratio, for which we investigate which pricing model is more likely to result in a higher retail price of the digital version and a greater profit for the publisher. For the case of different ratios, we perform a numerical comparison later in this section.

Analytic Comparison between Two Pricing Analyses When Two Retailers Adopt an Identical Revenue-Sharing Ratio We assume that the revenue-sharing ratios applying to two retailers are equal. Under the assumption, we analytically compare the retail prices of the digital version and the profits of the publisher under two pricing models.

Theorem 3 In the duopoly setting with an identical revenue-sharing ratio, when only the digital version of the book exists, if $s \geq (2-\beta)c_2/[c_2+(1-\beta)\eta_2]$, then the retail price of the digital version under the wholesale pricing model is greater than that under the agency pricing model, i.e., $p_{32} \geq p_{42}|_s$; otherwise, $p_{32} < p_{42}|_s$.

Moreover, if $s \geq (2-\beta)[(s\eta_2 - c_2)/(\eta_2 - c_2)]^2$, then the profit of the publisher under the wholesale pricing model is greater than that under the agency pricing model, i.e., $\pi_{3P} \geq \pi_{4P}|_s$); otherwise, $\pi_{3P} < \pi_{4P}|_s$. ■

The above theorem reveals that if the two retailers keep a sufficiently high revenue-sharing percentage (viz., the value of s is smaller than the cutoff level $(2-\beta)c_2/[c_2+(1-\beta)\eta_2]$), then the retail price under the agency pricing model is greater than that under the wholesale pricing model. Noting that the cutoff level is increasing in α_2 (decreasing in η_2), we conclude that, if the government implements a lower tax rate for the digital version, then the retail price under the agency pricing model is more likely to be smaller than that under the wholesale pricing model. Moreover, setting the value of β to zero, we find that the cutoff level $(2-\beta)c_2/[c_2+(1-\beta)\eta_2]$ in Theorem 3 is reduced to $2c_2/(c_2+\eta_2)$, which is the cutoff level for the comparison of the retail price in the monopoly setting, as shown by Theorem 1.

We also learn from Theorem 3 that if the retailers allocate a smaller percentage of their sales revenues to the publisher, then the publisher is more willing to choose the wholesale pricing model; otherwise, the publisher prefers to choose the agency pricing model. In addition, as the competition between the two retailers is more severe (viz., the value of β increases), the publisher is more likely to choose the wholesale pricing model, which means that the agency model in the competition environment makes the publisher worse.

Corollary 5 The retail price of the digital version under the agency pricing model is more likely to be above that under the wholesale pricing model in the duopoly setting than in the monopoly setting. ■

Numerical Comparison When Two Retailers Adopt Different Revenue-Sharing Ratios We next conduct numerical experiments to investigate the impact of different revenue-sharing ratios on two pricing models, using the parameter values in Section 3.3.2 but assuming that the revenue-sharing ratios for retailers 1 and 2 are $s_1 = 0.7$ and $s_2 = 0.8$, respectively. As our analytic results in Section 4.1.3 indicate, the tax rate for the digital version α_2 influences the retail price of the digital version as well as the publisher's and the retailer's profits under two pricing models. We accordingly consider two tax rates (i.e., $\alpha_2 = 10\%$ and $\alpha_2 = 20\%$); for each tax rate, we compare our results numerically.

The parameter $\beta \in [0, 1]$ characterizes the degree of substitutability between the two retailers, thus distinguishing the results in this section with those for the monopoly case in Section 3.3.2. It behooves us to present a sensitivity analysis regarding the impact of β on the retail price of the digital version as well as the publisher's and the retailer's profits under two pricing models. We increase the value of β from 0.5 to 0.9 with a step equal to 0.1, which means that the competition between two retailers is becoming more severe. For each value of β, we calculate the retail prices of the digital version for the two retailers as well as the publisher's and the retailer's profits under the wholesale and the agency pricing models when $\alpha_2 = 10\%$ and $\alpha_2 = 20\%$. The computational results are plotted as shown in Figure 3.

We learn from Figure 3(a) and (b) that, for different tax rates, the value of β impacts the retail

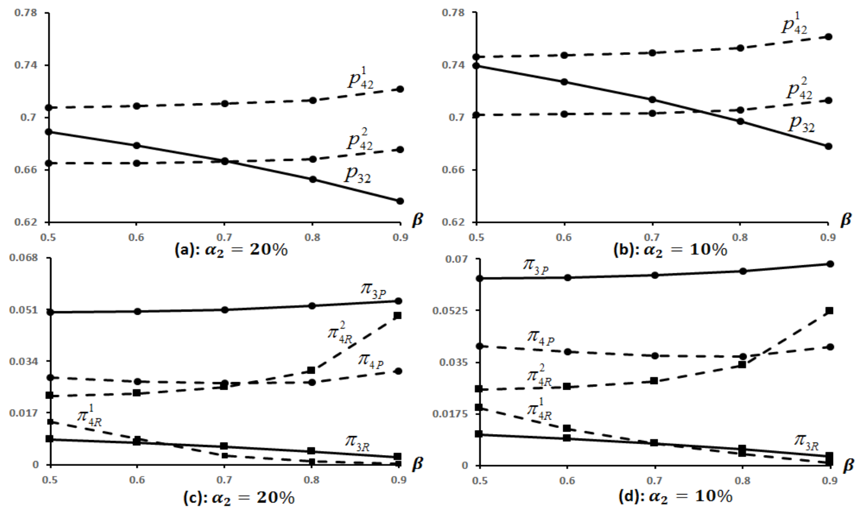

Figure 3: The comparison between the retail price of the digital version as well as the publisher's and the retailer's profits under the wholesale pricing model and those under the agency pricing models in the duopoly setting with only the digital version.

price of the digital version under each pricing model in a similar manner. Specifically, as the two retailers compete more severely, the retail price under the wholesale price decreases whereas the retail prices for the two retailers under the agency model increases. In addition, we find that the retail price for retailer 1 is always higher than that for retail 2, which exposes the fact that a retailer with a higher revenue-sharing ratio intends to determine a lower retail price of the digital version of the book. This happens mainly because the adoption of a higher ratio leads the retailer to share a smaller part of the sales revenue, thereby reducing the retail price for a decrease in the sales revenue. Furthermore, Figures 3(a) and (b) indicate that when the value of s is sufficiently high and the value of β is sufficiently small, the retail price of the digital version under the agency model is lower than that under the wholesale model. We thus conclude that, given a tax rate (revenue-sharing ratio), a sufficiently high revenue-sharing ratio (low tax rate) could make the retail price smaller under the agency model. This finding is similar to the result in Remark 1.

