Allocation of Cost Savings in a Three-Level Supply Chain with Demand Information Sharing: A Cooperative-Game Approach[1,2]

Mingming Leng
Department of Computing and Decision Sciences
Lingnan University
8 Castle Peak Road, Tuen Mun
Hong Kong

Mahmut Parlar[3]
DeGroote School of Business
McMaster University
Hamilton, Ontario, L8S 4M4
Canada

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Abstract

We analyze the problem of allocating cost savings from sharing demand information in a three-level supply chain with a manufacturer, a distributor and a retailer. To find a unique allocation scheme we use concepts from cooperative game theory. First, we compute analytically the expected cost incurred by the manufacturer and then use simulation to obtain expected costs for the distributor and the retailer. We construct a three-person cooperative game in characteristic-function form, and derive necessary conditions for the stability of each of five possible coalitions. To divide the cost savings between two members, or among three supply chain members, we use various allocation schemes. We present numerical analyses to investigate the impacts of demand autocorrelation coefficient, ρ, and the unit holding and shortage costs on the allocation scheme.

Key words: Supply chain management, information sharing, cooperative game theory, Nash arbitration scheme, constrained core, Shapley value, constrained nucleolus solution, simulation.

1 Introduction

In recent years, academics and practitioners have begun paying considerable attention to efficient management of supply chains involving material, information and financial flows. As Chopra and Meindl [7, p. 36] have indicated, the primary goal of an efficient supply chain is to meet customers' demands at the lowest cost. Hence, in order to achieve supply chain efficiency, each channel member is expected to pay attention to cost savings by, for example, collaborating on supply chain integration (SCI). As shown by Lee [16], information sharing plays a significant role in integrating a supply chain. Information shared by supply chain members mainly consists of demand information, inventory-related data, order status and production schedules (Lee and Whang [15]). As demonstrated in many articles, supply chain-wide information sharing can result in lower overall costs, whereas the lack of information sharing may have a negative impact on the supply chain performance. For example, the well-known phenomenon known as the "bullwhip effect" usually appears in a supply chain as a result of information distortion and can result in higher inventory levels, longer lead times and consequently lower supply chain profitability.

Many industries have experienced or hope to experience demonstrable benefits from information sharing. In a report on potential impacts of the e-commerce on the U.S. healthcare supply chain (HSC) prepared by the accounting and consulting firm Ernst and Young, it was indicated that cost savings generated by efficient information sharing could amount to US$2.6 billion; see, Hankin [11]. Another accounting and consulting firm Andersen also presented an industrial report [14] concerned with the value of e-commerce in HSC. In this study, Andersen obtained the same conclusion as Ernst and Young, and further estimated that information sharing could yield cost savings of US$3.9 billion. As discussed in Chopra and Meindl [7], Wal-Mart and Proctor & Gamble (P&G) also gained considerable benefits from sharing information on the point-of-sale (POS) data.

Demand data from ultimate customers, i.e., POS, is a most important piece of information that is worth sharing. As reported in [1], Dan DiMaggio, president of the UPS Supply Chain Solutions, has indicated that sharing sales data can help reduce inventories and accelerate fulfillment. Lee, So and Tang [17] (hereafter, LST) quantified the benefits of sharing demand information in a two-level supply chain involving a manufacturer and a retailer. LST assumed that customer demands faced by the retailer follow the one-period autoregressive model $AR(1)$, i.e.,

$$D_t = d + \rho D_{t-1} + \varepsilon_t, \tag{1}$$

where D_t represents the customer demand in period t, d is a positive constant, ρ is the autocorrelation parameter with $|\rho| \leq 1$, and ε_t is the error term that is i.i.d. with a symmetric distribution (e.g., normal) having mean 0 and variance σ^2. The demand process (1) for studying the bullwhip effect was adopted as early as 1987 by Kahn [12], and in recent years it has been frequently applied to the analysis of the bullwhip effect and information sharing. For example, Chen *et al.* [5] quantified the bullwhip effect, caused by demand forecasting and order lead times, in a two-stage supply chain with a manufacturer and a retailer who faces the demand process $AR(1)$. These authors also extended their analysis to multiple-stage supply chains with and without demand information sharing between the retailer and his upstream members, and they showed that the bullwhip effect can be reduced but cannot be completely eliminated by information sharing. Chen, Ryan and Simchi-Levi [6] considered two demand processes, $AR(1)$ and a demand process with a linear trend, and they quantitatively analyzed the bullwhip effect for two-stage supply chains consisting of a manufacturer and a retailer. In [6], the retailer was assumed to use the exponential smoothing and moving average forecasting techniques to update the mean and standard deviation of demand and thus the retailer's order-up-to point for each period. The authors demonstrated that using exponential smoothing results in a larger bullwhip effect than using the moving average, and also discussed several important managerial insights drawn from this research.

In our paper, we use (1) to model the demand process faced by the retailer, compute cost savings generated by information sharing and conduct a cooperative game analysis for the fair allocation of cost savings in a three-level supply chain. LST [17] provided empirical evidence to show that for most products the autocorrelation coefficient ρ is positive. When $\rho = 0$, the $AR(1)$ process is reduced to $D_t = d + \varepsilon_t$ which does not depend on the past demand information owned by the retailer. In this case, end-demand information sharing does not change the distributor's and the manufacturer's ordering decisions and does not reduce their costs. Thus, for our analysis we let $0 < \rho \leq 1$.

A number of papers have focused on the impact of information sharing on cost reduction in supply chains. For example, in two recent publications Simchi-Levi and Zhao ([29] and [30]) investigated the value of demand information sharing in a two-stage supply chain (including a manufacturer and a retailer) with production capacity constraints over a finite and an infinite time horizon, respectively. More specifically, in [29], the authors analyzed the value of information sharing between a retailer facing the i.i.d. demand and a manufacturer with a finite production capacity over a finite time horizon. Simchi-Levi and Zhao considered three strategies, i.e., no information sharing, information sharing with optimal policy, and information sharing with greedy policy, and studied the impact of information sharing on the manufacturer and found the optimal timing for information sharing. The authors concluded that by sharing demand information, the manufacturer can achieve a considerable reduction of inventory cost while assuring the same service level to the retailer. In [30], the authors examined the impact of information sharing on the manufacturer's cost and service level for the infinite horizon case. Allowing for time-varying cost functions, the paper characterized the manufacturer's optimal production-inventory policy with information sharing under both the discounted and average cost criteria, and identified situations under which information sharing is most beneficial.

There appear to be very few papers that have analyzed the information sharing problem from a game-theoretic point-of-view and investigated the problem of *allocating* cost savings generated by information sharing among channel members. Furthermore, the existing papers emphasizing allocation schemes only studied two-echelon supply chains; see, e.g., Raghunathan [26]. In this paper, we consider a three-level supply chain involving a manufacturer (M), a distributor (D) and a retailer (R), and we restrict our attention to allocating cost savings among supply chain members when they form a coalition for information sharing. In particular, when some supply chain members collaborate for demand information sharing and jointly achieve cost savings, we consider the question of fairly dividing the cost savings in order to keep them in the coalition. Under a fair allocation scheme, all members in a coalition are better off than before joining the coalition; otherwise, one or more supply chain members could leave the coalition. One may note that in the real business world, lack of trust between supply chain members could also prevent the members from joining the coalition. Since our paper focuses on the fair allocation of cost savings between supply chain members, we are assuming that all supply chain members trust one another. Under this assumption, once a fair allocation is made to all members in a coalition, these members would be willing to stay with the coalition and allocate total cost savings according to the fair scheme.

In the supply chain under study, the distributor procures final products from the manufacturer, and distributes the products to satisfy the orders placed by the retailer who then meets the demands of ultimate customers. We compute cost savings achieved through information sharing and construct a three-person cooperative game in characteristic-function form. In order to simplify the analysis, we assume that a single product is delivered down the supply chain to satisfy the end-demand. Moreover, we assume that the lead time between the manufacturer and the distributor and the lead time between the distributor and the retailer are both one period. During any time period, each channel member determines an order-up-to level to minimize his expected holding and shortage costs for the next period.

The paper is organized as follows. In Section 2 we discuss the information sharing and cost savings for different coalitional structures. Section 3 formulates a three-person information sharing game in characteristic-function form where we find necessary conditions for stability of every coalition, and discuss allocation schemes when a two-player coalition or the grand coalition is stable. In Section 4, we consider the implementation of the allocation schemes analyzed in the preceding section, and compute the side payments transferred between two players (when a two-player coalition is stable) and among three players (when the grand coalition is stable). In Section 5, we provide two numerical examples to find unique allocation schemes, and present sensitivity analyses to explore the impacts of the autocorrelation coefficient ρ, and the unit holding and shortage costs on the allocation schemes. The paper concludes in Section 6 with a brief summary and some remarks regarding future research.

2 Information Sharing and Cost Savings for Different Coalitional Structures

We define the demand information shared by a coalition as the demand data faced by the downstream member in the coalition. For example, the distributor and the manufacturer could form a two-player coalition, where the distributor is the immediate downstream member of the manufacturer. The distributor receives the orders placed by his immediate downstream, i.e., the retailer.

Hence, in this coalition the demand information shared by the distributor and the manufacturer is defined as the retailer's order quantity. When three players (i.e., all supply chain members) form a grand coalition, the information shared by them is the sales data at the retailer's level, i.e., the information on ultimate customers' demands, which is obtained by the retailer from the POS information.

In this paper we develop a cooperative game in characteristic-function form and analyze it to find the appropriate allocation scheme for "fairly" allocating expected cost savings. In order to find the characteristic-function values of various coalitions, we compute total cost savings for each possible coalition in which the participants share demand information faced by their downstream members. The joint cost savings of a coalition are equal to the sum of cost reductions incurred by all members in the coalition. In the supply chain under study, since the manufacturer is the most upstream member, we assume that any production quantity determined by the manufacturer will be realized by his own production schedule. This allows us to compute exactly the manufacturer's expected cost in closed-form.

However, the distributor's and the retailer's orders may not be completely fulfilled by the manufacturer and the distributor, respectively. Specifically, whether or not the order placed by the distributor for period t can be satisfied by the manufacturer depends on the order-up-to level chosen by the manufacturer at the end of the period $t-1$. For the retailer, the process would be much more complex, since the fulfillment of the retailer's order for the period $t+1$ relies on the *actual* order-up-to level at the distributor at the end of the period t, which also depends on the manufacturer's order-up-to level at the end of the period $t-1$. Although it may be possible to formulate our models with the (more realistic) assumption of less than 100% fill-rates, the resulting expressions become too intractable to analyze. For our game modeling and analysis, we use simulation to estimate expected costs of the retailer and the distributor.

In order to find a proper scheme of allocating the expected cost savings achieved by an information-sharing coalition, we follow the procedure below:

Step 1 Identify all possible information-sharing coalitional structures for the supply chain;

Step 2 Compute expected costs incurred by the manufacturer in different information-sharing coalitions;

Step 3 Use simulation to find expected costs incurred by the distributor and the retailer in different information-sharing coalitions;

Step 4 Develop a cooperative game in characteristic-function form in terms of the cost savings for all possible information-sharing coalitions;

Step 5 Analyze the cooperative game to find an appropriate solution representing an allocation scheme for the supply chain.

In the remainder of this section we consider Steps 1, 2 and 3, and in Section 3 we implement Steps 4 and 5. To illustrate Step 1, we refer to Figure 1 that depicts five different possible coalitional structures for information sharing between and among supply chain members. For example, Figure 1(1) corresponds to the coalitional structure $\{M, D, R\}$ where the supply chain members do not share end-demand information. For this case, the expected costs of the manufacturer, the distributor and the retailer are π^{M1}, π^{D1} and π^{R1}, respectively. Similarly, Figure 1(2) corresponds to the coalitional structure $\{M, (DR)\}$ where the distributor receives end-demand information from the retailer. (The fact that the demand information is sent from one supply chain member to another is indicated by the symbol $\|$ on the arrows in Figure 1.) For this case, the expected costs of the manufacturer, the distributor and the retailer are π^{M2}, π^{D2} and π^{R2}, respectively. The remaining parts (3)–(5) in Figure 1 have a similar interpretation.

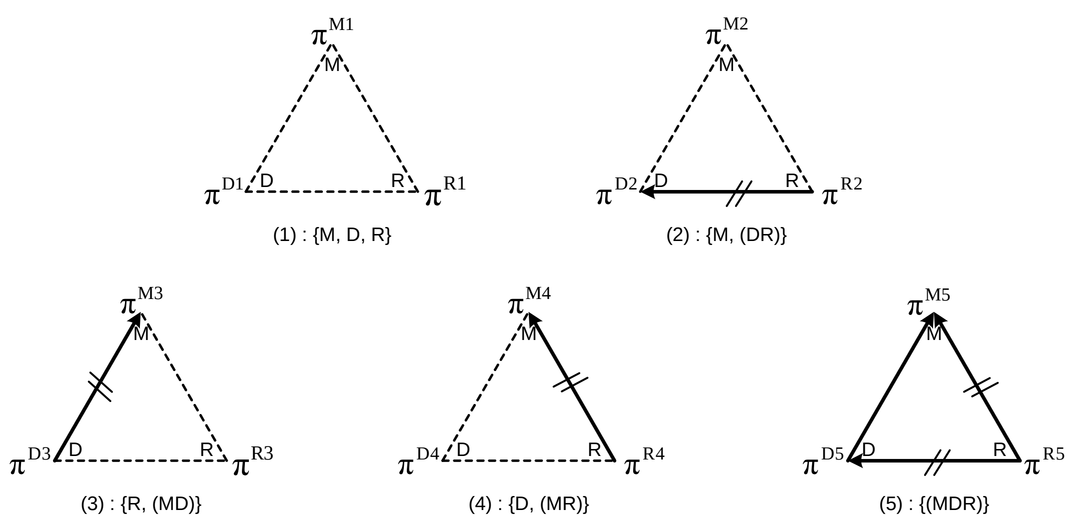

Figure 1: Information sharing possibilities for the three supply chain members M, D and R and the corresponding expected costs.

It is important to note here that in most supply chain collaboration cases, demand information is shared among consecutive echelons in a supply chain. Thus, a coalition such as between the retailer and the manufacturer (which are separated by the distributor in the echelon structure) is not commonly observed. However, such a coalitional structure can still be justified by considering the following example: The Seven-Eleven Japan's success originates from the fact that the stores transmit their POS transactions data to not only the headquarters but also wholesalers and manufacturers for better production schedule and new product development; see Lee and Whang [15]. Today's information technology (e.g., RFID[1]) enables the information sharing among all members in a supply chain. Thus, the retailer and the manufacturer may decide to collaborate for end-demand information sharing.

For Step 2, we compute the manufacturer's expected costs (i.e., $\pi^{M1}, \ldots, \pi^{M5}$) for all five coalitional structures shown in Figure 1. Calculation of these expected costs requires a knowledge of the distributor's ordering process faced by the manufacturer. Recall that the distributor makes his ordering decisions according to the order process of the retailer who uses the demand process (1) to calculate her optimal order quantities. Hence, to compute the manufacturer's expected costs, we identify the ordering processes of the distributor and the retailer. Our results for the manufacturer's expected costs are provided in Table 1. For detailed computations, see Appendix A where the retailer's ordering process is given in Lemma 1, the distributor's ordering process is given in Lemmas 2 and 3, and the manufacturer's expected costs are computed in Proposition 10.

