Game-Theoretic Analyses of Strategic Pricing Decision Problems in Supply Chains

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Abstract

We consider strategic pricing problems in which each firm chooses between a non-cooperative (individual pricing) strategy and a cooperative (price negotiation) strategy. We first analyze a monopoly supply chain involving a supplier and a retailer, and then investigate two competing supply chains each consisting of a supplier and a retailer. We find that a proper power allocation between the supplier and the retailer can make the two firms benefit from negotiating the wholesale and retail prices. When the supplier negotiates the wholesale price, the retailer's cooperative strategy can always induce supply chain coordination in the monopoly setting, whereas the two supply chains in the duopoly setting can be possibly coordinated only when the retailers determine their retail prices individually. In both the monopoly and duopoly settings, the wholesale price negotiation is a necessary part of the communications between supply chain members. When the supply chain competition intensifies, all firms are more likely to determine their prices individually rather than to negotiate their prices.

Keywords: Negotiation; generalized Nash bargaining solution; strategic pricing; supply chain.

1 Introduction

Many firms in today's supply chains are more closely connected than ever before, thus exhibiting greater motivations to cooperate with an aim to improve their profitability. A natural condition for successful cooperations is the fair allocation of profit surplus or cost savings within supply chains. In practice, the fair allocation usually results from the negotiation between the cooperating firms. A number of researchers have devoted their efforts to studying various negotiations in supply chain settings. Most of relevant publications are concerned with exogenous factors that may affect negotiation outcomes in supply chains. These factors mainly include competition (Dukes et al. 2006, Olczak 2011, Feng and Lu 2012, Feng and Lu 2013, Aydin and Heese 2014, Baron, Berman, and Wu 2016, and Nguyen 2017), uncertainty (Gurnani and Shi 2006, Leng and Parlar 2009, Feng et al. 2015, and Zheng and Negenborn 2015), and relative bargaining powers (Iyer and Villas-Boas 2003, Nagarajan and Bassok 2008, Lovejoy 2010, and Zhong et al. 2016).

In fact, negotiation outcomes may not only depend on exogenous factors but also result from the strategic decisions of consumers or a firm in supply chains. Any supply chain involves a number of negotiable issues, among which pricing negotiations are very common, because they can help determine a profit allocation. For example, the wholesale price serves to allocate the profit between a supplier and a retailer, as a higher wholesale price generates a greater profit for the supplier and a lower profit for the retailer, and a smaller wholesale price leads to an opposite result. Thus, the wholesale price naturally results from the negotiation. In practice, a retailer usually individually determines a retail price to maximize his profit. Nonetheless, there are also a number of real cases that indicate suppliers' engagement in the retail pricing decision processes. A supplier has a strong interest in the retail pricing decision, mainly because a higher retail price is very likely to reduce the supplier's sales (Bennett 2014). Furthermore, we learn from Bennett's report (2014) that commercial negotiations between manufacturers/suppliers and retailers necessarily include a discussion of wholesale prices and may also involve a discussion of retail prices such as recommended retail prices (RRPs) and resale/retail price maintenance (RPM). RRP is also known as the manufacturer's or supplier's suggested retail price, which has widely existed in retail markets for automobiles. RPM is the practice whereby the supplier and the retailer agree that the retailer sells the supplier's products at a certain retail price, or the practice whereby the two firms jointly set a minimum or maximum retail price. RPM has been used in a variety of retail markets for, e.g., jewelry, sports equipment, candy, biscuits, and many lines of clothing (jeans, shoes, socks, underwear, and shirts) (Krishnan and Winter 2007). Similar to Bennett (2014), Ertel (1999) revealed that, in today's society, each company exists in a complex network of relationships, and it has to negotiate not only purchasing and outsourcing contracts but also marketing arrangements with its suppliers or distributors. This implies that suppliers may engage in the decision-making process for retail price.

There are some real, specific examples for retail price negotiations between suppliers and retailers. Combs and Frei (1984) reported that, before the middle 1980s, the retailers in the Swiss market negotiated retail prices with the U.S. manufacturers of sports/leisure equipment. Sawyer (2018) disclosed that, in the United States, the National Association of Retail Druggists, the Proprietary Association of America, and the National Wholesale Druggists' Association had held conferences for manufacturers, wholesalers, and retailers to negotiate retail prices for compounding chemicals and botanicals as well as retail brands. In addition, Smith (2016) exposed that in recent years, researchers (especially, economists) have paid more attention to pricing relationships between retailers and suppliers, focusing on the retail prices that can be jointly achieved by retailers and suppliers in equilibrium.

Motivated by the above facts, we analyze supply chain problems in which a retailer and a supplier decide their retail and wholesale pricing strategies, respectively. The supplier needs to make a decision on whether to determine his wholesale price individually (corresponding to the supplier's non-cooperative strategy) or to negotiate the wholesale price with the retailer (corresponding to the supplier's cooperative strategy). After observing the supplier's decision, the retailer makes a decision on whether to determine his retail price individually (corresponding to the retailer's non-cooperative strategy) or to negotiate the retail price with the supplier (corresponding to the retailer's cooperative strategy). The supplier's non-cooperative and cooperative strategies and the retailer's non-cooperative strategy have been studied by researchers such as Iyer and Villas-Boas (2003), Feng and Lu (2012), Baron, Berman, and Wu (2016), and Zhong et al. (2016), whereas very few researchers have investigated the retailer's cooperative strategy although it has been adopted in practice. Furthermore, a very limited number of publications are concerned with the comparison between these two strategies in supply chains. The above observations show the necessity and importance of investigating whether the retailer/supplier should always choose a non-cooperative or cooperative strategy for her/his pricing decision. It is also important to find whether or not the supplier can benefit from the retailer's cooperative strategy. Accordingly, in this paper we mainly address the following questions: (i) under what conditions is the retailer/supplier willing to adopt her/his cooperative strategy? (ii) under what conditions does the supplier agree with the retailer to negotiate the retail price? and (iii) how do the two firms' strategic activities influence their profits?

To address the questions above, we first investigate the pricing strategies in a single two-echelon supply chain (a monopoly setting) consisting of a supplier and a retailer. We use the cooperative-game concept of generalized Nash bargaining (GNB) solution to characterize each negotiated price. Both the retailer and the supplier need to decide their pricing strategies. That is, each firm should determine whether to make an individually optimal pricing decision under her/his non-cooperative strategy or to negotiate the price with the other firm under her/his cooperative strategy. For the purpose of generality, the supplier's and the retailer's relative bargaining powers in different pricing strategies may differ.

In the monopoly setting, we first examine the retailer's retail pricing strategy when the wholesale price is negotiated as in many practices. Using a linear demand function, we solve the game under the retailer's non-cooperative strategy (for her retail price) and that under the retailer's cooperative strategy, and also compare our analytic results obtained from the two game analyses to draw the managerial implications regarding the conditions under which the retailer and the supplier are willing to adopt the retail price negotiation strategy. In addition, we study the supplier's wholesale pricing strategy under which the supplier decides whether to negotiate his wholesale price or to determine the wholesale price individually.