In addition, we observe from Figures 3(c) and (d) that the publisher enjoys a higher profit under the wholesale pricing model, which means that the publisher may prefers the wholesale model to the agency model. Moreover, an increase in the value of β helps raise the publisher's profit. That is, under the wholesale pricing model, the publisher benefits more from a higher competition between two retailers. However, under the agency model, the change in the publisher's profit exhibits a convex curve. This implies that under the agency model, the publisher has an incentive to adopt the model when the competition between the two retailers is sufficiently high or sufficiently low. The competition at the moderate level may make the publisher lose his incentive to consider the agency model. We also find from Figures 3(c) and (d) that the retailer with a sufficiently high revenue-sharing ratio can achieve a higher profit under the agency model than under the wholesale model.

4.1.4 The Agency Pricing Analysis in the Presence of the Most-Favored-Nation Clause

As reported by Johnson (2017), Apple's contracts with ebook publishers contain "Most-Favored-Nation" (MFN) clauses, which prevent the publishers from selling their ebooks at higher retail prices at Apple's iBookstore than other retailers. Next, we analyze the agency pricing model, assuming that the two retailers require the publisher to implement the MFN clause for the digital version of the book. This leads the two retailers to adopt a same retail price for the digital version; that is, $\hat{p}_2 \equiv \hat{p}_2^1 = \hat{p}_2^2$, where the symbol "$\hat{}$" denotes the retail price with the MFN as an ancillary restraint. Under the MFN clause, the order quantities of the two retailers are equal to $(1-\hat{p}_2)/(1+\beta)$, and the publisher's profit is

$$\hat{\pi}_{4P} = [(s_1+s_2)\hat{p}_2 - 2c_2]\frac{1-(1+\alpha_2)\hat{p}_2}{1+\beta}.$$

It thus follows that the optimal retail price of the ebook is $\hat{p}_{42} = [(s_1+s_2)\eta_2 + 2c_2]/[2(s_1+s_2)]$, which is independent of the parameter β which measures the competition between the two retailers. Moreover, the retail price $\hat{p}_{42}$ is dependent on the sum of the two revenue-sharing ratios rather than the value of any one ratio. We also find that $\hat{p}_{42}$ is decreasing in α_2 but the tax-inclusive retail price $(1+\alpha_2)\hat{p}_{42}$ is increasing in α_2.

4.2 The Pricing Analyses When the Digital and Print Versions Coexist

When the digital and print versions of the book exist in the market, we investigate the pricing decisions for the two versions under the wholesale and agency models.

4.2.1 Analysis of the Wholesale Pricing Model

In the duopoly setting, we calculate retailer j's $(j=1,2)$ and the publisher's profits as

$$\begin{aligned}
\pi_{5R}^j &= \frac{1}{J}\sum_{i=1}^{2}(p_i^j - w_i)[A - \overline{p}_i^j + \gamma\overline{p}_{3-i}^j + \beta\overline{p}_i^{3-j} - \beta\gamma\overline{p}_{3-i}^{3-j}];\\
\pi_{5P} &= \frac{1}{J}\sum_{j=1}^{2}\sum_{i=1}^{2}(w_i - c_i)[A - \overline{p}_i^j + \gamma\overline{p}_{3-i}^j + \beta\overline{p}_i^{3-j} - \beta\gamma\overline{p}_{3-i}^{3-j}],
\end{aligned}$$

where $A = (1-\gamma)(1-\beta)$ and $J = (1-\gamma^2)(1-\beta^2)$, as defined in Section 2. Given the wholesale prices w_i $(i=1,2)$, the two retailers make their retail prices in a simultaneous-move game. Solving the game gives Nash equilibrium in terms of w_i, which are used to replace the retail prices in π_{5P}. Then, maximizing the resulting π_{5P}, we can find the optimal wholesale prices w_{5i}, which can be then used to compute the Nash equilibrium-characterized retail prices p^j_{5i}.

Proposition 5 In the duopoly setting, when the digital and print versions coexist under the wholesale pricing model, we compute the optimal wholesale prices for version i ($i = 1, 2$) as

$$w_{5i} = \frac{D_{3-i}c_i + \beta\gamma\eta_i(\eta_i - \eta_{3-i})c_{3-i} + \eta_i[D_i + \beta\gamma\eta_{3-i}(\eta_{3-i} - \eta_i)]}{(2-\beta)[4\eta_i\eta_{3-i} - \gamma^2(\eta_i + \eta_{3-i})^2]}, \tag{16}$$

where $D_i \equiv (4 - 2\beta)\eta_i\eta_{3-i} - \gamma^2(\eta_i + \eta_{3-i})[\eta_{3-i} + (1-\beta)\eta_i]$. The retail price in Nash equilibrium for retailer j ($j = 1, 2$) is

$$p^j_{5i} = \eta_i \frac{(2-\beta)(w_{5i} - \gamma w_{5(3-i)} + A\eta_i)\eta_{3-i} + \gamma[(1-\beta)\eta_i + \eta_{3-i}](w_{5(3-i)} - \gamma w_{5i} + A\eta_{3-i})}{(2-\beta)^2\eta_i\eta_{3-i} - \gamma^2[\eta_{3-i} + (1-\beta)\eta_i][\eta_i + (1-\beta)\eta_{3-i}]}. \quad \blacksquare \tag{17}$$

The above proposition indicates that in Nash equilibrium, the two retailers make an identical retail price for each version of the book. Because the analytic pricing decisions in Proposition 5 are too complicated to be used for any further analysis, we subsequently investigate the case of an identical tax rate for two versions, similar to Section 3. When the tax rates for the digital and print versions are equal to α, $\eta_1 = \eta_2 = \eta = 1/(1+\alpha)$ and for version i ($i = 1, 2$), the optimal wholesale prices and the retail prices in Nash equilibrium are reduced to

$$w_{5i}|_\alpha = \frac{c_i + \eta}{2} \quad \text{and} \quad p_{5i}|_\alpha \equiv p^1_{5i}|_\alpha = p^2_{5i}|_\alpha = \frac{c_i + \eta + 2(1-\beta)\eta}{2(2-\beta)}. \tag{18}$$

The retail price $p_{5i}|_\alpha$ is decreasing in the degree of substitutability between the two retailers (i.e., β). In addition, the retail price for version i is dependent on the version's own unit production cost *but* independent of the book substitutability γ and others. As a result, the publisher's and the retailer's profits are

$$\pi_{5P}|_\alpha = \frac{(c_1 - c_2)^2 + 2(1-\gamma)[c_1c_2 - \eta(c_1 + c_2) + \eta^2]}{2(2-\beta)(1+\beta)(1-\gamma^2)\eta}, \tag{19}$$

and

$$\pi_{5R}|_\alpha = \frac{1-\beta}{2(2-\beta)}\pi_{5P}|_\alpha. \tag{20}$$

Theorem 4 When a single tax rate applies to both the digital and the print versions under the wholesale pricing model, in the duopoly setting the retail price of the digital version is independent of whether the print version exists. Moreover, the retail price of the digital version in the monopoly setting is greater than that in the duopoly setting. ■

The above theorem implies that under the wholesale pricing model, the competition between the two retailers results in a reduction in the retail price of the digital version, which benefits consumers.

