Demand Functions in Decision Modeling: A Comprehensive Survey and Research Directions[1, 2]

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Abstract: A variety of mathematical forms have been developed to characterize demand functions which depend on a firm's operational and marketing activities. Such demand functions are being increasingly used by researchers in economics and different functional areas of business. We provide a comprehensive survey of commonly-used demand models which depend on (i) price, (ii) rebate, (iii) leadtime, (iv) space, (v) quality, and, (vi) advertising. Our survey includes single-firm demand models in each category, as well as game theoretic multi-firm models involving strategic interaction among the firms. We observe that certain types of functional forms, such as, linear, power/iso-elastic, multinomial logit, and multiplicative competitive interaction, have been widely used to construct various demand models in all six categories, but that a large majority of publications deal with categories (i) and (v) of demand models. For each of the six categories, we survey relevant functional forms in the representative papers, and discuss the main properties, the advantages, the disadvantages, and comment on possible future research directions. We also present a discussions of the applications of these analytical demand models in empirical studies. The paper ends with a summary of our major findings.

Keywords: Demand function; price; rebate; lead time; space; quality; advertising; empirical study.

1 Introduction

In order to capture market share and increase sales, in highly competitive free markets of liberalized economies firms have to compete not only with their prices, but also with rebates, leadtimes, operating spaces, product and/or service quality, and advertising and other promotional expenditures. But, the degree of effectiveness of these competitive factors in increasing a firm's sales depends on how much of these factors influence the consumers' purchasing decisions.

As Lilien et al. (1992) have shown, a consumer usually progresses through the following five steps in her purchase cycle: In the first step, the consumer is aroused by internal stimuli and/or external drives. In the second step, the consumer searches for relevant information to identify a potential set

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of brands or products that satisfy her need. In the third step, the consumer evaluates the identified products according to her utilities associated with different product attributes such as price, quality, functionality, rebates, return policy, brand reputation, etc. In the fourth step, the consumer makes her purchase decision based on the comparison of the utilities of different products. In the final step, the consumer has post-purchase feelings such as whether or not she is satisfied with the product, and may request post-sales services such as product return/replacement, technical support services, etc. Thus, consumers are sensitive to firms' operational and marketing activities in each step and in order to be competitive, each firm has to make decisions on price, rebate, leadtime, quality, advertising etc., in a prudent manner and entice more potential consumers to buy its products or request its service.

In the past two decades, a considerable number of researchers in economics and different functional areas of business have studied the impact of various operational and marketing activities on the consumer demand. These researchers have developed a variety of mathematical functions to characterize the demand. Motivated by the prevalence of demand models in the literature, we present a comprehensive survey of the functional forms that have been used in such demand models. We focus our survey on various commonly-used demand models in six categories, which include (i) price-, (ii) rebate-, (iii) leadtime- (iv) space-, (v) quality-, and (vi) advertising-dependent demand models. For each model, we briefly describe the analysis in one or several representative publications, and discuss the main properties, the advantages, the disadvantages, and possible future research directions. For each category, we also compare relevant demand models to identify the conditions for the applicability of each model.

In this paper, we select representative publications mainly based on the following four criteria, which are listed in the order of their priority. First, we emphasize the review of the models that can be used to describe the demands of consumers who are sensitive to firms' decisions on price, rebate, leadtime, shelf space, product/service quality, and advertising. We do not consider the demand functions independent of a firm's decision, e.g., the demand functions in terms of a product's life-time, which have been reviewed by Goyal and Giri (2001). Second, our paper is focused on reviewing the demand models that have not been comprehensively surveyed in the extant literature. For example, we do not review the demand models in terms of a firm's initial and instantaneous inventory levels, because those models have been discussed by Urban (2005). Similarly, we do not consider the dynamic advertising models and empirical studies relating to advertising, because those have been comprehensively reviewed by Erickson (2003), Feichtinger et al. (1994), Huang, Leng, and Liang (2012), Mahajan et al. (1990), Sethi (1977), etc. Third, we mainly review early demand models, and very briefly mention or omit their extensions that do not exhibit any significant difference. Fourth, we have chosen to review the models that are simple but representative, in order to clearly present our survey and draw insightful conclusions.

We show in our review that the price-dependent demand models are the most commonly employed possibly because pricing strategy is the most effective tool that has been used to impact a firm's demand. Our comprehensive review includes, for each category, single firm demand function models. We also survey, where available, game theoretic multi-firm models involving strategic interaction. We have found that a large number of publications belong to the categories of price- and quality-dependent models, but fewer publications to other categories. This suggests that there may exist further research opportunities for using rebate-, leadtime-, space-, and advertising-dependent models.

The remainder of this paper is organized as follows: Sections 2 to 7 present a survey of the demand function models in six categories. Then, in Section 8, we review demand models for which empirical studies have been performed to estimate the model parameters. In Section 9, we summarize our survey, and discuss how the existing demand function models have helped analyze a variety of business and economics problems. For each category of demand models, we also provide a further discussion of possible research directions.

2 Price-Dependent Demand Models

In the past several decades, numerous theoretical demand models have been developed from various perspectives to investigate the impact of price on the consumer demand and to examine the firms' optimal pricing decisions for the settings with a monopolistic or multiple competitive firms. In this section, we first review demand models without price competition, and then survey those where there is price competition between two or among $n \geq 3$ firms.

2.1 Single Firm Price-Dependent Demand Models

These models are commonly used to characterize the demand for a product that only depends on the price of the product itself, which include the deterministic, stochastic, willingness-to-pay (WTP), and Poisson flow models. We divide these price-dependent models for a single firm into four subgroups according to the main factors that affect the consumer demand, in addition to the product price. Specifically, the stochastic demand models differ from the deterministic models because random factors may affect the consumer demand. Different from the deterministic models, the WTP models can be used to derive the demand function by considering the consumers' WTP, and the Poisson flow models incorporate both the consumers' WTP and their arrival time when they purchase.

2.1.1 Deterministic Models

In the simplest deterministic model, demand $d(p)$ is defined as a decreasing linear function of price p and written as $d(p) = a - bp$, where a, $b > 0$ and $0 \leq p \leq a/b$. Table 1 lists several linear models (LM) which extend this basic linear model. The simplest nonlinear (power) model is the iso-elastic model $d(p) = ap^{-b}$, where $a > 0$, $b > 1$. One of the advantages of this model is that it does not require a finite upper limit on price. The iso-elastic model is also called the constant elasticity model because the demand function exhibits a constant elasticity, i.e., $-b$. Table 1 lists several extensions of the power model (PM) and also summarizes the hybrid (HM), exponential (EM), logarithmic (LgM) and logit (LtM) models. In the basic exponential model (EM I) the demand response to a reduction in price follows an increasing returns to scale (see Hanssens and Parsons 1993). The standard logit model (LgtM I) is useful because its parameters can be estimated by regression methods in empirical applications.

Deterministic Demand Models	**Price-Dependent Deterministic Demand**
Linear Models (LM)	
LM I: Mills (1959), Petruzzi Dada (1999)	$d(p) = a - bp$, where $a, b > 0$
LM II: Eliashberg and Steinberg (1987)	$d(p) = a(t) - bp(t)$, where $a(t), b > 0$, $t \in [0, T]$
LM III: Chou and Parlar (2006)	$d(p) = a(t) - b(t)p(t)$, where $t \in [0, T]$
LM IV(Reference-price model): Fibich et al. (2003)	$d(p, r) = a - \delta p - \gamma(p - r)$, where $a, \delta, \gamma > 0$
Power Models (PM)	
PM I (Iso-elastic model): Karlin and Carr (1962), Petruzzi Dada (1999)	$d(p) = ap^{-b}$, where $a > 0, b > 1$
PM II (Algebraic model): Jeuland and Shugan (1988)	$d(p) = (ap + b)^{-\gamma}$, where $a, b > 0, \gamma > 1$
PM III (Linear-power model**)**: Song et al. (2008)	$d(p) = (a - bp)^{\gamma}$, where $a, b\gamma > 0, \gamma \in (-\infty, -1) \cup (0, \infty)$
PM IV: Chen et al. (2006)	$d(p) = a - bp^{\gamma}$, where $a, b > 0, \gamma \geq 1$
PM V: Ray et al. (2005)	$d(p) = (a - bp^{\beta})^{\gamma}$, where $a, b > 0, \beta \geq 1, \gamma \leq 1$
PM VI: Agrawal and Ferguson (2007)	$d(p) = a/[a + bp^{\gamma}]$, where $a, b, \gamma > 0$
Hybrid Model (HM): Lau and Lau (2003)	$d(p) = \tau(a_1 - bp) + (1 - \tau)a_2 p^{-\gamma}$, where $a_1 > 0, a_2 > 0$, $b > 0, \gamma > 1$, and $0 \leq \tau \leq 1$
Exponential Models (EM)	
EM I: Hanssens and Parsons (1993), Jeuland and Shugan (1988), Song et al. (2008)	$d(p) = a\exp(-\gamma p)$, where $a > 0, \gamma > 0$
EM II: Chen et al. (2006), Song et al. (2008)	$d(p) = \exp(a - bp)$, where $a > 0, b > 0$
EM III: Chen and Simchi-Levi (2004a)	$d(p) = a - \exp(\gamma p)$, where $a > 0, \gamma > 0$
Logarithmic Model (LgM): Chen et al. (2006), Ray et al. (2005)	$d(p) = \ln[(a - bp)^{\gamma}]$, where $a, b, \gamma > 0$
Logit Models (LtM)	
LtM I: Chen and Simchi-Levi (2012),	$d(p) = a\exp(-bp)/[1 + \exp(-bp)]$, where $a, b > 0$
LtM II: Phillips (2005)	$d(p) = 1/[1 + \exp[a + bp]]$

Table 1: Price-dependent deterministic demand functions.

Remark 1 The linear model is extensively used in the literature because it gives rise to explicit results for the optimal solution, and it is relatively easy to estimate its parameters in an empirical study. Moreover, the demand elasticity of the linear model [i.e., $d'(p)/[d(p)/p] = -bp/(a - bp)$] depends on the price decision variable p and it is always decreasing and concave in p. On the other hand, in most practical cases the assumption of a linear demand function and the requirement of a finite upper bound on the price do not correspond to reality. The advantage of the iso-elastic (power) model is that it can be transformed to a linear function by taking logarithms which makes the parameter estimation relatively easier. The other advantages of the power model include its ability to characterize the nonlinear effects arising in the market and the possibility of deriving it from the Cobb-Douglas production function. However, one of the drawbacks of the iso-elastic model is that the demand elasticity always equals the constant $-\gamma$. Another drawback of the iso-elastic model is that, when price approaches zero, demand approaches infinity, which is not realistic since market size is a finite quantity. See Oum (1989) for a detailed discussion of the advantages and drawbacks of the linear and iso-elastic demand models.

As shown in Table 1, numerous demand models have appeared in the literature, but there has been little justification for why one demand model should be chosen instead of the others. An exception to this is Lau and Lau (2003) who showed that, for a single-echelon system, using different types of demand functions leads to structurally very similar conclusions. However, for multi-echelon systems, even a small change in the specification of the demand function results in a significant change in the channel efficiency and profitability ratios, thus requiring a more prudent approach in the specification of the demand model. ⧫

2.1.2 Stochastic Models

The above deterministic demand models have been extended to the stochastic cases with frequent applications to the investigation of the newsvendor-type problems with pricing decisions. For a list of commonly-used stochastic models, see Table 2.

Stochastic Demand Models	Price-Dependent Stochastic Demand
General Model: Federgruen and Heching (1999), Kocabıyıkoğlu and Popescu (2011)	$D(p, \varepsilon)$
Additive Model: Mills (1959), Petruzzi and Dada (1999)	$D(p, \varepsilon) = d(p) + \varepsilon$
Multiplicative Model: Karlin and Carr (1962), Petruzzi and Dada (1999)	$D(p, \varepsilon) = d(p)\varepsilon$
Hybrid Model I: Petruzzi and Dada (1999), Young (1978)	$D(p, \varepsilon) = d_1(p)\varepsilon + d_2(p)$
Hybrid Model II: Chen and Simchi-Levi (2004a), Chen and Simchi-Levi (2004b)	$D(p, \varepsilon) = \varepsilon_1 d(p) + \varepsilon_2$
Power Model: Kocabıyıkoğlu and Popescu (2011)	$D(p, \varepsilon) = a\varepsilon/(\varepsilon + p^b)$
Exponential Model: Kocabıyıkoğlu and Popescu (2011)	$D(p, \varepsilon) = e^{\varepsilon - bp}$
Logarithmic Model: Kocabıyıkoğlu and Popescu (2011)	$D(p, \varepsilon) = \log_a(\varepsilon - bp)$
Logit Model: Agrawal and Ferguson (2007), Phillips (2005), Kocabıyıkoğlu and Popescu (2011)	$D(p, \varepsilon) = ae^{\varepsilon - bp}/(1 + e^{\varepsilon - bp})$

Table 2: Stochastic demand models commonly used in the literature.

We denote a general stochastic demand model as $D(p, \varepsilon)$, where p is the selling price and ε is a price-independent random variable with c.d.f. $F(\cdot)$ and p.d.f. $f(\cdot)$ defined on the range $[A, B]$ with mean $E(\varepsilon) = \mu$ and variance $\mathrm{Var}(\varepsilon) = \sigma^2$. For the general model, the variance captures the market risk. In the literature, the stochastic demand $D(p, \varepsilon)$ is usually specified either as an additive, a multiplicative, or an additive-multiplicative model (Young, 1978), and is commonly used for newsvendor-type problems with pricing (see, Federgruen and Heching, 1999 ; Petruzzi and Dada, 1999). More specifically, for the additive case (Mills, 1959), demand can be represented as,

$$D(p, \varepsilon) = d(p) + \varepsilon, \tag{1}$$

where $d(p)$ is the deterministic component of the random demand and is usually assumed to have the linear form $d(p) \equiv a - bp$ with $a, b > 0$. In the multiplicative case (Karlin and Carr, 1962), demand can be modeled as,

$$D(p, \varepsilon) = d(p)\varepsilon, \tag{2}$$

where the deterministic term $d(p)$ is often specified as the common power function $d(p) = ap^{-b}$ $(a > 0, b > 1)$.

Combining the additive and multiplicative models, Young (1978) proposed a hybrid additive-multiplicative demand function as, $D(p, \varepsilon) = d_1(p)\varepsilon + d_2(p)$, where $d_1(p)$ and $d_2(p)$ are two deterministic, non-increasing functions of p. When $d_1(p) = 1$, $D(p, \varepsilon)$ reduces to the additive function in (1), and when $d_2(p) = 0$, $D(p, \varepsilon)$ is transformed into the multiplicative model in (2).

Chen and Simchi-Levi (2004a) constructed another hybrid additive-multiplicative model that has two risk components, $D(p, \varepsilon) = \varepsilon_1 d(p) + \varepsilon_2$, where the two terms ε_1 and ε_2 are random variables and without loss of generality, they are assumed to have the mean values $E(\varepsilon_1) = 1$ and $E(\varepsilon_2) = 0$. When $\varepsilon_1 = 1$ and $\varepsilon_2 = 0$, this demand function reduces to the additive and multiplicative forms, respectively. A variety of functional forms for $d_1(p)$ and $d_2(p)$ have been used by different authors; a comprehensive list of these models in six groups is given in Table 1.

Remark 2 There exists significant differences between the optimal pricing and inventory decisions obtained by using the above additive and multiplicative models, which depend on how the demand randomness is incorporated into the demand function (see, Petruzzi and Dada, 1999). For example, the demand variance for the additive demand is independent of price but the coefficient of demand variation is increasing in price. In contrast to the additive model, the coefficient of variation of demand in the multiplicative model is independent of price but its variance is decreasing in price. To deal with this problem, Kocabıyıkoğlu and Popescu (2011) developed a unified framework for the newsvendor problem with pricing decision by introducing a new measure, the elasticity of lost-sales rate (LSR), which can be used for most relevant demand models in the marketing and operations studies, such as the additive linear model, multiplicative iso-elastic model, logit model, power model, exponential model, log model and economic willingness-to-pay model; see Table 2 in Kocabıyıkoğlu and Popescu (2011). ⧫

2.1.3 Willingness-To-Pay Models

One often needs to consider how the firm's pricing decision affects the purchase decision of individual customers who are usually assumed to have heterogeneous valuations. To that end, researchers have proposed the willingness-to-pay (WTP) model (see, Kalish, 1985), which characterizes consumers' heterogeneous willingness to buy from a firm whose selling price is p. Suppose that consumers have full information about the product. A consumer will buy if $p \leq V$, where V denotes a consumer's willingness-to-pay, which is a random variable with p.d.f. $f(v)$. Denoting the market size by Λ, Kalish (1985) developed the total demand model as $d(p) = \Lambda \int_{v \geq p} f(v)dv$. Kalish (1985) also extended the above model to incorporate the uncertainty about the product's experience attributes or the actual value of the product. The demand model in the uncertainty setting is given as $d(p) = \Lambda \int_{v \geq p/\delta} f(v)dv$, where $\delta \in (0, 1]$ denotes the reduction in the product's value due to its experience attributes. Thus, for a fixed price p, as δ becomes smaller, there will be fewer customers buying the product. In existing publications this model is used to investigate a firm's optimal pricing decisions, where a consumer's purchase decision is influenced by the firm's price as well as the product quality. This issue is further discussed in the quality-dependent demand model in Section 6.1.1.

