Negotiation-Sequence, Pricing, and Ordering Decisions in a Three-Echelon Supply Chain: A Coopetitive-Game Analysis

Feimin Zhong\textsuperscript{b}, Zhongbao Zhou\textsuperscript{c}, Mingming Leng\textsuperscript{d}

Submitted May 2020
Revised December 2020 and January 2021
Accepted February 2021
To appear in European Journal of Operational Research

\textsuperscript{a}The authors are grateful to the Editor (Professor Ruud Teunter) and two anonymous reviewers for their insightful comments that helped improve this paper.

\textsuperscript{b}School of Business Administration, Hunan University, Changsha, 410082, China. (Email: zhongfeimin@hnu.edu.cn)

\textsuperscript{c}School of Business Administration, Hunan University, Changsha, 410082, China. (Email: z.b.zhou@hnu.edu.cn)

\textsuperscript{d}Corresponding author. Faculty of Business, Lingnan University, Hong Kong. (Email: mm-leng@ln.edu.hk; Tel: +852 2616-8104; Fax: +852 2892-2442)
Abstract

We investigate a three-echelon supply chain in which a distributor at the middle echelon negotiates two wholesale price contracts with his upstream manufacturer and downstream retailer. In the first stage, the distributor decides on whether to first negotiate with the manufacturer or with the retailer; in the second (combined, noncooperative-cooperative, game) stage, the two negotiations are conducted sequentially. We find that the supply chain can be coordinated if the distributor first negotiates with the retailer. The distributor should choose the negotiation sequence for supply chain coordination, if he has a sufficiently large (small) relative bargaining power in the negotiation with the manufacturer (the retailer). We also extend our analysis to the cases in which the distributor and the manufacturer negotiate a buyback or two-part tariff contract, and draw similar outcomes when the distributor first negotiates with the retailer. In addition, under the two-part tariff contract, the distributor prefers to first negotiate with the retailer if the manufacturer has a sufficiently high disagreement payoff whereas, under the buyback contract, the distributor always prefers to first negotiate with the firm with a stronger bargaining power. Moreover, the two-part tariff (buyback) contract cannot (can) always coordinate the supply chain.

Keywords: Supply chain management; negotiation sequence, pricing; coopetitive game; generalized Nash bargaining solution.
1 Introduction

In a supply chain, negotiations provide a way for firms to collaborate with each other and achieve higher total profit. Moreover, negotiations aim at solving the conflicts on how the additional profits resulting from collaboration are allocated. The allocation of the total profit achieved by two firms is realized by the price that the downstream firm pays to the upstream firm. Thus, when the firms at two adjacent echelons interact, they naturally need to negotiate a price (see, e.g., Ertel 1999 and Zhong et al. 2016). Grayson et al. (2020) have found that any product flow from a manufacturer to a retailer usually involves one or more distributors, and any agreement on terms for transfer of ownership or possession may involve negotiations. This exposes the fact that, in practice, distributors may negotiate with both their upstream partners (e.g., suppliers and manufacturers) and their downstream partners (e.g., retailers).

Lemmons-Poscente (2006), who is the founder and president of International Speakers Bureau, also showed the existence and importance of negotiating both sides of a deal. A real example is about Ferguson plc (https://www.fergusonplc.com), which is the world’s leading value added distributor of plumbing and heating products and serves over 45,000 suppliers and 1 million customers. The company has negotiated with their suppliers and successfully obtained competitive prices from the suppliers because of its leading market position, and also consults with key downstream firms for a variety of business services. Thus, there is a natural question about the firm’s decision on the sequence of negotiations with his upstream and downstream partners: which negotiation should the firm first conduct? Different negotiation sequences may affect supply chain integration and thus influence supply chain as well as all firms’ profitability.

Extant publications do not consider such vertical negotiation-sequence decisions, we expect to contribute to the literature by analyzing the negotiation-sequence, wholesale pricing, and quantity decisions in a three-echelon supply chain. In the supply chain under our study, the distributor purchases a product from the manufacturer, and distributes the product to the retailer who faces a stochastic demand. The distributor needs to negotiate a wholesale price contract with the manufacturer, and also needs to negotiate another wholesale price contract with the retailer. The distributor should determine the sequence of the two negotiations and all the prices are then determined. Accordingly, we investigate a two-stage problem. In the first stage, the distributor decides on his negotiation sequence. In the second stage, the two negotiations are conducted sequentially to determine the wholesale prices and order quantities, which are characterized by using the concept of generalized Nash bargaining solution.

Our two-stage model involves both non-cooperative and cooperative analyses, which is somewhat similar to but differs from Hart and Moore’s multistage game (1990) and Brandenburger and Stuart’s biform game (2007). Specifically, Hart and Moore (1990) developed a multistage game under incomplete contracts, in which they characterized a unique Nash equilibrium for the non-cooperative part and used the Shapley value for the cooperative part. Brandenburger and Stuart (2007) defined a two-stage game, in which the players play a “simultaneous-move,” non-cooperative game in the first stage. The resulting strategies drawn in the first stage define a transferable-utility (TU) cooperative
game in the second stage, in which all players evaluate the TU cooperative game with respect to the confidence index. Differently, in our two-stage game, the distributor first makes a non-cooperative decision on the sequence of his two negotiations in the first stage. Then, in the second stage, the distributor conducts the two negotiations with his upstream and downstream partners to make quantity and pricing decisions, which are characterized by using the generalized Nash bargaining solution in a non-transferable utility (NTU) cooperative game. Therefore, our two-stage game differs from Hart and Moore’s multistage game and Brandenburger and Stuart’s biform game. Nonetheless, as our two-stage game problem also involves non-cooperative and cooperative games, we simply call our problem a “coopetitive game.”

According to our coopetitive game analyses, when the distributor first negotiates with the retailer, the supply chain can be coordinated. However, if the distributor first negotiates with the manufacturer, then a powerful retailer may reduce the supply chain efficiency, which differs from Zhong et al.’s finding (2016) that a powerful retailer can promote supply chain coordination. This occurs mainly because, in our paper, the distributor and the manufacturer negotiate both the wholesale price and order quantity, whereas Zhong et al. (2016) assumed that the neighboring firms only bargain over the wholesale price. We also study the buyback and two-part tariff contracts, and find that the buyback contract can always coordinate the supply chain whereas the two-part tariff contract cannot. In addition, the manufacturer’s sufficiently strong position and the retailer’s sufficiently weak position are necessary conditions for all the three firms to prefer to use the buyback contract, which can induce supply chain coordination.

A number of extant relevant publications have discussed exogenous factors that may affect bargaining outcomes. These factors include (i) relative bargaining powers (see, e.g., Iyer and Villas-Boas 2003, Nagarajan and Bassok 2005, Nagarajan and Bassok 2008, and Zhong et al. 2016), (ii) competition (see, e.g., Dukes et al. 2006), and (iii) uncertainty (see, e.g., Leng and Parlar 2009, and Zheng and Negenborn 2015). In addition, many researchers have studied the bargaining process; see the surveys by Bernstein and Nagarajan (2011), Ingene et al. (2012), and Nagarajan and Sosic (2008).

Very few publications have addressed the negotiation timing-related problem in the supply chain setting. Marx and Shaffer (2007) considered a two-echelon supply chain that consists of two competing suppliers and one retailer. Clark and Pereau (2009) studied a two-echelon supply chain with multiple suppliers and one retailer. Guo and Iyer (2013) considered a two-echelon supply chain including one supplier and two competing retailers. Different from them, our paper is concerned with an inside firm’s negotiation-sequence decision in a three-echelon supply chain. In addition, our paper is also related to the literature that focuses on buyback contract (or, return policy) and two-part tariff contract in supply chain models. There are a number of relevant review publications, which include Arshinder et al. (2011), Liu et al. (2015), Zhang and Zhou (2015), Guo et al. (2017), and Shen et al. (2019). Different from relevant publications, our paper is concerned with a distributor’s optimal negotiation sequence in a three-echelon supply chain under the two contracts.

The remainder of this paper is organized as follows. In Section 2, we describe a two-stage decision process. We solve the two-stage decision problem to find the distributor’s optimal negotiation sequence decision in Section 3. Then, we extend our analysis to two other contracts in Section 4.
This paper ends with a summary of major results in Section 5. We relegate the proofs of lemmas and theorems to Appendix A, where the proofs are given in the order that they appear in the main body of our paper.

2 Coopetitive Game: A Two-Stage Game Decision Process

The three-echelon \((n \geq 3)\) supply chain involves a manufacturer, a distributor, and a retailer. The manufacturer makes a product at a unit cost \(c\), which is then sold to the distributor at a wholesale price \(w_m\) with total quantity \(q_d\). The distributor then distributes the product to the retailer at a wholesale price \(w_d\). The retailer orders \(q_r\) units of the products and sells them at a retail price \(p\) \((p > c)\) in a market in which total demand \(D\) is a bounded, positive random variable with probability density function \(f\) and cumulative distribution function \(F\).

In the supply chain, the distributor and the manufacturer negotiate wholesale price \(w_m\) together with order quantity \(q_d\), and the manufacturer and the retailer negotiate wholesale price \(w_d\) and order quantity \(q_r\) \((q_r \leq q_d)\). Using the above, we can compute the manufacturer’s profit \(\pi_m\), the distributor’s profit \(\pi_d\), and the retailer’s expected profit \(\pi_r\) as 

\[
\pi_m = (w_m - c)q_d, \quad \pi_d = w_dq_r - w_mq_d, \quad \text{and} \quad \pi_r = pE[\min\{D, q_r\}] - w_dq_r,
\]

respectively. The supply chain-wide expected profit is \(\Pi = \pi_m + \pi_d + \pi_r = pE[\min\{D, q_r\}] - cq_r\). The globally-optimal order quantity (that maximizes \(\Pi\)) is obtained as 

\[
quasion = F^{-1}\left((p - c)/p\right),
\]

and the maximum supply chain-wide expected profit is thus computed as 

\[
\Pi^* = \int_{0}^{x^*} x f(x)dx.
\]

To facilitate our discussions, we let 

\[
T(x) = \int_{0}^{x} tf(t)dt,
\]

which is a strictly increasing function, and also find that \(\Pi^* = T(x^*)\).

As the distributor has two negotiations, he needs to address the question of which negotiation should be first conducted. We can accordingly characterize the decision-making process in the supply chain as a two-stage decision model.

Stage 1: Negotiation-Sequence Decision. The distributor determines his optimal negotiation sequence. That is, the distributor decides whether to first negotiate with the manufacturer or to first negotiate with the retailer.

Stage 2: Negotiated Wholesale Prices and Order Quantities. Based on the optimal negotiation sequence made in stage 1, the wholesale prices and the order quantities are negotiated. If the distributor first negotiates wholesale price \(w_m\) and order quantity \(q_d\) with the manufacturer, then he bargains with the retailer on the wholesale price contract \((w_d, q_r)\) based on the negotiated \((w_m, q_d)\). Otherwise, if the distributor first negotiates wholesale price \(w_d\) and order quantity \(q_r\) with the retailer, then the negotiated wholesale price \(w_m\) and order quantity \(q_d\) are based on the bargaining result \((w_d, q_r)\).