We also consider a duopoly setting in which two supply chains each consisting of a supplier and a retailer compete for customers in a market. In the presence of supply chain competition, we analyze the retailers' and also the suppliers' pricing strategies, similar to our study in the monopoly setting. We find that most results in the monopoly and duopoly settings are similar when the suppliers negotiate their wholesale prices with their retailers (which is common in reality). For example, a proper power allocation between the supplier and the retailer can lead the retail price negotiation to benefit the two firms, and wholesale price negotiation is a necessary part of the communications between supply chain members. Moreover, retail price negotiation can always induce supply chain coordination in the monopoly setting, whereas the two supply chains in the duopoly setting could be coordinated only when the retailers choose to determine their retail prices individually. We also obtain a number of managerial insights relating to supply chain competition. For example, the firms in the supply chain with the retail price and wholesale price negotiations are more likely to reach an agreement than those in the supply chain with the retailer's non-cooperative strategy. When the supply chain competition increases, both the suppliers and the retailers are more likely to determine their prices individually rather than to conduct negotiations.

Our paper is related to two streams of extant publications, which include (i) a stream of publications regarding supply chain negotiations with no competition and (ii) a stream of publications focused on supply chain negotiations in the competitive setting. We start with a review of the major publications in stream (i). These publications are mainly concerned with the impact of bargaining powers and uncertainty on supply chain negotiations. Regarding the impact of bargaining powers, a powerful upstream firm may increase the effect of double marginalization while a powerful downstream firm may promote supply chain coordination (see, Iyer and Villas-Boas 2003 and Zhong et al. 2016). Moreover, in an assembly supply chain, a powerful downstream assembler may prevent upstream suppliers from forming any coalition (Nagarajan and Bassok 2008). For the impact of uncertainty, Gurnani and Shi (2006) showed that, when the supply reliability estimation of a supplier and that of a retailer are private information, the supply chain profit reaches its maximum if and only if these estimates are not far apart. Zheng and Negenborn (2015) discussed the impact of market risk on the bargaining set and the disagreement point. They showed that, when the market risk increases, the bargaining set is wider (narrower) and the disagreement point is smaller (larger) when the retailer faces an elastic (fixed) demand. For specific discussions, see the surveys by Bernstein and Nagarajan (2011), Ingene, Taboubi, and Zaccour (2012), Jeuland and Shugan (1983), and Nagarajan and Sosic (2008). In our paper, we examine the impact of relative bargaining powers. Moreover, we consider a more general setting in which the retailer's relative bargaining power may change when she adopts different pricing strategies. This is significantly different from extant publications in which the relative bargaining power was assumed to be fixed and be independent of the retailer's pricing strategies.

We then review the publications in stream (ii). Dukes et al. (2006) studied a two-echelon supply chain in which there are two competing suppliers as well as two competing retailers. Each supplier can serve both retailers. The authors found that the retailer with a smaller retailing cost can obtain a smaller wholesale price. Different from this publication, in our competition model, we consider two competing supply chains rather than a single supply chain. The duopoly supply chains commonly exist in practice. For example, KFC only sells Pepsi-Cola in its supply chain, whereas McDonald's only sells Coca-Cola in its supply chain. Olczak (2011) compared the RPM and RRP strategies in a supply chain with one supplier and two competing retailers, which is different from our paper. Feng and Lu (2012) considered two competing supply chains, which is somewhat similar to the supply chain structure in our analysis of two competing supply chains but still differs from ours in decisions and research questions. Similar to Dukes et al. (2006), Feng and Lu (2013) showed that no sequential-move game can result from the negotiation in the supply chain. In a two-echelon supply chain in which two competing retailers buy from a common supplier, Guo and Iyer (2013) exposed that the supplier prefers to negotiate the wholesale prices with the two retailers simultaneously, if the retailers determine similar retail prices. Aydin and Heese (2014) studied a two-echelon supply chain with multiple competing suppliers and a single retailer. In addition, Baron, Berman, and Wu (2016) also considered two competing two-echelon supply chains with zero disagreement payoffs in negotiations. They showed that the manufacturer Stackelberg (MS) and vertical integration (VI) strategies are special cases of the wholesale price negotiation. Different from Baron, Berman, and Wu's paper (2016), we use general disagreement payoffs and find the retailers' and suppliers' pricing strategies.

2 Game-Theoretic Analyses of a Two-Echelon Supply Chain

We investigate strategic pricing decision problems in a two-echelon supply chain consisting of a supplier and a retailer. The supplier makes a product at unit acquisition cost c and sells it to the retailer at wholesale price w, and the retailer then serves customers at retail price p. For our analysis, we use a p-dependent linear demand function $D(p) = A - bp$, in which $A > bc$ and c denotes the supplier's unit acquiring cost. In the supply chain, the supplier's and the retailer's profits are computed as $\pi_s(w) = (w-c)D(p)$ and $\pi_r(w,p) = (p-w)D(p)$, respectively. The system-wide profit is $\Pi(p) = (p-c)D(p)$, which can be maximized to find the globally-optimal retail price as $p^*(c) = (A/b + c)/2$. We can compute the maximum system-wide profit as $\Pi^*(c) = b(A/b - c)^2/4$. These results will be used for discussions in our subsequent analyses.

In the supply chain, as the "leader," the supplier first makes his wholesale pricing strategy on whether to negotiate the wholesale price or determine it individually. Then, the retailer acts as the "follower" for her retail pricing strategy. In this leader-follower game, we first find the retailer's optimal retail pricing strategy (i.e., an optimal choice between the individually-optimal retail pricing decision and the retail price negotiation) for each of the following two cases: (i) the supplier makes his wholesale price individually, and (ii) the supplier negotiates the wholesale price with the retailer. Then, we compare the supplier's profits for his two choices (i.e., the individual wholesale pricing decision and the wholesale price negotiation) to find his optimal wholesale pricing strategy. Next, we begin by obtaining the retailer's optimal retail pricing strategy when the supplier chooses to negotiate the wholesale price.

Prior to analyzing our game, we provide a summary of all the major notations in Table 1.

Notation	**Definition**
NC_S (NC_R) and C_S (C_R)	the supplier's (retailer's) non-cooperative and cooperative pricing strategies, respectively.
superscripts nc_S, c_R, and nc_R	the cases in which the supplier and the retailer choose strategies (NC_S, NC_R), (C_S, C_R), and (C_S, NC_R), respectively.
$\lambda, \bar{\lambda}, \hat{\lambda} \in [0, 1]$	the supplier's relative bargaining powers when the supplier's and retailer's strategies are (C_S, NC_R), (C_S, C_R), and (NC_S, C_R), respectively.
d_s and d_r	the supplier's and retailer's disagreement points, respectively; i.e., their payoffs when they cannot reach an agreement.
w^x and p^x, $x \in \{nc_S, c_R, nc_R\}$	the wholesale and retail prices, respectively, for case x in the monopoly setting.
π_s^x and π_r^x, $x \in \{nc_S, c_R, nc_R\}$	the supplier's and retailer's profits, respectively, for case x in the monopoly setting.
$w_i^{x,y}$ and $p_i^{x,y}$, $i = 1, 2$ and $x, y \in \{nc_S, c_R, nc_R\}$	the wholesale and retail prices in the ith supply chain, respectively, when cases x and y occur in the first and second supply chains, respectively, in the duopoly setting.
$\pi_{s_i}^{x,y}$ and $\pi_{r_i}^{x,y}$, $i = 1, 2$ $x, y \in \{nc_S, c_R, nc_R\}$	the supplier's and retailer's profits in the ith supply chain, respectively, when cases x and y occur in the first and second supply chains, respectively, in the duopoly setting.

Table 1: List of major notations.