Remark 3 We note that if the random variable V follows the uniform distribution defined over $[A, B]$, then we find $d(p) = [\Lambda/(B-A)] \int_{p/\delta}^{B} dv = \Lambda[B - p/\delta]/(B-A)$ which is of the same form as the aggregate linear model LM I in Table 1. A similar result was obtained by Phillips (2005), who assumed that $\delta = 1$, as shown in Table 3. In fact, using different probability distributions of consumers's WTPs or utilities V (at the "micro-level") can lead to different aggregate demand functions (at the "macro-level"). In addition to the linear demand function resulting from the uniform distribution over $[A, B]$, we find that Kuo et al. (2008) and Phillips (2005) derived the aggregate demand functions assuming that V satisfies the exponential, logistic, Weibull, and Pareto distributions, as given in Table 3.

Our above discussion implies that which model (an aggregate model vs. a WTP model) we should choose mainly depends on whether or not to analyze the consumers' choices at the "micro-level." If we do not perform the analysis of the consumer choice, then we should not use a WTP model but directly assume an aggregate demand function that was reviewed in, e.g., Sections 2.1.1 and 2.1.2. Otherwise, we need to develop a WTP model and derive an aggregate demand function as discussed in this section. ⧫

Distribution	**Probability Density Function** $f(v)$	**Aggregate Demand Function** $d(p)$
Uniform(A, B): Phillips (2005)	$\begin{cases} \frac{1}{B-A}, & \text{if } A \le v \le B, \\ 0, & \text{otherwise.} \end{cases}$	$\frac{\Lambda(B-p)}{B-A}$ (Linear function)
Exponential(λ): Kuo et al. (2008)	$\begin{cases} \lambda \exp(-\lambda v), & \text{if } v \ge 0, \\ 0, & \text{otherwise.} \end{cases}$	$\Lambda \exp(-\lambda p)$ (Log-linear function)
Logistic(α, β): Phillips (2005)	$\frac{\exp[-(v-\alpha)/\beta]}{\beta\{1+\exp[-(v-\alpha)/\beta]\}^2}$	$\frac{\Lambda \exp[-(v-\alpha)/\beta]}{1+\exp[-(v-\alpha)/\beta]}$ (Logistic function)
Weibull(α, β): Kuo et al. (2008)	$\begin{cases} \frac{\alpha}{\beta}\left(\frac{v}{\beta}\right)^{\alpha-1} \exp\left[-\left(\frac{v}{\beta}\right)^{\alpha}\right], & \text{if } v \ge 0, \\ 0, & \text{otherwise.} \end{cases}$	$\Lambda \exp\left[-\left(\frac{p}{\beta}\right)^{\alpha}\right]$ (Exponential function)
Pareto(b): Phillips (2005)	$\begin{cases} bv^{-(b+1)}, & \text{if } v \ge 1, \\ 0, & \text{otherwise.} \end{cases}$	Λp^{-b} (Power function)

Table 3: A summary of commonly-used probability distributions of consumers' WTPs or utilities and the corresponding aggregate demand models.

2.1.4 Poisson Flow Models

In recent years, dynamic pricing and inventory models have been widely used in marketing and operations management; see, e.g., Bitran and Caldentey (2003) and Elmaghraby and Keskinocak (2003). In such models, one of the most important factors that influence the pricing decisions is demand; thus, in order to analyze the optimal pricing decisions, one needs to consider customers' arrival process and the changes in their willingness to pay (Elmaghraby and Keskinocak, 2003).

An early publication concerning the dynamic pricing decision for a perishable product was by Kincaid and Darling (1963), who developed a continuous time model in which the demand follows a Poisson process with intensity λ. A customer arriving at time t has a reservation price $V(t)$ for the product, which is assumed to be a random variable with c.d.f. $F(\cdot)$. Kincaid and Darling (1963) considered two cases. In the first case, the firm does not post the prices but receives offers from potential customers, which the firm may accept or reject. For this case, the customers are not strategic players. In the second case, customers are strategic, and each customer's net surplus drawn from his or her purchase is $U(p, t) = V(t) - p(t)$, where $p(t)$ is the price at time t. Thus, a customer buys if $V(t) \geq p(t)$. As a result, if customers' arrival rate is λ and each customer has an i.i.d. reservation price $V(t)$ with tail probability $\bar{F}(p)$, then the expected demand at price $p(t)$ is $\lambda\bar{F}(p(t))$ per unit time. In this setting, the firm can control the intensity of demand by choosing the price $p(t)$ at time t. For other similar models, see Bitran and Mondschein (1997), and Gallego and van Ryzin (1994).

Zhao and Zheng (2000) modeled the demand as a nonhomogeneous Poisson process with rate $\lambda(t)$ and allowed the probability distribution of the reservation price, $F(\cdot)$, to change over time. That is, consumers' arrival rate is only dependent on time t, and the purchase rate depends on customers' reservation prices. Assuming that customers whose reservation prices are higher than or equal to p will buy, Zhao and Zheng (2000) characterized the demand as a nonhomogeneous Poisson process with the arrival rate $\lambda(p, t) = \lambda(t)(1 - F(p, t))$.

Generalizing the model in Zhao and Zheng (2000), Xu and Hopp (2009) considered the price trades in the dynamic pricing setting from the martingale perspective. The authors assumed that a firm has a stock level of $n > 0$ items at time 0, which will be sold over the period $[0, T]$. Similar to Zhao and Zheng (2000), customers' arrival follows a nonhomogeneous Poisson process with intensity $\lambda(t)$ where $t \in [0, T]$. But, different from Zhao and Zheng (2000), Xu and Hopp (2009) assumed that each customer arriving at time t has a utility $U(p, t) = V(t) - \alpha(t)p$ where $V(t)$ is the customer's reservation price and $\alpha(t)$ is the price sensitivity parameter, and the customer purchases the product if $U(p, t) \geq 0$. The total demand at time t given the price p is thus computed as, $\lambda(p, t) = \lambda(t)[1 - F(\alpha(t)p, t)]$.

Remark 4 The general Poisson flow models—which take into account the customers' willingness-to-pay (WTP)—extend the early WTP models in several important directions. These models allow for, (i) the possibility of time-varying valuation, i.e., $V(t) \geq p(t)$, (ii) the time-dependent price $p(t)$, and (iii) the time-and-price dependent arrival rates $\lambda(p, t)$. Such generalizations make these models more useful in the context of, e.g., revenue management, while making the estimation of parameters a more challenging problem. ⧫

2.2 Competitive Multi-Firm Price-Dependent Demand Models

We now review price-dependent demand models when there is price competition among multiple firms where a price increase by one firm reduces the demand for its product but increases the demand for the competitors' products. Since the classic demand models for homogenous products were comprehensively discussed in the literature (see, for example, Tirole, 1988 ; Vives, 1999), this section only focuses on reviewing the demand models for firms' pricing competition with product differentiation.

Suppose that $n \geq 2$ firms serve a common market with n competing products. When firm $i = 1, 2, \ldots, n$ sets the price of its product as p_i, the demand for firm i's product is written as $d_i(\mathbf{p})$, where $\mathbf{p} \equiv (p_1, p_2, \ldots, p_n)$. As in most publications, we assume that n firms have complete market information. In addition, we find that the price-dependent demand $d_i(\mathbf{p})$ is commonly assumed to satisfy the following conditions (see, e.g., Bernstein and Federgruen, 2004 ; Chen, Yan, and Yao, 2004):

1. Firm i determines its price p_i on $\Lambda_i = [c_i, \bar{p}_i]$, where $\bar{p}_i$ denotes the maximum admissible value of p_i such that $d_i(\mathbf{p})|_{p_i=\bar{p}_i} = 0$, and c_i denotes firm i's unit acquisition cost.
2. $d_i(\mathbf{p})$ is continuous, bounded, and differentiable on the strategy space $\Lambda = \Lambda_1 \times \Lambda_2 \times \cdots \times \Lambda_n$.
3. $d_i(\mathbf{p})$ is decreasing in firm i's price p_i, i.e., $\partial d_i(\mathbf{p})/\partial p_i < 0$; and $d_i(\mathbf{p})$ is increasing in firm j's price p_j, i.e., $\partial d_i(\mathbf{p})/\partial p_j > 0$, for $j \neq i$.
4. $\sum_{j=1}^{n} \partial d_i(\mathbf{p})/\partial p_j < 0$, and $\sum_{j=1}^{n} \partial d_j(\mathbf{p})/\partial p_i < 0$, for $i = 1, 2, \ldots, n$. The former assumption assures that a *uniform* price increase by n firms results in a decrease in the demand for each firm's product. The latter assumption means that, as a firm increases its price, the aggregate demand for all firms' products is reduced.
5. The (negative) local price elasticity of $d_i(\mathbf{p})$, $e_i \equiv -[\partial d_i(\mathbf{p})/\partial p_i]/[d_i(\mathbf{p})/p_i]$, is increasing in p_i, i.e., $\partial e_i/\partial p_i \geq 0$. But, e_i is decreasing in p_j, i.e., $\partial e_i/\partial p_j \leq 0$, for $j \neq i$. Under the assumption of $\partial e_i/\partial p_i \geq 0$, the demand $d_i(\mathbf{p})$ has increasing price elasticity (IPE) in p_i; most price-dependent models (e.g., linear, power, etc.) satisfy this assumption. With the assumption $\partial e_i/\partial p_j \leq 0$ ($j \neq i$), Milgrom and Roberts (1990) showed that $\ln[d_i(\mathbf{p})]$ is supermodular in (p_i, p_j); and Chen, Yan and Yao (2004) proved that a newsvendor game—where n firms each jointly making its pricing and inventory decisions compete with their pricing strategies in a single-period setting—is supermodular.
6. $\partial e_i/\partial p_i + \sum_{j\neq i}(\partial e_i/\partial p_j) \geq 0$, for all $j \neq i$. Since, $\partial e_i/\partial p_i \geq 0$ and $\partial e_i/\partial p_j \leq 0$ ($j \neq i$), this assumption is equivalent to, $\partial e_i/\partial p_i \geq \sum_{j\neq i} |\partial e_i/\partial p_j|$ which says that the impact of firm i's own price on its elasticity dominates that of other firms' prices on firm i's elasticity which describes the substitution effect.

Letting $C_i(d_i(\mathbf{p}))$ denote the total acquisition cost incurred by firm i when it obtains $d_i(\mathbf{p})$ units of its products, we write the firm's profit as,

$$\pi_i(\mathbf{p}) = p_i d_i(\mathbf{p}) - C_i(d_i(\mathbf{p})), \tag{3}$$

which was first used by Bertrand (1883) to investigate a duopoly problem in which two firms make their pricing decisions to compete for market demand. Edgeworth (1922, 1925) extended the model (3) to investigate the problem where each firm with a limited capacity is assumed to serve only a subset of a market. If the lower-priced firm cannot satisfy the demand for its product, then the unserved consumers will buy from the high-priced firm. Such a demand model is called "Edgeworth demand" (see, e.g., Dixon, 1987), which was used by Edgeworth to show that the pure strategy equilibrium may not exist unless demand is highly elastic. Hotelling (1929) developed the demand systems with product differentiation based on heterogeneous tastes of consumers and under Hotelling's demand systems, Chamberlin (1933) and Robinson (1933) discussed imperfect competition with product differentiation.

When both $d_i(\mathbf{p})$ and $C_i(d_i(\mathbf{p}))$ in (3) are linear functions, it is easy to derive the equilibrium solution. However, when $d_i(\mathbf{p})$ and/or $C_i(d_i(\mathbf{p}))$ are nonlinear, then it may be difficult to find the analytic solution for the equilibrium, but conditions for the existence and uniqueness of the equilibrium are available. Cachon and Netessine (2004) indicated that the *quasi-concave method* and the *supermodular method* can be used for proving the existence of equilibrium solution; see Topkis (1979, 1998) for supermodularity. They also showed that the *algebraic argument*, the *contraction mapping*, the *univalent mapping*, and the *index theory approach* have been widely used to examine the uniqueness of the equilibrium. For additional discussions about the existence of equilibrium solutions for price competition games, see Vives (1999).

Remark 5 The demand model $d_i(\mathbf{p})$, for $i = 1, 2, \ldots, n$, can be derived from the representative consumer's utility (Anderson et al., 1992). Specifically, to derive the linear model, one usually needs to solve the following maximization problem (Anderson et al., 1992, and Vives, 1999):

$$\max_{\mathbf{q}} \Lambda(\mathbf{q}) \equiv u(\mathbf{q}) - \mathbf{p}^T\mathbf{q}, \tag{4}$$

where $u(\mathbf{q})$ is a representative consumer's utility, which is smooth and strictly concave; that is, the Hessian matrix of u, H_u, is negative definite; $\mathbf{q} \equiv (q_1, \ldots, q_n)$ is a vector representing the quantities purchased by a representative consumer, and $\mathbf{p}^T \equiv (p_1, \ldots, p_n)$ is firms' retail prices. The inverse demand functions can be obtained from the first-order conditions of $\Lambda(\mathbf{q})$**:** $p_i = \partial u_i(q)/\partial q_i$ for $q_i > 0$, $i = 1, 2, \ldots, n$; and inverting these inverse demand models, we find the direct demand functions. These demand models should have the symmetric cross effects: $\partial p_i/\partial q_j = \partial p_j/\partial q_i$, for $i \neq j$, and the downward-sloping property, $\partial p_i/\partial q_i < 0$. Furthermore, the utility must satisfy the condition, $\partial^2 u/\partial q_j \partial q_i = \partial p_i/\partial q_j \leq 0$, for $i \neq j$, if the products are substitutes, and $\partial^2 u/\partial q_j \partial q_i =$

$\partial p_i/\partial q_j \geq 0$, for $i \neq j$, if they are complements. ♦

2.2.1 Linear Model

We now present an example of the approach used in (Anderson et al., 1992; Vives, 1999) to obtain a linear demand function. Assume that n firms compete in a market, and each produces a differentiated good. We consider the following consumer's utility function,

$$u(\mathbf{q}) = \alpha \sum_{i=1}^{n} q_i - \frac{1}{2}\beta \sum_{i=1}^{n} q_i^2 - \gamma \sum_{j \neq i} q_i q_j, \quad \text{for } i = 1, 2, \ldots, n \text{ and } i \neq j, \tag{5}$$

where $\alpha > 0$, $\beta > 0$ denotes the impact of the price of product i on the demand for this product, and $\gamma > 0$ is interpreted as the substitutability measure. Note that the parameters β and γ should be set such that $\beta > \gamma$, because the demand for product i should be more sensitive to its own price changes than to that of the other product, and that the total market demand should not increase in the product prices.

Substituting this utility function into the optimization problem in (4) and solving, we obtain the following inverse linear demands and direct demands, respectively,

$$p_i(\mathbf{q}) = \alpha - \beta q_i - \gamma \sum_{j \neq i} q_j, \quad \text{and} \quad d_i(\mathbf{p}) = a_i - b_i p_i + \delta_i \sum_{j \neq i} p_j, \tag{6}$$

where $a_i = \alpha/[\beta + (n-1)\gamma]$, $b_i = [\beta + (n-2)\gamma]/[(\beta + (n-1)\gamma)(\beta - \gamma)]$, and $\delta_i = \gamma/[(\beta + (n-1)\gamma)(\beta - \gamma)]$.