For each two-firm negotiation, we characterize the negotiated result by using the generalized Nash bargaining (GNB) solution (Nash (1950), Nash 1953, Roth 1979). For a two-player bargaining problem, we can obtain the GNB solution by solving the following maximization problem:

\[
\max_{(y_1, y_2) \in \Omega} (y_1 - y_1^0)^{\lambda}(y_2 - y_2^0)^{1-\lambda}, \quad \text{s.t.} \quad y_1 \geq y_1^0 \quad \text{and} \quad y_2 \geq y_2^0,
\]
where $\lambda \in [0, 1]$ and $1 - \lambda$ denote players 1’s and 2’s relative bargaining powers, respectively; $\Omega$ is the bargaining set representing all possible profit outcomes; $y_i$ and $y_i^0$ correspond to player $i$’s profit and disagreement payoff, respectively, for $i = 1, 2$. In the supply chain, we denote the manufacturer’s, the distributor’s, and the retailer’s disagreement payoffs by $d_m \geq 0$, $d_d \geq 0$, and $d_r \geq 0$, respectively. In practice, if the negotiation between players 1 and 2 is unsuccessful, each player will leave for another business deal for which the player obtains a payoff. Such payoff is called the player’s “disagreement payoff” for the negotiation between players 1 and 2. For example, in the German market for coffee, Draganska et al. (2010) defined a supplier’s and a retailer’s disagreement payoffs in their negotiation as the payoffs if they sell other products in the market. Intuitively, the negotiation can end with an agreement if each player can obtain a higher profit than his disagreement payoff. Moreover, the player with a higher disagreement payoff holds a stronger position in the negotiation. Hence, the disagreement payoffs can be viewed as important determinants of the players’ bargaining positions. For our problem, when the distributor enters his second negotiation, he may incur losses if the players cannot reach an agreement in the second negotiation, because the breakup of the second negotiation may revoke the first agreement and the distributor may have a penalty due to his scrapping the first agreement. Accordingly, we characterize the distributor’s disagreement payoff in the second negotiation as $d_d - \Delta$, where $\Delta \in [0, d_d]$ denotes the distributor’s cost due to leaving the second negotiation, simply called the distributor’s “exit penalty.” When $\Delta = d_d$, the disagreement payoff is zero, which refers to the situation in which the distributor’s penalty is so high that he never profits from leaving the second negotiation. When $\Delta = 0$, the disagreement payoff is $d_d$, which represents the situation in which the distributor does not have any penalty if he backs out of the agreement reached in his first negotiation, and he can still choose an outside opportunity. In addition, when the distributor negotiates with the manufacturer (the retailer), his relative bargaining power is $\lambda_m$ ($\lambda_r$) and the manufacturer’s (the retailer’s) relative bargaining power is $1 - \lambda_m$ ($1 - \lambda_r$).

A firm’s relative bargaining power is based on the factors that may influence the firm’s bargaining process involving, e.g., negotiations tactics and procedures, information structure, different time preferences (see, Muthoo 1999). Moreover, Draganska et al. (2010) revealed that, in the German market for coffee, the bargaining powers between two firms may be dependent on their sizes, brand introductions, and service levels. Therefore, to find the relative bargaining power for a firm, one may first estimate its absolute bargaining position, which can be measured according to the firm’s overall performance in terms of all factors mentioned by Muthoo (1999) and Draganska et al. (2010). Then, as each firm’s relative bargaining power is not an inherent characteristic of the firm but depends on the bargaining partners (Draganska, Klapper and Villas-Boas 2010), we can use a proportional approach (Hamlen, Hamlen and Tschirhart 1980) to compute the firm’s relative bargaining power in his negotiation with another firm. For example, when firms $i$ ($i = 1, 2$) with estimated absolute bargaining powers $\alpha_i$ bargain, their relative bargaining powers could be simply estimated as $\lambda_i = \alpha_i / (\alpha_1 + \alpha_2)$, $i = 1, 2$. As Draganska et al. (2010) estimated, in Germany, Dallmayr, Idee, and Tchibo—three coffee manufacturers with highest selling prices—have the largest relative bargaining power (i.e., 0.66-0.67), and Jacobs—a manufacturer with top-selling brand—has the average bargaining power (i.e., 0.57).
Both the disagreement payoffs and the relative bargaining powers have significant impacts on the negotiation results. Nonetheless, the two concepts differ in that the disagreement payoffs represent the profits that the players can secure for certainty, whereas the relative bargaining powers mean the players’ abilities to gain additional surpluses to the disagreement payoffs (see, Svejnar 1986). For details regarding the GNB solution, see Binmore et al.’s discussion (1986).

In this paper, as the distributor individually decides on the negotiation sequence in the first stage and there are two Nash bargaining processes each involving a negotiation (joint decision) by the two firms in the second stage, we do not use any solution concept in game theory to describe the two-stage solutions as a whole. Instead, we simply call the individual decision in the first stage the distributor’s optimal solution and call each negotiation result in the second stage a GNB solution.

3 Analysis of the Three-Echelon Supply Chain

We find the negotiated pricing and quantity decisions and compute the resulting profits for all firms in the supply chain, when the distributor first negotiates with the retailer. We also obtain those when the distributor first negotiates with the manufacturer. Then, we compare the results and find the distributor’s optimal negotiation sequence. To solve the game, we use a backward induction. That is, we first obtain the results in the second negotiation, given a negotiation sequence and the results in the first negotiation. Then, using the results in the second negotiation, we compute the results in the first negotiation for a given negotiation sequence. Finally, we compare two possible negotiation sequences and find the optimal one for the distributor.

3.1 Supply Chain Analysis When the Distributor First Negotiates with the Retailer

The distributor and the retailer first bargain over wholesale price $w_d$ and order quantity $q_r$, and the distributor and the manufacturer then negotiate wholesale price $w_m$ and order quantity $q_d$. The following theorem characterizes the decision results of the supply chain.

Theorem 1 Suppose the distributor first negotiates with the retailer. If $\Pi^* \geq d_m + d_r + d_d + (1 - \lambda_m)\Delta/\lambda_m$, then all the firms can reach agreements on their two negotiations in the supply chain. The pricing and ordering decisions are

$$\begin{align*}
 w^R_d &= c + \lambda_r \frac{\Pi^*}{q_r^*} + (1 - \lambda_r) \frac{d_m + d_d}{q_r^*} - \lambda_r \frac{d_r}{q_r^*} + (1 - \lambda_r)(1 - \lambda_m) \frac{\Delta}{\lambda_m q_r^*}, \\
 w^R_m &= (1 - \lambda_m)w^R_d + \lambda_m c + \lambda_m \frac{d_m}{q_r^*} - (1 - \lambda_m) \frac{d_d - \Delta}{q_r^*}, \\
 q^R_r &= q^R_d = q_r^*.
\end{align*}$$
The resulting (expected) profits of the three firms are

\[
\begin{align*}
\pi_m^R &= \lambda_r (1 - \lambda_m) \left( \Pi^* - d_m - d_r - d_d - \frac{(1 - \lambda_m) \Delta}{\lambda_m} \right) + d_m + \frac{(1 - \lambda_m) \Delta}{\lambda_m}, \\
\pi_d^R &= \lambda_m \lambda_r \left( \Pi^* - d_m - d_r - d_d - \frac{(1 - \lambda_m) \Delta}{\lambda_m} \right) + d_d, \\
\pi_r^R &= (1 - \lambda_r) \left( \Pi^* - d_m - d_r - d_d - \frac{(1 - \lambda_m) \Delta}{\lambda_m} \right) + d_r.
\end{align*}
\]

Otherwise, if \( \Pi^* < d_m + d_r + d_d + (1 - \lambda_m) \Delta/\lambda_m \), then the distributor and the retailer cannot reach any agreement in their negotiation.

Theorem 1 indicates that when the supply chain members can reach agreements in their negotiations, the supply chain is coordinated as \( q_r^R = q_d^R = q_r^* \). When the distributor first negotiates with the retailer, a firm’s higher disagreement payoff can increase the firm’s own profit (i.e., \( \partial \pi_m^R / \partial d_m \geq 0 \), \( \partial \pi_d^R / \partial d_r \geq 0 \), \( \partial \pi_r^R / \partial d_d \geq 0 \)) but can reduce other firms’ profits (i.e., \( \partial \pi_m^R / \partial d_r \leq 0 \), \( \partial \pi_d^R / \partial d_r \leq 0 \), \( \partial \pi_r^R / \partial d_m \leq 0 \), \( \partial \pi_r^R / \partial d_d \leq 0 \)). However, as the value of \( \Delta \) increases, the manufacturer’s payoff increases whereas the distributor’s and the retailer’s profits decrease. The main reason is as follows: a larger value of \( \Delta \) means a lower disagreement of the distributor in the negotiation with the manufacturer. Therefore, for a higher \( \Delta \), the manufacturer obtains a larger proportion of the total supply chain profit; and, as the relative bargaining powers of the distributor and the retailer are unchanged, both the distributor and the retailer secure a less portion of the chain-wide profit. It thus follows that, if the distributor absorbs a higher cost that is generated by backing out of the first agreement due to the breakup of the second negotiation, i.e., the value of \( \Delta \) increases, then the manufacturer’s profit increases but the distributor’s profit decreases. In addition, a larger exit penalty (i.e., \( \Delta \)) can make the two negotiations less likely to reach agreements. Thus, setting a lower exit penalty would be helpful to inducing successful deals in the supply chain.

Moreover, the payoffs of the retailer, the distributor and the manufacturer are increasing in their own bargaining powers (i.e., \( \partial \pi_r^R / \partial \lambda_r \leq 0 \), \( \partial \pi_d^R / \partial \lambda_m \geq 0 \), \( \partial \pi_r^R / \partial \lambda_r \geq 0 \), \( \partial \pi_m^R / \partial \lambda_m \leq 0 \)).

3.2 Supply Chain Analysis When the Distributor First Negotiates with the Manufacturer

The distributor first bargains with the manufacturer over wholesale price \( w_m \) and order quantity \( q_d \), and then negotiates the wholesale price and quantity contract \((w_d, q_r)\) with the retailer. In this scenario, after the first negotiation, the distributor already buys the product. Hence, when the distributor negotiates with the retailer, the purchase cost \( w_m q_d \) is a sunk cost. As a consequence, the “actual profit” of the distributor is \( w_d q_r \) in negotiating with the retailer. This is a main difference (in addition to the negotiation sequence) from Section 3.1. The following theorem characterizes the negotiation decisions in the supply chain.
Theorem 2 Suppose the distributor first negotiates with the manufacturer. If
\[ c \leq \lambda_r p \] and \( \lambda_r T \left( F^{-1} \left( \frac{\lambda_r p - c}{\lambda_r p} \right) \right) \geq \lambda_r d_r + \lambda_r d_d + d_m + (1 - \lambda_r) \Delta, \]
then the two negotiations in the supply chain are successful, and the pricing and ordering decisions are obtained as
\[
\begin{align*}
q^M_r &= q^M_d = F^{-1} \left( \frac{\lambda_r p - c}{\lambda_r p} \right), \\
w^M_m &= c + (1 - \lambda_m) \frac{\lambda_r T (q^M_d) - \lambda_r d_r - \lambda_r d_d - (1 - \lambda_r) \Delta}{q^M_d} + \lambda_m \frac{d_m}{q^M_d}, \\
w^M_d &= \lambda_r p \frac{E \min \{D, q^M_d\} - \lambda_r \frac{d_r}{q^M_d}}{q^M_d} + (1 - \lambda_r) \frac{d_d - \Delta}{q^M_d}.
\end{align*}
\]

The resulting (expected) profits of the three firms are
\[
\begin{align*}
\pi^M_m &= (1 - \lambda_m) (\lambda_r T (q^M_d) - \lambda_r d_r - \lambda_r d_d - d_m - (1 - \lambda_r) \Delta) + d_m, \\
\pi^M_d &= \lambda_m (\lambda_r T (q^M_d) - \lambda_r d_r - \lambda_r d_d - d_m - (1 - \lambda_r) \Delta) + d_d, \\
\pi^M_r &= (1 - \lambda_r) \left( T (q^M_d) + c \frac{q^M_d}{\lambda_r} \right) + \lambda_r d_r - (1 - \lambda_r)(d_d - \Delta).
\end{align*}
\]

Otherwise, if \( c > \lambda_r p \) or \( \lambda_r T \left( F^{-1}((\lambda_r p - c)/(\lambda_r p)) \right) < \lambda_r d_r + \lambda_r d_d + d_m + (1 - \lambda_r) \Delta \), then the distributor and the manufacturer cannot reach any agreement in their negotiation.