4.2.2 Analysis of the Agency Pricing Model

We consider a duopoly setting in which the wholesale and agency pricing models are applied to the sales of the print and digital versions of the book, respectively. For this case, the publisher determines a wholesale price of the print version and the retail prices of the digital version for the two retailers, who then decide on their retail prices of the print version. Under the agency model for the digital version, as described in Section 4.1.2, the publisher and retailer j $(j = 1, 2)$ obtain $s_j \in (0, 1)$ and $1 - s_j$ of the retailer's sales revenue, respectively. Using (2), we can write retailer j's profit as

$$\pi_{6R}^j = \frac{1}{J}[(p_1^j - w_1)(A - \overline{p}_1^j + \gamma\overline{p}_2^j + \beta\overline{p}_1^{3-j} - \beta\gamma\overline{p}_2^{3-j}) + (1 - s_j)p_2^j(A - \overline{p}_2^j + \gamma\overline{p}_1^j + \beta\overline{p}_2^{3-j} - \beta\gamma\overline{p}_1^{3-j})],$$

and the publisher's profit as

$$\pi_{6P} = \frac{1}{J}\sum_{j=1}^{2}[(w_1 - c_1)(A - \overline{p}_1^j + \gamma\overline{p}_2^j + \beta\overline{p}_1^{3-j} - \beta\gamma\overline{p}_2^{3-j}) + (s_j p_2^j - c_2)(A - \overline{p}_2^j + \gamma\overline{p}_1^j + \beta\overline{p}_2^{3-j} - \beta\gamma\overline{p}_1^{3-j})].$$

For the print version of the book, the retail pricing decision problem is modeled as a "simultaneous-move" game, which can be solved to find the retail prices in Nash equilibrium. Then, we maximize the publisher's profit to find its optimal wholesale and retail prices of the digital version of the book.

Proposition 6 In the duopoly setting, when the pricing decisions for the print and digital versions are made under the wholesale and agency models, respectively, we compute the optimal wholesale price of the print version and the optimal retail price of the digital version for retailer j $(j = 1, 2)$ as

$$\begin{aligned}
w_{61} &= \frac{1}{D}[2(c_1 - \gamma c_2 + \eta_1 - \gamma\eta_1)A_{11} + (2 - \beta - \beta^2)(H_1 A_{21} + H_2 A_{31})], && (21)\\
p_{62}^j &= \frac{\eta_2}{D}[2(c_1 - \gamma c_2 + \eta_1 - \gamma\eta_1)A_{1(j+1)} + (2 - \beta - \beta^2)(H_1 A_{2(j+1)} + H_2 A_{3(j+1)})], && (22)
\end{aligned}$$

where

$$D \equiv \begin{vmatrix} 4 & -\gamma E_1 & -\gamma E_2 \\ (-2 + \beta + \beta^2)\gamma E_1 & -2s_1\eta_2 F_1 & \beta\eta_2 G \\ (-2 + \beta + \beta^2)\gamma E_2 & \beta\eta_2 G & -2s_2\eta_2 F_2 \end{vmatrix}$$

and A_{ij} is its corresponding cofactor. In D, $E_k \equiv \eta_1 - \eta_2 + 2s_k\eta_2$, $F_k \equiv (-4+\beta^2+2\gamma^2)\eta_1 + (1-s_k)(2-\beta^2)\gamma^2\eta_2$, $G \equiv s_2[(-4+\beta^2+3\gamma^2-\beta^2\gamma^2)\eta_1+\gamma^2\eta_2]+s_1[(-4+\beta^2+3\gamma^2-\beta^2\gamma^2)\eta_1+(1-2s_2)\gamma^2\eta_2]$, $H_k \equiv s_k(1-\gamma)(2-\beta+\gamma-\beta\gamma)\eta_1\eta_2 - c_1\gamma(\eta_1-\eta_2+s_k\eta_2)+c_2[(2-\gamma^2-\beta+\beta\gamma^2)\eta_1-(1-s_k)\gamma^2\eta_2]$.

For retailer j, we also calculate the retail prices of the print version in Nash equilibrium as

$$p_{61}^j = \frac{(2+\beta)(w_{61}+A\eta_1)+\gamma\{\overline{p}_{62}^j[(2-\beta^2)\eta_1+2(1-s_1)\eta_2]+\overline{p}_{62}^{3-j}\beta[(1-s_2)\eta_2-\eta_1]\}}{4-\beta^2}. \quad \blacksquare \tag{23}$$

The above proposition indicates that the pricing decisions are too complicated to be used for tractable analysis. Thus, similar to Section 4.1.2, we continue with our analysis when the two retailers implement an identical revenue-sharing ratio for the sales of the digital version in the market. That is, each retailer keeps $1-s$ of its revenue resulting from the sales of the digital version, and the publisher attains s of the total revenue of the two retailers. Moreover, we focus on the case of a single tax rate for the two versions of the book, similar to Section 3.

Theorem 5 In the duopoly setting, when the two retailers adopt an identical revenue-sharing ratio under the agency pricing model, if there is a single tax rate for the two versions of the book, then the two retailers charge an identical retail price for the digital version, which is the same as that in the monopoly setting. $\blacksquare$

According to the above theorem, we conclude that when the two retailer adopt an identical revenue-sharing ratio and there is an identical tax rate for the two book versions in the duopoly setting, the optimal retail price of the digital version is independent of the competition between the retailers and also independent of whether the print version exists or not. As a result, the publisher's and the retailer's profits are computed as

$$\begin{aligned}\pi_{6P} \quad = \quad & \frac{c_2^2[(2-\beta)(1-\gamma^2)+s\gamma^2]-2c_2 s[c_1\gamma+(1-\gamma)(2-\beta+\gamma-\beta\gamma)\eta]}{2s(2-\beta)(1+\beta)(1-\gamma^2)\eta} \\ & + \frac{c_1^2-2c_1(1-\gamma)\eta+(1-\gamma)[1-\gamma+s(2-\beta)(1+\gamma)]\eta^2}{2(2-\beta)(1+\beta)(1-\gamma^2)\eta},\end{aligned}$$

and

$$\begin{aligned}\pi_{6R}|_{s,\alpha} \quad \equiv \quad & \pi_{6R}^1|_{s,\alpha} = \pi_{6R}^2|_{s,\alpha} = \frac{c_2^2(-4+4s+4\gamma^2-4s\gamma^2+s^2\gamma^2)}{16s^2(1+\beta)(1-\gamma^2)\eta} \\ & + \frac{c_1^2-2c_1(1-\gamma)\eta+(1-\gamma)\eta^2[5+3\gamma-4s(1+\gamma)]-2c_2\gamma[c_1-(1-\gamma)\eta]}{16(1+\beta)(1-\gamma^2)\eta}.\end{aligned}$$

4.2.3 Comparison between Two Pricing Analyses When the Digital and Print Versions Coexist

Similar to Section 4.1.3, for tractability, we first only analytically compare the retail prices and the publisher's profits under the wholesale model and those under the agency model, when there are an identical revenue-sharing ratio and a single tax rate. For the case with different ratios and rates, we perform a numerical study later in this section.