Singh and Vives (1984), and Vives (1999) presented a special case of the above general demand model where two firms compete in a market, and each produces a differentiated good. The consumer's utility function similar to (5) is given as,

$$u(q_1, q_2) = \alpha_1 q_1 + \alpha_2 q_2 - (\beta_1 q_1^2 + 2\gamma q_1 q_2 + \beta_2 q_2^2)/2,$$

where $\alpha_i, \beta_i > 0$, and $\alpha_i \beta_j - \gamma \alpha_i > 0$, for $i = 1, 2$, and $i \neq j$. To ensure the strict concavity of the utility function, the parameters must satisfy, $\eta \equiv \beta_1 \beta_2 - \gamma^2 > 0$. Using this utility functions into the optimization problem in (4), we obtain the inverse and direct demand functions as $p_i = \alpha_i - \beta_i q_i - \gamma q_j$ and $d_i = a_i - b_i p_i + \delta p_j$, respectively, where $a_i = (\alpha_i \beta_j - \gamma \alpha_i)/\eta$, $b_i = \alpha_j/\eta$, and $\delta = \gamma/\eta$. Moreover, if $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2 = \gamma$, the two products are perfect substitutes, and when $\alpha_1 = \alpha_2$, the ratio $\gamma^2/(\beta_1 \beta_2)$ measures the degree of product differentiation.

Remark 6 As an alternative to the substitutability measure γ, some researchers in the marketing and operations areas have adopted the parameter δ (representing the cross-price effect) to measure the product substitutability. To show the difference between these substitutability measures, Lus and

Muriel (2009) have compared them, and showed that using δ instead of γ as the measure of the product substitutability may lead to unrealistic effects that cannot be supported empirically. ⧫

2.2.2 Attraction Model

Some researchers have proposed another novel demand model, known as the attraction model, which corresponds to each product's market share. The attraction model was developed axiomatically by Luce (1959) based on simple assumptions about consumer behavior, and has been extensively used to estimate the demand in the economics, marketing (Anderson, 1992; Mahajan and van Ryzin, 1998) and operations management (Bernstein and Federgruen, 2004; So, 2000). The attraction demand model is typically written as,

$$d_i(\mathbf{p}) = \frac{M v_i(p_i)}{v_0 + \sum_{j=1}^{n} v_j(p_j)}, \quad \text{for } i = 1, 2, \ldots, n, \tag{7}$$

where M is the potential number of customers in a market; v_0 is a non-negative constant which can be interpreted as the attractiveness of the no-purchase option and may be used to determine whether or not the total demand M is satisfied by n firms; and $v_i(p_i)$ $(i = 1, 2, \ldots, n)$ denotes firm i's attractiveness value that is a measure of the attractiveness of firm i's product. Note that the term $v_i(p_i)$ possesses the properties that $v_i(p_i) \geq 0$ and $\partial v_i(p_i)/\partial p_i < 0$, and it can be used to characterize firm i's market share under the assumption that the expected demand for firm i's product is proportional to the product's attractiveness.

In (7), $v_i(p_i)/[v_0 + \sum_{j=1}^{n} v_j(p_j)]$ is a typical logistic model (see, e.g., Bowerman and O'Connell, 2007) which characterizes the probability that a consumer chooses product i among n products. Thus, $d_i(\mathbf{p})$ is the expected demand for product i. When $v_0 = 0$, all firms satisfy the demand $\sum_{i=1}^{n} d_i(\mathbf{p}) = M$, which means that all customers are served by the firms. But, when $v_0 > 0$, the total demand of all firms is $\sum_{i=1}^{n} d_i(\mathbf{p}) < M$, which implies that some customers are lost. Note that in the above attraction model (7), each firm's market share should be nonnegative, and when $M = 1$ and $v_0 = 0$, the sum of all products' market shares equals unity, which is commonly known as *the logical-consistency requirements* (Cooper and Nakanishi, 1988).

In our review we find that a number of publications assumed specific functions for the term $v_i(p_i)$ in the attraction model of (7). Note that in many applications, the constant v_0 is assumed to be zero in order to be consistent with the logical-consistency requirements that the sum of all firms' market shares is equal to one (Cooper and Nakanishi, 1988). The commonly-used specific functions of $v_i(p_i)$ are summarized as follows:

1. **MultiNomial Logit (MNL) model** $v_i(p_i) = \exp(\alpha_i - \beta_i p_i)$: Here, $a_i > 0$ can be interpreted as the consumer's expected utility for the product, and $\beta_i > 0$ is the price sensitivity parameter. This model can be derived from the MNL consumer choice model (see, e.g., Guadagni and Little, 1983), and the representative consumer's utility (see Anderson et

al., 1992; Vives, 1999). Actually, MNL model gives the probability that a consumer would choose a product i among n available products, and thus, it is usually regarded as the expected demand for product i. Define the own-brand and cross-brand elasticities as follows, respectively, $e_i = [\partial d_i(\mathbf{p})/\partial p_i][p_i/d_i(\mathbf{p})]$ and $e_{i,j} = [\partial d_i(\mathbf{p})/\partial p_j][p_j/d_i(\mathbf{p})]$. Accordingly, we can obtain the two elasticities of the above MNL model as, $e_i = \beta_i[1 - d_i(\mathbf{p})]p_i$, and $e_{i,j} = -\beta_j d_j(\mathbf{p})p_j$.

2. **Multiplicative Competitive Interaction (MCI) model** $v_i(p_i) = \alpha_i p_i^{-\beta_i}$: Here, $\alpha_i > 0$ loosely represents the quality of product i; and $\beta_i > 1$ is a measure of consumers' price sensitivity for product i. For an application of the MCI model, see, e.g., Hadjinicola (1999) and So (2000). From the above definitions of the own-brand and cross-brand elasticities, we can derive these two elasticities for the MCI model as, $e_i = \beta_i[1 - d_i(\mathbf{p})]$, and $e_{i,j} = -\beta_j d_j(\mathbf{p})$.

In empirical studies, the MNL and MCI models are transformed into a linear model in order to estimate the model parameters (see Cooper and Nakanish, 1988). Recently, Gallego et al. (2006) considered Bertrand oligopoly price competition games in which firm i's profit function is given by (3), and the demand for firm i's product is determined by (7) with $M = 1$ and the attraction value $v_i(p_i)$ as either a linear model [$v_i(p_i) = \alpha_i - \beta_i p_i$, α_i, $\beta_i \geq 0$], the MNL, or the MCI model. Assuming that the cost function $C_i(d_i(\mathbf{p}))$ in (3) is convex, Gallego et al. showed the existence of a unique Nash equilibrium for the price competition game.

Remark 7 In the literature, the MNL and MCI models are the most commonly-used attraction models. The main difference between the MNL and MCI models is the assumed pattern of the own-brand elasticity with respect to price. According to, e.g., Cooper (1993), we define the market-share elasticity as the ratio of the relative change in a brand's market share corresponding to a relative change in the price of the brand. We find that the market-share elasticity of the MNL model increases up to a specific level and then declines, while the share elasticity of the MCI model is monotonically decreasing in price. Cooper (1993) suggested that the MCI model seems to be more appropriate than the MNL model when firms compete by determining their prices, because when price approaches zero, the market-share elasticity will more likely become very large. ⧫

2.3 Other Price-Dependent Models

In addition to the linear model and the attraction model, the following models have also been used to characterize the price-dependent demand in the case of two or more competing firms.

2.3.1 Hotelling Model and Its Extensions

Hotelling (1929) proposed a price competition model (called "the linear city model") with horizontal product differentiation, with the following features: Two competing firms produce identical products or offer the same service, and the firms differ from each other because of the space location, the post-sales service, or other aspects. Consumers may display preferences given the constraints of a product characteristic space, and perceive certain brands with common characteristics to be close substitutes, and differentiate these products from their unique characteristics. For a thorough review of the Hotelling models and their extensions, see Martin (1993) and Tirole (1988).

Incorporating firms' entry decisions, Salop (1979) extended the above "linear city model" to the "circular city model" (also see, e.g., Tirole, 1988) under the following assumptions: Consumers are located uniformly on a circle with a perimeter equal to 1. Multiple ($n \geq 2$) firms are located around the circle and all travel occurs along the circle; and each consumer buys one unit of the product and incurs a unit transport cost t. Moreover, each consumer is willing to buy at the lowest cost as long as the cost does not exceed the gross surplus obtained from the product.

2.3.2 Multiple Competing Firms

Here we survey three other models when multiple firms compete with their products.

Cobb-Douglas (constant elasticity log-linear) model This model is also called multiplicative demand model, which is given as,

$$d_i(\mathbf{p}) = a_i p_i^{-\beta_i} \left(\prod_{j \neq i} p_j^{\beta_{ij}} \right), \quad \text{for} \quad i = 1, 2, \ldots, n, \tag{8}$$

where $a_i > 0$; $\beta_i > 1$ denotes the absolute elasticity of firm i's demand with respect to its own price; $\beta_{ij} \geq 0$ ($j = 1, 2, \ldots, n$, $j \neq i$) denotes the cross elasticity with respect to firm j's price. In this Cobb-Douglas model, the price elasticities β_i and β_{ij} are both price-independent constants. For applications, see, for example, Allon and Federgruen (2007), and Bernstein and Federgruen (2004). Although the Cobb-Douglas model can capture the nonlinear phenomena in the market, it has the drawback that its demand elasticity always equals the power coefficient $-\beta_i$, $i = 1, 2, \ldots, n$. This model can be linearized by taking logarithms as, $\ln d_i(\mathbf{p}) = \ln a_i - \beta_i \ln p_i + \sum_{j \neq i} \beta_{ij} \ln p_j$, which is linear in the terms $\ln d_i(\mathbf{p})$, $\ln p_i$, $\ln p_j$, and $\hat{a}_i = \ln a_i$, for $i, j = 1, 2, \ldots, n$ and $j \neq i$. Such linearization of the Cobb-Douglas model facilitates the estimation of the parameters in an empirical study.

Constant expenditure model Constant expenditure model is $d_i(\mathbf{p}) = \gamma p_i^{-1} g(p_i) / \sum_{j=1}^{n} g(p_j)$, where $\gamma > 0$; and $g(p_i)$ is a positive, strictly decreasing function of p_i, (see, Allenby et al., 1998; Vives, 1999). Common specific forms of $g(p_i)$ include, (i) the constant elasticity of substitution model (CES) $g(p_i) = p_i^{-r}$, where $r > 0$; and (ii) the exponential function $g(p_i) = \exp(-\beta_i p_i)$, where $\beta_i > 0$. With a constant expenditure model, it easily follows that the ratio of the demands for two different firms i and j is given as, $d_i(\mathbf{p})/d_j(\mathbf{p}) = p_i^{-1} g(p_i) / p_j^{-1} g(p_j)$, which depends only on the prices p_i and p_j; that is, it is independent of the prices of other firms.

Remark 8 The MNL model of Section 2.2.2 and CES model have several identical properties: (i) The demand ratios of the MNL and CES models are given, respectively, as,

$$d_i(\mathbf{p})/d_j(\mathbf{p}) = \exp\left[(\alpha_i - \beta_i p_i)/(\alpha_j - \beta_j p_j)\right] \quad \text{and} \quad d_i(\mathbf{p})/d_j(\mathbf{p}) = (p_i/p_j)^{-1-\gamma},$$

for $i, j = 1, 2, \ldots, n$ and $j \neq i$. Accordingly, these two demand ratios are both independent of the number of firms, n, and all other firms' prices (besides p_i and p_j), (ii) the cross elasticities of the MNL and CES models are, $\eta^L_{ij} = \beta_j d_j(\mathbf{p}) p_j$ and $\eta^C_{ij} = r d_j(\mathbf{p}) p_j$, respectively, which show that the cross elasticities for both the MNL and CES models are identical for all firms, $i \neq j$. For the details, see Anderson et al. (1992).

Milgrom and Roberts (1990) considered the price competition games in which each firm's profit function is given by (3) and showed that, (i) the linear, (ii) the MNL, (iii) the Cobb-Douglas, and (iv) the constant expenditure models have the log-supermodularity property, i.e., $\partial^2[\ln d_i(\mathbf{p})]/(\partial p_i \partial p_j) \geq 0$, which assures the existence of a Nash equilibrium for the non-cooperative game involving each of these four demand models. Moreover, as discussed in Bernstein and Federgruen (2004) and Milgrom and Roberts (1990), the above four models also satisfy the condition that $\partial^2[\ln d_i(\mathbf{p})]/\partial p_i^2 > \sum_{j \neq i}\{\partial^2[\ln d_i(\mathbf{p})]/(\partial p_i \partial p_j)\}$, for $i = 1, 2, \ldots, n$; which assures the uniqueness of the Nash equilibrium. ⧫

2.4 General Remarks

In this section we reviewed different price-dependent demand models and compared their advantages and disadvantages. The linear models can lead to easily-obtainable analytic solutions, but they fail to take into account complex nonlinearities in the market demand. On the other hand, the nonlinear models may represent reality more closely, but (with the exception of the iso-elastic model) they are more difficult to obtain explicit analytical results.

For the case of price competition, both the linear and the attraction models lead to easily obtainable analytical result, but the attraction models have the added advantage that they can reflect the nonlinear effects arising in competitive phenomena. Because of this advantage, attraction models have recently become the preferred choice of many researchers in exploring price competition among firms. Other competitive demand models including the Cobb-Douglas model, and constant expenditure model are more difficult to analyze but one advantage of the Cobb-Douglas model is the ease with which one can estimate its parameters in empirical studies.

3 Rebate-Dependent Demand Models

In addition to the pricing strategy, manufacturers and retailers may offer a rebate (or, promotion) to the final customers as an attempt to stimulate the demand for their products. Of course, the manufacturer may also provide a rebate to its retailer based on the retailer's sales performance. In most of the published literature researchers have assumed that the rebates provided to the customers can directly influence the consumers' purchase decisions, and thus, the demand models are usually assumed to depend on the customer rebate. The rebate paid by the manufacturer to the retailer may also indirectly affect the consumers' demand because the retailer may pass a portion of the rebate to the final customers. But in most existing papers, this rebate is usually assumed to only affect the two firms' marginal profits, rather than the consumer demand. The aim of this section is to survey the demand functions which explicitly depend on the firms' rebate rewarded to the final customers.

3.1 Demand Models

Recently, several researchers have examined the effect of a firm's rebate to customers on its demand. Arcelus et al. (2005) used additive and multiplicative stochastic models to investigate the manufacturer's rebate as an incentive to stimulate the final customer's demand, and also compared the efficiency of this rebate to that of the promotion paid by the manufacturer to retailer. The additive demand model is given as, $D(p, R) = a_0 + \gamma R - bp + \varepsilon$, and the multiplicative demand model is written as, $D(p, R) = \alpha_0 R^{\delta} p^{-\beta} \varepsilon$, where R is the rebate amount, $\alpha_0 > 0$, γ and b are the rebate and price sensitivities, respectively, with $0 < \gamma < b$, and $0 < \delta < 1 < \beta$, and ε is a price- and rebate-independent random variable. Khouja (2006) used two rebate-dependent demand functions—similar to the deterministic components of the stochastic models of Arcelus et al. (2005)—to examine the manufacturer's mail-in rebate (MIR) policy; for similar models, see Sigué (2008), Cho et al. (2009), and Demirağ et al. (2010).

Chen et al. (2007) developed a multiplicative model to examine the effect of customers' redemption behavior on the manufacturer's MIR policy. But they assumed that there exists two customer segments in which the customers may or my not perceive the price reduction by the amount of rebate. Similar to Chen et al. (2007), Khouja and Zhou (2010) also examined the effect of the consumer's redemption behavior on the manufacturer rebate decision. They assumed that the demand is dependent on whether the consumers make their purchase and redemption decisions concurrently and whether the customer's redemption effort is larger than the net cost. Similar to Chen et al. (2007), Aydin and Porteus (2008) investigated the rebates paid by the manufacturer to the retailer and by the retailer to the customer, using a more general multiplicative model for demand defined as $D(p, R) = g(p - aR)\varepsilon$, where $g(\cdot)$ is a decreasing nonlinear function, and α is the rebate sensitivity constant. Yang et al. (2010) investigated the manufacturer's optimal mail-in rebate decision when he/she also provides a suggested retail price (MSRP). Yang et al. (2010) derived the demand model from the consumer's utility which depends on the manufacturer's rebate, the MSRP as well as the retail price.