One may note that in Table 1, we do not consider the supplier's and retailer's strategies (NC_S, C_R) in both the monopoly and duopoly settings. The main reason is that, according to our subsequent game analyses, if the supplier uses his non-cooperative strategy NC_S, then the retailer's best response is always her non-cooperative strategy NC_R. This finding indicates that (NC_S, C_R) cannot be an equilibrium result.

2.1 The Individually-Optimal Retail Pricing Decision

The supplier and the retailer first negotiate the wholesale price, and the retailer then determines the optimal retail price to maximize her individual profit. For the negotiated wholesale price, we compute the generalized Nash bargaining (GNB) solution (Nash 1953, Roth 1979), which is a common solution concept characterizing the negotiation result for two players who may have different bargaining powers. For the applications of the GNB solution in supply chain management, see the publications by, e.g., Nagarajan and Bassok (2008), Huang et al. (2013), Chan, Leng, and Liang (2014), and Huang et al. (2014).

To find the GNB solution, we need to solve the following maximization problem: $\max_w (\pi_s - d_s)^\lambda (\pi_r - d_r)^{1-\lambda}$, where $\lambda \in [0, 1]$ and $1 - \lambda$ denote the supplier's and the retailer's relative bargaining powers, respectively. For interpretations of the bargaining powers in the GNB solution, see, for example, Binmore, Osborne, and Rubinstein's discussion (1992). In the GNB model, (d_s, d_r) is the disagreement point; that is, $d_s \geq 0$ and $d_r \geq 0$ denote the supplier's and the retailer's payoffs, respectively, when they cannot reach any agreement and thus make no transaction. We first maximize the retailer's profit π_r for any given wholesale price w to find the retailer's optimal retail price p^{nc_R} (which is dependent on w). Then, we substitute p^{nc_R} into π_r and π_s to find the GNB solution w^{nc_R}. Accordingly, for all solutions and resulting profits, we add the superscripts "c_R" and "nc_R."

To visually show the results when the supplier uses strategy C_S and the retailer uses strategy NC_R, we plot Figure 1 in which the bargaining set of the supplier and the retailer is area (1). For details regarding the result, see Theorem 7 in online Appendix B.2, which indicates that, under strategy NC_R, if the supplier and the retailer can reach an agreement, then a firm's stronger relative bargaining power or larger disagreement point can lead the firm to achieve a higher profit. In addition, the supply chain-wide profit is decreasing in λ, which means that a sufficiently powerful retailer can help realize supply chain coordination whereas a sufficiently powerful supplier may increase the effect of double marginalization. We also find that the wholesale-pricing agreement can be achieved if and only if (d_r, d_s) is in areas (1) and (2) of Figure 1.

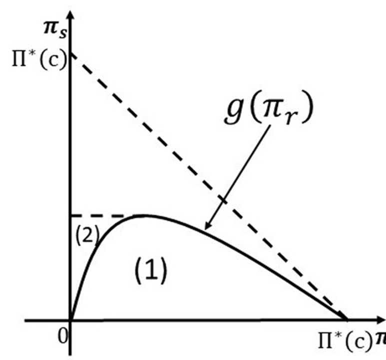

Figure 1: The bargaining set of (π_r, π_s).

2.2 The Negotiated Retail Pricing Decision

The supplier and the retailer bargain over both the wholesale price and retail price. The negotiated prices can be obtained by solving $\max_{w,p}(\pi_s - d_s)^{\bar{\lambda}}(\pi_r - d_r)^{1-\bar{\lambda}}$, where $\bar{\lambda} \in [0, 1]$ and $1 - \bar{\lambda}$ represent the supplier's and the retailer's relative bargaining powers when the retailer adopts strategy C_R, respectively. One may note that, for the retailer's C_R and NC_R strategies, we use different notations for the two firms' relative bargaining powers, which reflects the fact that the firms may be in different positions for these two cases. For generality, we do not assume that λ is larger or smaller than $\bar{\lambda}$ but allow any possible relation between λ and $\bar{\lambda}$.

Different from the retailer's strategy NC_R, the supply chain with the retailer's strategy C_R can be coordinated such that the negotiated solutions are identical to the globally-optimal solutions maximizing the system-wide profit; see Theorem 6 in online Appendix B.1. This occurs mainly because, when the retailer and the supplier bargain over both the wholesale and retail pricing decisions, they can jointly determine a globally optimal retail price to maximize the system-wide profit, and choose a wholesale price to split the total profit between the two firms. Therefore, the retail price is independent of the relative bargaining powers. Recall from Theorem 7 (in online Appendix B.2) that areas (1) and (2) in Figure 1 belong to the triangle $\{(\pi_r, \pi_s)|\pi_r \geq 0, \pi_s \geq 0, \pi_r + \pi_s \leq \Pi^*(c)\}$. Thus, the supply chain is more likely to reach an agreement under the retailer's strategy C_R than that under the retailer's strategy NC_R.

2.3 The Retailer's Optimal Retail Pricing Strategy

Using Theorems 6 and 7 (which are in online Appendices B.1 and B.2, respectively), we find that, when (d_r, d_s) is outside areas (1) and (2) in Figure 1, the supplier and the retailer cannot reach any agreement under the retailer's strategy NC_R but may willing to finish their transaction under the retailer's strategy C_R. Thus, the retailer should adopt strategy C_R and the supplier also benefits from the retailer's strategy C_R. When (d_r, d_s) stays inside areas (1) and (2) so that the two firms can reach an agreement under strategies C_R and NC_R, the retailer prefers strategy C_R to strategy NC_R, if $\pi_r^{c_R} \geq \pi_r^{nc_R}$, i.e.,

$$1 - \bar{\lambda} \geq \frac{\pi_r^{nc_R} - d_r}{\Pi^*(c) - d_r - d_s}, \text{ or, } \bar{\lambda} \leq \lambda_u \equiv 1 - \frac{\pi_r^{nc_R} - d_r}{\Pi^*(c) - d_r - d_s}. \tag{1}$$

That is, the retailer prefers strategy C_R, if her relative bargaining power $1 - \bar{\lambda}$ exceeds a threshold. The supplier benefits from the retailer's strategy C_R, if $\pi_s^{c_R} \geq \pi_s^{nc_R} = g(\pi_r^{nc_R})$, i.e.,

$$\bar{\lambda} \geq \lambda_l \equiv (g(\pi_r^{nc_R}) - d_s)/(\Pi^*(c) - d_r - d_s). \tag{2}$$

Since $\lambda_u - \lambda_l = 1 - (\pi_r^{nc_R} + g(\pi_r^{nc_R}) - d_s - d_r)/(\Pi^*(c) - d_r - d_s) \geq 0$, the retailer's strategy C_R can result in the Pareto improvement, if $\bar{\lambda} \in [\lambda_l, \lambda_u]$. This means that the retailer's strategy C_R can benefit both firms, if the supplier's relative bargaining power under the retailer's strategy C_R takes a proper value in $[\lambda_l, \lambda_u]$. The supplier's power outside the range cannot induce both firms to prefer the retailer's strategy C_R.

Theorem 1 *We find that* $\lambda_l \leq \lambda$, $\partial\lambda_u/\partial\lambda \geq 0$, $\partial\lambda_l/\partial\lambda \geq 0$, $\partial(\lambda_u - \lambda_l)/\partial\lambda \geq 0$, *and* $\partial\lambda_l/\partial d_s \leq 0$. *The range of interval* $[\lambda_l, \lambda_u]$ *is increasing in* λ, *which means that the retailer's cooperative strategy is more likely to result in the Pareto improvement when the supplier has a larger relative bargaining power under the retailer's non-cooperative strategy.*

The supplier prefers the retailer's strategy C_R, if $\bar{\lambda} \geq \lambda_l$. Hence, the supplier's larger disagreement point (i.e., a larger value of d_s) and/or the supplier's lower relative bargaining power under the retailer's strategy NC_R (i.e., a smaller value of λ) makes the supplier more likely to benefit from the retailer's strategy C_R.