Coalition	The Manufacturer's Expected Costs
$\{M, D, R\}$	$\pi^{M1} = \sigma\sqrt{(1+\rho)^2\left[(1+\rho)^2+4\rho^2\right]+\rho^4}\left[h^M k^M + \left(h^M + p^M\right) I\left(k^M\right)\right]$
$\{M, (DR)\}$	$\pi^{M2} = \sigma\sqrt{\left(1+\rho+\rho^2\right)^2+\rho^2(1+\rho)^2}\left[h^M k^M + \left(h^M + p^M\right) I\left(k^M\right)\right]$
$\{R, (MD)\}$	$\pi^{M3} = \sigma(1+\rho)\sqrt{\rho^2+(1+\rho)^2}\left[h^M k^M + \left(h^M + p^M\right) I\left(k^M\right)\right]$
$\{D, (MR)\}$	$\pi^{M4} = \sigma(1+\rho)^2\left[h^M k^M + \left(h^M + p^M\right) I\left(k^M\right)\right]$
$\{(MDR)\}$	$\pi^{M5} = \sigma\left(1+\rho+\rho^2\right)\left[h^M k^M + \left(h^M + p^M\right) I\left(k^M\right)\right]$
Notations	$k^M = \tilde{\Phi}^{-1}\left[p^M/\left(p^M+h^M\right)\right]$; $\tilde{\Phi}(\cdot)$ is the distribution function of the standard normal r.v.;
	h^M and p^M denote the unit holding cost and the unit shortage cost of the manufacturer;
	$I(z) = \int_z^\infty (x-z)d\tilde{\Phi}(x)$ is the unit normal loss function (see Porteus [25]).

Table 1: The minimum expected cost of the manufacturer for each coalitional structure.

Proposition 1 The manufacturer's expected costs π^{Mi}, $i = 1, \ldots, 5$, have the property that $\pi^{M5} < \pi^{M2} < \pi^{M4} < \pi^{M3} < \pi^{M1}$.

[1] See the VeriSign, Inc.'s report entitled "The EPCglobal Network: Enhancing the Supply Chain" at `http://www.verisign.com/stellent/groups/public/documents/white_paper/002109.pdf`. (URL last accessed in November 2007).

Proof. For a proof of this proposition and proofs of all subsequent Propositions 2 to 7, and Theorem 1, see Appendix C. ■

In Step 3, we use simulation to estimate expected costs for the distributor and the retailer. In particular, given a set of parameter values, we simulate each coalition in the three-stage supply chain for 30 runs with $N =$ 5,000 periods in each run to estimate the inventory-related costs at the distributor's and the retailer's levels.

We compute the distributor's cost for each period according to the product quantity received by him, which depends on the comparison between the distributor's desired order quantity and the manufacturer's available stock. Similarly, for each period, our computation of the retailer's cost is based on actual quantity received by the retailer, which is determined by identifying whether or not an order placed by the retailer can be fully satisfied by the distributor. This analysis is repeated for of the five coalitional structures $\{M, D, R\}$, $\{R, (MD)\}$, $\{D, (MR)\}$, $\{M, (DR)\}$ and $\{(MDR)\}$.

As an illustration, consider the coalitional structure $\{M, D, R\}$. To estimate expected costs for this coalition over N periods, we initialize the simulation at the end of period 0 and use the formulas (in Appendix A) to compute order-up-to levels for all three supply chain members under 100% fill-rate as S_0^{M1}, S_0^{D1} and S_0^R. We also assume that $D_{-1} = d$, which gives $D_0 = d+\rho d+\varepsilon_0 = (1+\rho)d+\varepsilon_0$. In each period we randomly generate the error term ε_t of (1) to simulate the demand D_t and compute the costs of the retailer and the distributor, and find their realized order-up-to levels. For example, for the retailer, at the end of period t, the remaining inventory is $RI_t^R = \max(0, S_{t-1}^R - D_t)$, and shortfall is $SF_t^R = \max(0, D_t - S_{t-1}^R)$. This gives the cost incurred by the retailer in coalition $\{M, D, R\}$ as $\pi_t^{R1} = h^R RI_t^R + p^R SF_t^R$. A similar calculation is performed for the distributor to find her cost as $\pi_t^{D1} = h^D RI_t^{D1} + p^D SF_t^{D1}$. This is repeated for all periods to compute the retailer's and the distributor's costs π_t^{R1} and π_t^{D1}, $t = 1, \ldots, N$. The above calculations give the average (expected) costs for the retailer and the distributor (π^{R1} and π^{D1}, respectively) as $\pi^{R1} = \left(\sum_{t=1}^N \pi_t^{R1}\right)/N$ and $\pi^{D1} = \left(\sum_{t=1}^N \pi_t^{D1}\right)/N$.

3 Modeling and Analysis of the Cooperative Game

In our game model, we consider the problem of "fairly" allocating cost savings between two players (if a two-player coalition is stable) or among three players (if the grand coalition is stable). In this paper, the definition of "fair allocation" is given as follows:

Definition 1 In the information-sharing cooperative game, a scheme for allocating cost savings among all supply chain members in a coalition is fair only if all members in the coalition accept the allocation scheme and are willing to stay in the coalition. ♦

3.1 An Information Sharing Cooperative Game in Characteristic-Function Form

Von Neumann and Morgenstern [32, Ch. VI] were the first to construct a theory of multi-person games where they assumed that various subgroups of players might join together to form coalitions. As we are considering a three-person game where the supply chain members can form coalitions in the process of sharing demand information, we construct our information sharing game in the characteristic value form. That is, we compute the characteristic values of all possible coalitions, i.e., $v(\varnothing)$, $v(M)$, $v(D)$, $v(R)$, $v(MD)$, $v(MR)$, $v(DR)$, $v(MDR)$.

In the theory of cooperative games, the "characteristic value" is the minimum amount that the coalition can attain using its own efforts only. In our paper the characteristic value of a coalition is defined as the cost savings that all players in the coalition could achieve under the worst conditions. We compute the characteristic values of all possible coalitions in Appendix B. Note that the characteristic value of an empty coalition is naturally zero, i.e., $v(\varnothing) = 0$. Our information-sharing cooperative game is thus presented as follows:

$$v(\varnothing) = 0, \quad v(M) = 0, \quad v(D) = \min(0, \pi^{D1} - \pi^{D4}), \quad v(R) = \min(0, \pi^{R1} - \pi^{R3}),$$
$$v(MD) = (\pi^{D1} - \pi^{D3}) + (\pi^{M1} - \pi^{M3}), \quad v(MR) = (\pi^{R1} - \pi^{R4}) + (\pi^{M1} - \pi^{M4}),$$
$$v(DR) = (\pi^{R1} - \pi^{R2}) + (\pi^{D1} - \pi^{D2}), \quad v(MDR) = (\pi^{R1} - \pi^{R5}) + (\pi^{D1} - \pi^{D5}) + \left(\pi^{M1} - \pi^{M5}\right).$$

3.2 Analysis of the Information-Sharing Cooperative Game

We now analyze the cooperative game to investigate the stability of each coalition and to allocate cost savings among players in a stable coalition. A coalition is called "stable" if its members have no incentive to leave the coalition. Naturally, if a coalition is unstable, then the coalition would disperse, so we focus our attention on the allocation schemes that assure stable coalitions. Since the characteristic values for all possible coalitions depend on expected costs, each coalition could be stable under different conditions. Next, we find necessary conditions for stability of every coalition.

Proposition 2 The necessary conditions for stability of each coalition in the cooperative game are given as follows:

1. The coalition $\{(MDR)\}$ is stable only if $v(MDR) \geq \max(v(M)+v(D)+v(R),\ (\pi^{R1}-\pi^{R3})+v(MD),\ (\pi^{D1}-\pi^{D4})+v(MR),\ (\pi^{M1}-\pi^{M2})+v(DR))$ and $v(MDR) \geq \omega^M+\omega^D+\omega^R$, where

$$\omega^M \equiv \begin{cases} \pi^{M1}-\pi^{M2} & \text{if } v(DR) \geq v(D)+v(R), \\ 0, & \text{if } v(DR) < v(D)+v(R), \end{cases}$$

$$\omega^D \equiv \begin{cases} \pi^{D1}-\pi^{D4}, & \text{if } v(MR) \geq v(R), \\ 0, & \text{if } v(MR) < v(R), \end{cases} \quad \text{and} \quad \omega^R \equiv \begin{cases} \pi^{R1}-\pi^{R3}, & \text{if } v(MD) \geq v(D), \\ 0, & \text{if } v(MD) < v(D). \end{cases}$$

2. The coalition $\{R,(MD)\}$ is stable only if $(\pi^{R1}-\pi^{R3})+v(MD) \geq v(MDR)$ and $v(MD) \geq v(D)$.

3. The coalition $\{D,(MR)\}$ is stable only if $(\pi^{D1}-\pi^{D4})+v(MR) \geq v(MDR)$ and $v(MR) \geq v(R)$.

4. The coalition $\{M,(DR)\}$ is stable only if $(\pi^{M2}-\pi^{M2})+v(DR) \geq v(MDR)$ and $v(DR) \geq v(D)+v(R)$.

5. The coalition $\{M,D,R\}$ is stable only if any other coalition is unstable. $\square$

3.2.1 Major Solution Concepts in the Theory of Cooperative Games

Since we don't need to consider the allocation problem when the coalition $\{M,D,R\}$ is stable, we next discuss the commonly-used solution concepts for the two-player games and the three-player games.

Solution Concepts in Two-Player Games When the necessary conditions for stability of a two-player coalition are satisfied, the coalition is stable if an allocation scheme is "fair" to each player. To assure fairness, we consider the scheme of allocating extra cost savings between two players. Here, the extra cost savings is defined as the difference between total cost savings generated by all members in a coalition and the sum of cost savings achieved by these members when they leave the coalition. In the theory of cooperative games, there are several commonly-used game concepts, e.g., egalitarian proposal, negotiation set, Nash arbitration scheme and Shapley value; see Cachon and Netessine [4], Leng and Parlar [18] and Straffin [31]. The egalitarian proposal suggests that two players in a cooperative game split extra cost savings equally. For our game model we can easily show that when the necessary conditions for a two-player coalition are satisfied, the negotiation set is non-empty and it includes many "fair" allocation schemes for two players. However, in our paper we are more interested in finding a unique allocation scheme. In addition to the egalitarian proposal discussed above, we use Nash arbitration scheme and Shapley value to investigate if any other "fair" unique allocation scheme can be found for our cooperative game.

The concept of Nash arbitration scheme was introduced by Nash [22]. Consider a two-player game with the status quo (x_0, y_0). This scheme suggests a unique solution (x, y) by solving the constrained nonlinear problem: max $(x - x_0)(y - y_0)$, subject to $x \geq x_0$ and $y \geq y_0$. Shapley value, developed by Shapley [28], is a solution concept for cooperative games, which provides a unique imputation and represents the payoffs distributed "fairly" by an outside arbitrator. For our game, Shapley value is interpreted as a scheme for allocating cost savings between two players or among three players. The unique Shapley values $\boldsymbol{\varphi} = (\varphi_1, \ldots, \varphi_n)$ are determined by $\varphi_i = \sum_{i \in T}(|T| - 1)!(n - |T|)![v(T) - v(T - i)]/n!$ where T denotes an information sharing coalition and $|T|$ is the size of T.

Solution Concepts in Three-Player Games We now discuss the allocation scheme when the necessary conditions for stability of the grand coalition $\{(MDR)\}$ are satisfied. In particular, if the necessary conditions given in Proposition 2 are satisfied, then for any other coalition we can always find a scheme of allocating cost savings generated by the grand coalition, so that all members would be better off if they form the grand coalition. Thus, starting from any coalition, three supply chain members ultimately decide to join the grand coalition under a fair allocation scheme. However, prior to finding a fair allocation scheme, they would not stay with the grand coalition. In order to make the grand coalition stable, we need to find the fair allocation scheme. The analysis for this case is much more complicated. Similar to our analysis for two-player coalitions, we define x_i as the allocated cost savings (i.e., payoffs) to the supply chain member $i = M, D, R$. A suitable solution representing the payoffs is the triple (x_M, x_D, x_R) with the following two properties: (i) *individual* *rationality*, i.e., $x_i \geqslant v(i)$ for all i; (ii) *collective rationality*, i.e., $x_M + x_D + x_R = v(MDR)$. The triple (x_M, x_D, x_R) satisfying these two properties is called an *imputation* for the game $\mathcal{G} = ([M, D, R], v)$; see Straffin [31].

From Definition 1, under a fair allocation scheme (imputation) none of the members should have any incentive to deviate from the grand coalition, which implies that the fair allocation scheme is undominated by any other possible scheme. Next, we discuss some commonly-used solution concepts in cooperative game theory, which can be classified into two categories—set-valued and unique-valued solution concepts.

Since we are interested in a unique allocation scheme, we shall briefly mention the set-valued solution concepts to show if the space of all possible fair allocation schemes is non-empty. In this category, each set-valued solution concept provides a set of fair allocation schemes that would make the grand coalition stable. For the imputation set, several solution concepts have been suggested, such as Aumann–Maschler bargaining set [2], kernel [9], the core [10] and the stable set [32]. Davis and Maschler [9] and Peleg [24] showed that Aumann–Maschler bargaining set is non-empty for all games, and the kernel is a subset of Aumann–Maschler bargaining set.

The stable set (a.k.a. von Neumann-Morgenstern solution) was introduced by von Neumann and Morgenstern [32]. This solution concept can suggest an allocation scheme that makes the grand coalition stable, but it is used only for essential games where $\sum_{i \in N} v(i) \neq v(N)$ for the coalition N. For inessential games, we cannot apply the concept to find an allocation scheme. Thus, for our paper, we don't consider the stable set. The *core* was first introduced by Gillies [10]. The core of an n-person cooperative game in characteristic form is defined as the set of all undominated imputations $(x_1, x_2, \ldots, x_n)$ such that for all coalitions $T \subseteq N = \{1, 2, \ldots, n\}$, we have $\sum_{i \in T} x_i \geqslant v(T)$; see also Owen [23] for a description of the core. Davis and Maschler [9] also demonstrated that if the core for a game is non-empty, then it must be contained in Aumann–Maschler bargaining set. Although allocation schemes suggested by the core assure the stability of the grand coalition, the core could be empty for some games, thus making it impossible to find an allocation scheme by using this concept. Even if the core is non-empty, we face the question of which allocation scheme should be used for dividing total cost savings among three players (supply chain members).

We now discuss the solution concepts of Shapley value and the nucleolus, each suggesting a unique allocation scheme. Shapley value can be computed easily by using a formula regardless of whether or not the core is empty. However, when the core is non-empty, Shapley value may not be in the core. Moreover, if the stability of the grand coalition depends on some conditions, then the allocation scheme in terms of Shapley value may make the grand coalition unstable.

An alternative solution concept known as the "nucleolus", which was proposed by Schmeidler [27], also defines an allocation scheme that minimizes the "unhappiness" of the most unhappy information sharing coalition. More specifically, let $e_T(\mathbf{x}) = v(T) - \sum_{i \in T} x_i$ denote the excess (unhappiness) of a coalition T with an imputation $\mathbf{x}$. This definition implies that the nucleolus can be found as follows: (i) First consider those coalitions T whose excess $e_T(\mathbf{x})$ is the largest for a given imputation $\mathbf{x}$. (ii) If possible, vary $\mathbf{x}$ to make this largest excess smaller. (iii) When the largest excess is made as small as possible, consider the next largest excess and vary $\mathbf{x}$ to make it as small as possible, etc. A commonly-used method of finding the nucleolus solution is to solve a series of linear programming (LP) problems; see Wang [33].