Analytic Comparison with an Identical Revenue-Sharing Ratio and a Single Tax Rate We analytically compare our results under the two pricing models, assuming that the two retailers adopt an identical revenue-sharing ratio and a single tax rate applies to the two book versions.

Theorem 6 For the case with an identical revenue-sharing ratio and a single tax rate, we find that for the comparison between the retail prices of the digital version, the result in Theorem 3 still holds. That is, if $s \geq (2-\beta)c_2/[c_2+(1-\beta)\eta]$, then the retail price under the wholesale model is greater than that under the agency model, i.e., $p_{52}|_{\alpha} \geq p_{62}|_{s,\alpha}$; otherwise, $p_{52}|_{\alpha} < p_{62}|_{s,\alpha}$.

Moreover, if $s[c_2^2+2c_2(1-\beta)\eta+\eta^2] \geq (2-\beta)(s^2\eta^2+c_2^2)$, then the profit of the publisher under the wholesale pricing model is greater than that under the agency model, i.e., $\pi_{5P}|_{\alpha} \geq \pi_{6P}|_{s,\alpha}$; otherwise, $\pi_{5P}|_{\alpha} < \pi_{6P}|_{s,\alpha}$. ■

The above theorem implies that if the retailer keeps a smaller allocation of the sales revenue of the digital version, and/or the tax rate for the digital version is reduced to that for the print version, then the retail price under the agency model is more likely to be less than that under the wholesale pricing model.

Numerical Comparison When the Two Retailers Adopt Different Revenue-Sharing Ratios and the Tax Rates for the Two Book Versions Are Different We relax the assumption of an identical revenue-sharing ratio and a single tax rate, and numerically compute the retail price of the digital version as well as the publisher's and the retailer's profits under the two pricing models. The results are then compared to investigate which pricing model may result in a higher retail price for the digital version. Using the parameter values as in our previous numerical study (in Sections 3.3 and 4.1.3), we perform two sensitivity analyses to examine the impact of β (i.e., the degree of substitutability between two retailers) and γ (i.e., the degree of substitutability between two book versions) on the retail price of the digital version and the profit of the publisher.

Impact of β We start with the sensitivity analysis for parameter β. We increase the value of β from 0.5 to 0.9 with a step equal to 0.1, similar to the sensitivity analysis in Section 4.1.3. For our numerical results, see Figure 4, where we learn from Figure 4(a) and (b) that the retail price of the digital version under the wholesale (agency) pricing model is decreasing (increasing) in β, similar to our result when only the digital version exists as in Section 4.1.3. The comparison between Figures 3 and 4 exposes the important insight that the existence of the print version helps reduce the retail price of the digital version. Moreover, Figures 4(a) and (b) indicate that a reduction in the tax rate for the digital version can lead the retail price to rise under both pricing models; furthermore, the increase in the retail price under the agency model is larger than that under the wholesale model. This means that the tax rate has a greater impact on the pricing decision under the agency model than under the wholesale model.

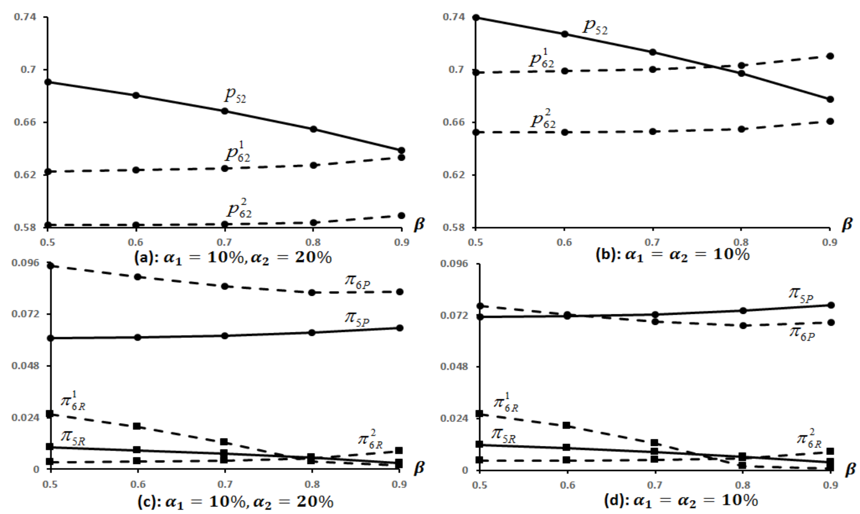

Figure 4: The impact of β on the retail price of the digital version as well as the publisher's and the retailer's profits in the duopoly setting with different revenue-sharing ratios for the two retailers and different tax rates for the two book versions.

We also note from Figures 4(c) and (d) that as the value of β increases, the publisher enjoys a higher profit under the wholesale pricing model, which is similar to the duopoly setting when only the digital version exists; see Figures 3(c) and (d). But, different from Figure 3, the existence of the print version may make the publisher obtain a higher profit under the agency model. This interesting result implies that the publisher benefits from making the two book versions. Moreover, Figures 4(c) and (d) indicate that when the value of β is smaller than a threshold, the retailer with a sufficiently small (large) revenue-sharing ratio obtains a higher (lower) profit under the agency model than under the wholesale model. But, when the value of β is larger than a threshold, the retailer with a sufficiently small (large) revenue-sharing ratio obtains a lower (larger) profit under the agency model.

Impact of γ We set $\beta = 0.7$ and increase the value of γ from 0.3 to 0.8 in steps of 0.1, and for each value we compute the retail price of the digital version as well as the publisher's and the retailer's profits under the two pricing models. We plot the results in Figure 5, where we learn from Figures 5(a) and (b) that the value of γ has a negligible impact on the retail price of the digital version under the wholesale pricing model. This implies that under the wholesale pricing model, the retail pricing decision is mainly dependent on the value of the tax rate for the digital version (α_2).