Arcelus and Srinivasan (2003) investigated the manufacturer's scanbacks and customer rebates by which the manufacturer intends to deter the retailer's forward-buying behavior. The demand under the rebate scheme is given as, $d(p, R) = \alpha_0(p - R)^\beta$, where R is the manufacturer's direct rebate to the customer, and $\alpha_0 > 0$, $\beta < -1$, $R > 0$. Considering a problem similar to that by Cho et al. (2009), Geng and Mallik (2011) examined a situation in which both the manufacturer and the retailer can offer an MIR to the end consumer. They used an additive framework to model the stochastic rebate-dependent demand $D(p, R_r, R_m) = a - b(p - \alpha R_r - \alpha R_m) + \varepsilon$, where R_r and R_m represent the retailer's and manufacturer's customer rebates, respectively; ε is the random error defined on $[\varepsilon_0, \varepsilon_1]$ with $\varepsilon_1 > \varepsilon_0 \geq 0$. In order to assure that the demand is positive, they assume that $a - bp + \varepsilon_0 \geq 0$.

Remark 9 Similar to the discussions for the linear and iso-elastic price-dependent demand model, one can easily derive the optimal results by using a linear rebate demand model (e.g., Geng and Mallik, 2011; Khouja, 2006), but such models do not reflect the nonlinearities observed in an actual rebate scheme. However, the power rebate-dependent models of Khouja (2006) and Arcelus and Srinivasan (2003) can characterize the nonlinear phenomenon, but the demand elasticity of the latter with respect to the rebate R, i.e., $[\partial d(p, R)/\partial R][R/d(p, R)] = \delta$, always equals a constant. Although the model of Arcelus and Srinivasan (2003) has a similar structure to that of Khouja (2006), the former's demand elasticity is, $[\partial d(p, R)/\partial R][R/d(p, R)] = \beta R/(p - R) > 0$ (when $P > R$), which is not a constant, but instead, depends on both the price and rebate. Moreover, when the rebate approaches zero, the power demands of Arcelus et al. (2005) and Khouja (2006) will also converge to zero, which is contradicts reality that even when there is no rebate offered, some customers would still purchase the products. Accordingly, the power model of Arcelus and Srinivasan (2003) may be more reasonable than that used by Khouja (2006). ♦

3.2 General Remarks

Our review above shows that when the rebate is offered by the manufacturer and/or the retailer to the customers, the demand function is assumed to be sensitive to both the retail price and the rebate level. Interestingly, we observe that the rebate-dependent demand models for supply chain analysis have only considered a supply chain with one manufacturer and one retailer. It appears to us that the rebate competition between/among manufacturers or retailers may be an interesting direction for future research. Our above review also shows that when considering the effect of firm's rebate on demand, most papers assumed that the demand is a linear or power function for the deterministic demands. Consequently, extending other price-dependent demand functions in Tables 1 to investigate the manufacturer's and/or the retailer's rebate policies may be worthy of devoting some attention in the future.

4 Leadtime-Dependent Demand Models

In addition to the pricing strategy, it is generally agreed that timely customer service is also a major determinant in gaining competitive advantage in today's markets. Geary and Zonnenberg (2000) reported that top performers among 110 organizations conducted initiatives not only to reduce costs and maintain reliability, but also to improve delivery speed and flexibility. Baker et al. (2001) also found that less than 10% of end consumers and less than 30% of corporate customers base their purchasing decisions on price only; for a substantial majority of purchasers both price and the delivery leadtime are crucial factors that determine their purchase decisions. Thus, in order to improve consumers' satisfaction and their competitiveness, firms should focus on fast, reliable delivery as well as the sale price.

Similar to Section 2, we first review leadtime-dependent demand models which do not involve competition. This is followed by a review of models where firms compete via their leadtimes.

4.1 Single Firm Price and Leadtime-Dependent Demand Models

When there is no leadtime competition, most authors have used the linear model, the Cobb-Douglas model, the multi-nomial logit (MNL) model and the willingness-to-pay model to characterize the leadtime-dependent demand.

4.1.1 Linear Model

Suppose that a firm determines a uniform guaranteed leadtime L, which applies to all customers served by the firm in a market. Palaka et al. (1998), and Pekgün et al. (2008) modeled the firm's operation as a simple $M/M/1$ queueing system with mean production rate μ (which is the capacity of the system) and mean arrival (demand) rate λ. With this assumption, Palaka et al. and Pekgün et al. assumed that the mean demand rate $d(p, L)$ is linearly dependent on the price p and the guaranteed delivery leadtime L, i.e.,

$$d(p, L) = a - \beta p - \gamma L, \tag{9}$$

where $a > 0$ represents the maximum attainable demand corresponding to zero price and zero leadtime; $\beta > 0$ denotes the price sensitivity of demand; and $\gamma > 0$ is the leadtime sensitivity of demand. In order to assure the positive demand for the firm, Palaka et al. (1998), and Pekgün et al. (2008) assumed that $d(c, k/\mu) = a - \beta c - \gamma k/\mu > 0$, where c denotes the firm's unit production cost, and $k \equiv 1 - \ln(1 - \alpha)$ with $\alpha \in [0, 1]$ as the firm's pre-determined (desired) service level. With suitable constraints, the optimal price and leadtime are determined. Liu, Parlar, and Zhu (2007) considered a demand model similar to that of Pekgün et al. (2008), and investigated a decentralized supply chain.

4.1.2 Cobb-Douglas Model

Another common leadtime-dependent demand function for a monopolist is the Cobb-Douglas model, which is written as,

$$d(p, L) = \lambda p^{-a} L^{-b}, \quad a, b > 0. \tag{10}$$

An example in So and Song (1998) illustrates the Cobb-Douglas model where a firm has a fixed capacity μ, and a constant unit production cost c. To find the optimal price and leadtime, the firm needs to solve the constrained maximization problem,

$$\max_{p>c,L} \pi(p, L) = \lambda(p - c)p^{-a} L^{-b}, \quad \text{s.t. } 1 - \exp\{-[\mu - d(p, L)]L\} \geq \alpha, \tag{11}$$

where $\lambda > 0$ represents the highest potential demand for the firm's product; $\alpha \in [0, 1]$ is the firm's desired service level, as discussed for the linear model in (9); $a > 0$ and $b > 0$ denote the price elasticity and the guaranteed delivery leadtime elasticity, respectively; the constraint is the same as that for the linear model in (9).

4.1.3 Multi-Nomial Logit (MNL) Model

Ho and Zheng (2004) considered a service firm which satisfies homogeneous customers' demands in a market with a fixed total demand rate Λ. The firm aims at maximizing its demand rate d, which is the product of the total demand rate Λ and the firm's market share $S(U)$ that is an MNL function of the homogeneous customer's utility U for the firm's service. That is, $d = \Lambda S(U) = \Lambda \exp(U)/[\exp(U)+A]$, where A denotes the sum of exponential values of other firms' utilities. For example, if there are two or more competing firms in the market and the firm considered i, then $A = \sum_{j \neq i} \exp(U_j)$. Such an MNL model has been widely used to characterize the market share; see, e.g., Lee and Cohen (1985), and McFadden (1980).

Ho and Zheng (2004) assumed that A is given, and utility U depends on (i) the firm's service delivery-time commitment (which is customers' maximum expectation), denoted by L; and (ii) the delivery quality by the firm as the probability that the firm's delivery time ℓ is not greater than its commitment L. As in the linear and Cobb-Douglas models, Ho and Zheng (2004) calculated the firm's service delivery quality as They develop the linear function to characterize the utility U as, $U = \beta_0 - \beta_L L + \beta_\omega \omega$, where $\beta_0 > 0$ reflects the homogeneous customers' utility drawn from the firm's leadtime-independent attributes; $\beta_L > 0$ and $\beta_\omega > 0$ reflect the customer's sensitivity to the firm's delivery leadtime commitment and that to the firm's service delivery quality, respectively. ω is the firm's service delivery quality that is determined as, $\omega = \Pr(\ell \leq L) = 1 - \exp[-(\mu - d)L]$, where μ is the firm's service rate.

4.1.4 Willingness-to-pay Model

The linear model, Cobb-Douglas (log-linear) model and the multi-nomial logit (MNL) model discussed above use aggregate demand functions to model the leadtime problem. Researchers have recently started using leadtime-dependent models based on the consumer's utility which is affected by the firm's guaranteed leadtime.

In this line of research, Li (1992) used a utility-based model to investigate a firm's pricing and quality investment problems, where the firm produces and sells a homogeneous good. Demand arises over time according to a Poisson process with intensity λ, and customers have homogeneous preference over the price p, the valuation of product (quality) q, and the product's delivery leadtime, L. The customer's utility function is $U(q, p, L) = u(q, p) - \gamma L$ where $u(q, p)$ represents the leadtime-independent utility function which is increasing in q and decreasing in p, and γ is the marginal decrease in utility for a unit increase in leadtime. Assuming that the customer's reservation price is $\bar{u}$, the customer will purchase from the firm if $E[U(q, p, L) \mid \mathcal{F}_t] \geq \bar{u}$, where $\mathcal{F}_t$ represents the information available to the customer at time t. From this condition, Li derived the firm's demand.

Zhao et al. (2012) studied a firm's optimal pricing and leadtime decisions by assuming that the consumer's utility is a linear function of the price as well as the quote lead times. They classified customers into two groups: lead time sensitive (LS), with proportion $\theta \in (0, 1)$ and price sensitive (PS) with proportion $1 - \theta$.

Remark 10 Our review of the leadtime-dependent models has revealed an interesting parallel between these models and the deterministic price-dependent models discussed in Section 2. In particular, the linear demand model LM I found in Table 1 is a simpler version of the leadtime-dependent model of (9), but the use of the latter involves several constraints in the optimization model. Similarly, the Cobb-Douglas model (10) of this section is a simplified version of the more general model (8) we reviewed in the context of price-dependent models with multiple competing firms. The multi-nomial logit model of this section was also encountered as a special case of the attraction models discussed in Section 2.2.2. Finally, the willingness-to-pay models of this section has an intimate connection to the similar model in Sections 2.1.3 and the Poisson flow model in Section 2.1.4. But different from the Poisson flow models, the willingness-to-pay models of this section do not consider the consumer's arrival time. Remarkably, what is missing in the leadtime-dependent models is the inclusion of a random factor ε that was present in all the models summarized in Table 2 in Section 2.1.2. This observation leads us to suggest that there may be several important open problems that incorporate stochastic demand environments with the leadtime-dependent demand scenarios. ⧫

4.2 Competitive Multi-Firm Leadtime-Dependent Demand Models

We now review leadtime-dependent demand models in the competing setting. The most common models include the linear model and the multi-nomial logit (MNL) model and the multiplicative competitive interaction (MCI) model.

4.2.1 Linear Model

The simplest demand model for this case was developed as a linear function. The extant relevant publications (e.g., Boyaci and Ray, 2003, 2006; Pekgün et al., 2009) assumed that each firm has a constant capacity, and that the demand faced by each firm is linearly dependent on both the price and the guaranteed delivery leadtime. That is, the linear demand model for firm i's product ($i = 1, 2$) is written as,

$$d_i(p_i, L_i; p_j, L_j) = a_i - b_i p_i - \alpha_i L_i + \beta_{ij} p_j + \gamma_{ij} L_j, \quad \text{for } i, j = 1, 2, \text{ and } i \neq j, \tag{12}$$

where $a_i > 0$ is the maximum potential demand in a market; $b_i > 0$ and $\beta_{ij} > 0$ are firm i's and firm j's ($j = 1, 2,\ j \neq i$) price sensitive parameters, respectively; and, $\alpha_i > 0$ and $\gamma_{ij} > 0$ denote firm i's and firm j's leadtime sensitive parameters, respectively. Since, compared with firm j's price p_j ($j = 1, 2$), firm i's price p_i ($i = 1, 2,\ i \neq j$) should have a higher impact on the demand for the firm's product, it is assumed that b_i is greater than β_{ij}, i.e., $b_i > \beta_{ij}$. Similarly, $\alpha_i > \gamma_{ij}$, for $i, j = 1, 2$, and $i \neq j$.

Using (12), we calculate the total demand satisfied by the two firms as, $\sum_{i=1}^{2} d_i(p_i, L_i; p_j, L_j) = \sum_{i=1}^{2} a_i - \sum_{i=1}^{2}[(b_i - \beta_{ji})p_i] - \sum_{i=1}^{2}[(\alpha_i - \gamma_{ji})L_i]$, from which we find that, as firm i increases its price p_i by \$1, the total demand is reduced by $(b_1 - \beta_{21})$ units. Pekgün et al. (2009) utilized the linear model in (12) to investigate the competition between two firms who determine their prices and leadtimes.

4.2.2 Multi-Nomial Logit (MNL) Model

In Section 4.1, we reviewed the MNL model proposed by Ho and Zheng (2004) to characterize the demand rate faced by a single firm. Ho and Zheng (2004) also investigated the duopolistic case where two firms compete for homogenous customers with their delivery-time commitments. Similar to the MNL model for a single firm in Section 4.1, we write firm i's demand rate function as,

$$d_i(\mathbf{L}) = \Lambda \frac{\exp(\beta_{0i} - \beta_L L_i + \beta_\omega \{1 - \exp[-(\mu_i - d_i)L_i]\})}{\sum_{j=1}^{2} \exp(\beta_{0j} - \beta_L L_j + \beta_\omega \{1 - \exp[-(\mu_j - d_j)L_j]\})}, \text{ for } i = 1, 2. \tag{13}$$

Ho and Zheng (2004) analyzed a two-player, simultaneous-move game in which firm i maximizes its demand rate d_i in (13). They showed that Nash equilibrium exists for the game.

The MNL model may be extended to address the oligopolistic competition where $n \geq 2$ firms compete for homogeneous customers' demands in a market with a fixed total demand rate D. As in Ho and Zheng (2004) and So (2000), we assume that firm i's demand rate d_i, which is calculated as the product of the total demand rate D and the firm's market share $S_i(u_i)$ that is an MNL function of the homogeneous customer's utility u_i for firm i's service. That is,

$$d_i(\mathbf{L}) = DS_i(u_i) = D\exp(u_i) \Big/ \Big[\sum_{j=1}^{n} \exp(u_j)\Big], \quad \text{for } i = 1, 2, \ldots, n. \tag{14}$$

where the utility u_i is assumed to have the form $u_i = \beta_i - \beta_{L_i} L_i + \beta_{\omega_i}\omega_i$, and $\omega_i \equiv [1 - e^{-(\mu_i - d_i)L_i}]$ is firm i's service quality; L_i is firm i's guaranteed service time; μ_i denotes firm i's service rate (capacity); and, $\beta_i > 0$, $\beta_{L_i} > 0$ and $\beta_{\omega_i} > 0$ are defined as the utility drawn from firm i's delivery time-independent attributes.

4.2.3 Multiplicative Competitive Interaction (MCI) Model

When $n \geq 3$ firms compete with their differentiated products, we find that the most common model for this case is the multiplicative competitive interaction (MCI). Recall that while in Section 2.2.2 (where MCI is discussed) demand is dependent on the price only, in this section demand for each firm depends on both the price and the delivery time.

So (2000) analyzed a multi-player, simultaneous-move game in which $n \geq 2$ service firms determine their prices and guaranteed delivery-times to maximize their operating profits. The firms serve a market with a fixed size $\Lambda > 0$, and compete for customers' demands that are sensitive to the prices, the delivery times, and other factors such as the firms' reputations or other service quality dimensions. Hence, any firm with the lowest price and the shortest time guarantee does not necessarily gain the entire market. So (2000) computed the demand for a firm's service as the product of the total market demand Λ and the firm's market share in the MCI function form. That is, the demand model for firm $i = 1, 2, \ldots, n$ is constructed as,

$$d_i(\mathbf{p}, \mathbf{L}) = \Lambda \left\{ \beta_i p_i^{-a} L_i^{-b} \Big/ \Big[\sum_{j=1}^{n} \left(\beta_j p_j^{-a} L_j^{-b}\right)\Big] \right\}, \tag{15}$$

where the term $\beta_i p_i^{-a} L_i^{-b} / [\sum_{j=1}^{n} (\beta_j p_j^{-a} L_j^{-b})]$ denotes firm i's market share. In (15), the attraction of firm i is denoted by $\beta_i p_i^{-a} L_i^{-b}$, where p_i and L_i are firm i's pricing and guaranteed time decisions, respectively; $\beta_i > 0$ represents the combined effect of the factors that are not related to the price and the delivery time; and, the parameters $a > 0$ and $b > 0$ are the price and time attraction factors of the market.

Since the constant market size Λ has a moderate value, So (2000) assumed that each firm's price is smaller than or equal to the maximum value $\bar{p}$ that can be charged for the available services in the market. In addition, as argued by So (2000), the demand model in (15) doesn't explicitly consider the impact of the reliability of delivering these time guarantees.