Compared with other concepts (e.g., the core and Shapley value), the nucleolus has several desirable properties: (1) A unique nucleolus solution always exists so that an allocation scheme can be devised for any game; (2) unlike Shapley value, if the core of a game is non-empty, the nucleolus solution is always in the core; (3) the solution always exists in Aumann-Maschler bargaining set for the grand coalition. For a discussion on these properties of the nucleolus solution, see Straffin [31, p. 150]. However, since the nucleolus solution is normally found by solving a series of linear programming problems, in general, it may be difficult to compute it analytically.

3.2.2 Solution of the Information-Sharing Cooperative Game

We now apply some of the relevant major solution concepts discussed in Section 3.2.1 to our game analysis. In particular, if a two-player coalition is stable, we use the Nash arbitration scheme and the Shapley value to find a unique allocation scheme; if the grand coalition is stable, we use the core to examine if a set of fair allocation schemes exists, and then use the concepts of Shapley value and the nucleolus to find a unique allocation solution.

Proposition 3 If a two-player coalition is stable, both Nash arbitration scheme and Shapley value suggest equal allocation of extra cost savings between two players, as given by the egalitarian proposal. As a result, a unique allocation scheme for each coalition is given as follows: If the coalition $\{i, (jk)\}$ (i, j, $k = M$, D, R, and $i \neq j \neq k$) is stable, then a unique allocation scheme for splitting the cost savings $v(jk)$ between players j and k is $(x_j, x_k) = ([v(jk) + v(j) - v(k)]/2, [v(jk) - v(j) + v(k)]/2)$. $\square$

When the grand coalition is stable, we first investigate whether or not the core is empty. As Proposition 1 implies, cooperation between the distributor and the retailer under the coalition structure $\{M, (DR)\}$ reduces the bullwhip effect and generates the cost savings of $\pi^{M1} - \pi^{M2}$ for the manufacturer. On the other hand, under the grand coalition $\{(MDR)\}$, the manufacturer receives an allocation of x_M. If the manufacturer joins the grand coalition and receives an allocation less than $\pi^{M1} - \pi^{M2}$, then the manufacturer could have an incentive to leave the grand coalition, which results in the coalition structure $\{M, (DR)\}$. Under coalition $\{M, (DR)\}$, the manufacturer has cost savings of $\pi^{M1} - \pi^{M2}$, and the distributor and the retailer share the cost savings $v(DR)$. If $v(DR) \geq v(D) + v(R)$, then the distributor and the retailer would prefer to stay in the two-player coalition (DR), and the manufacturer can thus enjoy cost savings of $\pi^{M1} - \pi^{M2}$. Otherwise, the distributor and the retailer could further deviate from the two-player coalition (DR), and the coalition structure $\{M, D, R\}$ forms and all members have zero cost savings. In conclusion, if $v(DR) \geq v(D) + v(R)$, then we must consider the constraint $x_M \geq \pi^{M1} - \pi^{M2}$. However, we don't need to involve this constraint if $v(DR) < v(D) + v(R)$, since $\{M, (DR)\}$ is unstable so that the manufacturer cannot obtain $\pi^{M1} - \pi^{M2}$ by leaving the grand coalition. Thus, whether or not the constraint $x_M \geq \pi^{M1} - \pi^{M2}$ should be considered depends on the condition $v(DR) \geq v(D) + v(R)$. For simplicity, we write the constraint as $x_M \geq \omega^M$, where ω^M was defined in Proposition 2. When the constraint is not satisfied, then the grand coalition $\{(MDR)\}$ would be unstable.

Similarly, since the distributor receives the savings ω^D when leaving the grand coalition, we must also consider the constraint for the stability of $\{(MDR)\}$ as $x_D \geq \omega^D$. If the retailer leaves the grand coalition, he receives the savings ω^R. The other constraint for the stability of $\{(MDR)\}$ is $x_R \geq \omega^R$.

McKelvey and Schofield [20] introduced the concept of *constrained core* to ensure stability of all coalitions in the constrained core. In our paper, we incorporate three constraints $x_M \geq \omega^M$, $x_D \geq \omega^D$ and $x_R \geq \omega^R$ to guarantee the coalitional stability in the three-level supply chain. As a result, for our cooperative game, the constrained core is defined as a set of all undominated imputations (x_M, x_D, x_R) such that, for all coalitions $T \subseteq N = \{M, D, R\}$, $\sum_{i \in T} x_i \geqslant v(T)$,

$x_M \geq \omega^M$, $x_D \geq \omega^D$ and $x_R \geq \omega^R$. This concept has been widely used in the economics, business and management fields; see, for example, Boyd, Prescott and Smith [3], Ligon and Thistle [19] and Montesano [21].

We now apply the constrained core to our game and obtain the following important result.

Theorem 1 The constrained core of the information sharing game in characteristic-function form is non-empty if and only if $2v(MDR) \geq v(MD) + v(DR) + v(MR)$. $\square$

Even though a non-empty core can suggest some fair allocation schemes, the following question still arises: Which allocation scheme should be used for the supply chain under study? The concept of constrained core cannot provide us with further help in our search for a unique imputation which also results in the stability of the grand coalition. Next, we search for a unique allocation scheme in terms of shapley value and the nucleolus.

Proposition 4 The allocation scheme in terms of Shapley value is given as follows: $\varphi_i = \{2v(MDR) + v(ij) + v(ik) - 2[v(jk) + v(j) + v(k)]\}/6$, for i, j, $k = M$, D, R, and $i \neq j \neq k$. However, if any one of the following conditions $\varphi_M \geq \omega^M$, $\varphi_D \geq \omega^D$ and $\varphi_R \geq \omega^R$ is not satisfied, the allocation scheme suggested by Shapley value makes the grand coalition unstable. $\square$

Since Proposition 4 indicates that Shapley value may not assure the stability of the grand coalition, we now use the nucleolus solution to suggest a fair allocation scheme. As we shall show below, when the core of our game is empty, we can compute the nucleolus solution analytically without resorting to linear programming. With the inclusion of three stability-assuring constraints (i.e., $\varphi_M \geq \omega^M$, $\varphi_D \geq \omega^D$ and $\varphi_R \geq \omega^R$), the result obtained is known as the *constrained nucleolus* solution which was introduced by Montesano [21]. Conceptually, this solution is the same as that of the (ordinary) nucleolus solution with the addition of constraints that assure stability of the coalition.

Proposition 5 If $2v(MDR) < v(MD) + v(DR) + v(MR)$, the constrained core is empty and we have the constrained nucleolus solution as $\nu_i = [v(ij) + v(ik) + v(MDR) - 2v(jk)]/3$, for i, j, $k = M$, D, R, and $i \neq j \neq k$, which satisfies three constraints $\nu_M \geq \omega^M$, $\nu_D \geq \omega^D$ and $\nu_R \geq \omega^R$, thus assuring the stability of the grand coalition. $\square$

If the constrained core is non-empty, then it would be very complicated to use the definition of nucleolus solution to obtain a closed-form formula. A commonly-used method of finding the nucleolus solution is to solve a series of linear programming (LP) problems. The first LP model is written as

$$\min \ u,$$

$$\text{s.t.} \qquad \textstyle\sum_{i \in T} x_i + u \geq v(T), \quad \text{for any } T \subseteq \{M, D, R\};$$

$$x_M \geq \omega^M, \quad x_D \geq \omega^D, \quad x_R \geq \omega^R; \text{ and } \quad x_M + x_D + x_R = v(MDR),$$

where u denotes the "unhappiness" of the most unhappy player. Solving the above LP, we can find a solution where the most unhappy player's allocation has reached a value which minimizes the player's unhappiness. For this case, substituting the value into the above LP problem, we solve the resulting LP problem to minimize the "unhappiness" of the second most unhappy player. We can find the constrained nucleolus solution after minimizing the unhappiness of all players. For a detailed discussion of the LP approach, see Wang [33].

4 Implementation of Unique Allocation Schemes

When only two players share information in a stable coalition (i.e., $\{M, (DR)\}$, $\{R, (MD)\}$ or $\{D, (MR)\}$), Proposition 3 presents schemes for allocating cost savings between two players. To implement these schemes, we employ the concept of *side payment*, which is defined as the amount transferred between two players so that both players obtain their fair allocations suggested by Proposition 3.

Proposition 6 The allocation schemes suggested by Proposition 3 to allocate cost savings between two players are implemented as follows:

1. If the coalition $\{M, (DR)\}$ is stable, then the side payment from the distributor to the retailer is $(\pi^{D1} - \pi^{D2}) - [v(DR) + v(D) - v(R)]/2$;
2. If the coalition $\{R, (MD)\}$ is stable, then the side payment from the manufacturer to the distributor is $(\pi^{M1} - \pi^{M3}) - [v(MD) - v(D)]/2$;
3. If the coalition $\{D, (MR)\}$ is stable, then the side payment from the manufacturer and the retailer is $(\pi^{M1} - \pi^{M4}) - [v(MR) - v(R)]/2$.

When a side payment computed above is negative, the absolute side-payment amount is transferred in a reverse direction. □

Next, we calculate the side payments transferred between any two players, when the grand coalition $\{(MDR)\}$ is stable. To that end, we first define α_i as player i's local cost savings after player i joins the grand coalition, $i = M, D, R$. For our problem, we have $\alpha_M = \pi^{M1} - \pi^{M5}$, $\alpha_D = \pi^{D1} - \pi^{D5}$ and $\alpha_R = \pi^{R1} - \pi^{R5}$. We also let x_i $(i = M, D, R)$ represent the allocation of total cost savings $v(MDR)$ to player i according to a fair allocation scheme. Note that $\alpha_M + \alpha_D + \alpha_R = x_M + x_D + x_R = v(MDR)$.

Proposition 7 When the grand coalition $\{(MDR)\}$ is stable, the side payments transferred between any two of three players are as follows:

1. If $\alpha_i - \beta_i \geq 0$, $\alpha_j - \beta_j \geq 0$ and $\alpha_k - \beta_k \leq 0$, i, j, $k = M$, D, R, and $i \neq j \neq k$, then the side payments are determined as follows: players i and j transfer the amount $(\alpha_i - \beta_i)$ and $(\alpha_j - \beta_j)$ to player k, respectively;
2. If $\alpha_i - \beta_i \geq 0$, $\alpha_j - \beta_j \leq 0$ and $\alpha_k - \beta_k \leq 0$, i, j, $k = M$, D, R, and $i \neq j \neq k$, then the side payments are determined as follows: player i transfers the amount $(\beta_j - \alpha_j)$ to player j and the amount $(\beta_k - \alpha_k)$ to player k. □

5 Numerical Examples and Sensitivity Analysis with Simulation

We first present two numerical examples and illustrate the application of cooperative game theory in allocating cost savings among three players (when the grand coalition is stable) or between two players (when a two player coalition is stable).

Example 1 In this example, we assume the following values for the parameters: For the end-demand process (1), we let $d = 100$, $\rho = 0.5$ and $\sigma = 20$. The "base" values of unit shortage penalty costs and unit holding costs are $(p^R, p^D, p^M) = (5, 3, 2)$ and $(h^R, h^D, h^M) = (2, 1.5, 1)$, respectively. We compute the manufacturer's expected costs as $\pi^{M1} = 22.71$, $\pi^{M2} = 22.27$, $\pi^{M3} = 22.70$, $\pi^{M4} = 22.70$, and $\pi^{M5} = 22.26$. For the distributor and the retailer, we simulate the system for 30 runs for a run length of $N =$ 5,000 periods, take the average of the results obtained in all runs in each coalition, and find the results as in Table 2.

Coalition	$\{M, D, R\}$	$\{M, (DR)\}$	$\{R, (MD)\}$	$\{D, (MR)\}$	$\{(MDR)\}$
Retailer	$\pi^{R1} = 77.73$	$\pi^{R2} = 62.97$	$\pi^{R3} = 69.40$	$\pi^{R4} = 68.22$	$\pi^{R5} = 62.98$
Distributor	$\pi^{D1} = 60.06$	$\pi^{D2} = 53.80$	$\pi^{D3} = 55.86$	$\pi^{D4} = 57.38$	$\pi^{D5} = 53.80$

Table 2: Simulation results for Example 1.

Using the results in Section 3.1, and those in Table 2, a cooperative game is constructed as follows: $v(\varnothing) = v(M) = v(D) = v(R) = 0$,$v(MD) = 12.31$, $v(MR) = 19.67$, $v(DR) = 21.62$, and $v(MDR) = 42.68$. Now, following Proposition 2, we find that the grand coalition $\{(MDR)\}$ is stable; and using Theorem 1, we also find that the constrained core is non-empty. To find a unique allocation scheme, we use the formula in Proposition 4 to compute Shapley value as $\varphi_M = \$12.35$; $\varphi_D = \$13.32$; $\varphi_R = \$17.01$. However, since $\varphi_M < \omega^M = \$17.71$, the unique allocation scheme suggested by Shapley value makes the grand coalition unstable. Hence, we use LP to compute the constrained nucleolus solution as $(\nu_M, \nu_D, \nu_R) = (\$17.71, \$11.505, \$13.465)$, which results in the stability of $\{(MDR)\}$. Next, we use Proposition 7 to implement the allocation scheme as follows: The distributor should receive a side payment of \$3.36 and \$1.285 from the manufacturer and the retailer, respectively. ◄

Under the 100% fill-rate at the distributor and the retailer levels, the expected costs are computed using the formulas in Appendix A as $\hat{\pi}^{D1} = \$51.74$, $\hat{\pi}^{R} = \$47.59$ (for the coalitions $\{M, D, R\}$, $\{R, (MD)\}$ and $\{D, (MR)\}$) and $\hat{\pi}^{D2} = \$49.09$ (for the coalitions $\{M, (DR)\}$ and $\{(MDR)\}$). Comparing these expected costs under 100% fill-rate assumption with those in Table 2, we find that the errors generated by this assumption are large, e.g., the percentage error for the retailer in the coalition $\{M, D, R\}$ is $(\pi^{R1} - \hat{\pi}^{R})/\hat{\pi}^{R} = 63.33\%$. (In our paper we define an error as "large" when the percentage error exceeds 5%.) Thus, we cannot use expected costs under the 100% fill-rate assumption to construct our cooperative game and find an allocation scheme.

We have performed 1,200 simulation experiments with different parameter values set as follows: First, we fixed $d = 100$ and $\sigma = 20$. Then, for each of 20 values of ρ (ranging from 0.01 to 0.1 in increments of 0.01, and from 0.1 to 1 in increments of 0.1) we varied the other parameters one-at-a-time while keeping the remaining ones at their base values given in Example 1. In particular, the p^R values were varied from 2 to 6.5 in increments of 0.5 [which we denote by $p^R = 2(0.5)6.5$] resulting in 10 values. Since ρ assumed 20 different values, the (ρ, p^R) combination resulted in $20 \times 10 = 200$ experiments. The other parameters were varied as $p^D = 1.5(0.5)6$, $p^M = 1(0.5)5.5$,

$h^R = 0.5(0.5)5$, $h^D = 0.5(0.25)2.75$ and $h^M = 0.2(0.2)2$, each resulting in 10 values. As a result of these 1,200 experiments, we found that the grand coalition is stable in a large number of cases (i.e., when $\rho > 0.03$), but the coalition $\{M, (DR)\}$ is stable only in a few cases (i.e., when $\rho \leq 0.03$). This demonstrates that, information sharing can achieve cost savings and improve supply chain performance. In order to illustrate the analysis for a stable two-player coalition, in the next example we assign a very small value to the autocorrelation parameter, i.e., $\rho = 0.02$.