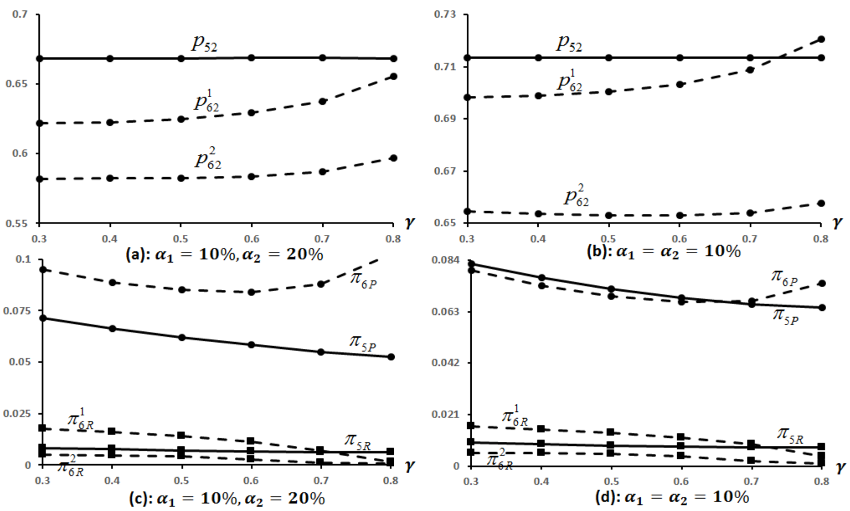

Figure 5: The impact of γ on the retail price of the digital version as well as the publisher's and the retailer's profits in the duopoly setting when the two retailers adopt different revenue-sharing ratios and the tax rates for the two book versions are different.

From Figures 5(c) and (d), we find that, different from the impact of β, the publisher's profit under the wholesale pricing model is smaller as a result of increasing the value of γ. In addition, if the tax rate for the digital version is significantly higher than that for the print version, then the profit of the publisher under the agency model is significantly greater than that under the wholesale model. Moreover, the retailer with a sufficiently high revenue-sharing ratio can achieve a higher profit under the wholesale model than under the agency model.

4.2.4 The Agency Pricing Analysis in the Presence of the MFN Clause

Similar to Section 4.1.4, we perform the agency pricing analysis when the "Most-Favored-Nation" (MFN) clause exists in the duopoly setting with the two book versions. For the analysis, the two retailers require the publisher to implement the MFN clause for the digital version, which results in an identical retail price for the digital version, viz., the publisher sets the retail price for the two retailers as $\hat{p}_2$, as defined in Section 4.1.4.

When the publisher may adopt different revenue-sharing ratios for the two retailers, we write retailer j's ($j = 1, 2$) and the publisher's profits as

$$\hat{\pi}^{j}_{6R} = (\hat{p}^{j}_{1} - \hat{w}_1)\zeta_1 + (1 - s_j)\hat{p}_2\zeta_2 \quad \text{and} \quad \hat{\pi}_P = \sum_{j=1}^{2}[(\hat{w}_1 - c_1)\zeta_1 + (s_j\hat{p}_2 - c_2)\zeta_2],$$

where $\zeta_1 \equiv [A-(1+\alpha_1)\hat{p}^{j}_{1} + \gamma(1+\alpha_2)\hat{p}_2 + \beta(1+\alpha_1)\hat{p}^{3-j}_{1} - \beta\gamma(1+\alpha_2)\hat{p}_2]/J$ and $\zeta_2 \equiv [A-(1+\alpha_2)\hat{p}_2 + \gamma(1+\alpha_1)\hat{p}^{j}_{1} + \beta(1+\alpha_2)\hat{p}_2 - \beta\gamma(1+\alpha_1)\hat{p}^{3-j}_{1}]/J$.

The optimal retail price of the digital version is calculated as

$$\begin{aligned}\hat{p}_{62} \quad = \quad & -\frac{B}{2(s_1+s_2)(4-\beta^2)(1-\beta)\eta_1 + \gamma^2[B\delta^2 + 2s_1s_2(2-\beta)B\eta_1 + Z(s_1-s_2)^2\eta_2]}\{-c_1\gamma\delta \\ & + c_2[(4-3\gamma^2-2\beta+2\beta\gamma^2)\eta_1 - \gamma^2\eta_2] + (1-\gamma)\eta_1[\gamma\delta + (1+\gamma)(s_1+s_2)(2-\beta)\eta_2]\},\end{aligned}$$

where $\delta \equiv \eta_1 - \eta_2$, $B \equiv -2+\beta+\beta^2$, and $Z \equiv 2+\beta-\beta^2$. If the tax rates for the two book versions are equal, i.e., $\alpha_1 = \alpha_2$, then $\eta_1 = \eta_2$ and $\delta = 0$; as a result, the optimal retail price $\hat{p}_{62}$ is independent of the unit production cost of the print version c_1 but is still dependent on the revenue-sharing ratios, the degree of substitutability between two retailers and that between two book versions. That is, under the MFN clause, how expensive the production cost of a print version is does not influence the optimal retail price of the digital version.

5 Summary and Future Research Directions

In this paper, we investigate a two-echelon supply chain involving a publisher and one/two retailers, who sell the digital version of a book to satisfy the price-dependent demand in a tax system. With the possible existence of the print version of the book, we perform the supply chain analysis under the wholesale and agency pricing models, and compare our results to draw managerial implications regarding the retail price of the digital version as well as the publisher's and the retailer's profits, which are expected to help practitioners make proper pricing decisions in a relevant market. The main insights are summarized as below.

1. In the monopoly and duopoly settings, a sufficiently high revenue-sharing ratio and a sufficiently low tax rate for the digital version can help make the retail price under the agency model smaller than that under the wholesale model.

2. The retail price of the digital version under the agency pricing model is more likely to be above

that under the wholesale pricing model in the duopoly setting than in the monopoly setting.

3. In the duopoly setting, as the competition between the two retailers becomes more severe, the retail price under the wholesale model decreases whereas the retail prices for two retailers under the agency model increase. Moreover, the existence of the print version helps reduce the retail price of the digital version.

4. In the duopoly setting, under the agency pricing model, the retail pricing decision for the digital version is significantly dependent on the degree of substitutability between the two book versions, whereas under the wholesale pricing model, the retail pricing decision is significantly dependent on the tax rate for the digital version.

5. In the duopoly setting, when the tax rate for the digital version is reduced, the publisher benefits more from the wholesale model than from the agency model.

6. In the duopoly setting, under the MFN clause, the retail price of the digital version is independent of the competition between the two retailers when the print version is unavailable, whereas the retail price is dependent on such a competition when the print version exists.