Remark 11 All three competitive models we have reviewed in this section have their parallels in Section 2.2 concerned with price competition. The multi-firm linear demand model in (6) is a special case of the multi-firm linear demand model with leadtime competition covered in this section.

Similarly, price-dependent multi-nomial logit (MNL) model of Section 2.2.2 is a variation of the leadtime-dependent MNL model of this section. Finally, the price-dependent MCI model of Section 2.2.2 is a special case of the MCI model of the current section. As explained in Remark 7, the main difference between the MNL and MCI models is the assumed pattern of the share elasticity with respect to price. Thus, even when we consider the leadtime-dependent models, the comments of Cooper (1993) related to the appropriateness of MNL vs. MCI also apply here. ♦

4.3 General Remarks

In this section we first reviewed four categories of leadtime-dependent models without leadtime competition. Roughly speaking, which model we should choose depends on whether we need to consider the effect of leadtime on consumers' choice. Specifically, if one expects to examine the effect of leadtime from an aggregate perspective, then the linear model, the Cobb-Douglas or the MNL models may be good options. However, if one intends to explore the effect of firm's leadtime on consumers' purchase choice, then the utility-based willingness-to-pay model may be appropriate. Note also that, as before, while one can obtain the optimal solutions relatively easily for the linear model, the optimal solutions for the Cobb-Douglas and MNL models may be less tractable.

For the case with leadtime competition, researchers have focussed on the linear, MCI and MNL models to describe the effect of leadtime on demand. These are aggregate models which do not reflect the effect of a firm's leadtime decisions on the consumers' utilities and thereby their purchase choice. It follows that, with increasing attention paid to consumer's behavior, it may be useful to extend the utility-based model in Section 4.1.4 to explore the leadtime competition.

5 Space-Dependent Demand Models

It has been observed by retailers that allocating a larger space to an item may increase the demand for that item (Urban, 2005). Motivated by this observation, a number of researchers have constructed space-dependent demand models which were used to investigate the space allocation problems for multiple-products. More specifically, when a retailer sells multiple products in a store, it needs to consider how the available shelf space should be efficiently allocated among the products to maximize profit. The space allocation problem for a firm could be a complex problem for the following reasons (see Corstjens and Doyle, 1981): First, from the revenue perspective, multiple products have widely-varying profit margins, space elasticities, and cross elasticities due to the complementary or substitutionary relationships among them. Second, from the cost perspective, the allocation of the shelf space among multiple products impacts the firm's operating costs; for example, the products often have different procurement, carrying, and out-of-stock costs.

A retailer's space allocation decision may, to some degree, influence its inventory level and hence its demand. As Urban (2005) has provided an extensive review of inventory-dependent demand models, we do not consider such models in our review. Instead, we survey commonly-used space-dependent demand models that were developed to determine the space allocation among $n \geq 2$ products. These models are classified into static (single-period) models to which the majority belong and dynamic (multi-period) demand models.

5.1 Static Space-Dependent Demand Models

In two early publications Anderson and Amato (1974) and Lee (1961) investigated two space allocation problems that only consider a positive space elasticity but ignore the cross-elasticities among different products in a store. In another early publication, Urban (1969) discussed the effects of the cross elasticity. The above publications used the Cobb-Douglas functions to capture the shelf space-dependent demand. A representative publication using the Cobb-Douglas model is by Corstjens and Doyle (1981), which was then extended by Bultez and Naert (1988) to construct a general shelf-space allocation model. We start our review with the general model, and then survey the Cobb-Douglas models.

5.1.1 General Model

Bultez and Naert (1988) consider a space allocation problem for a firm selling $n \geq 2$ products to customers, whose aggregate demand for product $i = 1, 2, \ldots, n$ is dependent on the space allocation in a general form. Letting $\mathbf{s} \equiv (s_1, s_2, \ldots, s_n)$ where s_i, $i = 1, 2, \ldots, n$, is the space allocated to product i, they use $d_i(\mathbf{s})$ to denote the demand for product i, which depends on the space s_j, $j = 1, 2, \ldots, n$, allocated to product j.

The demand $d_i(\mathbf{s})$ for product i is assumed to be a nonnegative, non-decreasing function of the space s_i (for $i = 1, 2, \ldots, n$), i.e., $d_i(\mathbf{s}) \geq 0$ and $\partial d_i(\mathbf{s})/\partial s_i \geq 0$. Consider the cross effects of other products that are also displayed at the firm's store, i.e., the sales of a product is impacted by both the space allocated to the product and the spaces allocated to other products. The cross effect of product j ($j = 1, 2, \ldots, n$, $j \neq i$) on the demand for product i is commonly reflected by using the cross-elasticity $\delta_{ij} \equiv [\partial d_i(\mathbf{s})/\partial s_j] \times [s_j/d_i(\mathbf{s})]$. Note that δ_{ij} may be positive or negative, depending on the nature of interactions taking place within the assortment. More specifically, if products i and j are substitutable, then $\delta_{ij} < 0$; but, if they are complementary products, then $\delta_{ij} > 0$.

5.1.2 Cobb-Douglas Models

Among the space-dependent demand models, the most commonly used is the Cobb-Douglas model that is written in the log-linear function form. As an early publication using this model, Corstjens and Doyle (1981) investigated an n product space allocation problem for a retailer who maximizes its profit that is dependent on the allocation scheme $\mathbf{s} = (s_1, s_2, \ldots, s_n)$ with s_i ($i = 1, 2, \ldots, n$) as the space allocated to product i. The Cobb-Douglas demand model developed by Corstjens and Doyle

(1981) incorporates both the individual space elasticity (i.e., "main effect") and the cross elasticities existing between different products in a store. That is, the demand for product i ($i = 1, 2, \ldots, n$) is given as,

$$d_i(\mathbf{s}) = \alpha_i (s_i)^{\beta_i} \left[\prod_{j=1, j\neq i}^{n} (s_j)^{\delta_{ij}} \right], \tag{16}$$

where $\alpha_i \geq 0$; $\beta_i \geq 0$ denotes the space elasticity of product i; and δ_{ij} means the cross-space elasticity between products i and j ($j = 1, 2, \ldots, n$, $j \neq i$). Note that δ_{ij} may be positive or may be negative, depending on whether products i and j are complementary or substitutable; and also note that δ_{ij} is not necessarily equal to δ_{ji}. Corstjens and Doyle (1981) provided a through analysis of the parameter estimation and find the optimal solution using a generalized geometric programming algorithm. Martínez-de-Albéniz and Roels (2010) recently ignored the cross elasticity and reduced the Cobb-Douglas model (16) to $d_i(\mathbf{s}) = \alpha_i (s_i)^{\beta_i}$, which could be a proper model to analyze the single-product problems.

Martín-Herrán et al. (2006) investigated a two-manufacturer, one-retailer supply chain with a space- and retail price-dependent demand model, which is simpler than (16). The two manufacturers (M_1 and M_2) with substitutable products determine their wholesale prices (w_i, $i = 1, 2$), and the retailer makes its space allocation (for the two manufacturers' products) and retail pricing decisions. The demand for the manufacturer M_i's product ($i = 1, 2$) was characterized by,

$$d_i(s_i; p_1, p_2) = \alpha (s_i)^{\gamma} (p_i)^{-\mu_i} (p_j)^{\varepsilon_i}, \quad \text{for } i, j = 1, 2,\ i \neq j, \tag{17}$$

where $\alpha > 0$; $0 < \gamma < 1$ is the constant space elasticity; p_i is the retail price for product i; and, μ_i and ε_i are the direct-price and cross-price elasticities of sales, respectively.

Urban (1998) investigated a retailer's space allocation problem, by extending Corstjens and Doyle's model in (16) to consider the demands for stocked and unstocked products in the assortment. Yang and Chen (1999) incorporated $L \geq 1$ space-independent marketing variables (e.g., price, advertising, promotion, etc.) into (16), and investigated the allocation of each of $m \geq 1$ shelves among $n \geq 1$ products.

5.2 Dynamic Space-Dependent Demand Model

Corstjens and Doyle (1983) extended the static space-dependent demand model in (16) to a dynamic (multi-period) space allocation model, assuming that the demand in period t depends on both the current space allocation decision and the demand in period $t-1$. The dynamic model in the Cobb-Douglas form was given as,

$$d_{it} = \alpha_i (s_{it})^{\beta_{it}} \left[\prod_{j=1, j\neq i}^{n} (s_{jt})^{\delta_{ijt}} \right] (d_{i,t-1})^{\lambda}, \text{ for } i = 1, 2, \ldots, n, \tag{18}$$

where d_{it} is the demand for product i in period t; $\alpha_i > 0$ is a time-independent parameter; and, β_{it} is the space elasticity of product i in period t. Moreover, in (18), s_{it} is the space allocated to product i in period t; δ_{ijt} is the cross elasticity between products i and j ($j = 1, 2, \ldots, n$, $j \neq i$) in period t; and λ is a constant elasticity, which measures how much of the goodwill effect in a period is retained in the next period, i.e., $\lambda = (\partial d_{it}/\partial d_{i,t-1})(d_{i,t-1}/d_{it})$.

5.3 General Remarks

Our review above reveals that most of the surveyed space-dependent demand models have used a Cobb-Douglas model to formulate the demand functions. The Cobb-Douglas model has the attractive property that its power coefficient can be interpreted as the demand elasticity with respect to the space. Consequently, the Cobb-Douglas model can characterize the nonlinear interactions observed in the actual market demand. However, as usual, the Cobb-Douglas model is usually less tractable in terms of obtaining explicit results for the optimal solution. For example, Corstjens and Doyle's (1981) model requires special purpose algorithms, but given the non-concavity of the objective function, the solution found may not even be globally optimal. In addition, the dynamic model in (18) can be viewed as an interesting and realistic extension of the Cobb-Douglas static model in (16). However, this dynamic model does not incorporate any state dynamics. A more general model where space is a state variable and the optimization is performed on an optimal control model may be more useful and realistic, but it may be more difficult to derive the optimal results. Martín-Herrán et al.'s (2006) optimal space and price decisions are based on the assumption that the difference between cross- and direct-price elasticities is independent of the brand, and the demand is only dependent on each brand's own space. Their model can be improved by investigating the optimal space allocation problem with a more realistic space-dependent demand model that is sensitive to both the space and price.

6 Quality-Dependent Demand Models

In this section, we review product quality- and service quality-dependent models to reflect the fact that the quality measurements for the manufacturing systems are different from those for the service systems. More specifically, the quality of a product is usually measured based on the product's performance, reliability, aesthetics, durability, etc., whereas the service quality mostly depends on convenience, assurance, time, etc.; see, e.g., Stevenson (2007). Therefore, the impacts of product quality on the demand should be different from those of service quality.

6.1 Product Quality-Dependent Demand Models

In recent years, many firms have focused on quality control of their products, and will continue with the quality improvement. However, in order to improve a product's quality, a firm manufacturing the product may have to make an R&D investment and incur a higher production cost. More specifically, as Banker et al. (1998) discussed, the investment in a quality improvement program may increase the fixed production cost due to three possible reasons: (i) a new investment for high precision, high reliability, or flexible equipment; (ii) organizational training and restructuring to implement a quality management scheme; or, (iii) additional effort to redesign the product or process to achieve the desired quality level.

To trade off the benefit and the cost, each firm (i.e., manufacturing system) may need to determine an optimal quality level for its product, which affect consumers' demands for the product.

6.1.1 Consumer's Utility-Based Models

Moorthy (1988) and Tirole (1988) considered a utility-based model where product quality is denoted by y. A consumer obtains the net surplus $u(p) = \theta y - p$ if he decides to purchase; and 0, otherwise, where $\theta > 0$ is a random variable with p.d.f. $f(\theta)$ and c.d.f. $F(\theta)$ which represents the consumers' valuations. This results in demand function, $d(p) = a[1 - F(p/y)]$.

Tirole (1988) extended the above model to investigate a utility-based demand for the case where two products (1 and 2) with different qualities y_1 and y_2 ($y_1 < y_2$) are sold in a market at prices $p_1 < p_2$. Each consumer buys either no product or only one of the two products. A consumer enjoys the surplus $u_1 = \theta y_1 - p_1$ if he or she buys a unit of product 1 with quality y_1, but obtains the surplus $u_2 = \theta y_2 - p_2$ if the consumer buys a unit of product 2 with quality y_2. In the surplus functions, $\theta > 0$ is a taste parameter that is randomly distributed over $[0, +\infty)$ with the p.d.f $f(\cdot)$ and the c.d.f. $F(\cdot)$. Denoting $\theta_i \equiv y_i/p_i$, for $i = 1, 2$, Tirole showed that when $\theta_1 \leq \theta_2$, the demand is $d_2(p_1, p_2) = a(1 - F(\theta_2))$, where a is the market size, and when $\theta_1 < \theta_2$, he found expressions for demand for low-quality and the high-quality products as, $d_1(p_1, p_2) = a[F((\hat{\theta}) - F(\theta_1)]$ and $d_2(p_1, p_2) = a[1 - F((\hat{\theta})]$, respectively, where $\hat{\theta} \equiv (p_2 - p_1)/(y_2 - y_1)$.

Zhao et al. (2009) defined a consumer's utility as a power function, i.e., $u(p) = y^{1/n} - p$. Such a model is useful to the research problems with product differentiation, because it possesses the following two properties: (i) the marginal utility has the constant elasticity $1/n - 1$ and (ii) the coefficient of relative risk aversion is $1 - 1/n$. Lauga and Ofek (2011) assumed that a product in a given category comprises two quality-related characteristics, y_1 and y_2, and developed their utility function as $u(p) = v + \theta_1 y_1 + \theta_2 y_2 - p$, where θ_1 and θ_2 represent the willingness-to-pay for qualities on characteristics y_1 and y_2, respectively. Desai (2001) incorporated both the quality (vertical) and the taste (horizontal) differentiations into a consumer's utility, assuming that each consumer may have his or her ideal product corresponding to a point on the $[0, 1]$ line segment. Similar to the Hotelling (1929) model, a consumer with valuation θ has a utility of $u(p) = \theta y - kt - p$, where t is the distance from a given product to the consumer's ideal point and k is the unit transportation cost. Chambers et al. (2006) assumed that consumers may differ in responding to price, and accordingly constructed the consumer utility function as $u(p) = y - \theta p$, where θ is the parameter reflecting consumers' heterogeneity in the price sensitivity. Chambers et al. showed that their utility model is equivalent to the Moorthy (1988) and Tirole (1988) model $u(p) = \theta y - p$.

6.1.2 Competitive Multi-Firm Linear Models

Since quality plays an important role in improving a firm's competitiveness, it would be interesting to determine the quality decisions for the firms that compete for consumers' demands. We start by discussing the linear quality-dependent demand model that was used by Banker et al. (1998) to jointly determine the prices and the quality levels for two or more competing firms.

Banker et al. (1998) first considered the duopolistic case in which two firms (i.e., firm $i = 1, 2$) compete in a common market with their pricing decisions (i.e., p_i, $i = 1, 2$) and quality levels (i.e., $y_i \geq 0$, $i = 1, 2$). The demand model for firm i is assumed to be linear in two firms' prices and quality levels, i.e.,

$$d_i(\mathbf{p}, \mathbf{y}) = k_i\alpha - \beta p_i + \gamma p_j + \lambda y_i - \mu y_j, \quad \text{for } i, j = 1, 2,\, i \neq j, \tag{19}$$

where $\mathbf{p} \equiv (p_1, p_2)$; $\mathbf{y} \equiv (y_1, y_2)$; $k_i\alpha > 0$ is the intrinsic demand potential parameter for firm i; and $\beta \in (0, 1]$ and $\lambda \in (0, 1]$ ($\gamma \in (0, 1]$ and $\mu \in (0, 1]$) are the demand responsiveness to firm i's (firm j's) price and quality, respectively. Moreover, β/γ and λ/μ are referred to as the *relative* responsiveness to price and quality, respectively, and they are both assumed to be greater than one, i.e., $\beta > \gamma$ and $\lambda > \mu$. As discussed by Tirole (1988), this is necessary because, if both firms were to raise their prices by \$1 or to decrease their quality levels by one unit, then the demand for each firm should be decreased.

In order to examine the impact of the number of competitors on quality, Banker et al. (1998) extended the two-firm demand model in (19) to a general case involving $n \geq 2$ firms. The general demand model was constructed as, $d_i(\mathbf{p}, \mathbf{y}) = \alpha_n - \beta_n p_i + \gamma_n \sum_{j \neq i} p_j + \lambda_n y_i - \mu_n \sum_{j \neq i} y_j$, for $i, j = 1, 2$, $i \neq j$.