Example 2 For a three-level supply chain with demand information sharing, we change the value of ρ from 0.5 to 0.02 but use the same values for the other parameters as in Example 1. Similar to Example 1, we compute the manufacturer's expected costs as $\pi^{M1} = 22.71$, $\pi^{M2} = 22.27$, $\pi^{M3} = 22.70$, $\pi^{M4} = 22.70$, and $\pi^{M5} = 22.26$, and we simulate the system to estimate expected costs for the retailer and the distributor; see the results in Table 3.

Coalition	$\{M, D, R\}$	$\{M, (DR)\}$	$\{R, (MD)\}$	$\{D, (MR)\}$	$\{(MDR)\}$
Retailer	$\pi^{R1} = 48.44$	$\pi^{R2} = 48.30$	$\pi^{R3} = 48.43$	$\pi^{R4} = 48.43$	$\pi^{R5} = 48.29$
Distributor	$\pi^{D1} = 33.50$	$\pi^{D2} = 33.49$	$\pi^{D3} = 33.48$	$\pi^{D4} = 33.48$	$\pi^{D5} = 33.49$

Table 3: Simulation results for Example 2.

For this example, we construct a cooperative game in characteristic form as

$$v(\varnothing) = v(M) = v(D) = v(R) = 0,$$

$$v(MD) = 0.03, \quad v(MR) = 0.03, \quad v(DR) = 0.16, \quad v(MDR) = 0.60.$$

For this game, we find that the coalition $\{M, (DR)\}$ is stable. Using Proposition 3, we allocate the cost savings $v(DR)$ between the distributor and the retailer as $x_D = x_R = v(DR)/2 = \$0.08$. To implement the allocation scheme, Proposition 6 suggests that the retailer transfers a side payment \$0.07 to the distributor. ◀

For Example 2, we compute the distributor's and the retailer's expected costs under 100% fill-rate as $\hat{\pi}^{D1} = 33.38$, $\hat{\pi}^{D2} = 33.38$, and $\hat{\pi}^{R} = 47.59$. For this example, the errors generated by the 100% fill-rate assumption are small. However, after performing 1,200 simulation experiments with the parameter values as explained above, we found that the errors are small (as in Example 2) when $\rho \leq 0.2$ but they are large (as in Example 1) when $\rho > 0.2$. Since the value of ρ is in the range $(0, 1]$, we can conclude that the errors are often large; thus, in this section we only use simulation to find expected costs of the distributor and the retailer.

5.1 The Impact of ρ on the Coalition Stability and the Allocation Scheme

We perform a sensitivity analysis to examine the effect of the autocorrelation coefficient ρ on the coalitional stability, total cost savings for the supply chain and allocations made to the members of the chain. In this sensitivity analysis, we first increase the value of ρ from 0.01 to 0.1 in increments of 0.01, and then increase ρ from 0.1 to 1.0 in increments of 0.1. The results are presented in Table 4 in Appendix D. Using the data in this table, we plot the allocations in Figure 2(a) and (b).

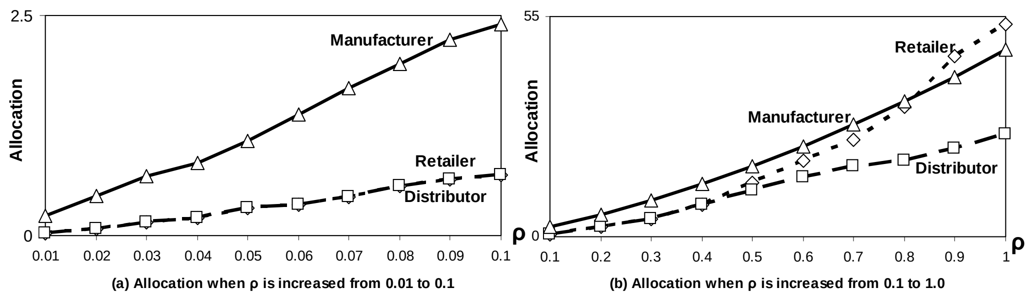

Figure 2: The impact of ρ on the allocation schemes in the three-level supply chain.

As Table 4 indicates, we find that, for a constant $\sigma = 20$, higher values of the parameter ρ generate higher total cost savings enjoyed by the entire supply chain. This result is expected since increasing ρ raises the value of historical data according to the end-demand model (1). However, when $\rho \leq 0.02$, we find that the value of information is not substantial so that the three members (especially, the manufacturer) would be unwilling to join the grand coalition $\{(MDR)\}$. But, the end-demand information is still worth sharing between the distributor and the retailer, so the coalition $\{M, (DR)\}$ becomes stable when ρ assumes very small values. When the grand coalition is stable for $\rho > 0.02$, we find that the core is always non-empty and the allocation scheme suggested by Shapley value makes the grand coalition unstable. In order to obtain a unique allocation scheme that achieves stability of $\{(MDR)\}$, we compute the constrained nucleolus solution, and use it to split total cost savings among three members.

Figure 2(a) indicates the allocations to three members when ρ is increased from 0.01 to 0.1 in increments of 0.01. We note that the allocations to the distributor and the retailer are equal when ρ is in this range. For a small value of ρ (0.01 or 0.02), the distributor and the retailer stay in the two-player coalition (DR), since they can obtain savings that make these two members better off than leaving the coalition. Moreover, the distributor and the retailer receive equal allocated savings, as suggested by Proposition 3. When ρ increases from 0.03 to 0.1, the grand coalition $\{(MDR)\}$ is stable, and the rise in the portion allocated to the manufacturer continues to be the fastest among three supply chain members. This is due to the fact that, for a larger ρ, the supply chain experiences larger cost savings; but, in order to entice the manufacturer to stay within the grand coalition (i.e., to keep it stable), the manufacturer receives higher allocations. In Figure 2(b) we plot the changes on the allocations to three members when ρ is increased from 0.1 to 1.0 in increments of 0.1. The savings allocated to the manufacturer continues to increase, but are less than those to the retailer when $\rho \geq 0.9$. This reflects the fact that, according to (1), the retailer's end-demand information plays a more important role in improving supply chain performance when the value of ρ is increased. Thus, increasing the value of ρ allocates higher values to the retailer. Especially, for those values ρ that are set to 0.9 or higher, the retailer obtains a higher allocation than other members. We find that, when ρ is greater than 0.4 but smaller than 0.9, the value of end-demand information held by the retailer increases but adds significant value to the grand coalition, thus making this coalition stable. As a result, the allocation to the retailer is higher than that to the distributor but is still lower than the allocation to the manufacturer.

5.2 The Impacts of Shortage and Holding Costs on the Coalition Stability and the Allocation Scheme

We now investigate the coalition stability and allocation schemes when the shortage and holding costs parameters of each supply chain member are varied around their base values of $(p^R, p^D, p^M) = (5, 3, 2)$ and $(h^R, h^D, h^M) = (2, 1.5, 1)$ as used in Example 1. Our computations reveal that, for all values of the parameters considered, the stable coalition is always $\{(MDR)\}$. This implies that the supply chain members' unit shortage and holding costs do not impact stability of the grand coalition. Additionally, we find that the allocation in terms of Shapley value also makes the grand coalition $\{(MDR)\}$ stable only when the manufacturer's unit shortage cost p^M is smaller than 1.2. Since, for most cases, only the constrained nucleolus solution results in stability of $\{(MDR)\}$, in our analysis we only use this solution concept in order to ensure consistency in our sensitivity analysis. We now present a discussion of the impact of each parameter on the allocation schemes as depicted in Figure 3.

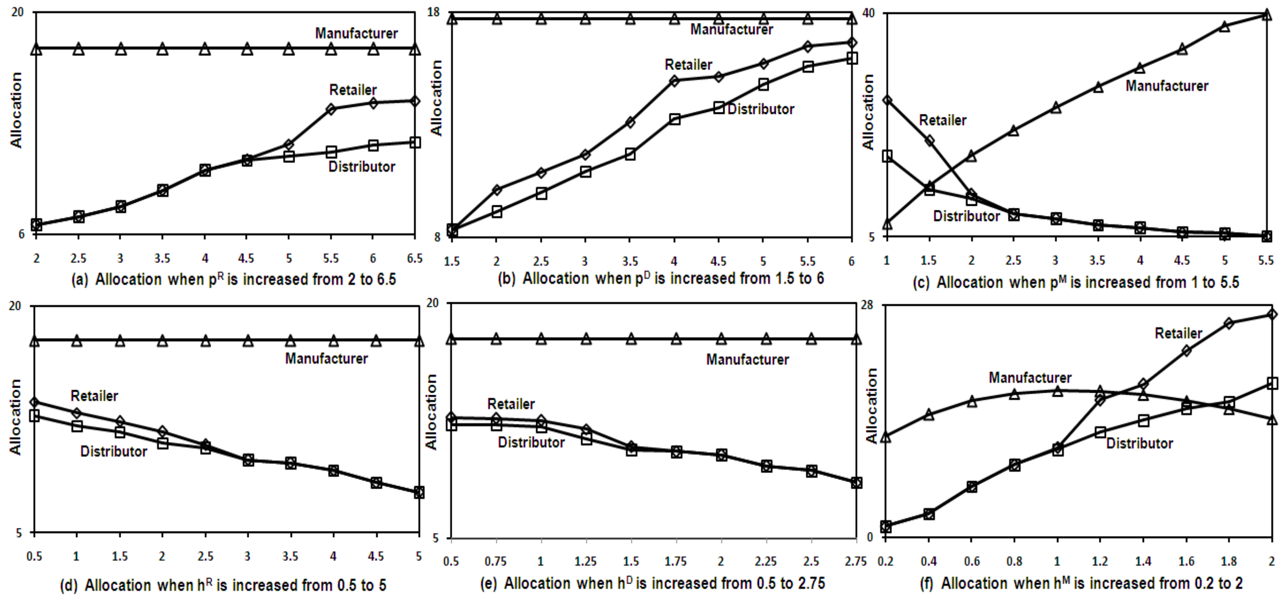

Figure 3: The impacts of p^i and h^i $(i = M, D, R)$ on the allocation schemes in the three-level supply chain.

5.2.1 The Impact of Unit Shortage Costs

We first examine the impact of the retailer's unit shortage cost p^R on the allocation of total cost savings and compute allocation schemes in terms of the constrained nucleolus solutions. While other parameter values are unchanged, the value of p^R is increased from 2 to 6.5 in increments of 0.5. Note that, since the unit shortage cost p^R would be, in general, greater than the unit holding cost $h^R = 2$, we let the minimum value of p^R equal to 2. From Figure 3(a), we find that the cost savings allocated to the manufacturer are constant. This result is justified as follows: As the retailer's unit shortage cost p^R increases and the unit holding cost h^R is constant, the retailer has an incentive to increase his order quantity to minimize total inventory-related cost, which may raise the distributor's shortage cost if the quantity available to the distributor is not increased. However, the manufacturer's cost doesn't change since he is located at the highest echelon in the three-level supply chain. For any value of p^R in the range $[2, 6.5]$, we find that, if the manufacturer leaves the grand coalition $\{(MDR)\}$, the coalition structure becomes $\{M, (DR)\}$ and the manufacturer's local cost savings becomes $\pi^{M1} - \pi^{M2} = \$17.71$. In order to keep the grand coalition $\{(MDR)\}$ stable, the allocation to the manufacturer should be no less than \$17.71. However, if the manufacturer receives more than \$17.71, the retailer and the distributor would obtain very low allocations and consequently leave the grand coalition $\{(MDR)\}$. Thus, as the constrained nucleolus solution suggests, the allocation to the manufacturer is equal to a constant \$17.71 for all values of the parameter p^R.

As the value of p^R increases, information sharing would generate more cost savings for this supply chain. Since the allocation to the manufacturer is the constant \$17.71, the allocations to the distributor and the retailer increase in p^R. However, when p^R is smaller than 5, the distributor and the retailer have equal allocations; when p^R is equal to or greater than 5, the retailer's end-demand information is so significant that the allocation to the retailer is greater than that made to the distributor.

Similarly, we find that, as the value of the distributor's unit shortage cost p^D increases from 1.5 to 6 in increments of 0.5, the allocation of total cost savings to the manufacturer is still the constant \$17.71, and the allocations to the retailer and the distributor increase. Moreover, total cost reduction is higher than that computed as the value of p^R changes; and the end-demand information sharing thus induces higher allocations of total cost savings to the retailer and the distributor. Different from Figure 3(a), Figure 3(b) indicates that the allocation to the retailer is always greater than that to the distributor when $p^D > 1.5$. This reflects the following fact: When p^D increases, the distributor intends to raise his order quantity, which leads to a larger possibility that the retailer's order can be satisfied by the distributor. As a result, the retailer can benefit from increasing the value of p^D.

To investigate the impact of the manufacturer's unit shortage cost p^M on the allocation, we increase the value of p^M from 1 to 5.5 in increments of 0.5. Figure 3(c) shows that as the value of p^M increases, the allocation to the manufacturer increases whereas the allocations to other members decrease. In particular, increasing the value of p^M entices the manufacturer to increase his order quantity, and the end-demand information sharing largely increases the cost savings incurred by the manufacturer. Since the distributor and the retailer benefit from increasing the value of p^M, they agree upon the significant allocation to the manufacturer to keep the coalition $\{(MDR)\}$ stable. However, even though the allocations to the retailer and the distributor decrease, these two members are still better off than leaving the grand coalition. Furthermore, when the value of p^M is smaller than 2, the retailer's end-demand information adds a more significant value to the grand coalition;

thus, the allocation to the retailer is highest among three supply chain members. However, when p^M is equal to greater than 2, the cost savings $\pi^{M1} - \pi^{M2}$ incurred by the manufacturer becomes very large. In order to prevent the manufacturer from leaving the grand coalition, the allocation to the manufacturer is increased to a high level, and the allocations to the retailer and the distributor are equal to a small value.

5.2.2 The Impact of Unit Holding Costs

We increase the value of the retailer's unit holding cost h^R from 0.5 to 5 in increments of 0.5, compute the allocation schemes in terms of constrained nucleolus solutions and plot Figure 3(d) to show the impact of h^R on the allocations of total cost savings. Similar to our analysis for Figure 3(a), we find that increasing the value of h^R has no impact on the allocation to the manufacturer. Additionally, when the retailer's unit holding cost h^R increases and the unit shortage cost is unchanged at $p^R = 5$, the retailer intends to decrease his order quantity, and the probability that the retailer's order can be satisfied by the distributor is increased. This implies that increasing the value of h^R may reduce the shortage cost incurred by the distributor. Therefore, compared with the increase of h^R, the end-demand information released by the retailer to the distributor is less important in improving the distributor's local performance, and thus cost savings generated by the information sharing is reduced, as shown by Figure 3(d).

When the value of h^R is smaller than 3, the end-demand information of the retailer is significantly important to supply chain improvement, so that the allocation of total cost savings to the retailer is higher than that to the distributor. However, when $h^R \geq 3$, the retailer's end-demand information becomes less important, and the distributor's cost savings generated by increasing the value of h^R raises. Thus, the allocations to the retailer and the distributor are equal.