Different from Theorem 1, when the distributor decides to first negotiate with the manufacturer, the supply chain cannot be coordinated and the supply chain-wide expected profit is
\[
T (q^M_d) + c (1 - \lambda_r) \frac{q^M_d}{\lambda_r} = p \int_0^{F^{-1}((\lambda_r p - c)/(\lambda_r p))} t f(t) dt + c \frac{1 - \lambda_r}{\lambda_r} F^{-1} \left( \frac{\lambda_r p - c}{\lambda_r p} \right).
\]
Since
\[
\frac{\partial}{\partial \lambda_r} \left[ T (q^M_d) + c (1 - \lambda_r) \frac{q^M_d}{\lambda_r} \right] = \frac{(1 - \lambda_r)c^2}{\lambda_r^2 p f \left( F^{-1}((\lambda_r p - c)/(\lambda_r p)) \right)} \geq 0,
\]
we find that if the retailer has a higher power (i.e., the value of \( 1 - \lambda_r \) is larger), then supply chain efficiency is lower. This result is different from Zhong et al.’s finding (2016) that a more powerful retailer can promote the efficiency of a multi-echelon supply chain. This occurs mainly because, in our model, the distributor negotiates both wholesale price and order quantity with the manufacturer and the purchase cost is thus deemed as a sunk cost, whereas Zhong et al. (2016) assumed that the negotiations in the middle echelon of the supply chain only determines wholesale prices, and thus the purchase cost of the downstream firm is not a sunk cost. The sunk cost of the distributor makes the retailer hold a very strong bargaining position, which deteriorates the supply chain performance. If the value of \( \lambda_r \) approaches 1, one can note that the supply chain coordination can be reached. This means that a distributor’s “full” power in the negotiation with the retailer can coordinate the supply chain.
There are real-world examples for both our model and Zhong et al.’s model (2016). First, negotiating both the price and the order quantity is very common in practice. According to Ertel’s relevant finding (1999), a supplier (e.g., a manufacturer) may reduce his wholesale price to induce a larger order in his negotiation with a buyer (e.g., a distributor). Fredman (2018) also mentioned that a buyer may have an incentive to place a larger order for a price discount from a supplier in the negotiation between the two firms. That is, the wholesale price and the order quantity may be negotiated together. Moreover, in reality, the distributors may hold the inventory using a “push” strategy, as considered in our paper. For example, the Mobicon Group Limited is an electronics distributor in Hong Kong, which carries the components (or products) on hand and becomes the main risk pooling point to its downstream firms (Ng and Chung 2008). Under the push strategy, the distributor’s order quantity is largely dependent on the negotiated wholesale price. The above observations can serve as real-world examples for our model in this paper, as they clearly indicate that it is possible for the distributor to negotiate both the wholesale price and the order quantity with the manufacturer, and the purchase cost is thus sunk. Different from our model, Zhong et al. (2016) only considered the wholesale price, which implies that the purchase cost of the downstream firm is not sunk. This is also possible in practice, as justified as follows: some distributors (e.g., some healthcare product manufacturers and their distributors; see, Ndung’U 2017) may use the pull strategy under which their order quantities are driven by the demand and are not largely dependent on the price negotiation. The above serves as an example to illustrate the setting assumed by Zhong et al. (2016).

Then, we examine the impacts of disagreement payoffs and bargaining powers on the three firms’ (expected) profits. Similar to Theorem 1, in most cases, when the distributor first negotiates with the manufacturer, a firm’s higher disagreement payoff can increase the firm’s own profit (i.e., \( \frac{\partial \pi^R_m}{\partial d_m} \geq 0, \frac{\partial \pi^R_d}{\partial d_d} \geq 0, \frac{\partial \pi^R_r}{\partial d_r} \geq 0 \)) but can reduce other firms’ profits (i.e., \( \frac{\partial \pi^R_m}{\partial d_r} \leq 0, \frac{\partial \pi^R_r}{\partial d_d} \leq 0, \frac{\partial \pi^R_m}{\partial d_r} \leq 0, \frac{\partial \pi^R_d}{\partial d_m} \leq 0, \frac{\partial \pi^R_r}{\partial d_d} \leq 0 \)). However, the retailer’s profit is independent of the manufacturer’s disagreement payoff. This occurs because the manufacturer’s disagreement payoff does not appear in the negotiation between the distributor and the retailer. Similar to Theorem 1, as the distributor’s loss caused by the breakup of the first agreement (i.e., \( \Delta \)) increases, the two negotiations are less likely to end with agreements. When the supply chain members can reach agreements in their negotiations, the retailer’s payoff (in the second negotiation) is increasing in \( \Delta \), whereas the distributor’s and the manufacturer’s profits (in the first negotiation) are decreasing in \( \Delta \). We also find

\[
\frac{\partial \pi^M_m}{\partial \lambda_m} \leq 0, \quad \frac{\partial \pi^M_d}{\partial \lambda_m} \geq 0, \quad \text{and} \quad \frac{\partial \pi^M_r}{\partial \lambda_r} \geq 0,
\]

which implies that a higher bargaining power of the manufacturer (distributor) can generate a higher profit for the manufacturer (distributor). However, we have

\[
\frac{\partial \pi^M_r}{\partial \lambda_r} = \frac{(1 - \lambda_r)e^2}{\lambda_r^2 p f(q_d^M)} - \left[ \frac{p}{\lambda_r^2} \int_0^{q_d^M} tf(t)dt + \frac{c q_d^M}{\lambda_r} - d_r - d_d + \Delta \right].
\]
When \( d_m = d_d = d_r = \Delta = 0 \), \( \lambda_r \rightarrow c/p \), and the support of \( D \) contains 0 (i.e., \( F(x) > 0 \) for all \( x > 0 \)), we may find that \( q_d^M \rightarrow 0 \) and \( \partial \pi_r^M / \partial \lambda_r \geq 0 \). Therefore, the retailer with a higher bargaining power cannot always obtain a higher (expected) profit. This occurs mainly because, when the bargaining power of the retailer increases, the retailer can enjoy a larger proportion of the expected supply-chain profit via negotiation, although the chain-wide profit decreases. Thus, when the decrease in the expected supply-chain profit has a larger impact on the retailer’s expected profit, the retailer is worse off from any increase in his bargaining power.

### 3.3 The Distributor’s Optimal Negotiation-Sequence Decision

We use Theorems 1 and 2 to analyze the distributor’s optimal negotiation sequence decision.

**Theorem 3** If \( \lambda_r < c/p \), the distributor first negotiates with the retailer. If \( \lambda_r \geq c/p \) and

\[
\lambda_m \geq \xi \equiv \frac{\lambda_r \Delta}{\lambda_r \Pi^* - \lambda_r T(q_d^M) + d_m - \lambda_r d_m + \Delta},
\]

then the distributor should first negotiate with the retailer; otherwise, if \( \lambda_m < \xi \), then the distributor should first negotiate with the manufacturer.

Theorem 3 shows that relative bargaining powers play a key role in determining the distributor’s optimal negotiation sequence. If the distributor has a sufficiently large relative bargaining power over the retailer and a sufficiently small relative bargaining power over the manufacturer, then the distributor should first negotiate with the manufacturer. Otherwise, it is better for the distributor to first negotiate with the retailer. The above reveals that powerful upstream firms (i.e., the manufacturer with a sufficiently large bargaining power \( 1 - \lambda_m \) and the distributor with a sufficiently large bargaining power \( \lambda_r \)) makes the supply chain worse off.

If \( \Delta = 0 \), we find that \( \xi = 0 \) and the distributor always first negotiates with the retailer, which leads to supply chain coordination. Setting a lower exit penalty in a negotiated contract can benefit the whole supply chain. This result suggests that the firms should not use penalties to improve the system-wide performance. We also examine the manufacturer’s and the retailer’s preferences on the distributor’s negotiation sequence decisions.

**Theorem 4** The manufacturer prefers the distributor to first negotiate with him if and only if \( \Pi^* < d_r + d_m + d_d + (1 - \lambda_r)\Delta / \lambda_r \). When supply chain negotiations can result in agreements under two negotiation sequences, i.e.,

\[
c \leq \lambda_r p, \quad T(q_d^M) \geq d_r + d_d + d_m / \lambda_r + (1 - \lambda_r)\Delta / \lambda_r, \quad \text{and} \quad \Pi^* \geq d_m + d_r + d_d + (1 - \lambda_m)\Delta / \lambda_m,
\]

the retailer expects the distributor to first negotiate with him, if and only if

\[
d_m + \Delta / \lambda_m \leq \Pi^* - T(q_d^M) - c q_d^M / \lambda_r.
\]
4 Extensions: Supply Chain Coordination with Contracts

We learn from Section 3 that the supply chain could not be coordinated when the distributor’s optimal negotiation sequence decision is to first negotiate with the manufacturer under the wholesale price contract. Such inefficiency occurs mainly because the distributor’s purchase cost is a sunk cost in his second negotiation with the retailer. In this section, we investigate supply chain coordination with contracts. To this end, we consider the following two contracts in the negotiation between the manufacturer and the distributor: a buyback contract and a two-part tariff contract, and address the problem of whether these two contracts can coordinate the supply chain. We also compare the (expected) profits of all firms under the different contracts.

Under the buyback contract, the distributor and the manufacturer negotiate a wholesale price \( w_m \) and a buyback price \( b_m \). The two firms do not negotiate the order quantity so that the distributor’s sunk cost could be avoided, and the distributor’s order quantity is the same as the retailer’s order quantity. Since the retailer’s unsold products have a salvage value when they are returned to the manufacturer, the distributor and the retailer use a drop shipping model, which is very common in practice. Under the two-part tariff contract, the distributor and the manufacturer negotiate a wholesale price \( w_m \) and a fixed payment \( K_m \). Similar to the buyback contract, the distributor and the manufacturer do not negotiate the order quantity, and the distributor’s and the retailer’s order quantities are identical. Note that, different from the transaction between the distributor and the manufacturer, the distributor and the retailer cannot implement a drop shipping system, because the unsold products of the retailer cannot bring any additional value to the distributor who thus has no incentive to bear the demand risk.

Next, we begin by investigating the supply chain under the buyback and two-part tariff contracts, when the distributor first negotiates with the retailer.