Balasubramanian and Bhardwaj (2004) developed two-firm, price- and quality level-dependent demand models that are similar to but simpler than that in (19). That is, the basic relevant model by Balasubramanian and Bhardwaj (2004) was given as, $d_i(\mathbf{p}, \mathbf{y}) = 1 - \beta(p_i - p_j) + \gamma(y_i - y_j)$, for $i, j = 1, 2$ and $i \neq j$. Then, Balasubramanian and Bhardwaj revised their basic model, and considered the demand model in the following function form: $d_i(\mathbf{p}, \mathbf{y}) = 1 - p_i + \beta p_j + \gamma y_i - \delta y_j$, for $i, j = 1, 2$ and $i \neq j$.

Remark 12 It is important to note that when F is uniform over $[A, B]$, we find $d(p) = a[1 - F(p/y)] = a[B - p/y]/(B - A)$ which is of the same form as the aggregate linear model LM I in Table 1. Interestingly, this was the same observation we made in Remark 3 when we discussed willingness-to-pay model by Kalish (1985). Thus, two apparently different problems lead to the same structure. ♦

6.2 Service Quality-Dependent Demand Models

Many firms compete with their service quality to gain larger market shares, in addition to traditional approaches such as pricing; see, e.g., Bernstein and Federgruen (2004). The quality of service is measured somewhat differently in manufacturing and service systems. For a manufacturing system, the typical measurements of service quality include, expected order delay, the probability that an order delay doesn't exceed a quoted leadtime, and the percentage of orders fulfilled accurately. For a service system, the measurements of service quality include expected customer waiting time, the probability that a customer receives his/her requested service within a guaranteed time window, and the probability that a customer doesn't leave before being served (Benjaafar et al., 2007).

Recall that, in Section 4, we have discussed the demand models that depend on the guaranteed delivery time, which involve delivery reliability constraints. In this section we consider the service quality-dependent demand models that are not concerned with the guaranteed delivery time for both the manufacturing and the service systems. We first review the models without quality competition, and then survey those for multiple competing firms.

6.2.1 Single Firm Demand Models

For the case of a single-firm, we review demand models that characterize the impacts of the firm's service quality on the aggregate demand for the firm's product or service. Our survey has revealed that existing publications have considered a general service quality-dependent model, a specific (square-root) model and a utility model.

General Model In an early publication, Desai and Srinivasan (1995) constructed a general service level-dependent demand model to analyze demand signalling by an informed franchisor (principal) to a risk-neutral franchisee (agent) whose effort cannot be monitored. The franchisor is one of two possible types: high-demand (H) and low-demand (L). The franchisor knows its "true" type, which is unknown to the franchisee. The demand $d(p, y)$ is assumed to be linearly dependent on the price p and be also dependent on the franchisee's service level y in a general function form; that is, $d_J(p, y) = T_J - p + g(y) + \varepsilon$, for $J = H, L$, where T_J is higher for the high-demand franchisor; $g(y)$ is the demand function only of the service level y, which is assumed to be weakly concave in y; and, ε denotes an exogenous r.v. with $E(\varepsilon) = 0$. Moreover, Desai and Srinivasan assumed that the franchisee's service cost $c(y)$ is a weakly convex function of the service level y. They developed a Stackelberg game model to analyze the two-part pricing scheme between a franchisor (as the decision leader) and a franchisee (as the decision follower).

Square-Root Model Desiraju and Moorthy (1997) developed a price- and service-dependent demand model to analyze a two-level supply chain involving a manufacturer and a retailer. The authors assumed that the demand faced by the retailer is linearly dependent on the price p and is also dependent on the service level y in a square-root function form. With these assumptions, the demand was given as, $d(p, y) = \alpha - p + \gamma\sqrt{y}$, for $\alpha > 0$ and $0 < \gamma < 2$. In this model, $\partial d(p, y)/\partial y > 0$, and $\partial^2 d(p, y)/\partial y^2 < 0$, which implies the demand is increasing in the service effort y but with an decreasing returns to scale.

6.2.2 Multi-Firm Demand Models for Multiple Competing Products

We now consider the multi-firm case in which $n \geq 2$ firms compete with their service qualities to increase the demand for their products or services. As our review indicates, most publications assumed that each firm jointly determines its price and service quality, thereby constructing price- and service quality-dependent demand models. Letting $\mathbf{p} \equiv (p_1, p_2, \ldots, p_n)$ and $\mathbf{y} \equiv (y_1, y_2, \ldots, y_n)$, we denote the demand for firm i by $d_i(\mathbf{p}, \mathbf{y})$, which is assumed to satisfy the monotonicity properties,

$$\frac{\partial d_i(\mathbf{p}, \mathbf{y})}{\partial p_i} \leq 0, \ \frac{\partial d_i(\mathbf{p}, \mathbf{y})}{\partial y_i} \geq 0; \ \frac{\partial d_i(\mathbf{p}, \mathbf{y})}{\partial p_j} \geq 0, \ \frac{\partial d_i(\mathbf{p}, \mathbf{y})}{\partial y_j} \leq 0, \quad \text{for } i, j = 1, 2, \ldots, n \text{ and } i \neq j. \tag{20}$$

These assumptions are reasonable because of the following reasons: if firm i increases its retail price p_i (service quality y_i), the demand $d_i(\mathbf{p}, \mathbf{y})$ for firm i's product or service is usually decreased (increased), but if firm j (a competitor of firm i) increases its price p_j (service quality y_j), then the demand $d_i(\mathbf{p}, \mathbf{y})$ is usually increased (decreased). The specific demand function forms include the attraction model, the linear model, and the log-separable model. Since Bernstein and Federgruen (2004) used the three models to investigate a problem, we next survey the analysis by Bernstein and Federgruen (2004) to illustrate these specific models.

Attraction Model This model characterizes the demand in the logistic function form, which is similar to that in Section 2.2.2, with the exception that the attraction model in this section is dependent on both the price $\mathbf{p}$ and the service level $\mathbf{y}$. The model is typically written as,

$$d_i(\mathbf{p}, \mathbf{y}) = M v_i(p_i, y_i) \bigg/ \left[v_0 + \sum_{j=1}^{n} v_j(p_j, y_j) \right], \quad \text{for } i = 1, 2, \ldots, n, \tag{21}$$

where M is the fixed market size; v_0 is a constant parameter; and $v_i(p_i, y_i)$ is firm i's attraction value that is dependent on its own price and service quality. Moreover, $v_i(p_i, y_i)$ is commonly assumed to be strictly decreasing in p_i but strictly increasing in s_i, i.e., $dv_i(p_i, y_i)/dp_i < 0$ and $dv_i(p_i, y_i)/dy_i > 0$.

The attraction values $v_i(p_i, y_i)$ in (21) were usually assumed by existing publications to have specific structures—i.e., multinomial logit (MNL) model, multiplicative competitive interaction (MCI) model, and multiple customer segments (MCS) model—as discussed in Section 2.2.2. For example, Bernstein and Federgruen (2004) considered the infinite-horizon competition among n retailers each facing a random demand in each period, characterizing the random demand faced by retailer

$i = 1, 2, \ldots, n$ as, $D_i(\mathbf{p}, \mathbf{y}) = d_i(\mathbf{p}, \mathbf{y})\varepsilon_i$, where $d_i(\mathbf{p}, \mathbf{y})$ is given as in (21) with the attraction value in the MNL function form (i.e., $v_i(p_i, y_i) = \exp[b_i(y_i) - \alpha_i p_i]$), and ε_i is a general continuous r.v. with a stationary distribution independent of the price $\mathbf{p}$ and the service level (fill rate) $\mathbf{y}$. Benjaafar et al. (2007) developed an attraction model that is only dependent on the service quality, which is given as, $d_i(\mathbf{y}) = v_i(y_i)/\sum_{j=1}^{n} v_i(y_j)$, for $i = 1, 2, \ldots, n$, where the attraction value $v_i(y_i)$ is a non-decreasing, concave function of y_i with $v_i(0) = 0$.

Linear Model The service quality-dependent demand model in a linear function form is the simplest one. Bernstein and Federgruen (2004) and (2007) constructed a linear model to characterize the demand for retailer i ($i = 1, 2, \ldots, n$). Similar to our discussion for the attraction demand model, we next review the model of Bernstein and Federgruen (2004), which was stated as,

$$d_i(\mathbf{p}, \mathbf{y}) = a_i - b_i p_i + \sum_{j \neq i} c_{ij} p_j + \beta_i y_i - \sum_{j \neq i} \gamma_{ij} y_j, \text{ for } a_i, b_i, c_{ij}, \beta_i, \gamma_{ij} > 0, \tag{22}$$

where all parameters are assumed to be positive constants, in order to assure the monotonicity properties in (20); and $b_i > \sum_{j \neq i} c_{ij}$. In addition, letting $k_i(y_i)$ denote retailer i's expected (end-of-period) inventory cost per unit of sales for a guaranteed service level y_i, Bernstein and Federgruen (2004) assumed that $k_i'(0.5) < \beta_i/b$, for $i = 1, 2, \ldots, n$, in order to assure that retailer i cannot be "better off" by offering a fill-rate of less than 50%.

Extending the linear model in (22) where the market base a_i is a constant, Allon and Federgruen (2007) considered a linear demand model where the market base is dependent on the service levels $\{y_i, i = 1, 2, \ldots, n\}$, which is written as, $d_i(\mathbf{p}, \mathbf{y}) = a_i(y_i) - b_i p_i + \sum_{j \neq i}(c_{ij} p_j) - \sum_{j \neq i} \alpha_{ij}(y_j)$, where the service-dependent market base $a_i(y_i)$ is assumed to be three times differentiable, increasing, and concave in the service level y_i, $\alpha_{ij}(y_j)$ is a non-decreasing and differentiable cross-term function. Moreover, such a demand function should satisfy the following two properties: (i) a uniform price increase by all firms cannot lead to an increase in the demand faced by each firm, i.e., $b_i > \sum_{j \neq i} c_{ij}$; and (ii) an price increase by a firm cannot result in an increase in the aggregate demand of all firms, i.e., $b_i > \sum_{j \neq i} c_{ji}$.

Log-Separable Model Bernstein and Federgruen (2004) assumed that, in the log-separable model, a regular system of price-dependent demand functions $\{q_i(\mathbf{p}), i = 1, 2, \ldots, n\}$ is scaled up or down as a function of the service levels y_i ($i = 1, 2, \ldots, n$) offered by the different retailers. That is, the price- and service level-dependent demand model in the log-separable function form is given as, $d_i(\mathbf{p}, \mathbf{y}) = \psi_i(\mathbf{y}) q_i(\mathbf{p})$, where $\psi_i(\mathbf{y})$ and $q_i(\mathbf{p})$ are assumed to satisfy the monotonicity properties in (20). Moreover, Bernstein and Federgruen (2004) assumed that the function $\psi_i(\mathbf{y})$ is log-supermodular in (y_i, y_j) ($i, j = 1, 2, \ldots, n$, $i \neq j$) and the function $q_i(\mathbf{p})$ is also log-supermodular in (p_i, p_j) ($i, j = 1, 2, \ldots, n$, $i \neq j$).

As Bernstein and Federgruen discussed in (2004), the function $\psi_i(\mathbf{y})$ may be normalized to one or may be proposed in a logit function form; and the function $q_i(\mathbf{p})$ may be the linear, the logit, the Cobb-Douglas, or the constant elasticity of substitution (CES) model. Note that the first three possible models for $q_i(\mathbf{p})$ has been discussed in Section 2; and the CES model, developed by Dixit and Stiglitz (1977), is typically given as, $q_i(\mathbf{p}) = \gamma(p_i)^{r-1}/\sum_{j=1}^{n}(p_j)^r, \quad$ for $r < 0$ and $\gamma > 0$.

6.3 General Remarks

This section surveyed the quality-dependent demand models. Our survey has revealed that most commonly-used product quality-dependent models include the utility-based model (for the single firm setting) and the linear model (for the competitive setting). In the utility-based model, the consumers are usually assumed to be heterogeneous on their valuations on the product quality. The linear model, on the other hand, is used to describe the effect of quality on demand from an aggregate perspective; that is, the consumers are assumed to have homogeneous valuations.

The present section also reviewed the commonly-used service quality-based demand models. In order to examine a monopolistic firm's optimal service problem, we can use the general and square-root model. Both of these two models are aggregate demand functions. But so far, few researchers have used the service quality-based demand model by considering the consumer's behavior, which may be worthy of further investigation. On the other hand, to capture the firms' optimal investment decisions on the service levels in a competitive setting, the demand functions used in the literature include the attraction model, the linear model and the log-separable model. The attraction model is a market share response model, which implies that an estimate of a brand's market share lies between 0 and 1, and naturally, the sum of the estimated market share for all brands at any given time equals one. Different from the attraction model, the linear and log-separable models reflect the actual demands in the market.

7 Advertising-Dependent Demand Models

Advertising is a common, effective marketing approach used to promote products to customers. As discussed by Banerjee and Bandyopadhyay (2003), the advertising competition and its impacts on consumer behavior and market performance usually have two major descriptions: (i) Advertising can be regarded as a channel that provides valuable information to customers who can then reduce informational product differentiation and make rational choices, (ii) advertising is also a device that persuades customers by means of intangible and/or psychic differentiators, thereby creating differentiation among products.

In practice, a firm may need to determine how much to spend on advertising and/or how to allocate its expenditure over time [that is, should advertising be static or dynamic (turned on and off) for a multi-period problem?] Thus, a proper advertising decision is important for a firm to succeed in a competitive market (Mesak and Mwans, 1998). In economics and marketing fields, models have been developed to characterize the impacts of the advertising on the demand in a static or dynamic setting, which is also the classification we use in our review.

In the past several decades, a number of dynamic models were proposed to examine the advertising-related problem; those models include the diffusion model, the Lanchester model, the Nerlove-Arrow model, the Vidale-Wolfe model, etc., which were reviewed in Erickson (2003), Feichtinger et al. (1994), Mahajan et al. (1990), and Sethi (1977). Thus, we limit our review to static demand models, which include, for the non-competitive case, the general and the log-separable (i.e., log-linear) models. For the competitive case, we discuss the multiplicative competitive interaction (MCI) and the multinomial logit (MNL) demand models.

7.1 Non-competitive Advertising-Dependent Demand Models

This section reviews the single-firm (non-competitive) demand models.

7.1.1 General Model

In an early paper, Dorfman and Steiner (1954) developed a formal theory of the optimal advertising problem for a monopolistic firm which wants to determine its price p and the advertising expenditure A for its product. They denote the market demand as, $d(p, A)$, with $\partial d/\partial p < 0 < \partial d/\partial A$, which implies that the demand is increasing in the advertising expenditure A but is decreasing in the price p. The monopolist's variable production cost is, $C(d)$, with $C' > 0$ so that the monopolist's profit is given as, $\Pi(p, A) = pd(p, A) - C(d) - \eta A$, where η is the cost of per unit advertisement. The optimal price and advertising decisions are determined by the first-order conditions, given the second-order conditions hold.

Pepall et al. (1999) presented two examples for the above-mentioned general demand $d(p, A)$. In the first example, they derived the demand model from the consumer's utility function. Assume that the consumers are vertically differentiated with respect to the effect of the advertising on their valuations. Accordingly, a consumer with type θ obtains the net surplus $u(p) = \theta g(A) - p$ if he or she purchases one unit of product, where $g(0) = 1$ and $g'(A) > 0$, and zero, otherwise. Letting $a > 0$ be the potential market size and assuming that the random variable θ is uniformly distributed over $[0, 1]$, the market demand is given as, $d(p, A) = a \int_{p/g(A)}^{1} 1 \cdot d\theta = a[1 - p/g(A)]$.

In the second example, a consumer may or may not receive the advertising, and thus, denote the probability that a consumer receives no advertising as, $(1 - 1/a)^A \approx e^{-A/a}$, for a large a. For this case, the demand is given as, $d(p, A) = a(1 - e^{-A/a})\hat{d}(p) \equiv G(A)\hat{d}(p)$, where $\hat{d}(p)$ is the impact of the price on demand with the properties such that, $\hat{d}'(p) < 0$ for $p \in [0, \bar{p}]$ and $d(\bar{p}) = 0$.