Since the supplier benefits from the retailer's strategy C_R if $\lambda_l \leq \bar{\lambda}$, we find that the supplier always prefers the retailer's strategy C_R to her strategy NC_R, if the retailer's pricing strategy has a sufficiently small impact on the two firms' relative bargaining powers (i.e., the difference between λ and $\bar{\lambda}$ is sufficiently small). However, the condition $\lambda \leq \lambda_u$ does not always hold; thus, the retailer may not always prefer to adopt strategy C_R under the condition.

Theorem 2 *When* (d_r, d_s) *belongs to areas (1) and (2) in Figure 1, there exists a* $d_t \geq 0$ *(which is dependent on* λ *and* d_s*) such that* $\lambda \leq \lambda_u$ *if* $d_r \geq d_t$.

When $\lambda \in [\lambda_l, \lambda_u]$, we learn that if the value of $\bar{\lambda}$ does not significantly differ from the value of λ, then the retailer prefers to use strategy C_R and the supplier also benefits from such strategy. Thus, as Theorem 2 shows, when the retailer's pricing strategy has a sufficiently small impact on the two firms' relative bargaining powers, both the retailer and the supplier are more likely to negotiate the retail price if the retailer has a larger disagreement point in the negotiation.

Remark 1 According to our discussions, we draw the following conclusions about the supplier's and the retailer's incentives to negotiate the retail price. First, the two firms prefer the retailer's strategy C_R when the supplier's relative bargaining power under strategy C_R takes a proper value in a certain range. Secondly, the two firms are more likely to negotiate the retail price when the supplier's relative bargaining power under strategy NC_R is larger. Third, the two firms' willingness to negotiate the retail price increases with the retailer's disagreement point, if their relative bargaining powers are independent of the retailer's pricing strategy.

The results above indicate that the supplier and the retailer do not expect the retailer to stick to either strategy C_R or strategy NC_R. This may help explain why, in practice, commercial negotiations between suppliers and retailers necessarily include a discussion of wholesale prices but may or may not involve a discussion of retail prices (Ertel 1999 and Bennett 2014). $\triangleleft$

2.4 The Supplier's Optimal Wholesale Pricing Strategy

In reality, the wholesale price negotiation is common. Nonetheless, we cannot conclude that the supplier always prefers to adopt the negotiation process for his wholesale price. Accordingly, in this section, we allows the supplier to choose between his non-cooperative and cooperative strategies.

When the supplier chooses to negotiate the wholesale price, the retailer needs to make her retail pricing strategy (i.e., C_R vs. NC_R). In Section 2.3, we have investigated the retailer's optimal retail pricing strategy when the wholesale price is negotiated. Next, we analyze another case in which the supplier determines his wholesale price individually. For this case, after observing the optimal wholesale price set by the supplier, the retailer also needs to make her retail pricing strategy (i.e., the choice between strategies C_R and NC_R). Similar to our previous discussion, if the retailer uses strategy C_R, then the relative bargaining powers of the supplier and the retailer are $\hat{\lambda}$ and $1-\hat{\lambda}$, respectively, where $\hat{\lambda}$ may differ from λ and $\bar{\lambda}$.

Theorem 3 *In Stackelberg equilibrium, the supplier's and the retailer's pricing strategies are obtained below.*

(i) If

$$\Pi^*(c)/2 > d_s \text{ and } \Pi^*(c)/4 > d_r, \tag{3}$$

and

$$\bar{\lambda} \in \left[0, \frac{\Pi^*(c)/2 - d_s}{\Pi^*(c) - (d_r + d_s)}\right] \cup [\lambda_u, 1], \tag{4}$$

then the supplier's optimal choice is NC_S, to which the retailer's best response is NC_R.

(ii) If the condition in (3) does not hold and $\bar{\lambda} \leq \lambda_u$, then the supplier's optimal choice is C_S, to which the retailer's best response is C_R.

(iii) If the condition in (3) does not hold and $\bar{\lambda} \geq \lambda_u$, then the supplier's optimal choice is C_S, to which the retailer's best response is NC_R.

(iv) If the condition in (3) holds but the condition in (4) does not hold, then the supplier's optimal choice is C_S, to which the retailer's best response is C_R.

Theorem 3 exposes that there are only three possibilities. The first possibility is that the supplier determines his wholesale price individually and the retailer responds by finding her retail price individually. In the second possibility, the supplier negotiates his wholesale price, and the retailer determines her retail price individually. In the third possibility, the supplier negotiates his wholesale price, and the retailer negotiates her retail price. We find that strategy (NC_S, C_R) cannot be a Stackelberg equilibrium. That is, if the supplier and the retailer decide to negotiate, then they always bargain over the wholesale price. This explains why the wholesale price negotiation is strictly necessary to the communications between supply chain members in practice (see, e.g., Bennett 2014). Using Theorem 3 we can also find the conditions under which, in Stackelberg equilibrium, the supplier prefers to adopt strategy C_S or NC_S.

Proposition 1 *When the condition in (3) holds, we find that if*

$$\bar{\lambda} \in \left[\frac{\Pi^*(c)/2 - d_s}{\Pi^*(c) - (d_r + d_s)}, \min\left\{\lambda_u, \frac{3\Pi^*(c)/4 - d_s}{\Pi^*(c) - (d_r + d_s)}\right\}\right], \tag{5}$$

then both the supplier and the retailer prefer strategy C_S to NC_S; otherwise, if $\bar{\lambda} \geq \lambda_u$ and λ is sufficiently large, then both firms prefer NC_S to C_S.

If the right-hand side of the condition in (4) is the whole interval $[0, 1]$ (i.e., $\lambda_u \leq (\Pi^*(c)/2 - d_s)/[\Pi^*(c) - (d_r + d_s)]$), then the supplier should always choose his non-cooperative strategy when he and the retailer are willing to complete their transaction in the supply chain.

Proposition 2 *If $d_s \leq g(\Pi^*(c)/2)$ and λ is sufficiently small, then $\lambda_u \leq (\Pi^*(c)/2 - d_s)/[\Pi^*(c) - (d_r + d_s)]$. That is, if the supplier's disagreement point and bargaining power under the retailer's non-cooperative strategy NC_R are both sufficiently small, then the supplier should always choose his non-cooperative strategy NC_S.*

This result occurs mainly because the supplier's small bargaining power and small disagreement point in his negotiation usually result in a small allocation of the system-wide profit to the supplier, who thus prefers to choose strategy NC_S. In the end, we analyze the supply chain with a demand function in general form to examine whether our major results obtained using the linear demand function are robust or not.