When the value of the parameter h^D is increased from 0.5 to 2.75 in increments of 0.25, we compute the allocations of total cost savings and plot Figure 3(e) to investigate the impact of h^D on the allocation scheme. Similar to our analysis on the impact of h^R, increasing the value of h^D has no impact on the manufacturer. However, when the value of h^D is close to the shortage cost $p^D = 3$, the importance of end-demand information is reduced. One may note that when $h^D = p^D$, information sharing doesn't impact the distributor's ordering decision; so no cost savings can be generated at the distributor level. Thus, when the value of h^D increases, total cost savings generated by the information sharing decreases, and the allocations to the retailer and the distributor are reduced. When $h^D < 1.75$, the end-demand information from the retailer is important, and consequently the allocation to the retailer is greater than that to the distributor. However, when the value of h^D is in the range $[1.75, 2.75]$, the information sharing cannot generate significant cost savings, so that the allocations to the distributor and the retailer are equal.

Next, we examine the impact of h^M on allocation schemes by increasing the value of h^M from 0.2 to 2 in increments of 0.2. When the value of h^M increases, the manufacturer intends to reduce his order quantity, and the distributor's order cannot be fulfilled with a larger probability, which further reduces the possibility that the retailer's order is satisfied by the distributor. Therefore, increasing the value of h^M may increase the shortage cost at each supply chain member, and thus, the information sharing among three supply chain members is more important to improving supply chain performance. From Figure 3(f) we find that, when the value of h^M increases in the range $[0.2, 1]$, the impact of h^M is not significant, so that the allocation to the manufacturer is increasing to reflect the fact that the retailer and the distributor entice the manufacturer to stay in the grand coalition. For this case, the retailer and the distributor equally share the remaining savings. However, when $h^M > 1$, the impact of h^M is so large that the end-demand information sharing plays a more important role in supply chain improvement. In order to motivate the retailer to release the demand information, the allocation to the retailer increases in a large magnitude. Since the cost savings $\pi^{M1} - \pi^{M2}$ (which is obtained by the manufacturer when leaving the grand coalition) is decreasing, the allocation to the manufacturer is also correspondingly reduced.

6 Conclusion

This paper developed an information sharing cooperative game in characteristic form and found an allocation scheme to share the cost savings arising from cooperation. More specifically, we considered a three-level supply chain involving a manufacturer, a distributor and a retailer. The three supply chain members cooperate with each other in sharing the demand information under positive lead-times. Such a collaboration results in a cost reduction in the supply chain. We investigated the scheme of splitting the cost savings among the supply chain members. In particular, we computed analytically the expected holding and shortage costs incurred at the manufacturer level, and used simulation to find expected costs of the distributor and the retailer. By using these costs, we found the characteristic values for all possible coalitions, and derived the necessary conditions for stability of every coalition. If a two-player coalition is stable, we presented a unique allocation scheme. When the grand coalition is stable, we showed that the constrained core of the game could be non-empty provided that a condition is satisfied. Next, we considered Shapley value to determine a unique allocation of cost savings but found that this allocation scheme could result in an unstable grand coalition (since at least one of three conditions required for stability is not satisfied). We then considered the nucleolus solution but in its computation we took into account three constraints that would keep the coalition stable. An analytic expression for the case of empty core was derived for solving the three-person game to find the constrained nucleolus solution. Our numerical study presented two examples to illustrate the modeling approach and the computations of allocation schemes, and also provided several sensitivity analyses to indicate the impacts of the autocorrelation coefficient ρ in model (1), the unit shortage and holding costs on the coalition stability and allocation schemes.

In our paper, we used the constrained core to examine whether the set of fair allocations is empty. However, a potential concern with using this concept as the stability criterion is that it is a myopic measure. A myopic solution assumes that the players don't consider the future; that is, they don't consider what might happen when they choose a solution. In addition, it is assumed that, when the players move to a myopic solution, there is no further deviation. Due to the limits of myopic solutions, two new concepts have been recently proposed to consider coalition formation as an ongoing, dynamic process with payoffs generated as the coalitions evolve. The concept of "largest consistent set" introduced by Chwe [8] is a solution concept that is used to analyze "farsighted" coalitions that consider the possibility of other coalitions forming in response to its actions. Equilibrium process of coalition formation introduced by Konishi and Ray [13] allow for the possibility of moving to another coalition by the expectation of a higher future payoff and it is related to the largest consistent set of Chwe [8]. A future research direction could be the applications of the farsighted solution concepts in supply chain analysis.

References

[1] Anonymous. Sharing sales data to rev up the supply chain. *Modern Materials Handling*, 57(14):9–10, December 2002.

[2] R. Aumann and M. Maschler. The bargaining set for cooperative games. In M. Dresher, L. S. Shapley, and A. W. Tucker, editors, *Advances in Game Theory*, pages 443–476. Princeton University Press, Princeton, 1964.

[3] J. H. Byod, E. C. Prescott, and B. D. Smith. Organizations in economic analysis. *Canadian Journal of Economics*, XXI(3):477–491, 1988.

[4] G. P. Cachon and S. Netessine. Game theoretic applications in supply chain analysis. In D. Simch-Levi, S. D. Wu, and Z.-J. Shen, editors, *Supply Chain Analysis in the eBusiness Era.* Kluwer, 2003.

[5] F. Chen, Z. Drezner, J. K. Ryan, and D. Simchi-Levi. Quantifying the bullwhip effect in a simple supply chain: The impact of forecasting, lead times, and information. *Management Science*, 46(3):436–443, March 2000.

[6] F. Chen, J. K. Ryan, and D. Simchi-Levi. The impact of exponential smoothing forecasts on the bullwhip effect. *Naval Research Logistics*, 47:269–286, 2000.

[7] S. Chopra and P. Meindl. *Supply Chain Management: Strategy, Planning and Operation.* Prentice-Hall, Upper Saddle River, 2001.

[8] M. S.-Y. Chwe. Farsighted coalitional stability. *Journal of Economic Theory*, 63:299–325, 1994.

[9] M. Davis and M. Maschler. The kernel of a cooperative game. *Naval Research Logistics Quarterly*, 12:223–259, 1965.

[10] D. B. Gillies. Solutions to general non-zero-sum games. In A. W. Tucker and R. D. Luce, editors, *Contributions to the Theory of Games*, pages 47–85. Princeton University Press, Princeton, New Jersey, 1959.

[11] R. A. Hankin. Bar coding in healthcare – A critical solution. *Business Briefing: Medical Device Manufacturing and Technology*, pages 2–4, 2002.

[12] J. A. Kahn. Inventories and the volatility of production. *The American Economic Review*, 77(4):667–679, September 1987.

[13] H. Konishi and D. Ray. Coalition formation as a dynamic process. *Journal of Economic Theory*, 110:1–41, 2003.

[14] R. G. Lacy, K. J. Connor, P. F. Crane, A. R. Gonce, K. A. Reid, and J. R. Harter. The value of eCommerce in the healthcare supply chain: Industry study report. Technical report, Andersen, June 2001.

[15] H. Lee and S. Whang. Information sharing in a supply chain. *International Journal of Technology Management*, 20(3/4):373–387, 2000.

[16] H. L. Lee. Creating value through supply chain integration. *Supply Chain Management Review*, September/October 2000.

[17] H. L. Lee, K. C. So, and C. S. Tang. The value of information sharing in a two-level supply chain. *Management Science*, 46(5):626–643, May 2000.

[18] M. Leng and M. Parlar. Game theoretic applications in supply chain management: a review. *INFOR*, 43(3):187–220, August 2005.

[19] J. A. Ligon and P. D. Thistle. The formation of mutual insurers in markets with adverse selection. *Journal of Business*, 78(2):529–555, 2005.

[20] R. D. McKelvey and N. Schofield. Generalized symmetry conditions at a core point. *Econometrica*, 55(4):923–933, July 1987.

[21] A. Montesano. Uncertainty with partial information on the possibility of the events. *Theory and Decision*, 51:183–195, 2001.

[22] J. Nash. The bargaining problem. *Econometrica*, 18:155–162, 1950.

[23] G. Owen. *Game Theory*. Academic Press, New York, 2nd edition, 1982.

[24] B. Peleg. Existence theorem for the bargaining set $M_1^{(i)}$. In M. Shubik, editor, *Essays in Mathematical Economics in Honor of Oskar Morgenstern*, pages 53–56. Princeton University Press, Princeton, 1967.

[25] E. Porteus. *Foundations of Stochastic Inventory Theory*. Stanford University Press, Stanford, California, 2002.

[26] S. Raghunathan. Impact of demand correlation on the value of and incentives for information sharing in a supply chain. *European Journal of Operational Research*, 146(3):634–649, May 2003.

[27] D. Schmeidler. The nucleolus of a characteristic function game. *SIAM Journal on Applied Mathematics*, 17:1163–1170, 1969.

[28] L. S. Shapley. A value for n-person games. In H. W. Kuhn and A. W. Tucker, editors, *Contributions to the Theory of Games II*, pages 307–317. Princeton University Press, Princeton, 1953.

[29] D. Simchi-Levi and Y. Zhao. The value of information sharing in a two-stage supply chain with production capacity constraints. *Naval Research Logistics*, 50(8):888–916, 2003.

[30] D. Simchi-Levi and Y. Zhao. The value of information sharing in a two-stage supply chain with production capacity constraints: The infinite horizon case. *Probability in the Engineering and Informational Sciences*, 18(2):247–274, April 2004.

[31] P. D. Straffin. *Game Theory and Strategy*. The Mathematical Association of America, Washington, D.C., 1993.

[32] J. von Neumann and O. Morgenstern. *Theory of Games and Economic Behaviour*. Princeton University Press, Princeton, 1944.

[33] J. Wang. *The Theory of Games*. Clarendon Press, Oxford, 1988.

Appendix A Computation of Each Supply Chain Member's Expected Cost under 100% Fill Rate

We define "end-demand" as the demand generated by the ultimate customer and assume that the end-demand is forecasted by the $AR(1)$ process defined by (1) as in LST [17]. We assume that the parameter values and structure of (1) are known to all three members in the supply chain. As in LST [17], σ is assumed to be significantly smaller than d. This assumption will help justify the assumption that each supply chain member's order-up-to level is always nonnegative. The end-demand information that would be shared by the members is the deterministic value of ε_t; that is, at the end of time period t, the retailer has already observed the end-demand realized in this period, i.e., D_t. Since D_{t-1} is also available, the retailer can find the exact value of the error term ε_t by simply computing $\varepsilon_t = [D_t - (d + \rho D_{t-1})]$. Next, at the end of period t, the retailer forecasts the end-demand D_{t+1} and places an order with the distributor to increase the inventory position to his order-up-to level S_t^R. Hence, when there is no sharing of the end-demand information, the distributor and the manufacturer cannot compute ε_t exactly since they have no knowledge of D_{t-1} and D_t. In this case, the distributor has to forecast the retailer's order by using the end-demand process (1), where ε_t is a normally-distributed random variable with mean 0 and variance σ^2.

A.1 Retailer's Expected Cost $\hat{\pi}^R$ under 100% Fill Rate

The demand process faced by the retailer is the $AR(1)$ model (1). The retailer's order-up-level at the end of time period t, denoted by S_t^R, is found by minimizing total expected cost function for period $t+1$ as follows. The mean m_t^R and the variance V_t^R of the demand D_{t+1}, conditional on the realized demand D_t, are obtained as

$$m_t^R = E(D_{t+1} \mid D_t) = E(d + \rho D_t + \varepsilon_{t+1}) = d + \rho D_t, \tag{2}$$

$$V_t^R = \mathrm{Var}(D_{t+1} \mid D_t) = \mathrm{Var}(\varepsilon_{t+1}) = \sigma^2. \tag{3}$$

Since ε_{t+1} is normally distributed with mean 0 and variance σ^2, the demand process (1) implies that demand D_{t+1} is a normally distributed random variable with mean $(d + \rho D_t)$ and variance σ^2.

Proposition 8 Assuming a 100% fill-rate, the retailer's optimal order-up-to level, S_t^R, at the end of period t and minimum expected cost, $\hat{\pi}^R$, for period $t+1$ are

$$S_t^R = d + \rho D_t + k^R \sigma \quad \text{and} \quad \hat{\pi}^R = \sigma \left[h^R k^R + \left(h^R + p^R\right) I\left(k^R\right) \right], \tag{4}$$

where $k^R = \tilde{\Phi}^{-1}\left[p^R / \left(p^R + h^R\right)\right]$; h^R and p^R denote unit holding cost and the unit shortage cost per time period at the retailer level, respectively; $\tilde{\Phi}(\cdot)$ is the distribution function of the standard normal r.v., and

$$I(z) = \int_z^{\infty} (x - z)\, d\tilde{\Phi}(x) \tag{5}$$

is the unit normal loss function (see Porteus [25, Chapter 1]).

Proof. Under the assumption of 100% fill-rate, the retailer's order can be fully satisfied by the distributor, so total expected cost $\hat{\pi}^R$ for period $t+1$ is given as

$$\begin{aligned}\hat{\pi}^R &= h^R \int_{-\infty}^{S_t^R} (S_t^R - D_{t+1})\phi(D_{t+1})\, dD_{t+1} + p^R \int_{S_t^R}^{\infty} (D_{t+1} - S_t^R)\phi(D_{t+1})\, dD_{t+1}, \\ &= h^R E\left[\left(S_t^R - D_{t+1}\right)^+\right] + p^R E\left[\left(D_{t+1} - S_t^R\right)^+\right], \end{aligned} \quad (6)$$

where $\phi(D_{t+1})$ is probability density function (p.d.f.) of the conditional normal random variable D_{t+1} with mean m_t^R and variance V_t^R. The optimal order-up-to level S_t^R that minimizes expected cost (6) can be obtained as

$$S_t^R = m_t^R + k^R \sqrt{V_t^R} = d + \rho D_t + k^R \sigma,$$

where $k^R = \tilde{\Phi}^{-1}\left[p^R / \left(p^R + h^R\right)\right]$, h^R and p^R respectively denote unit holding cost and the unit shortage cost per time period at the retailer level and $\tilde{\Phi}(\cdot)$ is the distribution function of the standard normal r.v.; see LST [17]. Using the optimal value of the order-up-to level S_t^R, we find the minimum expected cost as (4). ■

When the fill-rate is 100%, the retailer's order quantity Y_t^R at the end of period t is the difference between the desired order-up-to level S_t^R and the starting inventory $(S_{t-1}^R - D_t)$, i.e.,

$$Y_t^R = S_t^R - (S_{t-1}^R - D_t) = D_t + \rho\left(D_t - D_{t-1}\right). \quad (7)$$

Under the assumption that σ is significantly smaller than d and as justified by LST [17], one can show that $\Pr(Y_t^R < 0)$ is negligibly small; thus we assume that $Y_t^R \geq 0$.

Next, we proceed with the analysis of the distributor's ordering decisions.

A.2 Distributor's Expected Cost under 100% Fill Rate

Since the distributor is located in the middle of the three-level supply chain he could form a two-person coalition with the manufacturer to share his demand information (the retailer's order), or cooperate with the retailer to share end-demand information. We consider the following two cases for analyzing the distributor's ordering decisions and inventory-related costs under 100% fill-rate: (i) No information sharing with the retailer leading to $\hat{\pi}^{D1}$, and (ii) information sharing with the retailer leading to $\hat{\pi}^{D2}$.

Prior to analyzing the two cases, we first develop the $AR(1)$ model for characterizing the "demand process" (the retailer's order) faced by the distributor.

A.2.1 Retailer's Order Process Faced by the Distributor

Since the retailer is the distributor's immediate downstream "neighbor", orders placed by the retailer constitute the "demand" faced by the distributor. In order to analyze the distributor's ordering decisions, we now derive the $AR(1)$ model of the retailer's order process.