There are a number of future research directions that would be worth investigating. In our paper, we assumed that the tax rates are given. A nature question is about what the tax rate for the digital version should be. To address this question, we may need to compute the government's social welfare including tax revenue, consumer surplus, etc., and compare them under different pricing models. Moreover, in the paper we considered only a single book for which a publisher makes the digital version and possibly the print version. In reality, there may exist two or more books that are published by different publishers to compete for customers in a specific market; for example, there are many textbooks for each academic subject (e.g., Economics, Management Science, etc.). Therefore, it could be interesting to investigate a supply chain with two publishers and one/two retailers, where the two publishers compete with their substitutable books in a market. Each publisher may need to decide on whether the digital and print versions are made or only the digital version is produced. In addition, in our paper, we assumed that the unit cost of the print version and that of the digital version are common knowledge. It would behooves us to examine the case in which the cost information is the private knowledge of the publisher. Incorporating the information asymmetry into the supply chain analysis may be an interesting but challenging research direction.

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References

Dantas, D., Taboubi, S. & Zaccour, G. (2014). Which business model for e-book pricing?, *Economics Letters* **125**(1): 126–129.

Dobson, P. & Waterson, M. (1997). Countervailing power and consumer prices, *Economic Journal* **107**: 418–430.

Dobson, P. & Waterson, M. (2007). The competition effects of industry-wide vertical price fixing in bilateral oligopoly, *International Journal of Industrial Organization* **25**: 935–962.

Foros, O., Kind, H. J. & Shaffer, G. (2014). Turning the page on business formats for digital platforms: Does Apple's agency model soften competition. Working paper, Norweigian School of Economics, Norway.

Gans, J. S. (2012). Mobile application pricing, *Information Economics and Policy* **24(1)**: 52–59.

Gaudin, G. & White, A. (2014). On the antitrust economics of the electronic book industry. Working paper, School of Economics and Management, Tsinghua University, China.

Hu, Y. J. & Smith, M. D. (2016). The impact of ebook distribution on print sales: Analysis of a natural experiment. Working Paper, Georgia Institute of Technology, Atlanta, USA.

Hyatt, M. (2011). Why do eBook cost so much? `http://michaelhyatt.com/why-do-ebooks-cost-so-much.html` (URL last accessed on June 24, 2017).

International Publishers Association (2015). VAT/GST on books & e-books. `http://www.internationalpublishers.org/images/VAT2015.pdf` (URL last accessed on June 24, 2017).

Johnson, J. P. (2013). The agency and wholesale models in electronic content markets. Working Paper, Graduate School of Management, Cornell University, USA.

Johnson, J. P. (2017). The agency model and MFN clauses. To appear in *The Review of Economic Studies*.

McGrellis, K. (2011). E-books: Agency pricing and VAT. `http://www.mcgrellis.net/2011/01/e-books-agency-pricing-and-vat/` (URL last accessed on June 24, 2017).

Owen, L. (2014). Macmillan, too, returns to agency pricing with Amazon. `https://gigaom.com/2014/12/18/macmillan-too-returns-to-agency-pricing-with-amazon/` (URL last accessed on June 24, 2017).

Ruediger Wischenbart Content and Consulting (2014). Global eBook: A report on market trends and developments. `http://www.wischenbart.com/upload/1234000000358_04042014_final.pdf` (URL last accessed on June 24, 2017).

Singh, N. & Vives, X. (1984). Price and quantity competition in a differentiated duopoly, *The RAND Journal of Economics* **15**(4): 546–554.

Tan, Y. & Carrillo, J. E. (2014). The agency model for digital goods: Strategic analysis of dual channels in electronic publishing industry, *Proceedings of 2014 Portland International Conference on Management of Engineering and Technology (PICMET) Conference*.

Tan, Y. & Carrillo, J. E. (2017). Strategic analysis of the agency model for digital goods, *Production and Operations Management* **26**(4): 724–741.

Tan, Y., Carrillo, J. E. & Cheng, H. K. (2016). The agency model for digital goods, *Decision Sciences* **47**(4): 628–660.

Appendix A Proofs

Proof of Proposition 1. We first compute the retailer's best-response retail prices, given the wholesale price w_i. Differentiating π_{1R} with respect to p_1 and p_2, we obtain the first-order condition as

$$\begin{cases} 2\eta_1\overline{p}_1 - \gamma(\eta_1+\eta_2)\overline{p}_2 = w_1 - \gamma w_2 + \eta_1(1-\gamma), \\ 2\eta_2\overline{p}_2 - \gamma(\eta_1+\eta_2)\overline{p}_1 = w_2 - \gamma w_1 + \eta_2(1-\gamma). \end{cases}$$

Solving the above equations, we have the best response as

$$\overline{p}_i = \frac{w_i[2\eta_{3-i} - \gamma^2(\eta_1+\eta_2)] + w_{3-i}\gamma(\eta_i - \eta_{3-i}) + (1-\gamma)\eta_{3-i}[2\eta_{3-i} + \gamma(\eta_1+\eta_2)]}{4\eta_1\eta_2 - \gamma^2(\eta_1+\eta_2)^2}.$$

Substituting the retailer's best response into the publisher's profit and differentiating the resulting profit function w.r.t. w_1 and w_2, we obtain

$$\begin{cases} 4\eta_2 w_1 - 2\gamma(\eta_1+\eta_2)w_2 = \eta_2[2c_1 + 2\eta_1 - \gamma(\eta_1+\eta_2)] - c_2\gamma(\eta_1+\eta_2), \\ 4\eta_1 w_2 - 2\gamma(\eta_1+\eta_2)w_1 = \eta_1[2c_2 + 2\eta_2 - \gamma(\eta_1+\eta_2)] - c_1\gamma(\eta_1+\eta_2). \end{cases}$$

Solving the above gives the optimal wholesale price as $w_{1i} = (\eta_i + c_i)/2$, for $i = 1, 2$, which can be then used to attain the retail prices as given in (6). ■

Proof of Corollary 1. We can note from (6) that when $\alpha \equiv \alpha_1 = \alpha_2$, the retail price for the print version p_{11} and that for the digital version p_{12} are identical, and $\eta \equiv \eta_1 = \eta_2 = 1/(1+\alpha)$. We thus reduce the retail price and the tax-inclusive retail price for publishing version i to

$$p_{1i}|_\alpha = \frac{3\eta + c_i}{4} \quad \text{and} \quad \bar{p}_{1i}|_\alpha = \frac{3 + (1+\alpha)c_i}{4}.$$