Nguyen (1985) examined a firm's optimal advertising decision, assuming the random demand for the firm's product in the additive form as $d(A) = g(A) + \varepsilon$, where A is the firm's advertising level, and $g(A)$ is a constant sales response function with $g'(A) > 0$ and $g''(A) \leq 0$; and ε is a r.v. with mean zero and known variance. Nguyen identified the conditions under which whether the demand uncertainty affect the firm's advertising decisions. Lariviere and Padmanabhan (1997) considered the manufacturer's optimal slotting allowance decision when he introduces a new product and sells it through a downstream retailer, where the demand is influenced by both the price and retailer's marketing effort (advertising). They constructed an additive demand model as, $d(p, A) = a - \beta p + g(A)$, where $a > 0$ is the market size; $\beta > 0$ is the price sensitivity of demand; $g(A)$ is the response function of sales with the properties that $g(A) \geq 0$, $g'(A) > 0$, and $g''(A) < 0$.

7.1.2 Log-Separable Model

Recent publications have considered the advertising cooperation between a manufacturer and a retailer, where the manufacturer and the retailer decide to invest on the national advertising (A_1) and local advertising (A_2), respectively. Moreover, the manufacturer decides whether to pay for a portion or the entire advertising cost for a retailer (see, e.g., SeyedEsfahani et al., 2011; Xie and Neyret, 2009). Similar to the second demand model of Pepall et al. (1999), they assumed that the demand has the multiplicative form, $d(A_1, A_2, p) = ad_1(A_1, A_2)d_2(p)$, where a is the base demand, $d_1(A_1, A_2)$ and $d_2(p)$ capture the impact of the advertising investments and retail price on the demand, respectively. In the published literature, the advertising effect $d_1(A_1, A_2)$ is usually assumed to have a power form, $\alpha - \beta(A_1)^{-\gamma}(A_2)^{-\delta}$ (e.g., Szmerekovsky and Zhang, 2009; Xie and Neyret, 2009), or a square-root function, $k_1\sqrt{A_1} + k_2\sqrt{A_2}$ (e.g., Xie and Wei, 2009; SeyedEsfahani et al., 2011). The effect of price on demand, $d_2(p)$, is usually assumed to be a linear function, $a - bp$ (e.g., Xie and Neyret, 2009; Xie and Wei, 2009), or a nonlinear function having the iso-elastic form, $p^{-\gamma}$, $\gamma > 1$ (e.g., Szmerekovsky and Zhang, 2009).

Remark 13 All of the aforementioned general and separable demand models are assumed to be increasing and concave in the advertising expenditure, which is the common "advertising saturation effect." Note that in the stochastic model of Nguyen (1985), the randomness is incorporated into the demand function as an additive form. Accordingly, similar to the discussions for the stochastic price-dependent models in Section 2.1.2, one can also investigate the firm's advertising decision where the demand error is modeled as a multiplicative form, and also compare these results with those obtained by using the additive demand model. Moreover, one can also use the general framework of Kocabıyıkoğlu and Popescu (2011) to investigate the firm's optimal advertising decision.

Our review above also indicates that the demand models for the advertising cooperation in supply chain analysis is usually modeled in the separable form, where the researchers characterize the effect of advertising by the power model, $d_1(A_1, A_2) = \alpha - \beta(A_1)^{-\gamma}(A_2)^{-\delta}$, or the square-root function, $d_1(A_1, A_2) = k_1\sqrt{A_1} + k_2\sqrt{A_2}$. Although these two models are both increasing and concave in the advertising expenditures A_1 and A_2, there exists an important difference between the two models.

Specifically, in the power model, the function $d_1(A_1, A_2)$ has an upper bound α, i.e., $d_1(A_1, A_2) < \alpha$, but when the advertising levels A_1 or A_2 are sufficiently small, the function $d_1(A_1, A_2)$ assumes negative values, which is not realistic. On the other hand, demand in the square-root model increases without bound as the advertising levels A_1 or A_2 increases, which is also not realistic because the market size should be bounded from above for any advertising level. Finally, when the two firms' advertising levels are zero, i.e., $A_1 = A_2 = 0$, the square-root function $d_1(A_1, A_2)$, and hence, the consumers' demand $d(A_1, A_2, p)$ in Section 7.1.2 will be also zero, which is another drawback of the square-root model. ♦

7.2 Competitive Multi-Firm Advertising-Dependent Demand Models

In this section we review two important competitive models.

7.2.1 Multi-Nomial Logit (MNL) Model

Gruca and Sudharshan (1991) used an MNL model to examine the multi-firm competition problem in which $n \geq 2$ firms determine their prices $\mathbf{p} = (p_1, p_2, \ldots, p_n)$ and marketing (advertising) expenditures $\mathbf{A} = (A_1, A_2, \ldots, A_n)$. Their MNL demand model is given as, $d_i = V_i / \sum_{j=1}^{n} V_j$, where $V_i = a_i \exp(\alpha_i p_i + \beta_i A_i)$; and p_i and A_i is the price and advertising effort of firm i, respectively. Gruca and Sudharshan identified the conditions for the existence of the Nash equilibrium.

Extending the model of Gruca and Sudharshan (1991), Basuroy and Nguyen (1998) assumed that the attraction value of firm i's product is dependent on both the price p_i as well as the advertising expenditure A_i, and is calculated as, $V_i(p_i, A_i) = \exp[f_i(p_i) + g_i(A_i)]$, for $i = 1, 2, \ldots, n$, where $f_i(p_i)$ and $g_i(A_i)$ denote the price response function and the function relating awareness to advertising expenditure, respectively. Moreover, in order to assure the existence of a Nash equilibrium, Basuroy and Nguyen (1998) assumed that $f_i'(\cdot) < 0$, $f_i''(\cdot) = 0$; and $g_i''(\cdot) < 0$, for $i = 1, 2, \ldots, n$; and used some examples in the marketing literature to support their assumption that $g_i''(\cdot) < 0$. In particular, the functions $f_i(p_i)$ and $g_i(A_i)$ were specified as, $f_i(p_i) = -\beta p_i$ ($\beta > 0$) and $g_i(A_i) = \gamma \ln A_i$ ($\gamma > 0$), where Basuroy and Nguyen investigated the long-run market equilibrium.

7.2.2 Multiplicative Competitive Interaction (MCI) Model

A common advertising-dependent model is given in the MCI function form as discussed by Mills (1961). Assume that there are $n \geq 2$ firms competing in a market by determining their advertising expenditures. Mills considered the MCI demand model, $d_i = aV_i / \sum_{j=1}^{n} V_j$, where a is the potential market size, and $V_i = \eta_i A_i^{\beta}$, for $i = 1, 2, \ldots, n$, and $\beta > 0$ measures the market share sensitivity with respect to the advertising expenditure; and $\eta_i > 0$ measures the relative consumer preference for firm i. Mills developed a finite algorithm to determine the Nash equilibrium among n firms. For another application of the MCI demand model, see Karnani (1983) who considered the general "marketing activity" rather than only the advertising expenditure. Cooper and Nakanishi (1988) considered a MCI model in which the demand is dependent on both the firms' retail prices, the advertising expenditures as well as the distribution efforts; that is, $V_i = p_i^{\alpha} A_i^{\beta} B_i^{\gamma}$, for $i = 1, 2, \ldots, n$, where p_i, A_i and B_i are the retail price, advertising expenditure, and distribution effort of firm i.

Remark 14 We note that the MNL and MCI models are the two commonly-used attraction models for the advertising competition among multiple firms. Similar to our discussions in Remark 7, we know that the main difference between the MNL and MCI models results from the share elasticity with respect to the advertising level. Specifically, the share elasticity of the MNL model increases up to a specific level and then declines in advertising, while the share elasticity of the MCI model is monotonically decreasing in advertising. But different from the price-dependent models, Cooper (1993) suggested that the MNL model seems to be more appropriate than the MCI model when firms compete by deciding their advertising levels. ⧫

7.3 General Remarks

This section reviewed the advertising-dependent demand models commonly used in the literature. The survey of the demand models without competition reveals that the demand can be derived from the consumer's utility function or modeled as an aggregate model. Which model we choose depends on whether we attempt to consider the consumer's choice. The above review also shows that most of the advertising-dependent models belong to the class of aggregate models, except for the first demand example of Pepall et al. (1999).

For the competitive case, we have seen that most models only consider the advertising competition among individual firms, not from the supply chain perspective where each chain consists of manufacturer(s) and retailer(s). A potential research topic could incorporate the competitive decisions where the manufacturer makes the national advertising decisions and the retailer is responsible for local advertising and the retail price decisions.

8 Empirical Studies of Demand Models

Our survey in the preceding sections indicates that a considerable number of demand models have been developed to investigate various problems arising in economics, marketing, and operations fields. Many of these models have been empirically tested to forecast the aggregate market demand of potential consumers to help decision makers. In this section, we focus on the review of the empirical literature involving demand functions, examine the suitability of those demand functions in the empirical studies, and also present our suggestions regarding the applications of demand functions in the empirical research.

We note that there is a large number of published empirical studies concerning advertising models which have examined the static or dynamic impact of advertising effort on the demand; see, for example, the reviews by Erickson (2003), Feichtinger et al.(1994), Huang, Leng, and Liang (2012), Leeflang et al. (2000), Mahajan et al. (1990), and Sethi (1977). Since these papers have already reviewed advertising-dependent demand models, we do not review such papers in this section.

8.1 Empirical Studies of Price-Dependent Demand Models

A number of researchers have used the linear, exponential, power, MNL, and MCI models to forecast, measure, and test the effect of retail price on the market demand. Genesove and Mullin (1998) conducted an empirical study for the sugar industry, adopting a demand model in the form $d(p) = \beta(\alpha - p)^{\gamma}$, which can be, (i) a linear model when $\gamma = 1$, (ii) a log-linear model when $\alpha = 0$ and $\gamma < 0$, (iii) an exponential model when $\alpha, \gamma \to \infty$ and α/γ constant, (iv) a quadratic model when $\gamma = 2$. The non-linear functional forms (ii), (iii), and (iv) were transformed into $\ln d = \ln(-\beta) + \gamma \ln p$, $\ln d = \ln \beta + \gamma p/\alpha$, and $\ln d = \ln \beta + 2\ln(\alpha - p)$, respectively. Genesove and Mullin (1998) considered both a high season for sugar—which starts at the end of May and reaches its peak in September—and a low season which includes other months. For the two seasons, the authors estimated the parameters, and calculated standard errors of the estimates (SEEs) to test the performance of the above four demand models.

The estimated parameters with corresponding standard errors given in parenthesis are provided as follows: (i) $(\beta\alpha)_H = 10.74$ (1.57) and $\beta_H = 1.36$ (0.36)/$(\beta\alpha)_L = 13.37$ (1.90) and $\beta_L = 2.30$ (0.48) in the linear model; (ii) $[\ln(-\beta)]_H = 3.17$ (0.40) and $\gamma_H = -1.10$ (0.28)/$[\ln(-\beta)]_L = 4.19$ (0.65) and $\gamma_L = -2.03$ (0.48) in the log-linear model; (iii) $(\ln \beta)_H = 2.70$ (0.29) and $(\gamma/\alpha)_H = -0.26$ (0.07)/$(\ln \beta)_L = 3.52$ (0.48) and $(\gamma/\alpha)_L = -0.53$ (0.12) in the exponential model; (iv) $(\ln \beta)_H = -2.48$ (0.54) and $\alpha_H = 11.88$ (2.03)/$(\ln \beta)_L = -1.20$ (0.47) and $\alpha_L = 7.72$ (0.86) in the quadratic model. Note that the SEEs are significantly small compared with the estimates. Moreover, the above estimates indicates that for each model the demand curve in both the high and the low seasons is downward sloping. Thus, Genesove and Mullin (1998) concluded from both the statistic and practical perspectives that the four demand models can "reasonably" describe the market sales in two seasons. Furthermore, we note that in each season, the exponential model should be the best-fitting one among the four models, because the SEEs for the exponential model are the smallest. This may imply that, for the sugar industry, the exponential model could be highly applicable to empirical study.

In addition to the above, the logit demand functions have also been widely applied in empirical studies. Sudhir (2001) analyzed a distribution channel where two manufacturers sell through a common retailer to consumers, using a logit model and a multiplicative model. In the logit model, the demand for the brand i product ($i = 1, 2$) in period t was characterized as $d_{it} = \exp(U_{it})/[1 + \sum_{k=1}^{2} \exp(U_{kt})]$, where $U_{it} = \beta_{01} + \beta_f f_{it} + \beta_d y_{it} + \beta_{dp} y_{it} p_{it} + \beta_{fp} f_{it} p_{it} + \beta_{pi} p_{it} + \xi_{it}$. Note that in U_{it}, β_{0i} is the intrinsic attraction of brand i, f_{it}, y_{it}, and p_{it} denote the feature, the display, and the retail price for brand i in period t, and ξ_{it} is the unobservable component of utility. In the multiplicative demand model the demand for brand i in period t is, $d_{it} = a_i(p_{it})^{b_{ii}}(p_{jt})^{b_{ij}}(f_{it})^{f_{ii}}(f_{jt})^{f_{ij}}(y_{it})^{y_{ii}}(y_{jt})^{y_{ij}}$, where b_{ii} (f_{ii}, y_{ii}) and b_{ij} (f_{ij}, y_{ij}) denote the own-brand price (feature, display) elasticity and the cross-brand price (feature, display) elasticity of the demand for brand i, respectively. Sudhir adopted the above two demand models to empirically investigate the yogurt and the peanut butter categories for two of the biggest stores (i.e., stores 1 and 2) in a particular market, and computed log-likelihoods of the best-fitting logit and multiplicative demand models. Since there are a large number of estimated parameters in the two best-fitting demand models, we do not write the best-fitting estimates but refer readers to Tables 5a and 5b in the publication by Sudhir (2001). We note from Sudhir (2001) that in the yogurt category, the best-fitting logit and multiplicative models for store 1 have the log-likelihoods of -196.90 and –343.09, respectively; and those for store 2 have the log-likelihoods of -247.27 and –423.90, respectively. In the peanut butter category, the log-likelihoods of the best-fitting logit and multiplicative models for store 1 are -240.66 and –320.14, respectively; and those for store 2 are -212.96 and –421.39. According to the above log-likelihood statistics, Sudhir (2001) concluded that the logit demand model can fit the empirical data much better compared to the multiplicative demand model.

We also find a number of publications concerning the MNL model for the empirical studies. Besanko et al. (1998) theoretically and empirically found the prices in Nash equilibrium for multiple manufacturers and multiple retailers, using the MNL model $d_{jt} = e^{\alpha(v_{jt}-p_{jt})}/(1+\sum_{k=1}^{n} e^{\alpha(v_{kt}-p_{kt})})$, for $j = 1, \ldots, n$, where v_{jt} is the "average" consumer's maximum willingness to pay for brand j at time t and p_{jt} is the price of product j at time t. Moreover, for the empirical study, the authors used the data in the yogurt and the catsup product categories for a 102 week period in 1986–1988, which the AC Nielsen Company collected from nine stores (belonging to a single "Everyday Low Price" (EDLP) chain) in Springfield, Missouri. Besanko et al. (1998) found that the absolute value of empirical estimates of the demand elasticity with respect to price are greater than 1 except for the Yoplait brand (in the yogurt category), which is consistent with the theoretical results. For other MNL models in the empirical studies, see, e.g., Khan and Jain (2005) and Nevo (2000). A few researchers (Parks, 1969; Nakanishi and Cooper, 1982) performed their empirical studies by using the MCI models, which also belong to the category of attraction models which we reviewed in Section 2.2.2. Nakanishi and Cooper (1982) showed that the empirical estimation with the MCI model is better than that with the MNL model, which is in agreement with our Remark 7.

Remark 15 The linear and the logit models (mainly the MNL model) are the most common in the price-related empirical studies. The wide applications of the linear model may be ascribed to the easy estimation of the parameters in such a model. The logit model has broad applications possibly because of the following two facts: (i) taking the logarithm transformation can convert the logit model into a linear model and (ii) the logit model can properly describe the nonlinear phenomenon in practice, as discussed by, e.g., Sudhir (2001). In addition, we learn from Genesove and Mullin

(1998) and Sudhir (2001) that standard error of the estimate and the log-likelihood statistic should be useful to the selection of demand models. We also learn that the best-fitting demand models may vary in different industries. For example, for the sugar industry, the exponential demand model may be highly applicable to empirical study, whereas for the yogurt and peanut butter industries, the logit demand model may be highly applicable to empirical study. ⧫

8.2 Empirical Studies of Coupon-Dependent Demand Models

Our review indicates that very few papers have empirically examined the validation of the coupon-dependent demand models reviewed in Section 3. Leone and Srinivasan (1996) developed an exponential demand model to empirically explore the impact of the face value of a manufacturer's coupon on the manufacturer's profit in a duopoly setting. In their model two manufacturers making products of different brands compete in a market consisting of two customer categories—i.e., the coupon prone (CP) category in which the customers are prone to the coupon and the coupon indifferent (CI) segment in which the customers are indifferent to the coupon.