Lemma 1 Assuming a 100% fill-rate, the retailer's order process faced by the distributor is a one-period autocorrelated process

$$Y_{t+1}^R = d + \rho Y_t^R + \varepsilon_{t+1}^R, \tag{8}$$

where $\varepsilon_{t+1}^R = (1+\rho)\,\varepsilon_{t+1} - \rho\varepsilon_t$.

Proof. The $AR(1)$ model (1) implies $D_{t-1} = (D_t - d - \varepsilon_t)/\rho$. Replacing D_{t-1} in (7) with $(D_t - d - \varepsilon_t)/\rho$ gives $Y_t^R = D_t + \rho\,(D_t - D_{t-1}) = d + \rho D_t + \varepsilon_t$ so that the retailer's order quantity at the end of $t+1$ is $Y_{t+1}^R = d + \rho D_{t+1} + \varepsilon_{t+1}$. Since, from (1), we have $D_{t+1} = d + \rho D_t + \varepsilon_{t+1}$, we can write Y_{t+1}^R as $Y_{t+1}^R = (1+\rho)\,d + \rho^2 D_t + (1+\rho)\,\varepsilon_{t+1}$. In order to express Y_{t+1}^R in terms of Y_t^R, we combine the expressions for Y_{t+1}^R and Y_t^R to obtain $Y_{t+1}^R = d + \rho Y_t^R + \varepsilon_{t+1}^R$ where $\varepsilon_{t+1}^R = (1+\rho)\,\varepsilon_{t+1} - \rho\varepsilon_t$. ■

Since the error term is $\varepsilon_{t+1}^R = (1+\rho)\,\varepsilon_{t+1} - \rho\varepsilon_t$, where ε_t is normally distributed with zero mean and variance σ^2, we find that ε_{t+1}^R is also a normally distributed random variable with the mean

$$E\left(\varepsilon_{t+1}^R\right) = (1+\rho)\,E\left(\varepsilon_{t+1}\right) - \rho E\left(\varepsilon_t\right) = 0,$$

and the variance

$$\operatorname{Var}\left(\varepsilon_{t+1}^R\right) = (1+\rho)^2 \operatorname{Var}\left(\varepsilon_{t+1}\right) + \rho^2 \operatorname{Var}\left(\varepsilon_t\right) = \left[(1+\rho)^2 + \rho^2\right]\sigma^2.$$

A.2.2 Distributor's Expected Costs Under Different Coalitions

Assuming that the distributor's order can be fully satisfied by the manufacturer, we compute the distributor's expected costs for the following two cases: (i) no information sharing from the retailer and (ii) information sharing from the retailer. The distributor now decides the order-up-to level S_t^D at the end of the period t by forecasting the retailer's order quantity for the period $t+1$. In particular, we find the conditional mean and the variance of Y_{t+1}^R, and then compute S_t^D that minimizes the distributor's expected costs (i.e., the holding and shortage costs) for the period $t+1$.

Proposition 9 Assuming a 100% fill-rate, the distributor's optimal order-up-to level (S_t^D) at the end of period t and minimum expected cost ($\hat{\pi}^D$) for the period $t+1$ are

$$(S_t^D, \hat{\pi}^D) = \begin{cases} (S_t^{D1}, \hat{\pi}^{D1}), & \text{under the coalitions } \{M, D, R\}, \{R, (MD)\} \text{ and } \{D, (MR)\}, \\ (S_t^{D2}, \hat{\pi}^{D2}), & \text{under the coalitions } \{M, (DR)\} \text{ and } \{(MDR)\}, \end{cases}$$

where

$$
\begin{aligned}
S_t^{D1} &= d + \rho Y_t^R + k^D \sqrt{V_t^{D1}}, \quad \text{and} \quad \hat{\pi}^{D1} = \sqrt{V_t^{D1}} \left[h^D k^D + \left(h^D + p^D\right) I\left(k^D\right) \right], \\
S_t^{D2} &= d + \rho Y_t^R - \rho \varepsilon_t + k^D \sqrt{V_t^{D2}}, \quad \text{and} \quad \hat{\pi}^{D2} = \sqrt{V_t^{D2}} \left[h^D k^D + \left(h^D + p^D\right) I\left(k^D\right) \right], \\
\sqrt{V_t^{D1}} &= \sigma \sqrt{(1+\rho)^2 + \rho^2}, \quad \text{and} \quad \sqrt{V_t^{D2}} = \sigma (1+\rho),
\end{aligned}
$$

and k^D, h^D, p^D are defined similarly to k^R, h^R and p^R, respectively; ε_t is the realized value of the error term in the $AR(1)$ demand process (1), and Y_t^R is the size of the order placed by the retailer at the end of the period t.

Proof. Under the assumption of 100% fill-rate, we find the distributor's optimal solution and compute the corresponding expected cost as follows:

1. When the distributor doesn't share the demand information from the retailer under the coalitions $\{M, D, R\}$, $\{R, (MD)\}$ and $\{D, (MR)\}$, we know from (8) that the retailer's order quantity is an $AR(1)$ process such that $Y_{t+1}^R = d + \rho Y_t^R + (1+\rho)\varepsilon_{t+1} - \rho\varepsilon_t$, where Y_t^R is known to the distributor but ε_{t+1} and ε_t are unknown. Due to no information sharing with the retailer, the distributor has to consider ε_{t+1} and ε_t as two i.i.d. normal random variables with mean zero and variance σ^2. We compute the conditional mean m_t^{M1} and the conditional variance V_t^{M1} of the retailer's order quantity Y_{t+1}^R as

$$
\begin{aligned}
m_t^{D1} &= E(Y_{t+1}^R \mid Y_t^R) = d + \rho Y_t^R, \\
V_t^{D1} &= \mathrm{Var}(Y_{t+1}^R \mid Y_t^R) = \left[(1+\rho)^2 + \rho^2 \right] \sigma^2.
\end{aligned}
$$

   Thus, similar to the analysis in Section A.1, the distributor's expected cost for $t+1$ is

$$
\hat{\pi}^{D1} = h^D E\left[\left(S_t^{D1} - Y_{t+1}^R\right)^+ \right] + p^D E\left[\left(Y_{t+1}^R - S_t^{D1}\right)^+ \right].
$$

   The optimal order-up-to level that minimizes $\hat{\pi}^{D1}$ is found as $S_t^{D1} = m_t^{D1} + k^D\sqrt{V_t^{D1}}$. Simplifying $\hat{\pi}^{D1}$ we obtain $\hat{\pi}^{D1} = \sigma\sqrt{[(1+\rho)^2 + \rho^2]} \left[h^D k^D + \left(h^D + p^D\right) I\left(k^D\right) \right]$.

2. Under the coalitions $\{M, (DR)\}$ and $\{(MDR)\}$, when the retailer discloses end-demand information (that is, the realized value of the error term ε_t) to the distributor, the conditional mean m_t^{D2} and the conditional variance V_t^{D2} of the retailer's order quantity Y_{t+1}^R are found as

$$
\begin{aligned}
m_t^{D2} &= E(Y_{t+1}^R \mid Y_t^R, \varepsilon_t) = d + \rho Y_t^R - \rho\varepsilon_t, \\
V_t^{D2} &= \mathrm{Var}(Y_{t+1}^R \mid Y_t^R, \varepsilon_t) = (1+\rho)^2 \sigma^2.
\end{aligned}
$$

   In this case, the distributor's total expected cost for period $t+1$ is

$$
\hat{\pi}^{D2} = h^D E\left[\left(S_t^{D2} - Y_{t+1}^R\right)^+ \right] + p^D E\left[\left(Y_{t+1}^R - S_t^{D2}\right)^+ \right].
$$

As a consequence, the distributor's optimal order-up-to level that minimizes $\hat{\pi}^{D2}$ becomes $S_t^{D2} = m_t^{D2} + k^D \sqrt{V_t^{D2}}$. Finally, simplifying $\hat{\pi}^{D2}$ we have that

$$\hat{\pi}^{D2} = \sigma\left(1+\rho\right)\left[h^D k^D + \left(h^D + p^D\right) I\left(k^D\right)\right].$$

■

Next, we compute the order size of the distributor under the assumption of 100% fill-rate. When the distributor joins one of the coalitions $\{M, D, R\}$, $\{R, (MD)\}$ and $\{D, (MR)\}$, the size of order that the distributor places with the manufacturer at the end of period t, denoted by Y_t^{D1}, is the difference between the desired order-up-to level S_t^{D1} and the starting inventory $(S_{t-1}^{D1} - Y_t^R)$, i.e.,

$$Y_t^{D1} = S_t^{D1} - (S_{t-1}^{D1} - Y_t^R) = Y_t^R + \rho\left(Y_t^R - Y_{t-1}^R\right). \tag{9}$$

When the distributor joins one of the coalitions $\{M, (DR)\}$ and $\{(MDR)\}$, the order quantity Y_t^{D2} is computed as the difference between the desired order-up-to level S_t^{D2} and the starting inventory $(S_{t-1}^{D2} - Y_t^R)$, i.e.,

$$Y_t^{D2} = S_t^{D2} - (S_{t-1}^{D2} - Y_t^R) = Y_t^R + \rho\left(Y_t^R - Y_{t-1}^R\right) - \rho(\varepsilon_t - \varepsilon_{t-1}). \tag{10}$$

A.3 Manufacturer's Expected Cost

We now focus our attention on the ordering decisions at the manufacturer's level. In the three-level supply chain under study, the manufacturer is located at the highest echelon that is farthest from the ultimate customer. We assume that since the manufacturer produces the final product in the supply chain, the production quantity determined by him is fully realized. The demand faced by the manufacturer is the order received from the distributor. But the manufacturer may also share with the distributor the information regarding the orders received from the retailer, and/or share with the retailer the information regarding the end-demand. This results in five possible cases for sharing information between the manufacturer and the other members of the supply chain as discussed previously and depicted in Figure 1 leading to π^{M1}, π^{M2}, π^{M3}, π^{M4} and π^{M5}. Thus, we examine each situation to find the manufacturer's ordering decisions and compute the corresponding expected costs. Prior to analyzing the five cases, we investigate the distributor's order process.

A.3.1 Distributor's Order Process Faced by the Manufacturer

Based on whether or not the distributor receives end-demand information from the retailer, we model two different order processes for the distributor.

No information sharing between the distributor and the retailer The results in this subsection will be used later to compute π^{M1}, π^{M3} and π^{M4}. We know from (9) that the distributor's order size is $Y_t^{D1} = Y_t^R + \rho\left(Y_t^R - Y_{t-1}^R\right)$ and from (8) that the retailer's order process is

$Y_{t+1}^R = d + \rho Y_t^R + (1+\rho)\,\varepsilon_{t+1} - \rho\varepsilon_t$. We will use these results to characterize the distributor's order process in the $AR(1)$ form.

Lemma 2 Assuming a 100% fill-rate, the distributor's order process faced by the manufacturer, when the distributor and the retailer don't share end-demand information, is a one-period autocorrelated process

$$Y_{t+1}^{D1} = d + \rho Y_t^{D1} + (1+\rho)^2\,\varepsilon_{t+1} - 2\rho\,(1+\rho)\,\varepsilon_t + \rho^2\varepsilon_{t-1}, \tag{11}$$

where ε_{t+1}, ε_t and ε_{t-1} are unknown to the distributor.

Proof. First, we write

$$Y_t^{D1} = Y_t^R + \rho(Y_t^R - Y_{t-1}^R) = d + \rho Y_t^R + (1+\rho)\,\varepsilon_t - \rho\varepsilon_{t-1}. \tag{12}$$

For the case of no information sharing, the order size of the distributor for period $t+1$ is thus $Y_{t+1}^{D1} = d + \rho Y_{t+1}^R + (1+\rho)\,\varepsilon_{t+1} - \rho\varepsilon_t$. By using the retailer's order process, Y_{t+1}^{D1} can be expressed in terms of Y_t^R, i.e.,

$$\begin{aligned} Y_{t+1}^{D1} &= d + \rho\left[d + \rho Y_t^R + (1+\rho)\,\varepsilon_{t+1} - \rho\varepsilon_t\right] + (1+\rho)\,\varepsilon_{t+1} - \rho\varepsilon_t \\ &= (1+\rho)\,d + \rho^2 Y_t^R + \varepsilon_{t+1}^{D1}. \end{aligned} \tag{13}$$

where $\varepsilon_{t+1}^{D1} = (1+\rho)^2\,\varepsilon_{t+1} - \rho\,(1+\rho)\,\varepsilon_t$. Combining (12) and (13) gives the $AR\,(1)$ process of distributor's order in (11). ■

Information sharing between the distributor and the retailer The results in this subsection will be used later to compute π^{M2} and π^{M5}.

Lemma 3 Assuming a 100% fill-rate, the distributor's order process faced by the manufacturer, when the distributor and the retailer share end-demand information, is a one-period autocorrelated process

$$Y_{t+1}^{D2} = d + \rho Y_t^{D2} + \varepsilon_{t+1}^{D2}, \tag{14}$$

where $\varepsilon_{t+1}^{D2} = \left(\rho^2 + \rho + 1\right)\varepsilon_{t+1} - \rho\,(1+\rho)\,\varepsilon_t$ and the forecasting error ε_t is known to the distributor but ε_{t+1} is still unknown to both the retailer and distributor.

Proof. Similar to our analysis for the case of no information sharing, we re-write the distributor's order process in the setting of information sharing from (10) as $Y_t^{D2} = Y_t^R + \rho\left(Y_t^R - Y_{t-1}^R\right) - \rho(\varepsilon_t - \varepsilon_{t-1})$. Using (8) to replace Y_t^R in the distributor's order process results in $Y_t^{D2} = d + \rho Y_t^R + \varepsilon_t$, and analogously, $Y_{t+1}^{D2} = d + \rho Y_{t+1}^R + \varepsilon_{t+1}$. In order to write the order process in the $AR\,(1)$ form, we express Y_{t+1}^{D2} in terms of Y_t^{D2} and the forecasting errors and find (14). ■

With the $AR\,(1)$ process of the distributor's order, we can compute the manufacturer's ordering decisions and the corresponding costs for five cases mentioned previously.

A.3.2 Manufacturer's Expected Costs Under Different Coalitions

We now compute the manufacturer's expected costs under five coalitions depicted in Figure 1. Similar to the distributor and the retailer, the manufacturer now decides the order-up-to level at the end of the period t by forecasting the distributor's order quantity for the period $t+1$. Specifically, using the means and the variances of the quantity of order placed by the distributor, the manufacturer computes optimal order-up-to level that minimizes his expected costs for the period $t+1$.