Proposition 3 *If we use the general demand function $D(p)$ that is a non-negative and decreasing function (i.e., $D(p) \geq 0$ and $D'(p) < 0$) with $D(T) = 0$ where T is the maximum retail price, then the* *results in Theorems 1, 2, 3, 6 (in online Appendix B.1), and 7 (in online Appendix B.2) hold. That is, our results could be robust, as they are not sensitive to the linear demand assumption.*

3 The Strategic Pricing Decisions in Two Supply Chains

We analyze two competing supply chains each consisting of a supplier and a retailer, with an aim to explore the wholesale and retail pricing strategies in the presence of the competition between supply chains. The suppliers in the two supply chains, denoted by S_i $(i = 1, 2)$, make substitutable products at unit acquisition cost c, and sell their products to the retailers, denoted by R_i, at wholesale prices w_i. The retailers then compete for customers in a market at retail prices p_i. For ease of exposition, we consider the linear demand functions for retailers R_1 and R_2 as $D_1(p_1; p_2) = A - bp_1 + \alpha\,(p_2 - p_1)$ and $D_2(p_2; p_1) = A - bp_2 + \alpha\,(p_1 - p_2)$, respectively, where $\alpha \geq 0$ characterizes the substitutability of the two products (measuring the competition between the two retailers). Note that, if firms S_{3-i} $(i = 1, 2)$ and R_{3-i} cannot reach an agreement, the demand faced by R_i is $D_i(p_i; 0) = A - bp_i + \alpha\,(0 - p_i) = A - (b + \alpha)\,p_i$. To ensure $D_i(p_i; 0) > 0$, we reasonably assume that $A > c(b + \alpha)$. We start by computing the maximum total profit of the two supply chains, which serves as a benchmark for our subsequent analyses. The total profit is $\bar{\Pi}(c) \equiv (p_1 - c)(A - bp_1) + (p_2 - c)(A - bp_2) - \alpha(p_1 - p_2)^2$, which is maximized when $p_1 = p_2 = (A/b + c)/2$. We can then find the maximum total profit as $\bar{\Pi}^*(c) = (A - cb)^2/(2b) = 2\Pi^*(c)$.

In each supply chain, we first consider the case that wholesale price w_i $(i = 1, 2)$ results from the negotiation between the supplier and the retailer. It should be noted that, if $A = b + \alpha = 1$ and $c = 0$, then our demand functions are exactly the same as in Baron, Berman, and Wu (2016) as well as McGuire and Staelin (1983). Then, we investigate the supplier's optimal wholesale pricing strategy, similar to Section 2.4.

3.1 The Retailers' Optimal Retail Pricing Strategies in the Duopoly Setting

Similar to our analysis for a single supply chain in Section 2, we consider both strategy NC_R and strategy C_R for each retailer. In strategy NC_R, supplier S_i and retailer R_i have their relative bargaining powers λ $(0 \leq \lambda \leq 1)$ and $1 - \lambda$, respectively; and, in strategy C_R, the two firms hold relative bargaining powers $\bar{\lambda}$ $(0 \leq \bar{\lambda} \leq 1)$ and $1 - \bar{\lambda}$. For both cases, the disagreement payoffs of the retailers and the suppliers in both supply chains are d_r and d_s, respectively.

Next, we begin by investigating the following scenario: the two suppliers adopt strategy C_S, and the two retailers decide on their pricing strategies (i.e., the choice between C_R and NC_R) concurrently. Then, under the strategies, the retailers and the suppliers make their wholesale and retail pricing decisions. We specify the timing of pricing decisions for the following three cases, which correspond to three possible combinations of the two retailers' pricing strategies.

1. If each retailer chooses strategy C_R, then the firms in the two supply chains conduct the wholesale- and retail-price negotiations "simultaneously" (with no communication). For this case, we solve the GNB model for each supply chain to find its negotiated wholesale and retail prices as a response to any given wholesale and retail prices in the other supply chain. Then, using the best-response GNB results, we compute the negotiated pricing decisions for each supply chain.

2. If retailer R_i $(i = 1, 2)$ chooses strategy C_R whereas retailer R_j $(j = 3 - i)$ chooses strategy NC_R, then the two supply chains make their pricing decisions in the following sequence: First, firms S_j and R_j negotiate wholesale price w_j. Secondly, observing negotiated wholesale price w_j, retailer R_j determines an optimal retail price p_j to maximize her individual profit. In supply chain i, supplier S_i and retailer R_i negotiate their wholesale and retail prices (w_i, p_i). As there is no communication between the two supply chains, retailer R_j's individual pricing decision and the pricing negotiations in supply chain i take place "simultaneously."

   We follow the backward induction process to find the pricing decisions. First, for a given value of wholesale price w_j, we obtain retail price p_j and the negotiated results (w_i, p_i) in Nash equilibrium, by, for a prior observed w_j, computing firm R_j's optimal retail price in response to (w_i, p_i) as well as the best negotiation results (w_i, p_i) in response to p_j. As a consequence, p_j and (w_i, p_i) are functions of w_j. Then, substituting the w_j-dependent results of p_j and (w_i, p_i) into the GNB model for S_j and R_j, we find the negotiated result of wholesale price w_j.

3. If each retailer chooses strategy NC_R, then the firms in the two supply chains negotiate their wholesale prices "simultaneously;" and, after observing the negotiated wholesale prices, the two retailers determine their individually optimal retail prices "simultaneously."

When both retailers adopt strategy C_R, we present the resulting prices in Theorem 8 in online Appendix B.3. We compute all firms' profits as $\pi_{r_1}^{c_R,c_R} = \pi_{r_2}^{c_R,c_R} = d_r + (1-\bar{\lambda})\{(b+\alpha)[(A-bc)/(2b+\alpha)]^2 - d_r - d_s\}$ and $\pi_{s_1}^{c_R,c_R} = \pi_{s_2}^{c_R,c_R} = d_s + \bar{\lambda}\{(b+\alpha)[(A-bc)/(2b+\alpha)]^2 - d_r - d_s\}$. The total (system-wide) profit of the two supply chains is $\Pi_{c_R,c_R} \equiv 2(b+\alpha)[(A-bc)/(2b+\alpha)]^2$, which is smaller than the maximum total profit $\bar{\Pi}^*(c)$. That is, the prices in Theorem 8 cannot result in the maximum total profit of the supply chains. Moreover, the total profit Π_{c_R,c_R} is decreasing in α. That is, the competition between the two supply chains makes the total profit lower. We also find that the profit loss compared with the coordinated system is $1 - \Pi_{c_R,c_R}/\bar{\Pi}^*(c) = [\alpha/(\alpha+2b)]^2$, which implies that the two supply chains can be coordinated if there is no competition between them (i.e., $\alpha = 0$).

When both retailers adopt strategy NC_R, the supplier and the retailer in each supply chain can reach an agreement if (d_r, d_s) is in areas (1) and (2) of Figure 2. For the results, see Theorem 9 in online Appendix B.4. We compute the retailers' and the suppliers' profits in both supply chains as $\pi_{r_1}^{nc_R,nc_R} =$

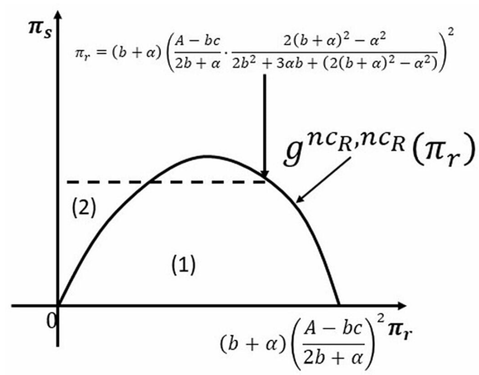

Figure 2: The bargaining set when both retailers choose the non-cooperative strategy.