Using the above, we can prove the corollary. ■

Proof of Proposition 2. The proof is similar to that of Proposition 1. ■

Proof of Corollary 2. When $\alpha \equiv \alpha_1 = \alpha_2$, $\eta \equiv \eta_1 = \eta_2 = 1/(1+\alpha)$ and the retail price and the tax-inclusive retail price of the digital version are reduced to

$$p_{22}|_\alpha = \frac{s\eta + c_2}{2s} \quad \text{and} \quad \bar{p}_{22}|_\alpha = \frac{s + (1+\alpha)c_2}{2s},$$

which can be used to draw the results in this corollary. ■

Proof of Theorem 1. From the proofs of Corollaries 1 and 2, we obtain the retail prices of the digital version under the wholesale and the agency pricing models as $p_{12}|_\alpha = [3 + c_2(1+\alpha)]/[4(1+\alpha)]$ and $p_{22}|_\alpha = [s + c_2(1+\alpha)]/[2s(1+\alpha)]$. Therefore, if $[3+c_2(1+\alpha)]/[4(1+\alpha)] \geq [s+c_2(1+\alpha)]/[2s(1+\alpha)]$, or $s \geq 2(1+\alpha)c_2/[1+(1+\alpha)c_2]$, then $p_{12}|_\alpha \geq p_{22}|_\alpha$. Otherwise, $p_{12}|_\alpha < p_{22}|_\alpha$. This theorem is proved. ■

Proof of Theorem 2. The theorem follows the comparison between $\pi_{1P}|_\alpha$ in (7) and $\pi_{2P}|_\alpha$ in (10). ■

Proof of Proposition 3.

Given the wholesale price w, we attain the retail prices for the digital version in Nash equilibrium as

$$p_2^1 = p_2^2 = \frac{w + (1-\beta)\eta}{2-\beta}.$$

Substituting the above into the publisher's profit in (12) and maximizing the resulting profit for the optimal wholesale price, we have w_{32} as shown in (13), which can be then used to find the optimal retail price as p_{32}. This proposition is thus proved. ■

Proof of Corollary 3. According to (13), we find that the retail price p_{32} is increasing in α_2. As implied by (14), the sales q_{32} is also increasing in α_2. Moreover, we calculate the tax-inclusive retail price as $\bar{p}_{32} = (1+\alpha_2)p_{32} = [2(1-\beta)+1+(1+\alpha_2)c_2]/[2(2-\beta)]$, which is increasing in α_2. ■

Proof of Proposition 4. Under the agency pricing model, the retail price is determined by the publisher. Differentiating the publisher's profit with respect to p_2^1 and p_2^2, we have

$$\begin{cases} 2\eta_2 s_1 \overline{p}_2^1 - \eta_2\beta(s_1+s_2)\overline{p}_2^2 = (1-\beta)(\eta_2 s_1 + c), \\ 2\eta_2 s_2 \overline{p}_2^2 - \eta_2\beta(s_1+s_2)\overline{p}_2^1 = (1-\beta)(\eta_2 s_2 + c). \end{cases}$$

Solving the above gives p_{42}^j as in (15). ■

Proof of Corollary 4. Using (15), we can find the optimal retail price and the publisher's profit as given in this corollary. Moreover, we can find from the retail price and the profit that $p_{42}^j|_s$ is decreasing in s and $\pi_{4P}|_s$ is decreasing in β. ■

Proof of Theorem 3. The comparison between the retail price in (13) and the profit in (14) under the wholesale pricing model with that in Corollary 4 under the agency pricing model yields this Theorem. ■

Proof of Corollary 5. Differentiating $(2-\beta)c_2/[c_2+(1-\beta)\eta_2]$ w.r.t. β gives

$$\xi \equiv \frac{-c_2[c_2+(1-\beta)\eta_2] + (2-\beta)c_2\eta_2}{[c_2+(1-\beta)\eta_2]^2}.$$

The value of ξ must be positive if $(2-\beta)c_2\eta_2 > c_2[c_2+(1-\beta)\eta_2]$, which is satisfied when $(1+\alpha_2)c_2 < 1$. From (14), we find that $(1+\alpha_2)c_2$ must be smaller than 1 so that the sales are positive. Therefore, $\xi > 0$, which means that the cutoff level $(2-\beta)c_2/[c_2+(1-\beta)\eta_2]$ is increasing in β. Therefore, the condition $s > (2-\beta)c_2/[c_2+(1-\beta)\eta_2]$ in Theorem 3 is more difficult to be satisfied than the condition $s > 2c_2/(c_2+\eta_2)$ in Theorem 1. This corollary is thus proved. ■

Proof of Proposition 5. Given the publisher's wholesale price w_i, we compute two retailers' best responses. Differentiating retailer 1's profit with respect to its retail prices yields

$$\begin{cases} 2\eta_1\overline{p}_1^1 - \beta\eta_1\overline{p}_1^2 - \gamma(\eta_1+\eta_2)\overline{p}_2^1 + \beta\eta_1\gamma\overline{p}_2^2 = (1-\beta)(1-\gamma) + w_1 - \gamma w_2, \\ 2\eta_2\overline{p}_2^1 - \beta\eta_2\overline{p}_2^2 - \gamma(\eta_1+\eta_2)\overline{p}_1^1 + \beta\eta_2\gamma\overline{p}_1^2 = (1-\beta)(1-\gamma) + w_2 - \gamma w_1; \end{cases}$$

and differentiating retailer 2's profit w.r.t. its retail prices gives

$$\begin{cases} 2\eta_1\overline{p}_1^2 - \beta\eta_1\overline{p}_1^1 - \gamma(\eta_1+\eta_2)\overline{p}_2^2 + \beta\eta_1\gamma\overline{p}_2^1 = (1-\beta)(1-\gamma) + w_1 - \gamma w_2, \\ 2\eta_2\overline{p}_2^2 - \beta\eta_2\overline{p}_2^1 - \gamma(\eta_1+\eta_2)\overline{p}_1^2 + \beta\eta_2\gamma\overline{p}_1^1 = (1-\beta)(1-\gamma) + w_2 - \gamma w_1. \end{cases}$$

To obtain Nash equilibrium, we need to solve the following equations

$$\begin{cases} 2\eta_1\overline{p}_1^1 - \beta\eta_1\overline{p}_1^2 - \gamma(\eta_1+\eta_2)\overline{p}_2^1 + \beta\eta_1\gamma\overline{p}_2^2 = (1-\beta)(1-\gamma) + w_1 - \gamma w_2, \\ 2\eta_2\overline{p}_2^1 - \beta\eta_2\overline{p}_2^2 - \gamma(\eta_1+\eta_2)\overline{p}_1^1 + \beta\eta_2\gamma\overline{p}_1^2 = (1-\beta)(1-\gamma) + w_2 - \gamma w_1, \\ 2\eta_1\overline{p}_1^2 - \beta\eta_1\overline{p}_1^1 - \gamma(\eta_1+\eta_2)\overline{p}_2^2 + \beta\eta_1\gamma\overline{p}_2^1 = (1-\beta)(1-\gamma) + w_1 - \gamma w_2, \\ 2\eta_2\overline{p}_2^2 - \beta\eta_2\overline{p}_2^1 - \gamma(\eta_1+\eta_2)\overline{p}_1^2 + \beta\eta_2\gamma\overline{p}_1^1 = (1-\beta)(1-\gamma) + w_2 - \gamma w_1. \end{cases}$$