**Lemma 1** Suppose the distributor first negotiates with the retailer and negotiates the buyback or two-part tariff contract with the manufacturer. If \( \Pi^* \geq d_r + d_m + d_d + (1 - \lambda_m) \Delta / \lambda_m \), then all the firms can reach agreements on their negotiations in the supply chain. The resulting (expected) profits are

\[
\begin{align*}
\pi^{R, bb}_{m} &= \pi^{R, tpt}_{m} = \lambda_r (1 - \lambda_m) \left( \Pi^* - d_m - d_r - d_d - \frac{(1 - \lambda_m) \Delta}{\lambda_m} \right) + d_m + \frac{(1 - \lambda_m) \Delta}{\lambda_m}, \\
\pi^{R, bb}_{d} &= \pi^{R, tpt}_{d} = \lambda_m \lambda_r \left( \Pi^* - d_m - d_r - d_d - \frac{(1 - \lambda_m) \Delta}{\lambda_m} \right) + d_d, \\
\pi^{R, bb}_{r} &= \pi^{R, tpt}_{r} = (1 - \lambda_r) \left( \Pi^* - d_m - d_r - d_d - \frac{(1 - \lambda_m) \Delta}{\lambda_m} \right) + d_r.
\end{align*}
\]

Otherwise, if \( \Pi^* < d_r + d_m + d_d + (1 - \lambda_m) \Delta / \lambda_m \), then the distributor and the retailer cannot reach any agreement in their negotiation.

Lemma 1 exposes that, when the distributor first negotiates with the retailer, all firms’ (expected) profits are the same regardless of which contract the distributor and the manufacturer negotiate. The reason is as follows: the retailer’s order quantity—which determines the whole supply chain’s
expected profit—is obtained when the distributor first negotiates with the retailer. Hence, the supply chain’s expected profit is maximized in the first negotiation, and the (expected) profits of all firms are determined based on their bargaining powers and disagreement payoffs, regardless of which contract they use. Subsequently, we analyze the negotiation results when the distributor first negotiates with the manufacturer.

4.1 Analysis of the Buyback Contract When the Distributor First Negotiates with the Manufacturer

We analyze the supply chain when the distributor first bargains with the manufacturer over the buyback contract, and then negotiates with the retailer.

**Theorem 5** Suppose the distributor first negotiates the buyback contract with the manufacturer. If
\[ \Pi^* \geq d_r + d_m + d_d + (1 - \lambda_r)\Delta/\lambda_r, \]
then the negotiations in the supply chain can end successfully with the three firms’ (expected) profits as follows:

\[
\begin{align*}
\pi_{m}^{M,bb} &\equiv (1 - \lambda_m)[\Pi^* - d_r - d_m - d_d - (1 - \lambda_r)\Delta/\lambda_r] + d_m, \\
\pi_{d}^{M,bb} &\equiv \lambda_r\lambda_m[\Pi^* - d_r - d_m - d_d - (1 - \lambda_r)\Delta/\lambda_r] + d_d, \\
\pi_{r}^{M,bb} &\equiv \lambda_m(1 - \lambda_r)[\Pi^* - d_r - d_m - d_d - (1 - \lambda_r)\Delta/\lambda_r] + d_r + (1 - \lambda_r)\Delta/\lambda_r.
\end{align*}
\]

Otherwise, if \( \Pi^* < d_r + d_m + d_d + (1 - \lambda_r)\Delta/\lambda_r \), then the distributor and the manufacturer cannot reach any agreement.

We find that the distributor prefers to first negotiate with the retailer if and only if \( \lambda_r \leq \lambda_m \). Moreover, the retailer prefers to join the distributor’s first negotiation, if and only if
\[
\Pi^* \geq \left( \frac{1}{\lambda_r} + \frac{1}{\lambda_m} + \frac{\lambda_m}{1 - \lambda_m} \right) \Delta + d_d + d_r + d_m;
\]
and the manufacturer prefers to be in the distributor’s first negotiation, if and only if
\[
\Pi^* \geq \left( \frac{1}{\lambda_r} + \frac{1}{\lambda_m} + \frac{\lambda_r}{1 - \lambda_r} \right) \Delta + d_d + d_r + d_m.
\]

Lemma 1 and Theorem 5 show that, when the distributor and the retailer choose the drop shipping model and the distributor and the manufacturer negotiate the buyback contract, the supply chain can always be coordinated. This is mainly ascribed to the fact that the buyback contract does not specify the order quantity when the distributor and the manufacturer negotiate, thus inducing the distributor and the retailer to use a drop shipping system because such model may help reduce the distributor’s sunk cost of buying from the manufacturer. Moreover, the distributor prefers to first negotiate with the firm with a stronger bargaining power. Both the manufacturer and the retailer prefer the distributor to first negotiate with themselves if the maximum expected supply chain-wide profit (i.e., \( \Pi^* \)) is sufficiently large.
4.2 Analysis of the Two-Part Tariff Contract When the Distributor First Negotiates with the Manufacturer

We investigate the supply chain when the distributor first bargains with the manufacturer over the two-part tariff contract, and then negotiates with the retailer.

**Lemma 2** When \(d_r \leq pE[D]\), we let \(q_S(\lambda_r, d_r)\) denote a solution of the following optimization problem:

\[
\Lambda(q) \equiv \max_{q \geq T^{-1}(d_r)} [\lambda_r T(q) + (p - pF(q) - c)q];
\]

that is,

\[
S(\lambda_r, d_r) \equiv \Lambda(q_S(\lambda_r, d_r)) = \lambda_r T(q_S(\lambda_r, d_r)) + (p - pF(q_S(\lambda_r, d_r))) - c)q_S(\lambda_r, d_r).
\]

Then, \(T(q_S(\lambda_r, d_r)) \geq d_r\) and \(S(\lambda_r, d_r) \leq \Pi^* - (1 - \lambda_r) d_r\).

We then compute the negotiation results when the distributor first negotiates with the manufacturer, and obtain the optimal negotiation sequence for the distributor.

**Theorem 6** Suppose the distributor first negotiates the two-part tariff contract with the manufacturer. If \(d_r \leq pE[D]\) and \(S(\lambda_r, d_r) \geq \lambda_r d_r + \lambda_r d_d + d_m + (1 - \lambda_r) \Delta,\) then the negotiations in the supply chain are successful and the (expected) profits of the three firms are

\[
\begin{cases}
\pi_{m,tpt}^M \equiv (1 - \lambda_m)(S(\lambda_r, d_r) - \lambda_r d_r - \lambda_r d_d - d_m - (1 - \lambda_r) \Delta) + d_m, \\
\pi_{d,tpt}^M \equiv \lambda_m(S(\lambda_r, d_r) - \lambda_r d_r - \lambda_r d_d - d_m - (1 - \lambda_r) \Delta) + d_r, \\
\pi_{r,tpt}^M \equiv (1 - \lambda_r)[T(q_S(\lambda_r, d_r)) - d_d + \Delta] + \lambda_r d_r.
\end{cases}
\]

Otherwise, if \(d_r > pE[D]\) or \(S(\lambda_r, d_r) < \lambda_r d_r + \lambda_r d_d + d_m + (1 - \lambda_r)\Delta,\) then the distributor and the manufacturer cannot reach any agreement.

If supply chain negotiations can succeed under any negotiation sequence, i.e., \(d_r \leq pE[D],\) \(S(\lambda_r, d_r) \geq \lambda_r d_r + \lambda_r d_d + d_m + (1 - \lambda_r) \Delta,\) and \(\Pi^* \geq d_r + d_m + d_d + (1 - \lambda_m) \Delta / \lambda_m,\) then (1) the distributor prefers to first negotiate with the retailer if and only if \((1 - \lambda_r)d_m + (1 - \lambda_r / \lambda_m) \Delta \geq S(\lambda_r, d_r) - \lambda r \Pi^*,\) (2) the retailer prefers to join the distributor’s first negotiation if and only if \(d_m + \Delta / \lambda_m \leq \Pi^* - T(q_S(\lambda_r, d_r)),\) and (3) the manufacturer prefers the distributor to first negotiate with the retailer if and only if \((1 - \lambda_r)d_m + (1 + (1 - \lambda_r)/\lambda_m) \Delta \geq S(\lambda_r, d_r) - \lambda r \Pi^*.\)

Different from the buyback contract, the two-part tariff contract cannot always induce supply chain coordination. This occurs mainly because, although the two-part tariff contract can reduce the distributor’s sunk cost resulting from the inventory, the fixed payment \(K_m\) is deemed as a sunk cost when the distributor negotiates with the retailer. Hence, the buyback contract is more efficient than the two-part tariff and wholesale price contracts in the three-echelon supply chain. Moreover, the distributor prefers to first negotiate with the retailer, if and only if the manufacturer has a sufficiently high disagreement payoff (i.e., sufficiently great outside opportunity). In addition, if the distributor’s
optimal negotiation sequence is to first negotiate with the retailer, then the manufacturer agrees with
the distributor’s optimal negotiation sequence.

According to Lemma 1 and Theorems 5 and 6, under either the buyback contract or the two-part
tariff contract, the distributor’s larger exit penalty (i.e., $\Delta$) may reduce the possibility of reaching
agreements in both negotiations. This result coincides with our finding when the wholesale price
contract is used.

4.3 Contract Preference

We learn from our previous analyses that both the wholesale price and two-part tariff contracts cannot
coordinate the supply chain, if the distributor decides to first negotiate with the manufacturer. It
behoves to discuss whether all of the three firms have an incentive to choose the buyback contract
for supply chain coordination.

**Theorem 7** If $\lambda_r \geq \lambda_m$ and $\Pi^* \geq d_r + d_d/\lambda_r + d_m$, then all the three firms prefer the buyback
contract to the others.

Theorem 7 shows that the manufacturer’s larger bargaining power than the retailer (i.e., $1 - \lambda_m \geq
1 - \lambda_r$) is a necessary condition for all the three firms to prefer to choose the buyback contract, which
may further induce supply chain coordination. Moreover, the condition $\Pi^* \geq d_r + d_d/\lambda_r + d_m$ means
that the bargaining power of the retailer (i.e., $1 - \lambda_r$) should be sufficiently small. Thus, a sufficiently
weak position of the retailer is also necessary for all three firms to choose the buyback contract.

5 Summary and Concluding Remarks

In this paper we consider a negotiation-sequence problem for a distributor who is located in the middle
of a three-echelon supply chain also involving a manufacturer in the first tier and a retailer in the
third tier. The distributor needs to negotiate a wholesale price contract with the manufacturer and
bargain over another wholesale price contract with the retailer. It naturally behooves the distributor
to choose an optimal negotiation sequence that maximizes his individual profit. However, there are
very few publications that have investigated the negotiation-sequence problems. Moreover, those
publications consider such a problem only for a two-echelon supply chain, in which a dominant firm
has the power to determine a sequence for all negotiations in the supply chain horizontally. For
example, Guo and Iyer (2013) studied a two-echelon supply chain involving a manufacturer and two
competing retailers. The manufacturer needs to make a decision on the sequence of negotiating with
these two retailers.

Different from the extant publications, in our model, the distributor needs to determine a sequence
of the two negotiations with the manufacturer and the retailer vertically. The two possible sequence
decisions in our problem have existed in practice. One may note that it is common for a distributor
to use a push strategy by holding an inventory before negotiating with the retailer. To that end, the
distributor should negotiate with the manufacturer first. Nonetheless, in reality, some distributors
may use the pull strategy, under which the distributors are very likely to negotiate with the retailer first. For example, as the Smart Insights (2013) reported, Stormhoek (a South African winery) and its distributors had changed their traditional business model from a push to a pull strategy, due to a challenge competing for supermarket shelf space. As a result, the sales of Stormhoek wines increased from 50,000 cases of wine a year in 2004 to 200,000 cases in 2006. In addition, Ndung’U (2017) exposed that healthcare product manufacturers and their distributors have been increasingly using both push and pull strategies.