$\pi_{r_2}^{nc_R,nc_R} = (b+\alpha)[(A-b\hat{w})/(2b+\alpha)]^2$ and $\pi_{s_1}^{nc_R,nc_R} = \pi_{s_2}^{nc_R,nc_R} = (\hat{w}-c)(b+\alpha)[(A-b\hat{w})/(2b+\alpha)]$. The total profit of the two supply chains is

$$\begin{aligned} 2(b+\alpha)\left(\frac{A-b\hat{w}}{2b+\alpha}\right)\left(\frac{A+(b+\alpha)\hat{w}}{2b+\alpha}-c\right) &\leq \max_{w\geq c} 2(b+\alpha)\left(\frac{A-bw}{2b+\alpha}\right)\left(\frac{A+(b+\alpha)w}{2b+\alpha}-c\right) \\ &= \frac{(A-bc)^2}{2b} = 2\Pi^*(c). \end{aligned}$$

Hence, the two supply chains can be coordinated when both retailers use strategy NC_R and the negotiated wholesale prices in the two chains are $\hat{w} = [\alpha A + (2b+\alpha)bc]/[2b(b+\alpha)]$. Moreover, when the two supply chains can be coordinated, the retailers' and suppliers' profits are $\pi_{r_1}^{nc_R,nc_R} = \pi_{r_2}^{nc_R,nc_R} = (A-bc)^2/[4(b+\alpha)]$ and $\pi_{s_1}^{nc_R,nc_R} = \pi_{s_2}^{nc_R,nc_R} = \alpha(A-bc)^2/[4b(b+\alpha)]$, respectively. Thereby, if $d_r < (A-bc)^2/[4(b+\alpha)]$, $d_s < \alpha(A-bc)^2/[4b(b+\alpha)]$, and λ is properly chosen, then the two supply chains can be coordinated when both retailers adopt strategy NC_R. This result differs from that in the monopoly setting in which only the retailer's strategy C_R can coordinate the supply chain. The main reason is that, in the duopoly setting, coordinating a single chain under the retailer's strategy C_R may intensify the competition between two supply chains and thus reduce the total profit of each supply chain. One can note that there are two different coordination scenarios. One is the coordination of a single supply chain, in which the two firms make integrated pricing decisions (corresponding to the retailer's strategy C_R). The other is the coordination of the two supply chains, whose total profit reaches its maximum.

When one retailer adopts strategy C_R whereas the other one chooses strategy NC_R, both supply chains can reach agreements if (d_r, d_s) is in areas (1) and (2) of Figure 3. If (d_r, d_s) is in area (3) of Figure 3, only the supply chain with its retailer's strategy C_R can reach an agreement. Our results are presented in Theorem 10 (which is given in online Appendix B.6). We can observe that the supply chain with its retailer's strategy C_R is more likely to reach an agreement than the supply chain with its retailer's strategy NC_R. This happens mainly because strategy C_R allows the supplier and the retailer to choose a retail price but results in a higher competition between the two supply chains. For this case, the wholesale price plays a role in allocating the supply chain profit between the supplier and the retailer, who may thus possibly choose an appropriate wholesale price such that both of them can gain more than their disagreement points.

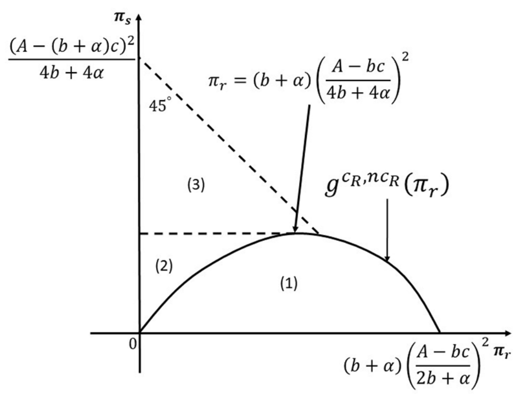

Figure 3: One retailer uses the cooperative strategy whereas the other uses the non-cooperative strategy.

Prior to further discussions, we summarize our results in Theorems 8 (in online Appendix B.3), 9 (in online Appendix B.4), and 10 (in online Appendix B.6) in Table 2.

Proposition 4 *When the value of α is sufficiently large and it increases, the upper boundaries of areas (1), (2), and (3) in Figure 3 move downward and the union of areas (1) and (2) in Figure 2 shrinks. That is, the supply chains competition can significantly reduce the likelihood for the firms in both supply chains to reach agreements for any given pricing strategies of the retailers.*

Next, we consider the Nash equilibrium for the retailers when they choose their pricing strategies simultaneously. Since the 2×2 Nash game of the two retailers is symmetric, the pure Nash equilibrium always exists for any given parameters. When (d_r, d_s) is in area (3) of Figure 4, (C_R, C_R) is the unique