We then obtain the two retailers' best responses as

$$\overline{p}_1^1 = \overline{p}_1^2 = \frac{(2-\beta)(w_1 - \gamma w_2 + A\eta_1)\eta_2 + \gamma(\eta_1 - \beta\eta_1 + \eta_2)(w_2 - \gamma w_1 + A\eta_2)}{(2-\beta)^2\eta_1\eta_2 - \gamma^2(\eta_1+\eta_2-\beta\eta_1)(\eta_1+\eta_2-\beta\eta_2)},$$

and

$$\overline{p}_2^1 = \overline{p}_2^2 = \frac{(2-\beta)(w_2 - \gamma w_1 + A\eta_2)\eta_1 + \gamma(\eta_2 - \beta\eta_2 + \eta_1)(w_1 - \gamma w_2 + A\eta_1)}{(2-\beta)^2\eta_1\eta_2 - \gamma^2(\eta_1+\eta_2-\beta\eta_1)(\eta_1+\eta_2-\beta\eta_2)}.$$

Substituting the responses into the publisher's profit and maximizing it, we have the optimal wholesale price as in (16), which is then substituted into the retailers' responses, we obtain the optimal retail price as in (17). This proposition is thus proved. ■

Proof of Theorem 4. From (13) and (18), we find that the retail price of the digital version when the print version does not exist $p_{32}|_\alpha$ is the same as that when the print version exists $p_{52}|_\alpha$. Moreover, from the proof of Corollary 1, we have the retail price of the digital version as $p_{12}|_\alpha = (3\eta + c_2)/4$, which is greater than $p_{52}|_\alpha$ in (18) if $c_2(1+\alpha) < 1$. As argued in the proof of Corollary 5, the inequality $c_2(1+\alpha) < 1$ always holds. Thus, this theorem is proved. ■

Proof of Propsition 6. To find the Nash equilibrium-characterized retail prices of the print version for two retailers, we obtain retailer j's $(j = 1, 2)$ best response, given the wholesale price w_1 and the retail price of the digital version p_2^j, as

$$p_{61}^j(w_1, p_2^j) = \frac{(2+\beta)(w_1 + A\eta_1)}{4-\beta^2} + \frac{\gamma(1+\alpha_2)\{p_2^j[(2-\beta^2)\eta_1 + 2(1-s_1)\eta_1] + \beta p_2^{3-j}[(1-s_2)\eta_2 - \eta_1]\}}{4-\beta^2}. \tag{24}$$

Substituting two retailers' best response into the publisher's profit, we differentiate the resulting profit with respect to w_1, $(1+\alpha_2)p_2^1$, and $(1+\alpha_2)p_2^2$, and obtain the first-order conditions as follows:

$$\begin{cases} 4w_1 - \gamma\sum_{j=1}^2[E_j(1+\alpha_2)p_2^j] = 2[c_1 - \gamma c_2 + (1-\gamma)\eta_1], \\ (2-\beta-\beta^2)H_1 = (-2+\beta+\beta^2)\gamma E_1 w_1 - 2s_1\eta_2 F_1(1+\alpha_2)p_2^1 + \beta\eta_2 G(1+\alpha_2)p_2^2, \\ (2-\beta-\beta^2)H_2 = (-2+\beta+\beta^2)\gamma E_2 w_1 + \beta\eta_2 G(1+\alpha_2)p_2^1 - 2s_2\eta_2 F_2(1+\alpha_2)p_2^2, \end{cases}$$

where $G \equiv s_2[(-4+\beta^2+3\gamma^2-\beta^2\gamma^2)\eta_1 + \gamma^2\eta_2] + s_1[(-4+\beta^2+3\gamma^2-\beta^2\gamma^2)\eta_1 + (1-2s_2)\gamma^2\eta_2]$; for $k = 1, 2$, $E_k \equiv \eta_1 - \eta_2 + 2s_k\eta_2$, $F_k \equiv (-4+\beta^2+2\gamma^2)\eta_1 + (1-s_k)(2-\beta^2)\gamma^2\eta_2$, and $H_k \equiv s_k(1-\gamma)(2-\beta+\gamma-\beta\gamma)\eta_1\eta_2 - c_1\gamma(\eta_1-\eta_2+s_k\eta_2) + c_2[(2-\gamma^2-\beta+\beta\gamma^2)\eta_1 - (1-s_k)\gamma^2\eta_2]$.

Using Cramer's rule we have the optimal wholesale price for the print version and the optimal retail price for the digital version, as given in (21) and (22), respectively. Substituting the optimal solutions into (24) yields the retail prices of the print version in Nash equilibrium as given in (23). ■

Proof of Theorem 5. When the revenue-sharing ratios for two retailers are equal, we can use (22) to find that the two retailers have the same optimal retail price of the digital version as given below.

$$p_{62}|_s \equiv p_{62}^1|_s = p_{62}^2|_s = \frac{c_1\gamma(\eta_2-\eta_1) + c_2[(4-3\gamma^2-2\beta+2\beta\gamma^2)\eta_1 - \gamma^2\eta_2] + C_0}{4s(2-\beta)\eta_1(1-\gamma^2) - \gamma^2(\eta_1-\eta_2)^2},$$

where $C_0 \equiv (1-\gamma)\eta_1\{2s(2-\beta)\eta_2 + \gamma[\eta_1 - (1-4s+2s\beta)\eta_1]\}$. If the government decides to reduce the tax rate of the digital version to that of the print version, viz., a single tax rate α applies to two versions of the book, then the optimal retail price of the digital version for the two retailers is

$$p_{62}|_{s,\alpha} \equiv p_{62}^1 = p_{62}^2 = \frac{c_2 + s\eta}{2s},$$

which is the same as $p_{22}|_\alpha = (c_2 + s\eta)/(2s)$ as given in Corollary 2 for the analysis of the agency model in the monopoly setting. Moreover, p_{62} is the same as $p_{42}|_s = (c_2 + s\eta_2)/(2s)$ as given in Corollary 4 for the analysis of the agency model in the duopoly setting with only the digital version. ■

Proof of Theorem 6. Using (18) and the proof of Theorem 5, we can obtain the result regarding the retail price of the digital version. Then, using (19) and the proof of Theorem 5, we can attain the result for the profit of the publisher. ■