The exponential demand model for manufacturer i's product ($i = 1, 2$) at time t in the CP category is $d_{ict} = \exp\left[\beta_0 + \beta_1/(p_{it} - \hat{v}_i\beta_2^{\tau})\right]$, where p_{it} is the price of brand i at time t, $\hat{v}_{it}$ is the value of the coupon for brand i that is valid at time t, and τ is the number of weeks since the coupon was dropped. In d_{ict}, the parameters β_0, β_1, and β_2 were empirically estimated. Note that β_2 captures the decaying effect of the coupon's face value on brand sales in the CP category. Using the least squares method, Leone and Srinivasan (1996) found that $\beta_0 = 1.8098$, $\beta_1 = 713.1394$, and $\beta_2 = 1.0000$.

Kumar and Swaminathan (2005) extended the demand model of Leone and Srinivasan (1996) to a generalized coupon-sales response model, which is given as,

$$d_{irt} = \exp\left[\beta_0 + \beta_1/(p_{irt} - \gamma_i\hat{v}_i\beta_2^{\tau_i}) - \sum_{i=1}^{3} U_i + \beta_5 T_{irt} + \beta_7\hat{T}_{irt} + \varepsilon_{irt}\right], \text{ for } i = 1, \ldots, n, \quad (23)$$

where d_{irt} and p_{irt} denote the demand and the price for brand i in store $r = 1, \ldots, \bar{r}$ at time t, respectively; $U_1 \equiv \sum_{j\neq i}[\beta_{3j}/(p_{jrt} - \delta_j\hat{v}_j\beta_{4j}^{\tau_j})]$, $U_2 \equiv \sum_{j\neq i}\beta_{6j}T_{jrt}$, and $U_3 \equiv \sum_{j\neq i}\beta_{8j}T_{jrt}$. In (23), $\hat{v}_i$ is the value of the coupon for brand i; T_{irt} and $\hat{T}_{irt}$ are two dummy variables that represent the display activity and the featuring activity for brand i in store r at time t, respectively; and, γ_i and δ_j are the face values of the coupons for brands i and j at time t, respectively. All β's with numbered subscripts are parameters that were estimated in the empirical study.

The demand model in (23) can characterize the fact that the coupon strategy may vary over the coupon expiration period, and can thus be regarded as a general model capturing the competition among multiple brands. Kumar and Swaminathan compared their general model in (23) with (i) the linear model $d_{ist} = \beta_{0i} + \beta_{1i}(p_{ist} - \gamma_{st}\hat{v}_i\beta_{2i}^{\tau}) + Z$, (ii) the semi-log model $\log d_{ist} = \beta_{0i} + \beta_1(p_{ist} - \gamma_{st}\hat{v}_i\beta_{2i}^{\tau}) + Z$, and (iii) the double-log model $\log d_{ist} = \beta_{0i} + \beta_1[\log(p_{ist} - \gamma_{st}\hat{v}_i\beta_{2i}^{\tau})] + Z$. Note that $Z \equiv -\sum_{i=1}^{3} U_i + \beta_5 T_{irt} + \beta_7 \hat{T}_{irt} + \varepsilon_{irt}$. Using a nonlinear least squares (NLS) method, Kumar and Swaminathan (2005) estimated the parameters in the generalized model in (23) and other three models, and also found that if the single couponing strategy (i.e., $\gamma_i = 1$ and $\delta_j = 1$) is used, then the range of adjusted R^2 for the model in (23) is from 0.69 to 0.81, and the range for other three models is from 0.54 to 0.62. If the double couponing strategy (i.e., $\gamma_i = 2$ and $\delta_j = 2$) applies, then the range of adjusted R^2 for the model in (23) and other three models are from 0.72 to 0.81 and from 0.52 to 0.59. According to the above, Kumar and Swaminathan (2005) concluded that the generalized model is better than other three models.

From our review above, we conclude that, the exponential and the double-log models have been used to conduct the empirical studies regarding the impact of the coupon on the demand. In future, we may consider the empirical studies with other coupon-dependent models specified in Section 3, which include the power model by Arcelus and Srinivasan (2003), the logit model by Aydin and Porteus (2008), etc. In addition, the NLS method and the adjusted R^2 test can be used to select a proper demand model.

8.3 Empirical Studies of Space-Dependent Demand Models

In recent years, very few publications have appeared that deal with the empirical studies concerning space-dependent demand models. Desmet and Renaudin (1998) examined the monthly sales responsiveness to the shelf space allocated for product category c in 125 stores—belonging to store type l—during 11 months. They adopted the demand function $d_c = \exp(\alpha_{c0} + \sum_{i=1}^{125} \delta_{ci} \hat{s}_i + \sum_{j=1}^{11} \gamma_{cj} m_j) \times s^{\beta_{cl}}$, where α_{c0} is a constant. Note that d_c represents the turnover share of product category c, which is calculated as the ratio of the turnover volume of product category c to the total turnover volume of all product categories in 125 stores during 11 months. In the above demand function, s is the share of the space allocated to product category c—i.e., the ratio of the space allocated to product category c to the total space available to all product categories in 125 stores during 11 months; and β_{cl} is the space elasticity of product category c in stores of type l.

Moreover, in Desmet and Renaudin's demand function (1998), $\hat{s}_i$ ($i = 1, 2, \ldots 125$) denotes store i's specific characteristic value that distinguishes this store from other 124 stores according to the differences in all stores' locations and competitive environment; and δ_{ci} is the parameter reflecting the impact of $\hat{s}_i$ on product category c. Thus, the term $\delta_{ci}\hat{s}_i$ determines the component of the demand for the products in category c at store i that possesses specific characteristic value $\hat{s}_i$. We also learn that m_j ($j = 1, 2, \ldots, 11$) represents a specific characteristic value reflecting retailers' seasonal space allocations in month j, and the parameter γ_{cj} denotes the impact of m_j on product category c. It thus follows that the term $\gamma_{cj} m_j$ is the component of the demand for product category c in month j that possesses specific seasonal value m_j.

Desmet and Renaudin (1998) considered the distribution of space elasticities for three different stores: plus store, standard store, and essential store, and found that the average value of the space elasticity (weighted by the number of stores of each type) is 0.2138. It was shown that the space elasticities for each product category vary considerably from -0.44 to 0.80. This demonstrates that the space elasticities for some categories may be negative; for example, the elasticities are -0.12 and -0.13 for the fashion category in plus and standard stores, respectively. Such results are contrary to (i) the model assumptions in Section 5.1.2, where the space elasticity is always assumed to be positive and (ii) earlier relevant empirical studies (e.g., Desmet and Renaudin, 1998), where the space elasticity varies between 0.15 and 0.8.

For another relevant paper, see Van Dijk et al. (2004) who incorporated the spatial interaction between shelf space and the error term into their empirical study. In future work, it may be useful to consider other theoretical demand models surveyed in Section 5, which include, e.g., the demand models with both the spacing and the pricing decision variables.

8.4 General Remarks

We learn that many empirical studies assume linear demand functions and apply econometrics techniques to estimate the parameters in their linear models. However, as our review of the preceding sections has revealed, majority of extant demand functions (e.g., the attraction model, the constant expenditure model, the Cobb-Douglas model, etc.) are nonlinear. Nevertheless, a number of nonlinear functions can be transformed into linear ones for the empirical studies. For example, taking the logarithm of the Cobb-Douglas demand model in (8), we obtain the double-log linear regression to estimate the Cobb-Douglas parameters. As in Besanko et al. (1998), the MultiNomial Logit (MNL) model—which is an attraction model, see Section 2.2.2—can be also transformed into a double-log function. For those non-linear models that cannot be transformed into linear functions, one can also empirically estimate parameter values by using nonlinear methods that are provided by, e.g., Hanssens and Parsons (1993). Our review also indicates that standard error of the estimate, the log-likelihood statistic, and the NLS method with the adjusted R^2 test should be helpful to measuring the goodness of fit of demand models and identifying a proper demand model for empirical study.

Majority of extant publications are focused on the empirical studies with price-dependent demand models, which were shown to be accurate in capturing the actual market demand. However, our selection of the best-fitting model may be dependent on research problems. For example, to investigate the impact of the channel power on the interactions between a manufacturer and a retailer, Kadiyali et al. (2000) chose a logit model rather than a linear or double-log model. Sudhir (2001) showed that the logit model is much better than the multiplicative model for the problem of investigating a manufacturer's pricing decision in the presence of a strategic retailer. Kumar and Swaminathan (2005) developed a generalized space-dependent model to estimate the space elasticity, and concluded that this general model is much better than the linear, semi-log, and double-log models in fitting the data. Since extant publications mostly consider linear and logit demand models, it may be important to conduct empirical studies with other nonlinear models (e.g., Cobb-Douglas, power, etc.) and examine the validity of those nonlinear models in practice.

We also observed that researchers seldom considered the empirical studies with the demand models that depend on the delivery leadtime and the product's quality. It thus follows that, in the future, one may need to empirically analyze leadtime- or quality-related problems. These problems include, e.g., if and how the leadtime- and quality-dependent demand models in Sections 4 and 6 can be used to accurately describe the actual demand.

9 Summary and Concluding Remarks

In this paper, we surveyed the commonly-used mathematical models that were developed to characterize the dependency of demand on various factors such as price, rebate, lead time, space, quality, and advertising. Such demand functions have been increasingly useful in investigating many important problems in business and economics, because consumers are usually sensitive to firms' operational and marketing activities and their demands would thus change as firms implement different strategies.

We learn from our survey that some functional forms (e.g., linear, MNL, MCI, power/iso-elastic) have been widely used to construct various demand function models. For example, the MNL function was utilized to model the price-, the leadtime-, the quality-, and the advertising-dependent demands. To understand clearly the applications of each functional form that we surveyed in our paper, we summarize the distribution of these function forms in the following six categories: (i) price-, (ii) rebate-, (iii) leadtime-, (iv) space-, (v) quality-, and (vi) advertising-dependent demand models; see Table 4. Note from Table 4 that, except for the lead-time category, a number of publications in other categories have not assumed any specific demand models but considered general mathematical function forms that were assumed to possess the convexity or concavity property. Additionally, the linear, the MNL, the MCI, and the power/iso-elastic demand models have been extensively used to analyze various problems, whereas other specific functional forms (e.g., exponential, the log-separable, the square-root, etc.) have not been very popular.

Mathematical Function Form	**Factor that Affects Demand**					
	Price	**Rebate**	**LeadTime**	**Space**	**Quality**	**Advertising**
General	•	•		•	•	•
Cobb-Douglas	•			•		
Constant Expenditure	•					
Exponential	•					
Linear	•	•	•		•	
Log-Separable					•	•
Logarithmic	•					
MCI	•		•		•	•
MCS	•					
MNL	•		•		•	•
Power/Iso-elastic	•	•	•	•		•
Square-Root					•	
Transcendental Logarithmic	•					
Utility-Based	•	•	•		•	

Table 4: Distribution of mathematical functional forms in six categories of dependent demand models: price-, rebate-, leadtime-, space-, quality-, and advertising-dependent demand models.

We also find that a large number of publications belong to the categories of price- and quality-dependent demand models; but, a small number of publications considered the demand models in other categories. This implies that there may be open problems in the areas of rebate-, leadtime-, space-, and advertising-/marketing effort-dependent models and that, in the future, researchers may need to pay more attention to these models. Our findings on each category of demand models are summarized as follows.

1. **Price-dependent demand models**. In our review we found that this category has been the most popular and most interesting to academic researchers, possibly because pricing is the most common marketing tool that has been used to affect consumers' purchase decisions. Moreover, we note that many researchers jointly considered pricing and at least one other decision, e.g., leadtime, quality, advertising in their models. But few publications have considered joint pricing and space decisions, which may be worth investigating in the future.
2. **Rebate-dependent demand models.** These demand models have recently attracted the attention of researchers in marketing and operations fields, as rebates have been widely used by many firms to improve their sales. Our review indicates that extant rebate-dependent models were developed to characterize the demand faced by a monopolistic firm or a supply chain including only one manufacturer and one retailer. Rebate competition between two, or among three or more, firms or supply chains may thus provide interesting research avenues in the future.
3. **Leadtime-dependent demand models**. This category includes the demand models that depend on the firm's delivery leadtime. More specifically, the leadtime-dependent demand models are generally used to analyze inventory-related problems. However, very few publications have considered the joint impact of the leadtime and other decisions (e.g., stocking decision) or factors (e.g., shelf space) that affect the inventory cost. It would thus be interesting to incorporate both the leadtime and the stocking decision or shelf space into the demand function as potential research topics.
4. **Space-dependent demand models**. We have found that such models were developed only for the operations of retail systems. In fact, space allocation decision is not only important to the management of tangible products but also to the delivery of effective service. For example, if the space available in a service system (e.g., restaurant, hotel) is small, then customers may be unlikely to request the service. It would thus be necessary to consider space-dependent demand for service systems.
5. **Quality-dependent demand models**. Quality is important to both the manufacturing and service systems in today's business world. But, as our review indicates, there are relatively few models that characterize the impact of product quality on the demand, whereas most relevant publications considered the service level for manufacturing systems and the service quality for service systems. Thus, researchers may need to consider other function forms (e.g., Cobb-

Douglas, MNL, MCI) to address product quality-related problems.

6. Advertising-dependent demand models: Marketing activities influence consumers' purchase decisions. Our review has revealed that many publications have focused on the impacts of advertising on the demand. Even though some papers have considered general marketing activity or effort which influence demand (e.g., Karnani, 1983; Lariviere and Padmanabhan, 1997), it may be useful to develop models to investigate demand functions that depend on other specific marketing activities such as mail-in rebate, interest-free loan, etc. It may also be useful to examine online demand that depends on other, no-advertising type, marketing activities such as free shipping, word-of-mouth (i.e., the opinions that past online buyers write online).

In addition to the above observations for each category, we find from our survey that the majority of publications assumed general or specific functions (listed in Table 4) to investigate a variety of problems. Those functions may not properly characterize the consumer behavior. As a response, a number of researchers have developed utility-based approaches to investigate a variety of problems in the categories of price-, rebate-, leadtime-, and quality-dependent demand models, as indicated by Table 4.

Among these four categories, the quality-dependent models provide a wide variety of promising research problems since quality is an important determinant of consumers' purchasing decisions. For example, an important research direction should be concerned with the empirical analysis of consumers' quality-dependent utilities. For such an empirical study, we need to collect and analyze the data capturing consumers' attitudes toward quality and other factors affecting their purchasing decisions, and the data analysis may help explore more insights regarding the impact of quality on consumer behavior and firms' optimal decisions on their quality levels. In addition, one may consider the following quality-related research direction. As Section 6.1.1 indicates, most of extant utility models are linear functions of quality levels, whereas very few publications considered non-linear function forms—for example, Zhao et al. (2009) used a power utility function. Since non-linear utility functions should be more realistic than the linear functions, in the future we may need to consider other common non-linear functions such as MCI and MNL to characterize the consumer utility. The analysis of the non-linear utility functions should result in more interesting insights that may be different from those generated from the analysis of the linear utility functions.

We also find from Section 6.1.1 that the existing utility functions incorporate the impact of the quality and the price. But actually, in addition to the quality, consumers' decisions may be dependent on the leadtime, the rebate, the space, the advertising, etc. It would thus be interesting to consider utility functions in terms of quality and other factors that do not include price as a decision variable. Moreover, the quality category only includes a few relevant papers in a monopoly or duopoly setting. In future, it may be also interesting to investigate the oligopoly case in which $n > 3$ firms compete in the market by determining their product quality levels. Furthermore, for the oligopoly case, we could incorporate the price sensitivity and the taste differentiation into the consumers' utility functions, which may generate new insights. For the applications of the price sensitivity and the taste differentiation in the monopoly or duopoly setting, see Chambers et al. (2006) and Desai (2001), respectively.

In conclusion, we find that the demand functions in Table 4 have been shown to be useful in analyzing a variety of business- and economics-related problems. Such models may be used for the investigation of new problems, and they may even create possibilities for developing new types of demand models.

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