Scenarios	$(C_R,\, C_R)$	$(NC_R,\, NC_R)$	$(C_R,\, NC_R)$	$(C, -)$
(w_1, w_2)	$w_i^{c_R,c_R} = c + \frac{\bar{\lambda}(A-bc)}{2b+\alpha} + (2b+\alpha) \times \frac{\left[(1-\bar{\lambda})\,d_s - \bar{\lambda} d_r\right]}{(b+\alpha)(A-bc)}$, for $i=1,2$.	$w_i^{nc_R,nc_R}$ $(i=1,2)$ is obtained by solving (13).	$\begin{cases} w_1^{c_R,nc_R} = c + \frac{\bar{\lambda}}{2}\left(\frac{A+\alpha p_2^{c_R,nc_R}}{b+\alpha} - c\right) \\ \quad + \frac{2\left[(1-\bar{\lambda})\,d_s - \bar{\lambda} d_r\right]}{A+\alpha p_2^{c_R,nc_R} - (b+\alpha)\,c}; \\ w_2^{c_R,nc_R} = \frac{(2b+3\alpha)\,A + (b+\alpha)\,\alpha c}{2b^2+4\alpha b+\alpha^2} \\ \quad - \frac{(2b+\alpha)(2b+3\alpha)\sqrt{\pi_{r_2}^{c_R,nc_R}/(b+\alpha)}}{2b^2+4\alpha b+\alpha^2}. \end{cases}$	$\bar{w}_1^{c,nc} = c + \frac{\bar{\lambda}(b+\alpha)\left[A-(b+\alpha)\,c\right]}{4(b+\alpha)^2} + \frac{(1-\bar{\lambda})\,d_s - \bar{\lambda} d_r}{A-(b+\alpha)\,c}.$
(p_1, p_2)	$p_i^{c_R,c_R} = \frac{A+(b+\alpha)\,c}{2b+\alpha}$, for $i=1,2$.	$p_i^{nc_R,nc_R} = \frac{A+(b+\alpha)\,w_i^{nc_R,nc_R}}{2b+\alpha}$, for $i=1,2$.	$\begin{cases} p_1^{c_R,nc_R} = \frac{A}{2b+\alpha} + \frac{2(b+\alpha)^2 c}{(2b+\alpha)(2b+3\alpha)} \\ \quad + \frac{(b+\alpha)\,\alpha w_2^{c_R,nc_R}}{(2b+\alpha)(2b+3\alpha)}, \\ p_2^{c_R,nc_R} = \frac{A}{2b+\alpha} + \frac{2(b+\alpha)^2\, w_2^{c_R,nc_R}}{(2b+\alpha)(2b+3\alpha)} \\ \quad + \frac{(b+\alpha)\,\alpha c}{(2b+\alpha)(2b+3\alpha)}. \end{cases}$	$\bar{p}_1^{c,nc} = \frac{1}{2}\left(\frac{A}{b+\alpha} + c\right).$
(π_{s_1}, π_{s_2})	$\pi_{s_i}^{c_R,c_R} = d_s + \bar{\lambda}(b+\alpha)\left(\frac{A-bc}{2b+\alpha}\right)^2 - \bar{\lambda}(d_r+d_s)$, for $i=1,2$.	$\pi_{s_i}^{nc_R,nc_R} = (b+\alpha)\left(\frac{A-bw_i^{nc_R,nc_R}}{2b+\alpha}\right) \times \left(w_i^{nc_R,nc_R} - c\right)$, for $i=1,2$.	$\begin{cases} \pi_{s_1}^{c_R,nc_R} = d_s - \bar{\lambda}(d_r+d_s) \\ \quad + \bar{\lambda}\left[\frac{b+\alpha}{4}\left(\frac{A+\alpha p_2^{c_R,nc_R}}{b+\alpha} - c\right)^2\right], \\ \pi_{s_2}^{c_R,nc_R} = g^{c_R,nc_R}(\pi_{r_2}^{c_R,nc_R}). \end{cases}$	$\pi_{s_1}^{c,nc} = d_s - \bar{\lambda}(d_r+d_s) + \bar{\lambda}(b+\alpha)\left[\frac{A-(b+\alpha)\,c}{2(b+\alpha)}\right]^2.$
(π_{r_1}, π_{r_2})	$\pi_{r_i}^{c} = d_r + (1-\bar{\lambda})(b+\alpha)\left(\frac{A-bc}{2b+\alpha}\right)^2 - (1-\bar{\lambda})(d_r+d_s)$, for $i=1,2$.	$\pi_{r_i}^{nc} = (b+\alpha)\left(\frac{A-bw^{nc,nc}}{2b+\alpha}\right)^2$, for $i=1,2$.	$\begin{cases} \pi_{r_1}^{c,nc} = d_r - (1-\bar{\lambda})(d_r+d_s) \\ \quad + \frac{(1-\bar{\lambda})(b+\alpha)}{4}\left(\frac{A+\alpha p_2^{c,nc}}{b+\alpha} - c\right)^2, \\ \pi_{r_2}^{c,nc} \text{ is obtained by solving (17).} \end{cases}$	$\pi_{r_1}^{c,nc} = \bar{\lambda} d_r - (1-\bar{\lambda})\,d_s + (1-\bar{\lambda})(b+\alpha)\left[\frac{A-(b+\alpha)\,c}{2(b+\alpha)}\right]^2.$
Profit Loss	$\left[\frac{\alpha}{\alpha+2b}\right]^2$	$1 - 2(b+\alpha)\left(\frac{A-bw^{nc,nc}}{2b+\alpha}\right) \times \frac{\left(\frac{A+(b+\alpha)\,w^{nc,nc}}{2b+\alpha} - c\right)}{\bar{\Pi}^*(c)}.$	$\frac{\pi_{r_2}^{c,nc} + g^{c,nc}(\pi_{r_2}^{c,nc})}{\bar{\Pi}^*(c)} + \frac{b+\alpha}{4\bar{\Pi}^*(c)}\left(\frac{A+\alpha p_2^{c,nc}}{b+\alpha} - c\right)^2.$	$\frac{d_r+d_s}{\bar{\Pi}^*(c)} + \frac{b+\alpha}{\bar{\Pi}^*(c)}\left(\frac{A-(b+\alpha)\,c}{2b+2\alpha}\right)^2.$

Table 2: A summary of major analytic results that are obtained when at least one supply chain does not break up in the duopoly setting. In scenario (C, C), both retailers choose the cooperative strategy to negotiate their retail prices with the suppliers. In scenario (NC, NC), both retailers choose the non-cooperative strategy to determine their retail prices individually. In scenario (C, NC), the retailer in a supply chain chooses the cooperative strategy whereas the retailer in the other supply chain chooses the non-cooperative strategy. In scenario (C, –), the retailer in a supply chain chooses the cooperative strategy but the other supply chain breaks up. In addition, "Profit Loss" means the percentage of the system-wide profit loss (i.e., the difference between the maximum total profit of the two supply chain and the total profit in terms of equilibrium solutions) in the maximum total profit of the two supply chain.

pure Nash equilibrium of the retailers. When (d_r, d_s) is in area (2) of Figure 4, there are two possible pure Nash equilibria of the retailers: (C_R, C_R) and (NC_R, NC_R). If (d_r, d_s) falls in area (1) of Figure 4, then all of four pure strategy combinations (i.e., (C_R, C_R), (C_R, NC_R), (NC_R, C_R), and (NC_R, NC_R)) are possible pure Nash equilibria. If (d_r, d_s) belongs to area (5) of Figure 4, then there are three possible pure Nash equilibria of the retailers, which are (NC_R, C_R), (C_R, NC_R), and (NC_R, NC_R). When (d_r, d_s) is in area (4) of Figure 4, (NC_R, NC_R) is the unique pure Nash equilibrium.

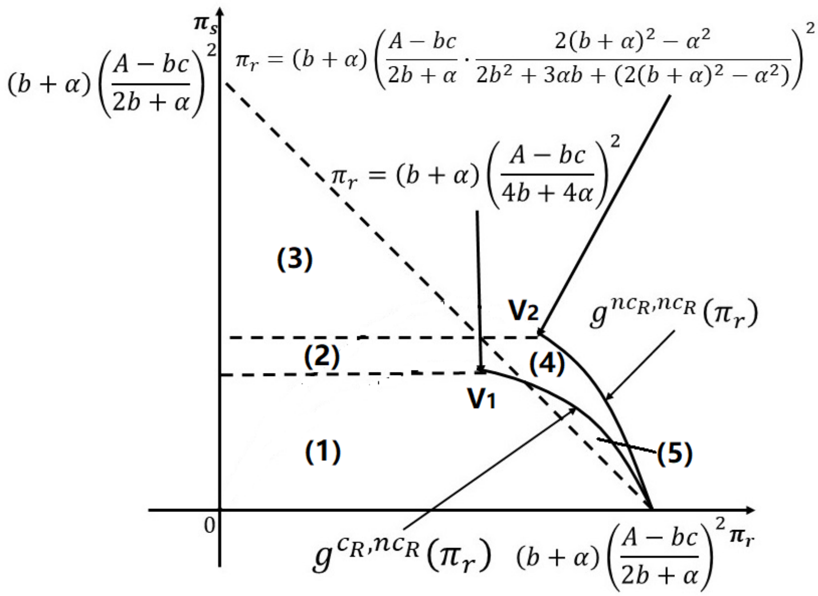

Figure 4: The pure Nash equilibria for the two retailers.