We find that the supply chain can be coordinated if the distributor decides to first negotiate with the retailer. When the distributor first negotiates with the manufacturer, the retailer’s stronger bargaining power in his pricing negotiation may deteriorate the supply chain-wide performance. Considering the distributor’s optimal negotiation sequence decision, we show that the distributor prefers to first negotiate with the retailer if he has a sufficiently large relative bargaining power in the negotiation with the manufacturer, or he has a sufficiently small relative bargaining power in the negotiation with the retailer.

We also extend our analyses to the cases in which the distributor and the manufacturer negotiate the buyback or two-part tariff contracts. There are a large number of publications that focus on the supply chain negotiation under a buyback or two-part tariff contract. For example, Iyer and Villas-Boas (2003) and Milliou et al. (2003) compared the performances of different contracts. Symeonidis (2008), Feng and Lu (2013), Lee et al. (2016), and Pinopoulos (2017) focused on supply chain competitions. He and Zhao (2012) examined the effects of both supply and demand uncertainties. Gaudin (2016) studied the effects of cost fluctuations on the wholesale and retail prices. Kitamura et al. (2017) considered a situation under which a supplier deters another supplier’s entry of an upstream market. Haruvy et al. (2020) exposed that concessions in supply chain negotiations have a critical impact on supply chain efficiency. Milliou and Petrakis (2007) and Gabrielsen and Roth (2009) investigated the two-part tariff contract in the presence of multiple upstream manufacturers. Different from all the relevant publications, our paper is concerned with a distributor’s optimal negotiation sequence in a three-echelon supply chain under the two contracts.

Our analyses of the buyback and two-part tariff contracts expose that the buyback contract can always coordinate the supply chain, whereas the two-part tariff contract may not coordinate the supply chain. Under the buyback contract, the distributor prefers to first negotiate with the firm with a higher bargaining power. In addition, the manufacturer’s larger bargaining power than the retailer is a necessary condition for all the firms to willingly adopt the buyback contract.

**Acknowledgement 1** The first two authors (i.e., Feimin Zhong and Zhongbao Zhou) were supported by the National Natural Science Foundation of China (Nos. 71801086, 71771082) and the Natural Science Foundation of Human Province (Nos. 2019JJ50083, 2017JJ1012). The third author (Mingming Leng) was supported by the General Research Fund (GRF) of the Hong Kong Research Grants Council under Research Project No. LU13500020.
References


Thus, the negotiation result is the solution of the following problem if

\[\pi_d = w_d q_r - w_m q_d\] and \[\pi_m = (w_m - c)q_d.\]

Thus, the negotiation result of the distributor and the manufacturer is the solution of the following problem:

\[
\max_{w_m,q_d} \quad (w_d q_r - w_m q_d - d_d + \Delta)^{\lambda_m} ((w_m - c)q_d - d_m)^{1 - \lambda_m}
\]
\[
s.t. \quad w_d q_r - w_m q_d \geq d_d - \Delta, \quad (w_m - c)q_d \geq d_m, \quad \text{and} \quad q_d \geq q_r.
\]

Since \[\pi_d + \pi_m = w_d q_r - c q_d \leq (w_d - c)q_r\], the negotiation can succeed if \((w_d - c)q_r \geq d_m + d_d - \Delta\). Moreover, if \((w_d - c)q_r \geq d_m + d_d - \Delta\), then the above objective is less or equal to \([((w_d - c)q_r - d_m - d_d + \Delta)^{\lambda_m} (1 - \lambda_m)^{1 - \lambda_m}\). This can occur only when we choose the following maximizer (which is in terms of \[w_d\] and \[q_r\]): \((\hat{w}_m(w_d, q_r), \hat{q}_d(w_d, q_r)) = ((1 - \lambda_m)w_d + \lambda_m c + \lambda_m d_m/q_r - (1 - \lambda_m)(d_d - \Delta)/q_r, d_d)\).

Now we consider the first negotiation between the distributor and the retailer. Their (expected) profits are

\[
\begin{cases}
\pi_d = w_d q_r - \hat{w}_m(w_d, q_r)\hat{q}_d(w_d, q_r) = \lambda_m((w_d - c)q_r - d_m) + (1 - \lambda_m)(d_d - \Delta), \\
\pi_r = pE \min\{D, q_r\} - w_d q_r.
\end{cases}
\]

Thus, the negotiation result is the solution of the following problem if \((w_d - c)q_r \geq d_m + d_d - \Delta\):

\[
\max_{w_d, q_r} \quad \lambda_m((w_d - c)q_r - d_m) + (1 - \lambda_m)(d_d - \Delta) - d_d)^{\lambda_r}\{pE \min\{D, q_r\} - w_d q_r - d_r\}^{1 - \lambda_r}
\]
\[
s.t. \quad (w_d - c)q_r - d_m + (1 - \lambda_m)(d_d - \Delta)/\lambda_m > d_d/\lambda_m, \quad pE \min\{D, q_r\} - w_d q_r > d_r.
\]

Since \[\pi_d/\lambda_m + \pi_r = pE \min\{D, q_r\} - c q_r - d_m + (1 - \lambda_m)(d_d - \Delta)/\lambda_m \leq \Pi^* - d_m + (1 - \lambda_m)(d_d - \Delta)/\lambda_m\], the negotiation can succeed if \[\Pi^* \geq d_m + d_r + d_d + (1 - \lambda_m)\Delta/\lambda_m.\] Moreover, if \[\Pi^* \geq d_m + d_r + d_d + (1 - \lambda_m)\Delta/\lambda_m.\] then the above objective is less or equal to \[\lambda_m^{\lambda_r}(1 - \lambda_r)^{\lambda_r}(\Pi^* - d_m - d_r - d_d - (1 - \lambda_m)\Delta/\lambda_m),\] which holds when we choose the maximizer \((w^R_d, q^R_r) = (c + \lambda_r \Pi^*/q^R_r + (1 - \lambda_r)(d_m + d_d)/q^R_r - \lambda_r d_r/q^R_r + (1 - \lambda_r)(1 - \lambda_m)\Delta/(\lambda_m q^R_r), q^R_r),\) and the condition for the second negotiation to
succeed (i.e., \((w_d^R - c)q_r^R \geq d_m + d_d - \Delta\)) is also satisfied. Therefore, we have

\[(w_m^R, q_d^R) = (\tilde{w}_m(w_d^R, q_r^R), \tilde{q}_d(w_d^R, q_r^R)) = ((1 - \lambda_m)w_d^R + \lambda_m c + \lambda_m d_m / q_r^* - \lambda_m (d_d - \Delta) / q_r^*, q_r^*).\]

The profits of the three firms are then computed as given in this theorem.

In conclusion, the first negotiation can succeed, if and only if \(\Pi^* \geq d_m + d_r + d_d + (1 - \lambda_m)\Delta / \lambda_m.\) If the first negotiation ends successfully, then the second negotiation can also succeed. ■

**Proof of Theorem 2.** For any given negotiation result \((w_m, q_d)\), the (expected) profits of the distributor and the retailer are \(\pi_d = w_d q_r\) and \(\pi_r = pE \min\{D, q_r\} - w_d q_r\), respectively. Thus, the negotiation result is the solution of the following problem:

\[
\max_{w_d, q_r} \{w_d q_r - d_d + \Delta\}^{\lambda_r} \{pE \min\{D, q_r\} - w_d q_r - d_r\}^{1 - \lambda_r}
\]

\[\text{s.t. } w_d q_r > d_d - \Delta, pE \min\{D, q_r\} - w_d q_r > d_r, \text{ and } q_r \leq q_d.\]

Since \(\pi_d + \pi_r = pE \min\{D, q_r\} \leq pE \min\{D, q_d\}\), the negotiation can succeed only if \(pE \min\{D, q_d\} \geq d_r + d_d - \Delta\). Moreover, if \(pE \min\{D, q_d\} \geq d_r + d_d - \Delta\), then the above objective is no greater than \(\{pE \min\{D, q_d\} - d_r - d_d + \Delta\} \lambda_r (1 - \lambda_r)^{1 - \lambda_r}\), which can occur when we choose the following unique maximizer (in terms of \((w_m, q_d)\)):

\[\tilde{w}_d(w_m, q_d), \tilde{q}_d(w_m, q_d) = (\lambda_r pE \min\{D, q_d\} / q_d - \lambda_r d_r / q_d + (1 - \lambda_r)(d_d - \Delta) / q_d, q_d).\]

We consider the negotiation between the distributor and the manufacturer, whose profits are

\[
\begin{cases}
\pi_d = \tilde{w}_d(w_m, q_d) \tilde{q}_r(w_m, q_d) - w_m q_d \\
\pi_r = \lambda_r [pE \min\{D, q_d\} - d_r] + (1 - \lambda_r)(d_d - \Delta) - w_m q_d \\
\pi_m = (w_m - c)q_d.
\end{cases}
\]

Thus, the negotiation result is the solution of the following problem if \(pE \min\{D, q_d\} \geq d_r + d_d - \Delta\):

\[
\max_{w_m, q_d} \{\lambda_r [pE \min\{D, q_d\} - d_r] + (1 - \lambda_r)(d_d - \Delta) - w_m q_d - d_d\}^{\lambda_m} \{w_m q_d - d_d\}^{1 - \lambda_m}
\]

\[\text{s.t. } \lambda_r [pE \min\{D, q_d\} - d_r] + (1 - \lambda_r)(d_d - \Delta) - w_m q_d > d_d, \text{ and } (w_m - c)q_d > d_m.\]

Since

\[
\pi_d + \pi_m = \lambda_r pE \min\{D, q_d\} - c q_d - \lambda_r d_r + (1 - \lambda_r)(d_d - \Delta)
\]

\[\leq \max_{q_d} \lambda_r pE \min\{D, q_d\} - c q_d - \lambda_r d_r + (1 - \lambda_r)(d_d - \Delta)
\]

\[= \begin{cases}
-\lambda_r d_r + (1 - \lambda_r)(d_d - \Delta) \text{ if } c \geq \lambda_r p \\
\lambda_r T(F^{-1}(\lambda_r p / (\lambda_r p)) - \lambda_r d_r + (1 - \lambda_r)(d_d - \Delta) \text{ if } c < \lambda_r p
\end{cases}
\]

the negotiation can be successful if \(c \leq \lambda_r p \text{ and } \lambda_r T(F^{-1}(\lambda_r p / (\lambda_r p)) - \lambda_r d_r + (1 - \lambda_r)(d_d - \Delta) \geq d_d + d_m\). In addition, if \(c \leq \lambda_r p \text{ and } \lambda_r T(F^{-1}(\lambda_r p / (\lambda_r p)) - \lambda_r d_r + (1 - \lambda_r)(d_d - \Delta) \geq d_d + d_m\),
then the above objective is no larger than
\[
\lambda_m (1 - \lambda_m)^{1-\lambda_m} \left( \lambda_r T \left( F^{-1} \left( \frac{\lambda_r p - c}{\lambda_r p} \right) \right) - \lambda_r d_r - \lambda_r d_d - d_m - (1 - \lambda_r)\Delta \right),
\]
which can happen by using the following unique maximizer
\[
\begin{align*}
\begin{cases}
    w^M_m = c + \frac{(1 - \lambda_m) \left[ \lambda_r T(F^{-1}((\lambda_r p - c)/(\lambda_r p))) - \lambda_r d_r - \lambda_r d_d - (1 - \lambda_r)\Delta \right] + \lambda_m d_m}{F^{-1}((\lambda_r p - c)/(\lambda_r p))}, \\
    q^M_d = F^{-1}((\lambda_r p - c)/(\lambda_r p));
\end{cases}
\end{align*}
\]
and the condition for the second negotiation to succeed (i.e., \(pE \min \{D, q^M_d \} = (\lambda_r T(q^M_d) + c q^M_d) / \lambda_r \geq d_r + d_d - \Delta\)) is also satisfied. Therefore, we can obtain the negotiated decisions of \((w^M_d, q^M_r)\) as
\[
(w^M_d, q^M_r) = (\hat{w}_d(w^M_m, q^M_d), \hat{q}_r(w^M_m, q^M_d)) = \left( \lambda_r p \frac{E \min \{D, q^M_d \}}{q^M_d} - \lambda_r \frac{d_r}{q^M_d} + (1 - \lambda_r)\frac{d_d - \Delta}{q^M_d}, q^M_d \right).
\]
The profits of the three firms are then calculated as in this theorem.