Proposition 10 The manufacturer's optimal order-up-to level (S_t^M) at the end of period t and minimum expected cost (π^M) for the period $t+1$ are

$$(S_t^M, \pi^S) = \begin{cases} (S_t^{M1}, \pi^{M1}), & \text{under the coalition } \{M, D, R\}, \\ (S_t^{M2}, \pi^{M2}), & \text{under the coalition } \{M, (DR)\}, \\ (S_t^{M3}, \pi^{M3}), & \text{under the coalition } \{R, (MD)\}, \\ (S_t^{M4}, \pi^{M4}), & \text{under the coalition } \{D, (MR)\}, \\ (S_t^{M5}, \pi^{M5}), & \text{under the coalition } \{(MDR)\}, \end{cases}$$

where

$$\begin{aligned}
S_t^{M1} &= d + \rho Y_t^{D1} + k^M \sqrt{V_t^{M1}}, \text{ and } \pi^{M1} = \sqrt{V_t^{M1}} \left[h^M k^M + \left(h^M + p^M\right) I\left(k^M\right)\right], \\
S_t^{M2} &= d + \rho Y_t^{D2} + k^M \sqrt{V_t^{M2}}, \text{ and } \pi^{M2} = \sqrt{V_t^{M2}} \left[h^M k^M + \left(h^M + p^M\right) I\left(k^M\right)\right], \\
S_t^{M3} &= d + \rho Y_t^{D1} - \rho(1+\rho)\varepsilon_t + \rho^2 \varepsilon_{t-1} + k^M \sqrt{V_t^{M3}}, \text{ and } \pi^{M3} = \sqrt{V_t^{M3}} \left[h^M k^M + \left(h^M + p^M\right) I\left(k^M\right)\right], \\
S_t^{M4} &= d + \rho Y_t^{D1} - 2\rho(1+\rho)\varepsilon_t + \rho^2 \varepsilon_{t-1} + k^M \sqrt{V_t^{M4}}, \text{ and } \pi^{M4} = \sqrt{V_t^{M4}} \left[h^M k^M + \left(h^M + p^M\right) I\left(k^M\right)\right], \\
S_t^{M5} &= d + \rho Y_t^{D2} - \rho(1+\rho)\varepsilon_t + k^M \sqrt{V_t^{M5}}, \text{ and } \pi^{M5} = \sqrt{V_t^{M5}} \left[h^M k^M + \left(h^M + p^M\right) I\left(k^M\right)\right],
\end{aligned}$$

$$\begin{aligned}
\sqrt{V_t^{M1}} &= \sigma \sqrt{(1+\rho)^2 \left[(1+\rho)^2 + 4\rho^2\right] + \rho^4}, \ \sqrt{V_t^{M2}} = \sigma \sqrt{(1+\rho+\rho^2)^2 + \rho^2 (1+\rho)^2}, \\
\sqrt{V_t^{M3}} &= \sigma (1+\rho) \sqrt{\rho^2 + (1+\rho)^2}, \ \sqrt{V_t^{M4}} = \sigma (1+\rho)^2 \text{ and } \sqrt{V_t^{M5}} = \sigma \left(1+\rho+\rho^2\right),
\end{aligned}$$

and k^M, h^M, p^M are defined similarly to k^R, h^R, p^R, respectively.

Proof. Using the distributor's ordering processes (11) and (14), we compute the manufacturer's expected costs under five different coalitions.

1. Under the coalition $\{M, D, R\}$, the manufacturer's expected cost in period $t+1$ is

$$\pi^{M1} = h^M E\left[\left(S_t^{M1} - Y_{t+1}^{D1}\right)^+\right] + p^M E\left[\left(Y_{t+1}^{D1} - S_t^{M1}\right)^+\right].$$

The manufacturer's optimal order-up-to level at the end of time period t is found by minimizing π^{M1} which gives $S_t^{M1} = m_t^{M1} + k^M \sqrt{V_t^{M1}}$, where,

$$\begin{aligned} m_t^{M1} &= E\left(Y_{t+1}^{D1} \mid Y_t^{D1}\right) = d + \rho Y_t^{D1}, \\ V_t^{M1} &= \mathrm{Var}\left(Y_{t+1}^{D1} \mid Y_t^{D1}\right) = \sigma^2 \left\{(1+\rho)^2 \left[(1+\rho)^2 + 4\rho^2\right] + \rho^4\right\}. \end{aligned}$$

Simplifying π^{M1} we have

$$\pi^{M1} = \sigma \sqrt{(1+\rho)^2 \left[(1+\rho)^2 + 4\rho^2\right] + \rho^4} \left[h^M k^M + \left(h^M + p^M\right) I\left(k^M\right)\right].$$

2. Under the coalition $\{M, (DR)\}$, the manufacturer's expected cost in period $t+1$ is

$$\pi^{M2} = h^M E\left[\left(S_t^{M2} - Y_{t+1}^{D2}\right)^+\right] + p^M E\left[\left(Y_{t+1}^{D2} - S_t^{M2}\right)^+\right].$$

Using this expression we find the optimal order-up-to level at the end of period t as $S_t^{M2} = m_t^{M2} + k^M \sqrt{V_t^{M2}}$, where

$$\begin{aligned} m_t^{M2} &= E\left(Y_{t+1}^{D2} \mid Y_t^{D2}\right) = d + \rho Y_t^{D2}, \\ V_t^{M2} &= \mathrm{Var}\left(Y_{t+1}^{D2} \mid Y_t^{D2}\right) = \sigma^2 \left[\left(1+\rho+\rho^2\right)^2 + \rho^2 (1+\rho)^2\right]. \end{aligned}$$

Simplifying π^{M2} we have

$$\pi^{M2} = \sigma \sqrt{\left(1+\rho+\rho^2\right)^2 + \rho^2 (1+\rho)^2} \left[h^M k^M + \left(h^M + p^M\right) I\left(k^M\right)\right].$$

3. Under the coalition $\{R, (MD)\}$, the manufacturer's expected cost for period $t+1$ is

$$\pi^{M3} = h^M E\left[\left(S_t^{M3} - Y_{t+1}^{D1}\right)^+\right] + p^M E\left[\left(Y_{t+1}^{D1} - S_t^{M3}\right)^+\right].$$

Minimizing π^{M3} we find the optimal order-up-to level as $S_t^{M3} = m_t^{M3} + k^M \sqrt{V_t^{M3}}$, where

$$\begin{aligned} m_t^{M3} &= E(Y_{t+1}^{D1} \mid Y_t^{D1}, \varepsilon_t^R) = d + \rho Y_t^{D1} - \rho \varepsilon_t^R = (1+\rho)\, d + \rho Y_t^{D1} - \rho\left(Y_t^R - \rho Y_{t-1}^R\right) \\ V_t^{M3} &= \mathrm{Var}(Y_{t+1}^{D1} \mid Y_t^{D1}, \varepsilon_t^R) = (1+\rho)^4 \sigma^2 + \rho^2 (1+\rho)^2 \sigma^2 = \sigma^2 (1+\rho)^2 \left[(1+\rho)^2 + \rho^2\right]. \end{aligned}$$

Using S_t^{M3}, the manufacturer's minimum expected cost is found as

$$\pi^{M3} = \sigma (1+\rho) \sqrt{\rho^2 + (1+\rho)^2} \left[h^M k^M + \left(h^M + p^M\right) I\left(k^M\right)\right].$$

4. Under the coalition $\{D, (MR)\}$, the manufacturer's expected cost is

$$\pi^{M4} = h^M E\left[\left(S_t^{M4} - Y_{t+1}^{D1}\right)^+\right] + p^M E\left[\left(Y_{t+1}^{D1} - S_t^{M4}\right)^+\right].$$

Minimizing π^{M4} we find the optimal order-up-to level as $S_t^{M4} = m_t^{M4} + k^M \sqrt{V_t^{M4}}$, where

$$\begin{aligned} m_t^{M4} &= E(Y_{t+1}^{D1} \mid Y_t^{D1}, \varepsilon_t, \varepsilon_{t-1}) = d + \rho Y_t^{D1} - 2\rho(1+\rho)\varepsilon_t + \rho^2 \varepsilon_{t-1}, \\ V_t^{M4} &= \mathrm{Var}(Y_{t+1}^{D1} \mid Y_t^{D1}, \varepsilon_t, \varepsilon_{t-1}) = (1+\rho)^4 \sigma^2. \end{aligned}$$

Using S_t^{M4} we obtain

$$\pi^{M4} = \sigma(1+\rho)^2 \left[h^M k^M + \left(h^M + p^M\right) I\left(k^M\right)\right].$$

5. Under the coalition $\{(MDR)\}$, the manufacturer's expected cost for $t+1$ is

$$\pi^{M5} = h^M E\left[\left(S_t^{M5} - Y_{t+1}^{D2}\right)^+\right] + p^M E\left[\left(Y_{t+1}^{D2} - S_t^{M5}\right)^+\right].$$

The distributor's order process is now (14) rather than (11). In a manner analogous to the previous analyses, we compute the manufacturer's optimal order-up-to level as $S_t^{M5} = m_t^{M5} + k^M \sqrt{V_t^{M5}}$ where

$$\begin{aligned} m_t^{M5} &= E\left(Y_{t+1}^{D2} \mid Y_t^{D2}, \varepsilon_t\right) = d + \rho Y_t^{D2} - \rho(1+\rho)\varepsilon_t, \\ V_t^{M5} &= \mathrm{Var}\left(Y_{t+1}^{D2} \mid Y_t^{D2}, \varepsilon_t\right) = \sigma^2 \left[\left(1+\rho+\rho^2\right)^2\right]. \end{aligned}$$

Using S_t^{M5}, we have

$$\pi^{M5} = \sigma\left(1+\rho+\rho^2\right)\left[h^M k^M + \left(h^M + p^M\right) I\left(k^M\right)\right].$$

■

Appendix B Computation of the Characteristic Values for All Coalitions

We now compute the characteristic values of all possible coalitions, i.e., $v(M)$, $v(D)$, $v(R)$, $v(MD)$, $v(MR)$, $v(DR)$ and $v(MDR)$. First, the characteristic value of an empty coalition is naturally zero, i.e., $v(\varnothing) = 0$. Next, consider the single-player coalitions. When the retailer does not share demand information with other members of the supply chain, his characteristic value $v(R)$ depends on whether or not the distributor and manufacturer share demand information. If they don't share information, then the retailer receives no cost savings, i.e., $v(R) = 0$. Otherwise, the retailer's cost savings is $v(R) = \pi^{R1} - \pi^{R2}$. Since the characteristic function value $v(i)$, $(i = M, D, R)$ represents the amount (cost savings) that member i could achieve under the worst possible conditions (Straffin [31, p. 131]) if the retailer does not share demand information with any other member, we obtain his characteristic value as $v(R) = \min(0, \pi^{R1} - \pi^{R2})$. Similarly, the distributor's characteristic value is obtained as $v(D) = \min(0, \pi^{D1} - \pi^{D4})$. However, the manufacturer the situation is different, since the manufacturer is the most upstream member in the supply chain so that the fill-rate faced the manufacturer is 100%. When the distributor and retailer share information, the bullwhip effect is reduced and the manufacturer experiences a cost savings of $\pi^{M1}-\pi^{M2}$ which is positive according to Proposition 1. On the other hand if the distributor and retailer do not share information, then the manufacturer has no cost savings. As the characteristic function value $v(M)$ is the cost savings that the manufacturer could achieve under the worst possible conditions if he does not share demand information with any other member, we obtain $v(M) = 0$.

Now, we consider the two-member coalitions and the grand coalition.

$v(MD)$: **The value of the coalition** (MD) The characteristic value of the coalition involving only the manufacturer and the distributor is total expected cost that both members could save when only they share information. Therefore, in order to compute $v(MD)$, we calculate the cost savings incurred at the manufacturer and the distributor levels. In this case, the retailer doesn't release end-demand information to any other member. When the distributor shares his information with the manufacturer, the distributor achieves the expected cost savings of $\pi^{D1} - \pi^{D3}$. The manufacturer's expected cost savings are computed as $\pi^{M1} - \pi^{M3}$, thus we have that $v(MD) = (\pi^{D1} - \pi^{D3}) + (\pi^{M1} - \pi^{M3})$.

$v(MR)$: **The value of the coalition** (MR) In this case, the retailer's expected cost savings are $\pi^{R1}-\pi^{R4}$, and the manufacturer's expected cost savings are computed as $\pi^{M1}-\pi^{M4}$, so that $v(MR) = (\pi^{R1} - \pi^{R4}) + (\pi^{M1} - \pi^{M4})$.

$v(DR)$: **The value of the coalition** (DR) Since the retailer's expected cost savings are $\pi^{R1} - \pi^{R2}$, and the distributor's expected cost savings are $\pi^{D1} - \pi^{D2}$, we have $v(DR) = (\pi^{R1} - \pi^{R2}) + (\pi^{D1} - \pi^{D2})$.

$v(MDR)$: **The value of the grand coalition** (MDR) In the grand coalition, all members can enjoy cost savings from sharing the end-demand information. We obtain $v(MDR) = (\pi^{R1} - \pi^{R5}) + (\pi^{D1} - \pi^{D5}) + \big(\pi^{M1} - \pi^{M5}\big)$.

Appendix C Proofs

Proof of Proposition 1. A straightforward comparison of π^{Mi}, $i = 1, \ldots, 5$ in Table 1 yields the property for the manufacturer's expected costs. For example, it is easy to see from Table 1 that since $\rho \neq 0$, $\pi^{M5} < \pi^{M2}$, i.e., the manufacturer incurs a smaller cost when he is a partner in the grand coalition $\{(MDR)\}$ than under the coalitional structure $\{M, (DR)\}$. ■

Proof of Proposition 2. In the three-player cooperative game there are five possible coalitions, i.e., $\{M, D, R\}$, $\{R, (MD)\}$, $\{D, (MR)\}$, $\{M, (DR)\}$ and $\{(MDR)\}$. A coalition will be stable if leaving the coalition makes a player worse off. We begin by analyzing the stability of the grand coalition $\{(MDR)\}$.

1. The grand coalition $\{(MDR)\}$ would be stable only if the following two criteria are satisfied:
   (a) The total cost savings incurred by all members in the grand coalition are no less than those achieved in any other coalitions. Specifically, the following conditions must be satisfied for stability of the grand coalition: $v(MDR) \geq (\pi^{R1} - \pi^{R3}) + v(MD)$, $v(MDR) \geq (\pi^{D1}-\pi^{D4})+v(MR)$, $v(MDR) \geq (\pi^{M1}-\pi^{M2})+v(DR)$; $v(MDR) \geq v(M)+v(D)+v(R)$.

Otherwise, if a coalition generates higher cost savings, we cannot find an allocation scheme to make the grand coalition stable.

(b) None of the players in the grand coalition has an incentive to leave the coalition. When the manufacturer leaves the coalition, he has two possible cost savings. If $v(DR) \geq v(D) + v(R)$, then the distributor and the retailer would prefer to stay in the two-player coalition (DR), so the manufacturer incurs the savings $(\pi^{M1} - \pi^{M2})$; otherwise, the two-player coalition (DR) is changed to two single-player coalitions, and the manufacturer's cost savings is reduced to zero. Summarizing the above gives the manufacturer's cost savings (after leaving the grand coalition) as

$$\omega^M = \begin{cases} \pi^{M1} - \pi^{M2} & \text{if } v(DR) \geq v(D) + v(R), \\ 0, & \text{if } v(DR) < v(D) + v(R). \end{cases}$$

Similarly, we can write ω^D and ω^R as

$$\omega^D = \begin{cases} \pi^{D1} - \pi^{D4}, & \text{if } v(MR) \geq v(R), \\ 0, & \text{if } v(MR) < v(R), \end{cases} \quad \text{and } \omega^R = \begin{cases} \pi^{R1} - \pi^{R3}, & \text{if } v(MD) \geq v(D), \\ 0, & \text{if } v(MD) < v(D). \end{cases}$$

Thus, the second condition for the stability of the grand coalition is $v(MDR) \geq \omega^M + \omega^D + \omega^R$, which assures that no player has an incentive to leave the coalition.

2. The coalition $\{R, (MD)\}$ would be stable only if the following two criteria are satisfied:

(a) Similar to our analysis for the stability of grand coalition, total cost savings incurred by all members in this coalition are no less than those achieved in the grand coalition. Specifically, the following conditions must be satisfied: $(\pi^{R1} - \pi^{R3}) + v(MD) \geq v(MDR)$.