We thus conclude that the first negotiation can succeed if and only if \(c \leq \lambda_r p\) and \(\lambda_r T(F^{-1}((\lambda_r p - c)/(\lambda_r p))) - \lambda_r d_r + (1 - \lambda_r)(d_d - \Delta) \geq d_d + d_m\). If the first negotiation succeed, then the second negotiation is also successful. ■

**Proof of Theorem 3.** When \(\lambda_r \leq c/p\), the distributor and the manufacturer cannot reach any agreement if their negotiation first takes place. Hence, the distributor’s optimal decision is always to first negotiate with the retailer.

When \(\lambda_r > c/p\) and \(\lambda_m \geq \lambda_r \Delta/(\lambda_r \Pi^* - \lambda_r T(q^M_d) + d_m - \lambda_r d_m + \Delta)\), the distributor and the manufacturer cannot reach any agreement if they first negotiate and \(\lambda_r T(q^M_d) \geq \lambda_r d_r + \lambda_r d_d + d_m + (1 - \lambda_r)\Delta\). Otherwise, if \(\lambda_r T(q^M_d) \geq \lambda_r d_r + \lambda_r d_d + d_m + (1 - \lambda_r)\Delta\), then
\[
\begin{align*}
\Pi^* - d_m - d_r - d_d - \frac{1 - \lambda_m}{\lambda_m} \Delta \\
\geq \Pi^* - d_m - d_r - d_d - \Pi^* + T(q^M_d) - \frac{d_m}{\lambda_r} + d_m - \frac{\Delta}{\lambda_r} + \Delta \\
= T(q^M_d) - d_r - d_d - \frac{d_m}{\lambda_r} - \frac{(1 - \lambda_r)\Delta}{\lambda_r} \geq 0.
\end{align*}
\]
Thus, the supply chain members under both negotiation sequences can reach agreements. Note that
\[
\pi^R_d \geq \pi^M_d \iff \lambda_m \geq \lambda_r \Delta/(\lambda_r \Pi^* - \lambda_r T(q^M_d) + d_m - \lambda_r d_m + \Delta),
\]
which means that the distributor’s optimal negotiation sequence is to first negotiate with the retailer, if \(\lambda_r T(q^M_d) \geq \lambda_r d_r + \lambda_r d_d + d_m + (1 - \lambda_r)\Delta\). Therefore, the distributor should first negotiate with the retailer when \(\lambda_r \geq c/p\) and \(\lambda_m \geq \lambda_r \Delta/(\lambda_r \Pi^* - \lambda_r T(q^M_d) + d_m - \lambda_r d_m + \Delta)\).

When \(\lambda_r \geq c/p\) and \(\lambda_m \leq \lambda_r \Delta/(\lambda_r \Pi^* - \lambda_r T(q^M_d) + d_m - \lambda_r d_m + \Delta)\), the distributor and the retailer cannot reach any agreement if they first negotiate and \(\Pi^* < d_m + d_r + d_d + (1 - \lambda_m)\Delta/\lambda_m\).
Otherwise, if \( \Pi^* \geq d_m + d_r + d_d + (1 - \lambda_m)\Delta/\lambda_m \), we find

\[
T(q^M_d) - d_r - d_d - \frac{d_m}{\lambda_r} - \frac{(1 - \lambda_r)\Delta}{\lambda_r} = \Pi^* - d_m - d_r - d_d - \Pi^* + T(q^M_d) + d_m - \frac{d_m}{\lambda_r} - \frac{(1 - \lambda_r)\Delta}{\lambda_r} \\
\geq \frac{(1 - \lambda_m)\Delta}{\lambda_m} - \Pi^* + T(q^M_d) + d_m - \frac{d_m}{\lambda_r} - \frac{(1 - \lambda_r)\Delta}{\lambda_r} \\
\geq 0.
\]

Thus, under any negotiation sequence, supply chain members can reach an agreement. As

\[
\pi^R_d \leq \pi^M_d \Leftrightarrow \lambda_m \leq \lambda_r \Delta/(\lambda_r \Pi^* - \lambda_r T(q^M_d) + d_m - \lambda_r d_m + \Delta),
\]

the distributor’s optimal negotiation sequence is to first negotiate with the manufacturer if \( \Pi^* \geq d_m + d_r + d_d + (1 - \lambda_m)\Delta/\lambda_m \). Therefore, the distributor should first negotiate with the manufacturer when \( \lambda_r \geq c/p \) and \( \lambda_m \leq \lambda_r \Delta/(\lambda_r \Pi^* - \lambda_r T(q^M_d) + d_m - \lambda_r d_m + \Delta) \). ■

**Proof of Theorem 4.** For any given negotiation sequence, if the supply chain cannot reach any agreement, then the other negotiation sequence is weakly better to both the retailer and the manufacturer. Hence, we only need to consider the case that the supply chain negotiations succeed under both negotiation sequence. For this case, we have

\[
\pi^R_m - \pi^M_m = (1 - \lambda_m) \left[ \lambda_r (\Pi^* - T(q^M_d)) + (1 - \lambda_r) d_m + (1 - \lambda_r) \frac{\Delta}{\lambda_m} + \Delta \right] \\
\geq (1 - \lambda_m) \lambda_r (\Pi^* - T(q^M_d)) \\
= (1 - \lambda_m) \lambda_r (T(q^*_r) - T(q^M_d)) \\
\geq 0,
\]

and \( \pi^R_r - \pi^M_r = (1 - \lambda_r)[\Pi^* - T(q^M_d) - d_m - \Delta/\lambda_m + \frac{c q_d^M}{\lambda_r}] \). Hence, the manufacturer prefers the distributor to negotiate with him first, if and only if the supply chain negotiations cannot succeed when the distributor first negotiates with the retailer, i.e., \( \Pi^* < d_r + d_d + d_m/\lambda_r + (1 - \lambda_r)\Delta/\lambda_r \). When supply chain negotiations succeed under both negotiation sequences, the retailer prefers to join the distributor’s first negotiation, if and only if \( \pi^R_r - \pi^M_r \geq 0 \), i.e., \( d_m + \Delta/\lambda_m \leq \Pi^* - T(q^M_d) - \frac{c q_d^M}{\lambda_r} \). ■

**Proof of Lemma 1.**

We begin by analyzing the buyback contract \((w_m, b_m)\). For a given negotiation result \((w_d, q_r)\), the profits of the distributor and the manufacturer are

\[
\begin{align*}
\pi_d &= w_d E \min\{D, q_r\} + b_m E(q_r - D)^+ - w_m q_r, \\
\pi_m &= (w_m - c)q_r - b_m E(q_r - D)^+.
\end{align*}
\]

Thus, the negotiation between the distributor and the manufacturer ends with the negotiation result
as the solution of the following problem:

$$\begin{align*}
\max_{w_m, b_m} & \quad [w_d E \min\{D, q_r\} + b_m E(q_r - D)^+ - w_m q_r - d_d + \Delta]^{\lambda_m} \\
& \times \left[ (w_m - c) q_r - b_m E(q_r - D)^+ - d_m \right]^{1-\lambda_m} \\
\text{s.t.} & \quad w_d E \min\{D, q_r\} + b_m E(q_r - D)^+ - w_m q_r \geq d_d - \Delta, \\
& \quad (w_m - c) q_r - b_m E(q_r - D)^+ \geq d_m.
\end{align*}$$

Since $\pi_d + \pi_m = w_d E \min\{D, q_r\} - c q_r$, the negotiation can end successfully if $w_d E \min\{D, q_r\} - c q_r \geq d_m + d_d - \Delta$. Moreover, if $w_d E \min\{D, q_r\} - c q_r \geq d_m + d_d - \Delta$, then the above objective is no larger than $[w_d E \min\{D, q_r\} - c q_r - d_m - d_d + \Delta]^{\lambda_m(1 - \lambda_m)^{1-\lambda_m}}$, which holds when we choose the following maximizer in terms of $(w_d, q_r)$:

$$(\hat{w}_m(w_d, q_r), \hat{b}_m(w_d, q_r)) = \left( (1 - \lambda_m)(w_d E \min\{D, q_r\} - d_d + \Delta)/q_r + \lambda_m d_m/q_r + \lambda_m c, 0 \right).$$

Next, we consider the first negotiation between the distributor and the retailer. Their (expected) profits are

$$\begin{align*}
\pi_d &= w_d E \min\{D, q_r\} + \hat{b}_m(w_d, q_r) E(q_r - D)^+ - \hat{w}_m(w_d, q_r) q_r \\
&= \lambda_m [w_d E \min\{D, q_r\} - c q_r - d_m] + (1 - \lambda_m)(d_d - \Delta),
\end{align*}$$

and $\pi_r = (p - w_d) E \min\{D, q_r\}$, respectively. Thus, the negotiation result of the distributor and the retailer is the solution of the following problem:

$$\begin{align*}
\max_{w_d, q_r} & \quad \{\lambda_m[w_d E \min\{D, q_r\} - c q_r - d_m] + (1 - \lambda_m)(d_d - \Delta) - d_d\}^{\lambda_r} \\
& \times \left[ (p - w_d) E \min\{D, q_r\} - d_r \right]^{1-\lambda_r} \\
\text{s.t.} & \quad \lambda_m[w_d E \min\{D, q_r\} - c q_r - d_m] + (1 - \lambda_m)(d_d - \Delta) \geq d_d, \\
& \quad (p - w_d) E \min\{D, q_r\} \geq d_r.
\end{align*}$$

Since $\pi_d/\lambda_m + \pi_r = p E \min\{D, q_r\} - c q_r - d_m + (1 - \lambda_m)(d_d - \Delta)/\lambda_m \leq \Pi^* - d_m + (1 - \lambda_m)(d_d - \Delta)/\lambda_m$, the negotiation can succeed if $\Pi^* \geq d_r + d_m + d_d + (1 - \lambda_m)\Delta/\lambda_m$. In addition, if $\Pi^* \geq d_r + d_m + d_d + (1 - \lambda_m)\Delta/\lambda_m$, then the above objective is no greater than $\lambda_m^{\lambda_r} \lambda_r^{\lambda_r} (1 - \lambda_r)^{1-\lambda_r} (\Pi^* - d_m - d_r - d_d - (1 - \lambda_m)\Delta/\lambda_m)$, which can be achieved by the following maximizer

$$(\hat{w}_r(q_r), \hat{q}_r) = \left( p - d_r + (1 - \lambda_r)(\Pi^* - d_m - d_r - d_d - (1 - \lambda_m)\Delta/\lambda_m), q_r^{\star} \right).$$

The profits of all firms are thus computed as in this lemma. We also find that the profits are the same as those in Theorem 1. Thus, we conclude that the buyback contract and the wholesale price contract can lead to an identical result.

Then, we investigate the two-part tariff contract $(w_m, K_m)$. For a given negotiation result $(w_d, q_r)$, the profits of the distributor and the manufacturer are $\pi_d = (w_d - w_m) q_r - K_m$ and $\pi_m = (w_m - c) q_r + K_m$, respectively. Thus, the negotiation result of the distributor and the manufacturer is the
solution of the following problem:
\[
\max_{w_m,q_d} \quad ((w_d - w_m)q_r - K_m - d_d + \Delta)\lambda_m ((w_m - c)q_r + K_m - d_m)^{1-\lambda_m} \\
\text{s.t.} \quad (w_d - w_m)q_r - K_m \geq d_d - \Delta, (w_m - c)q_r + K_m \geq d_m.
\]

Since \(\pi_d + \pi_m = (w_d - c)q_r\), the negotiation can succeed if \((w_d - c)q_r \geq d_m + d_d - \Delta\). We also find that, if \((w_d - c)q_r \geq d_m + d_d - \Delta\), then the above objective is no more than \(\lambda_m (1 - \lambda_m)^{1-\lambda_m}[(w_d - c)q_r - d_m - d_d + \Delta]\), which can be achieved by the following maximizer (in terms of \((w_d, q_r)\)):
\[
(w_m(w_d, q_r), \hat{K}_m(w_d, q_r)) = ((1 - \lambda_m)w_d + \lambda_m c + \lambda_m d_m/q_r - (1 - \lambda_m)(d_d - \Delta)/q_r, 0).
\]

In the first negotiation between the distributor and the retailer, the firms’ (expected) profits are
\[
\pi_d = (w_d - \hat{w}_m(w_d, q_r))q_r - \hat{K}_m(w_d, q_r) = \lambda_m [(w_d - c)q_r - d_m] + (1 - \lambda_m)(d_d - \Delta), \\
\pi_r = pE \min\{D, q_r\} - w_d q_r.
\]
which are the same as those in the proof of Theorem 1. Therefore, the two-part tariff contract and the wholesale price contract can also generate the same result. ■

**Proof of Theorem 5.** For any given negotiation result \((w_m, b_m)\), the distributor’s and the retailer’s (expected) profits are
\[
\pi_d = w_d E \min\{D, q_r\} + b_m E(q_r - D)^+ - w_m q_r \quad \text{and} \quad \pi_r = (p - w_d) E \min\{D, q_r\}.
\]
Thus, the negotiation result is the solution of the following problem:
\[
\max_{w_d, q_r} \quad \{w_d E \min\{D, q_r\} + b_m E(q_r - D)^+ - w_m q_r - d_d + \Delta\}^{\lambda_r}\{p - w_d) E \min\{D, q_r\} - d_r\}^{1-\lambda_r}, \\
\text{s.t.} \quad w_d E \min\{D, q_r\} + b_m E(q_r - D)^+ - w_m q_r \geq d_d - \Delta, (p - w_d) E \min\{D, q_r\} \geq d_r.
\]
Since
\[
\pi_d + \pi_r = pE \min\{D, q_r\} + b_m E(q_r - D)^+ - w_m q_r \leq \max_q \{pE \min\{D, q\} + b_m E(q - D)^+ - w_m q\} \leq (p - b_m) \int_0^{F^{-1}((p-w_m)/(p-b_m))} x f(x) dx,
\]
the negotiation can achieve an agreement only if
\[
(p - b_m) \int_0^{F^{-1}((p-w_m)/(p-b_m))} x f(x) dx \geq d_r + d_d - \Delta.
\]
Moreover, if the above condition holds, then the above objective is no larger than
\[
\lambda_r^{\lambda_r} (1 - \lambda_r)^{1-\lambda_r} \left[ (p - b_m) \int_0^{F^{-1}((p-w_m)/(p-b_m))} x f(x) dx - d_r - d_d + \Delta \right],
\]

which can be achieved by the following unique maximizer in terms of \((w_m, b_m)\):

\[
\left(\hat{w}_d(w_m, b_m), \hat{q}_r(w_m, b_m)\right) = \left(\frac{d_r + (1 - \lambda_r) \left(\int_0^{F^{-1}\left(p - w_m\right)/p - b_m} x f(x)dx - d_r - d_d + \Delta\right)}{p - \frac{d_r + (1 - \lambda_r) \left(\int_0^{F^{-1}\left(p - w_m\right)/p - b_m} x f(x)dx - d_r - d_d + \Delta\right)}{E \min \left\{D, F^{-1}\left(p - w_m\right)\right\}}, F^{-1}\left(p - w_m\right)\right).
\]

We then consider the negotiation between the distributor and the manufacturer, whose profits are computed as

\[
\pi_d = \hat{w}_d(w_m, b_m) E \min \left\{D, \hat{q}_r(w_m, b_m)\right\} + b_m E (\hat{q}_r(w_m, b_m) - D)^+ - w_m q_r
\]

\[
= \lambda_r \left(\int_0\hat{q}_r(w_m, b_m) x f(x)dx - d_r\right) + (1 - \lambda_r)(d_d - \Delta),
\]

\[
\pi_m = (w_m - c)\hat{q}_r(w_m, b_m) - b_m E (\hat{q}_r(w_m, b_m) - D)^+
\]

\[
= [p[1 - F(\hat{q}_r(w_m, b_m))] - c]\hat{q}_r(w_m, b_m) + b_m \int_0\hat{q}_r(w_m, b_m) x f(x)dx.
\]

Thus, if \(p - b_m \int_0^{F^{-1}(p - w_m)/(p - b_m)} x f(x)dx \geq d_r + d_d - \Delta\), then the negotiation result is the solution of the following problem:

\[
\max_{w_m, b_m} \quad \left\{\lambda_r \left(\int_0\hat{q}_r(w_m, b_m) x f(x)dx - d_r\right) + (1 - \lambda_r)(d_d - \Delta) - d_d\right\}^{\lambda_m}
\]

\[
\times \left\{[p[1 - F(\hat{q}_r(w_m, b_m))] - c]\hat{q}_r(w_m, b_m) + b_m \int_0\hat{q}_r(w_m, b_m) x f(x)dx - d_m\right\}^{1-\lambda_m}
\]

\[
\text{s.t.} \quad [p[1 - F(\hat{q}_r(w_m, b_m))] - c]\hat{q}_r(w_m, b_m) + b_m \int_0\hat{q}_r(w_m, b_m) x f(x)dx \geq d_m, \\
\lambda_r \left(\int_0\hat{q}_r(w_m, b_m) x f(x)dx - d_r\right) + (1 - \lambda_r)(d_d - \Delta) \geq d_d, \\
(p - b_m) \int_0^{F^{-1}(p - w_m)/(p - b_m)} x f(x)dx \geq d_r + d_d - \Delta.
\]

Since

\[
\frac{\pi_d}{\lambda_r} + \pi_m
\]

\[
= [p[1 - F(\hat{q}_r(w_m, b_m))] - c]\hat{q}_r(w_m, b_m) + p \int_0\hat{q}_r(w_m, b_m) x f(x)dx - d_r + \frac{(1 - \lambda_r)(d_d - \Delta)}{\lambda_r}
\]

\[
\leq \max_q \left\{[p[1 - F(q)] - c]q + p \int_0^q x f(x)dx\right\} - d_r + \frac{(1 - \lambda_r)(d_d - \Delta)}{\lambda_r}
\]

\[
= \Pi^* - d_r + \frac{(1 - \lambda_r)(d_d - \Delta)}{\lambda_r},
\]

the negotiation succeed if \(\Pi^* - d_r + (1 - \lambda_r)(d_d - \Delta)/\lambda_r \geq d_d/\lambda_r + d_m\). In addition, if \(\Pi^* \geq d_r + d_m + d_d + (1 - \lambda_r)\Delta/\lambda_r\), then the above objective is no longer than \([\Pi^* - d_r - d_m - d_d - (1 -
\( \lambda_r \Delta / \lambda_r \lambda_m^m (1 - \lambda_m)^{1 - \lambda_m} \), which can hold when we use the following unique maximizer

\[
\begin{align*}
\hat{w}_m &= c + (p - c)(1 - \lambda_m)[\Pi^* - d_r - d_m - d_d - (1 - \lambda_r)\Delta / \lambda_r] + d_m] / \Pi^*, \\
\hat{b}_m &= p[(1 - \lambda_m)[\Pi^* - d_r - d_m - d_d - (1 - \lambda_r)\Delta / \lambda_r] + d_m] / \Pi^*.
\end{align*}
\]

The profits of the three firms are computed as in this theorem.

Next, we consider the distributor’s optimal negotiation sequence as well as the manufacturer’s and the retailer’s preferences on the distributor’s sequence decision. When supply chain negotiations can end successfully in any negotiation sequence, we have

\[
\begin{align*}
\pi_{mm}^{M, bb} &= (1 - \lambda_m)[\Pi^* - d_r - d_m - d_d - (1 - \lambda_r)\Delta / \lambda_r] + d_m, \\
\pi_{mm}^{M, bb} &= \lambda_r \lambda_m [\Pi^* - d_r - d_m - d_d - (1 - \lambda_r)\Delta / \lambda_r] + d_d, \\
\pi_{mm}^{M, bb} &= \lambda_m (1 - \lambda_r)[\Pi^* - d_r - d_m - d_d - (1 - \lambda_r)\Delta / \lambda_r] + d_r + (1 - \lambda_r)\Delta / \lambda_r, \\
\pi_{mm}^{R, bb} &= \lambda_r (1 - \lambda_m)[\Pi^* - d_m - d_d - (1 - \lambda_m)\Delta / \lambda_m] + d_m + (1 - \lambda_m)\Delta / \lambda_m, \\
\pi_{mm}^{R, bb} &= \lambda_m \lambda_r [\Pi^* - d_m - d_d - (1 - \lambda_m)\Delta / \lambda_m] + d_d, \\
\pi_{mm}^{R, bb} &= (1 - \lambda_r)[\Pi^* - d_m - d_d - (1 - \lambda_m)\Delta / \lambda_m] + d_r.
\end{align*}
\]

Hence, it is easy to see that the distributor prefers to first negotiate with the retailer if and only if \( \lambda_r \leq \lambda_m \), the retailer prefers the distributor to first negotiate with him if and only if

\[
\Pi^* \geq \left( \frac{1}{\lambda_r} + \frac{1}{\lambda_m} + \frac{\lambda_m}{1 - \lambda_m} \right) \Delta + d_d + d_r + d_m,
\]

and the manufacturer prefers the distributor to first negotiate with him if and only if

\[
\Pi^* \geq \left( \frac{1}{\lambda_r} + \frac{1}{\lambda_m} + \frac{\lambda_r}{1 - \lambda_r} \right) \Delta + d_d + d_r + d_m.
\]

**Proof of Lemma 2.** Since \( q_S(\lambda_r, d_r) \) satisfies the constraint \( q_S(\lambda_r, d_r) \geq T^{-1}(d_r) \) and \( T \) is an increasing function, we find that \( T(q_S(\lambda_r, d_r)) \geq d_r \), and

\[
S(\lambda_r, d_r) = \lambda_r T(q_S(\lambda_r, d_r)) + (p - pF(q_S(\lambda_r, d_r)) - c)q_S(\lambda_r, d_r)
\]

\[
= T(q_S(\lambda_r, d_r)) + pq_S(\lambda_r, d_r)(1 - F(q_S(\lambda_r, d_r))) - c q_S(\lambda_r, d_r) - (1 - \lambda_r) T(q_S(\lambda_r, d_r))
\]

\[
= pE \min \{ D, q_S(\lambda_r, d_r) \} - c q_S(\lambda_r, d_r) - (1 - \lambda_r) T(q_S(\lambda_r, d_r))
\]

\[
\leq \Pi^* - (1 - \lambda_r) T(q_S(\lambda_r, d_r)) \leq \Pi^* - (1 - \lambda_r) d_r.
\]

The lemma is thus proved. ■

**Proof of Theorem 6.** For any given negotiation result \( (w_m, K_m) \), the (expected) profits of the distributor and the retailer are \( \pi_d = (w_d - w_m) q_r \) and \( \pi_r = pE \min \{ D, q_r \} - w_d q_r \), respectively.
Thus, the negotiation result is the solution of the following problem:

$$\max_{w_d, q_r} \{(w_d - w_m)q_r - d_d + \Delta\}^{\lambda_r} \{pE \min\{D, q_r\} - w_dq_r - d_r\}^{1 - \lambda_r}$$

s.t. \( (w_d - w_m)q_r > d_d - \Delta \) and \( pE \min\{D, q_r\} - w_dq_r > d_r \).