(b) Each player in the two-player coalition (MD) will be worse off if s/he leaves the coalition. When a player leaves, the manufacturer and the distributor would have the cost savings $v(M)$ and $v(D)$. In order to keep the two players in the coalition, we must have the condition $v(MD) \geq v(M) + v(D) = v(D)$.

The analysis for the coalitions $\{D, (MR)\}$ and $\{M, (DR)\}$ is similar.

3. Obviously, the coalition $\{M, D, R\}$ is stable if any other coalition is unstable.

■

Proof of Proposition 3. We first use Nash arbitration scheme to compute the allocation scheme. If the coalition $\{M, (DR)\}$ is stable, the Nash arbitration scheme of allocating the cost savings between the distributor and the retailer is computed by solving

$$\max\ [x_D - v(D)][x_R - v(R)], \quad \text{s.t.} \quad x_D \geq v(D), \quad x_R \geq v(R).$$

Since $x_D + x_R = v(DR)$, the problem can be reduced to max $[x_D - v(D)][-x_D + v(DR) - v(R)]$, s.t. $x_D \geq v(D)$ and $x_R \geq v(R)$. Ignoring the constraints, we find the first-order derivative of the objective function w.r.t x_D as $-2x_D + v(DR) - v(R) + v(D)$. Equating it to zero and solving the resulting equation for x_D, we have that $x_D = [v(DR) + v(D) - v(R)]/2$. The value of x_R is obtained as $v(DR) - x_D = [v(DR) - v(D) + v(R)]/2$. Note that the constraints ($x_D \geq v(D)$, $x_R \geq v(R)$) are satisfied by the solution. As a result, the Nash arbitration scheme is the same as the egalitarian proposal. Similarly, we can find the Nash arbitration scheme for the other two-player coalitions.

Next, when the coalition $\{M, (DR)\}$ is stable, we use Shapley value to allocate the cost savings $v(DR)$ between the distributor and the retailer. Using the above formula, we have

$$\begin{aligned}\varphi_D &= \frac{0!1![v(D) - v(\phi)]}{2!} + \frac{1!0![v(DR) - v(R)]}{2!} = \frac{v(DR) + v(D) - v(R)}{2}, \\ \varphi_R &= \frac{0!1![v(R) - v(\phi)]}{2!} + \frac{1!0![v(DR) - v(D)]}{2!} = \frac{v(DR) - v(D) + v(R)}{2}.\end{aligned}$$

Thus, the allocation in terms of Shapley value is also the same as that suggested by the egalitarian proposal. When we consider all stable two-player coalitions (i.e., $\{i, (jk)\}$, i, j, $k = M$, D, R, and $i \neq j \neq k$), we reach the allocation scheme. ■

Proof of Theorem 1. In a non-empty constrained core, an imputation (x_M, x_D, x_R) exists such that for all $T \subseteq \{M, D, R\}$, $\sum_{i \in T} x_i \geq v(T)$, $x_M \geq \omega^M$, $x_D \geq \omega^D$ and $x_R \geq \omega^R$. Proposition 2 indicates that $x_M + x_D + x_R = v(MDR) \geq \omega^M + \omega^D + \omega^R$, which assures that we can always find an allocation satisfying the three constraints. Since $x_M + x_D + x_R = v(MDR)$, the inequality $x_M + x_D \geq v(MD)$ is equivalent to the inequality $x_R \leq v(MDR) - v(MD)$. Similarly, $x_M \leq v(MDR) - v(DR)$ and $x_D \leq v(MDR) - v(MR)$. Summing the three inequalities, we obtain that $x_M + x_D + x_R \leq 3v(MDR) - [v(MD) + v(DR) + v(MR)]$. Using $x_M + x_D + x_R = v(MDR)$, we find the condition $2v(MDR) \geq v(MD) + v(DR) + v(MR)$ under which the constrained core is non-empty. For example, under the condition we examine an arbitrary imputation $(\hat{x}_M, \hat{x}_D, \hat{x}_R)$, where $\hat{x}_M = \omega^M + \zeta^M$ (in which $0 \leq \zeta^M \leq v(MDR) - v(DR) - \omega^M$), $\hat{x}_D = \omega^D + \zeta^D$ (in which $0 \leq \zeta^D \leq v(MDR) - v(MR) - \omega^D$), $\hat{x}_R = \omega^R + \zeta^R$ (in which $0 \leq \zeta^R \leq v(MDR) - v(MD) - \omega^R$) and $\sum \zeta = v(MDR) - \sum \omega^i$. According to Proposition 2, the imputation $(\hat{x}_M, \hat{x}_D, \hat{x}_R)$ always exists and the constrained core is non-empty if the condition $2v(MDR) \geq v(MD) + v(DR) + v(MR)$ is satisfied. ■

Proof of Proposition 4. The $(\varphi_M, \varphi_D, \varphi_R)$ values easily follow by using the formula $\varphi_i = \sum_{i \in T} (|T| - 1)!(3 - |T|)![v(T) - v(T - i)]/3!$ where T denotes an information sharing coalition, $|T|$ is the size of T and $i = M, D, R$. To assure stability of the grand coalition $\{(MDR)\}$, the three conditions $\varphi_M \geq \omega^M$, $\varphi_D \geq \omega^D$ and $\varphi_R \geq \omega^R$ must be satisfied. Otherwise, the grand coalition is unstable. ■

Proof of Proposition 5. Theorem 1 indicates that the constrained core is empty if $2v(MDR) < v(MD) + v(DR) + v(MR)$. We now obtain the solution for the game with an empty core. As we described above, to find the nucleolus solution, we should make the excesses for all coalitions as small as possible. These excesses are given as follows:

$$\begin{aligned}
e_{(R)}(\mathbf{x}) &= v(R) - x_R = \min(0, \pi^{R1} - \pi^{R3}) - x_R, \\
e_{(D)}(\mathbf{x}) &= v(D) - x_D = \min(0, \pi^{D1} - \pi^{D4}) - x_D, \\
e_{(M)}(\mathbf{x}) &= v(M) - x_M = -x_M, \\
e_{(DR)}(\mathbf{x}) &= v(DR) - (x_D + x_R) = (\pi^{R1} - \pi^{R2}) + (\pi^{D1} - \pi^{D2}) - (x_D + x_R), \\
e_{(MR)}(\mathbf{x}) &= v(MR) - (x_M + x_R) = (\pi^{R1} - \pi^{R4}) + (\pi^{M1} - \pi^{M4}) - (x_M + x_R), \\
e_{(MD)}(\mathbf{x}) &= v(MD) - (x_M + x_D) = (\pi^{D1} - \pi^{D3}) + (\pi^{M1} - \pi^{M3}) - (x_M + x_D), \\
e_{(MDR)}(\mathbf{x}) &= v(MDR) - (x_M + x_D + x_R) = 0.
\end{aligned}$$

Since $2v(MDR) < v(MD) + v(DR) + v(MR)$ and $x_M + x_D + x_R = v(MDR)$, we have

$$\begin{aligned}
&[v(DR) - (x_D + x_R)] + [v(MR) - (x_M + x_R)] + [v(MD) - (x_M + x_D)] \\
= \; & e_{(DR)}(\mathbf{x}) + e_{(MR)}(\mathbf{x}) + e_{(MD)}(\mathbf{x}) > 0,
\end{aligned}$$

which implies that at least one of the three excesses ($e_{(DR)}(\mathbf{x})$, $e_{(MR)}(\mathbf{x})$ and $e_{(MD)}(\mathbf{x})$) is positive. Since $\min(0, \pi^{R1} - \pi^{R3}) \leq 0$ and $\min(0, \pi^{D1} - \pi^{D4}) \leq 0$, $e_{(R)}(\mathbf{x})$, $e_{(D)}(\mathbf{x})$ and $e_{(M)}(\mathbf{x})$ are non-positive. In order to reach the nucleolus solution, we first minimize the largest excess ("unhappiness"). Thus, for this case we minimize the largest excess of $e_{(DR)}(\mathbf{x})$, $e_{(MR)}(\mathbf{x})$ and $e_{(MD)}(\mathbf{x})$. If $e_{(DR)}(\mathbf{x})$ is the maximum excess, we raise imputation x_D and/or x_R. This lowers the imputation x_M and increases the excesses $e_{(MR)}(\mathbf{x})$ and $e_{(MD)}(x)$. A similar analysis is followed when $e_{(MR)}(x)$ or $e_{(MD)}(x)$ is the largest. When we ignore the constraints $\nu_M \geq \omega^M$, $\nu_D \geq \omega^D$ and $\nu_R \geq \omega^R$, we reach the nucleolus solution once $e_{(DR)}(\mathbf{x})$, $e_{(MR)}(\mathbf{x})$ and $e_{(MD)}(\mathbf{x})$ are equal. Thus, solving $e_{(DR)}(\mathbf{x}) = e_{(MR)}(\mathbf{x}) = e_{(MD)}(\mathbf{x})$ gives

$$\begin{aligned}
\nu_M &= \frac{v(MD) + v(MR) + v(MDR) - 2v(DR)}{3}, \; \nu_D = \frac{v(MD) + v(DR) + v(MDR) - 2v(MR)}{3}, \\
\nu_R &= \frac{v(DR) + v(MR) + v(MDR) - 2v(MD)}{3}.
\end{aligned}$$

Since $2v(MDR) < v(MD) + v(DR) + v(MR)$, we have

$$\begin{aligned}
\nu_M &> v(MDR) - v(DR) \geq \pi^{M1} - \pi^{M2} = \omega^M \text{ (in the coalition } \{M, (DR)\}), \\
\nu_D &> v(MDR) - v(MR) \geq \pi^{D1} - \theta^{D4} = \omega^D \text{ (in the coalition } \{D, (MR)\}), \\
\nu_R &> v(MDR) - v(MD) \geq \pi^{R1} - \pi^{R3} = \omega^R \text{ (in the coalition } \{R, (MD)\}).
\end{aligned}$$

Thus, (ν_M, ν_D, ν_R) is the constrained nucleolus solution when the constrained core is empty. ■

Proof of Proposition 6. When the coalition $\{M, (DR)\}$ is stable, the distributor and the retailer jointly have the cost savings of $v(DR) = (\pi^{R1} - \pi^{R2}) + (\pi^{D1} - \pi^{D2})$, where the manufacturer's and the retailer's local savings are $(\pi^{R1} - \pi^{R2})$ and $(\pi^{D1} - \pi^{D2})$, respectively. From Proposition 3, these two players have the allocations as $x_D = [v(DR)+v(D)-v(R)]/2$ and $x_R = [v(DR)-v(D)+v(R)]/2$. Since the distributor achieves the savings $\pi^{D1} - \pi^{D2}$ before the allocation and has the savings x_D after the allocation, the side payment transferred from the distributor to the retailer is computed as $(\pi^{D1} - \pi^{D2}) - x_D$. After such a transfer, the distributor has the allocation x_D, and the retailer has the allocation $(\pi^{R1} - \pi^{R2}) + (\pi^{D1} - \pi^{D2}) - x_D = v(DR) - x_D = x_R$. Note that, if the side payment is negative, the retailer transfers the amount $\left|(\pi^{D1} - \pi^{D2}) - x_D\right|$ to the distributor. The side payments for the other two situations can be found in a similar manner. ■

Proof of Proposition 7. Since $\alpha_M + \alpha_D + \alpha_R = x_M + x_D + x_R$, we have $(\alpha_M - x_M) + (\alpha_D - x_D) + (\alpha_R - x_R) = 0$, where each term in the LHS of the last equality represents the difference between the cost savings obtained by a player before and after the allocation. Among these three differences, at least one is non-positive and at least one is non-negative. There are two possibilities as follows:

1. If two differences are non-negative and one difference is non-positive, then the players with non-negative differences transfer their differences to the player with a non-positive difference. For example, if the manufacturer and the distributor have the non-negative differences $(\alpha_M - x_M)$ and $(\alpha_D - x_D)$, respectively, then they transfer the amounts $(\alpha_M - x_M)$ and $(\alpha_D - x_D)$ to the retailer, so that the retailer receives total side payment amounting to $(\alpha_M - x_M) + (\alpha_D - x_D)$.
2. If two differences are non-positive and one difference is non-negative, then the player with a non-negative difference transfers the difference to the other two players. For example, if the manufacturer has the non-negative difference $(\alpha_M - x_M)$, the manufacturer transfers the amount of $(x_D - \alpha_D)$ to the distributor, and the amount of to $(x_R - \alpha_R)$ the retailer. Note that $\alpha_M - x_M = [v(MDR) - \alpha_D - \alpha_R] - [v(MDR) - x_D - x_R] = (x_D - \alpha_D) + (x_R - \alpha_R)$.

The proposition follows after considering all six possibilities. ■

Appendix D Sensitivity Analysis of the Parameter ρ

When the value of ρ is increased from 0.01 to 1.0, we find the stable coalition and compute Shapley value and the constrained nucleolus solution, as shown in Table 4.

Sensitivity analysis when the value of ρ is increased from 0.01 to 0.1 in increments of 0.01.										
		Total	When a two-player coalition is stable			When the grand coalition $\{(MDR)\}$ is stable				
	Stable	Cost	Allocation scheme			Is core	Does Shapley value	Constrained nucleolus solution		
ρ	coalition	Savings	Retailer	Distributor	Manufacturer	empty?	make $\{(MDR)\}$ stable?	Retailer	Distributor	Manufacturer
0.01	$\{M,(DR)\}$	0.28	0.03	0.03	0.22	—	—	—	—	—
0.02	$\{M,(DR)\}$	0.61	0.08	0.08	0.45	—	—	—	—	—
0.03	$\{M,(DR)\}$	0.96	—	—	—	No	No	0.15	0.15	0.68
0.04	$\{M,(DR)\}$	1.22	—	—	—	No	No	0.20	0.20	0.82
0.05	$\{M,(DR)\}$	1.7	—	—	—	No	No	0.31	0.31	1.08
0.06	$\{M,(DR)\}$	2.08	—	—	—	No	No	0.35	0.35	1.38
0.07	$\{(MDR)\}$	2.56	—	—	—	No	No	0.44	0.44	1.68
0.08	$\{(MDR)\}$	3.08	—	—	—	No	No	0.565	0.565	1.95
0.09	$\{(MDR)\}$	3.49	—	—	—	No	No	0.635	0.635	2.22
0.10	$\{(MDR)\}$	3.77	—	—	—	No	No	0.685	0.685	2.4
Sensitivity analysis when the value of ρ is increased from 0.1 to 1.0 in increments of 0.1.										
0.10	$\{(MDR)\}$	3.77	—	—	—	No	No	0.685	0.685	2.4
0.20	$\{(MDR)\}$	10.31	—	—	—	No	No	2.37	2.37	5.57
0.30	$\{(MDR)\}$	18.16	—	—	—	No	No	4.5	4.5	9.16
0.40	$\{(MDR)\}$	29.26	—	—	—	No	No	8.025	8.025	13.21
0.50	$\{(MDR)\}$	42.68	—	—	—	No	No	13.465	11.505	17.71
0.60	$\{(MDR)\}$	56.57	—	—	—	No	No	19.05	14.88	22.64
0.70	$\{(MDR)\}$	69.77	—	—	—	No	No	24.295	17.475	28
0.80	$\{(MDR)\}$	85.05	—	—	—	No	No	32.34	18.92	33.79
0.90	$\{(MDR)\}$	107	—	—	—	No	No	44.98	21.92	40.01
1.00	$\{(MDR)\}$	125.3	—	—	—	No	No	53.19	25.45	46.66

Table 4: The impacts of the parameter ρ on the coalitional stability and allocation schemes.