Since \( \pi_d + \pi_r = pE \min\{D, q_r\} - w_mq_r \leq \max_{q_r} \{pE \min\{D, q_r\} - w_mq_r\} = T(F^{-1}((p - w_m)/p)) \), the negotiation can succeed if \( T(F^{-1}((p - w_m)/p)) \geq d_r + d_d - \Delta \). Moreover, if \( T(F^{-1}((p - w_m)/p)) \geq d_r + d_d - \Delta \), then the above objective is no larger than \( T(F^{-1}((p - w_m)/p)) - d_r - d_d + \Delta \lambda_r(1 - \lambda_r)^{1 - \lambda_r} \), which can be achieved by the following unique maximizer:

$$\left(\hat{w}_d(w_m, K_m), \hat{q}_r(w_m, K_m)\right) = \left( w_m + \lambda_r T(F^{-1}((p - w_m)/p)) - d_r \right) \frac{1 - \lambda_r}{F^{-1}((p - w_m)/p)} + \frac{(1 - \lambda_r)(d_d - \Delta)}{F^{-1}((p - w_m)/p)}, F^{-1}((p - w_m)/p) \right).$$

Noting that

$$T(x) = p \int_0^x t f(t) dt \leq p \int_0^\infty t f(t) dt = pE[D],$$

we find that \( d_r + d_d - \Delta \leq pE[D] \) is a necessary condition for supply chain members to reach agreements.

Next, we consider the negotiation between the distributor and the manufacturer, whose profits are

$$\left\{ \begin{array}{l} \pi_d = (\hat{w}_d(w_m, K_m) - w_m) \hat{q}_r(w_m, K_m) - K_m \\ \pi_m = (w_m - c) \hat{q}_r(w_m, K_m) + K_m = (w_m - c) F^{-1}((p - w_m)/p) + K_m. \end{array} \right.$$ 

Thus, if \( T(F^{-1}((p - w_m)/p)) \geq d_r \), the negotiation result is the solution of the following problem:

$$\max_{w_m, K_m} \left[ \lambda_r \left( T \left( F^{-1} \left( \frac{p - w_m}{p} \right) \right) - d_r \right) + (1 - \lambda_r)(d_d - \Delta) - K_m - d_d \right]^{\lambda_m}$$

$$\times \left[ (w_m - c) F^{-1} \left( \frac{p - w_m}{p} \right) + K_m - d_m \right]^{1 - \lambda_m}$$

s.t. \( \lambda_r \left( T \left( F^{-1} \left( \frac{p - w_m}{p} \right) \right) - d_r \right) + (1 - \lambda_r)(d_d - \Delta) - K_m \geq d_d, \)

\( (w_m - c) F^{-1} \left( \frac{p - w_m}{p} \right) + K_m \geq d_m \),

\( T \left( F^{-1} \left( \frac{p - w_m}{p} \right) \right) \geq d_r. \)
Since $T(F^{-1}((p - w_m)/p)) \geq d_r$, i.e., $\bar{q}_r(w_m, K_m) \geq T^{-1}(d_r)$, we find

\[\{\lambda_r[T(F^{-1}((p - w_m)/p)) - d_r] + (1 - \lambda_r)(d_d - \Delta) - K_m\} + \{(w_m - c)F^{-1}((p - w_m)/p) + K_m\}\]

\[\lambda_rT(F^{-1}((p - w_m)/p)) + (w_m - c)F^{-1}((p - w_m)/p) + (1 - \lambda_r)(d_d - \Delta) - \lambda_r d_r\]

\[= \lambda_rT(\bar{q}_r(w_m, K_m)) + (p - pF(\bar{q}_r(w_m, K_m)) - c)\bar{q}_r(w_m, K_m) + (1 - \lambda_r)(d_d - \Delta) - \lambda_r d_r\]

\[\leq \max_{q \geq T^{-1}(d_r)}[\lambda_rT(q) + (p - pF(q) - c)q] + (1 - \lambda_r)(d_d - \Delta) - \lambda_r d_r\]

\[= \lambda_rT(q_S(\lambda_r, d_r)) + (p - pF(q_S(\lambda_r, d_r)) - c)q_S(\lambda_r, d_r) + (1 - \lambda_r)(d_d - \Delta) - \lambda_r d_r\]

\[= S(\lambda_r, d_r) + (1 - \lambda_r)(d_d - \Delta) - \lambda_r d_r,\]

and the negotiation is successful if $S(\lambda_r, d_r) + (1 - \lambda_r)(d_d - \Delta) - \lambda_r d_r \geq d_d + d_m$. Moreover, if $S(\lambda_r, d_r) \geq \lambda_r d_r + \lambda_r d_d + d_m + (1 - \lambda_r)\Delta$, then the above objective is no more than $[S(\lambda_r, d_r) - \lambda_r d_r - \lambda_r d_d - d_m - (1 - \lambda_r)\Delta]\lambda_m(1 - \lambda_m)^{1 - \lambda_m}$, which can be achieved by the following unique maximizer

\[
\begin{cases}
\hat{w}_m^{M,tpt} = p - pF(q_S(\lambda_r, d_r)), \\
K_m^{M,tpt} = (1 - \lambda_m)(S(\lambda_r, d_r) - \lambda_r d_r - \lambda_r d_d - (1 - \lambda_r)\Delta) + \lambda_m d_m - (w_m^{M,tpt} - c)q_S(\lambda_r, d_r);
\end{cases}
\]

and the condition for the second negotiation to succeed—i.e., $T(F^{-1}((p - w_m^{M,tpt})/p)) = T(q_S(\lambda_r)) \geq T(T^{-1}(d_r)) = d_r$—is also satisfied. Therefore,

\[
(\hat{w}_d(w_m, K_m), \hat{q}_r(w_m, K_m))
= \left(\frac{w_m^{M,tpt} + \lambda_r[T(F^{-1}((p - w_m^{M,tpt})/p)) - d_r]}{F^{-1}((p - w_m^{M,tpt})/p)}, \frac{(1 - \lambda_r)(d_d - \Delta)}{F^{-1}((p - w_m^{M,tpt})/p)}, F^{-1}((p - w_m^{M,tpt})/p)\right)
\]

\[
= \left(p - pF(q_S(\lambda_r, d_r)) + \frac{\lambda_rT(q_S(\lambda_r, d_r)) - d_r}{q_S(\lambda_r, d_r)} + \frac{(1 - \lambda_r)(d_d - \Delta)}{q_S(\lambda_r, d_r)}, q_S(\lambda_r, d_r)\right).
\]

The profits of the three firms are computed as in this theorem. We then conclude that the first negotiation succeeds if and only if $d_r \leq pE[D]$ and $S(\lambda_r, d_r) \geq \lambda_r d_r + \lambda_r d_d + d_m + (1 - \lambda_r)\Delta$. If the first negotiation is successful, then the second negotiation can also succeed.

We then examine the distributor’s optimal negotiation sequence, as well as the manufacturer’s and the retailer’s preferences on the distributor’s negotiation sequence. When the supply chain negotiations can succeed in any negotiation sequence, we have

\[
\begin{cases}
\pi_d^{M,tpt} - \pi_d^{R,tpt} = \lambda_m[S(\lambda_r, d_r) - \lambda_r \Pi^* - (1 - \lambda_r)d_m - \Delta + \lambda_r \Delta/\lambda_m], \\
\pi_m^{M,tpt} - \pi_m^{R,tpt} = (1 - \lambda_r)[T(q_S(\lambda_r, d_r)) - \Pi^* + d_m + \Delta/\lambda_m], \\
\pi_d^{M,tpt} - \pi_r^{R,tpt} = (1 - \lambda_m)[S(\lambda_r, d_r) - \lambda_r \Pi^* - (1 - \lambda_r)d_m - \Delta - (1 - \lambda_r)\Delta/\lambda_m].
\end{cases}
\]

Therefore, when the supply chain negotiations can succeed in both negotiation sequences, i.e.,

\[d_r \leq pE[D], S(\lambda_r, d_r) \geq \lambda_r d_r + \lambda_r d_d + d_m + (1 - \lambda_r)\Delta, \text{ and } \Pi^* \geq d_r + d_m + d_d + (1 - \lambda_m)\Delta/\lambda_m,
\]

\[27\]
we find the following results.

1. The distributor prefers to first negotiate with the retailer if and only if \((1 - \lambda_r)d_m + (1 - \lambda_r/\lambda_m)\Delta \geq S(\lambda_r, d_r) - \lambda_r\Pi^*\).

2. The retailer prefers the distributor to first negotiate with him if and only if \(d_m + \Delta/\lambda_m \leq \Pi^* - T(q_S(\lambda_r, d_r))\).

3. The manufacturer prefers the distributor to first negotiate with the retailer if and only if
\[
(1 - \lambda_r)d_m + (1 + (1 - \lambda_r)/\lambda_m)\Delta \geq S(\lambda_r, d_r) - \lambda_r\Pi^*.
\]
This theorem is thus proved.

**Proof of Theorem 7.** If \(\lambda_r \leq \lambda_m\), then the distributor first negotiates with the retailer under the buyback contract. When the other two contracts apply, the distributor chooses the optimal negotiation sequence, and his profit is \(\max\{\pi^R_d, \pi^M_d\} \geq \pi^R_d = \pi^{R,bb}_d\), or \(\max\{\pi^{R,tpt}_d, \pi^{M,tpt}_d\} \geq \pi^{R,tpt}_d = \pi^{R,bb}_d\). Therefore, the distributor cannot benefit from the buyback contract. Thus, the condition \(\lambda_r \geq \lambda_m\) is necessary for all three firms to prefer to use the buyback contract. Note that, when \(\lambda_r \geq \lambda_m\), the distributor first negotiates with the manufacturer under the buyback contract. The negotiations in the supply chain are successful under the buyback contract if \(\Pi^* \geq d_r + d_d + d_m + (1 - \lambda_r)\Delta/\lambda_r\). That is, \(\Pi^* \geq d_r + d_d + d_m + (1 - \lambda_r)\Delta/\lambda_r\) is another necessary condition for all the three firms to prefer to use the buyback contract. Thus, the theorem is proved.