In Figure 4, there are two important points V_1 and V_2. Since

$$(b+\alpha)\left(\frac{A-bc}{4b+4\alpha}\right)^2 + g^{c_R,nc_R}\left((b+\alpha)\left(\frac{A-bc}{4b+4\alpha}\right)^2\right) \leq (b+\alpha)\left(\frac{A-bc}{2b+\alpha}\right)^2,$$

point V_1 is always in the triangle formed by unifying areas (1), (2), and (3) in Figure 4; and point V_2 can stay either in or outside the triangle, which depends on the value of the parameters. In addition, the slopes of g^{c_R,nc_R} and g^{nc_R,nc_R} at $\pi_r = (b+\alpha)\left((A-bc)\,/\,(2b+\alpha)\right)^2$ is smaller than -1, and $g^{nc_R,nc_R}(\pi_r) \geq g^{c_R,nc_R}(\pi_r)$, which implies that areas (4) and (5) in Figure 4 are never empty. That is, the coordination of each supply chain can lead both supply chains to dissolve, whereas, under no coordination in any supply chain, the firms in each supply chain may reach an agreement, if the supplier's disagreement is sufficiently small and the retailer's disagreement is sufficiently high. Moreover, we find that, if the value of α is sufficiently high, then the supplier's profit at point V_2 is greater than $(b + \alpha)\left((A - bc) / (2b + \alpha)\right)^2$, i.e.,

$$g^{nc_R, nc_R}\left((b+\alpha)\left(\frac{A-bc}{4b+4\alpha} \times \frac{2b^2+4\alpha b+\alpha^2}{4b^2+7\alpha b+\alpha^2}\right)^2\right) \geq (b+\alpha)\left(\frac{A-bc}{2b+\alpha}\right)^2,$$

which exposes that area (3) in Figure 4 disappears. This result indicates that when the competition between two supply chains is higher, the retailers are more likely to choose strategy NC_R (which cannot result in the coordination of each supply chain) rather than strategy C_R (which can achieve the coordination of each supply chain). That is, a sufficiently high competition between supply chains is less likely to induce the coordination of each supply chain. Our result coincides with a result obtained by McGuire and Staelin (1983).

Our previous analyses show that the supply chain competition can lower the total profits in both supply chains, when the retailers do not change their pricing strategies. However, when the supply chain competition is higher, the upper boundaries of areas (1), (2), and (3) in Figure 4 move downward. Thus, if (d_r, d_s) is initially in area (1) and both retailers use strategy C_R, then, as the value of α increases, (d_r, d_s) may move into area (4) or (5). That is, the retailers may change their strategy C_R (which dissolves both supply chains) to strategy NC_R (which can enable transactions). Moreover, the profits in both supply chains may increase due to the change in the value of α. Therefore, the supply chain competition may not always hurt the supply chains, since a sufficiently high competition may force the retailers to use strategy NC_R (which does not intensify the competition) and may thus benefit both supply chains.

We can also note that pure Nash equilibrium (C_R, NC_R) exists, if all firms' outside opportunities have a sufficiently low value such that (d_r, d_s) is in area (1) of Figure 4, and the suppliers have moderate bargaining powers in their negotiations with the retailers such that $\lambda_u^{c_R, c_R} \leq \bar{\lambda} \leq \lambda_l^{nc_R, nc_R}$ (for the expression of $\lambda_u^{c_R, c_R}$ and $\lambda_l^{nc_R, nc_R}$, see Theorem 12). When a retailer uses strategy C_R, the corresponding supply chain decides a centralized retail price decision that maximizes the total chain profit, and uses the wholesale price to allocate the supply chain profit between the retailer and the supplier. When a retailer uses strategy NC_R, the corresponding supply chain makes a decentralized retail price decision. Hence, as our result shows, it is possible that one supply chain uses the coordinated retail pricing decision, whereas the other supply chain uses the decentralized retail pricing decision. This result differs from a result found by McGuire and Staelin (1983) that there is no Nash equilibrium when a supply chain uses the centralized strategy whereas the other uses the decentralized strategy. The main reason for the difference is that we allow wholesale price negotiations between the suppliers and the retailers, whereas McGuire and Staelin (1983) assumed that the wholesale price is only determined by the supplier when a decentralized retail pricing strategy is used.

Theorem 4 *If (d_r, d_s) is in area (1) of Figure 4 and both retailers use strategy C_R, then the suppliers can benefit from strategy C_R, if*

$$\bar{\lambda} \geq \lambda_l^{c_R,c_R} \equiv \frac{g^{c_R,nc_R}(\pi_{r_2}^{c_R,nc_R}) - d_s}{(b+\alpha)[(A-bc)/(2b+\alpha)]^2 - d_r - d_s}.$$

Moreover, we find that $\lambda_u^{c_R,c_R} \geq \lambda_l^{c_R,c_R}$, $\lambda \geq \lambda_l^{c_R,c_R}$, $\partial\lambda_u^{c_R,c_R}/\partial\lambda \geq 0$, $\partial\lambda_l^{c_R,c_R}/\partial\lambda \geq 0$, $\partial\lambda_l^{c_R,c_R}/\partial d_s \leq 0$, and $\partial\left(\lambda_u^{c_R,c_R} - \lambda_l^{c_R,c_R}\right)/\partial\lambda \geq 0$. The range of $[\lambda_l^{c_R,c_R}, \lambda_u^{c_R,c_R}]$ is increasing in λ. This means that the retailers' cooperative strategies are more likely to result in the Pareto improvement when the suppliers have larger relative bargaining powers under the retailers' non-cooperative strategy.

Theorem 4 indicates that there may be a Nash equilibrium when the coordinated retail pricing decisions are chosen in both supply chains. This differs from the result obtained by Baron, Berman, and Wu (2016) who found that no Nash equilibrium exists when both supply chains use the coordinated pricing strategies. The main reason is that we consider a binary decision space for each retailer (i.e., C_R or NC_R, given any relevant bargaining powers), whereas Baron, Berman, and Wu (2016) investigated the supply chains' decisions on the relative bargaining powers. Comparing it with Theorem 1, we reveal that the supply chain competition does not change the impacts of λ on the retailers' pricing strategies. That is, the two retailers' C_R strategies are more likely to result in the Pareto improvement, if the suppliers have larger relative bargaining powers under the retailers' strategy NC_R. We also expose that a supplier's larger disagreement point (i.e., a larger value of d_s) makes the supplier more likely to benefit from his retailer's strategy C_R. Whether λ is in the interval $[\lambda_l^{c_R,c_R}, \lambda_u^{c_R,c_R}]$ or not is dependent on the relation between λ and $\lambda_u^{c_R,c_R}$.

Theorem 5 *When (d_r, d_s) is in area (1) of Figure 4, there exists a $d_t^{c_R,c_R} \geq 0$ (which depends on λ and d_s) such that $\lambda \leq \lambda_u^{c_R,c_R}$, if $d_r \geq d_t^{c_R,c_R}$.*

The result in Theorem 5 is similar to that in Theorem 2. Moreover, using Theorems 4 and 5, we find that the results in Remark 1 hold when two supply chains compete.

3.2 The Suppliers' Optimal Wholesale Pricing Strategies in the Duopoly Setting

Similar to Section 3.1, we consider the following three different situations for the strategic pricing decisions. First, both suppliers choose strategy C_S (i.e., (C_S, C_S)), which has been discussed in Section 3.1. Secondly, both suppliers choose strategy NC_S (i.e., (NC_S, NC_S)). Thirdly, one supplier S_i ($i = 1, 2$) chooses strategy NC_S whereas the other supplier S_{3-i} chooses strategy C_S (i.e., (NC_S, C_S) or (C_S, NC_S)). Similar to our analysis for the monopoly setting, regardless of whether a supplier chooses strategy NC_S or C_S, the corresponding retailer should adopt her strategy C_R or NC_R. If the supplier chooses strategy C_S, then the corresponding retailer chooses strategy C_R or NC_R before the wholesale price is negotiated. Otherwise, if the supplier chooses strategy NC_S, then the corresponding retailer adopts strategy C_R or NC_R after observing the supplier's wholesale price. The supplier's and the retailer's relative bargaining powers are $\hat{\lambda}$ and $1-\hat{\lambda}$, if the supplier and the retailer adopt strategies NC_S and C_R, respectively. Below is a description of the decision sequence.

If strategy (NC_S, NC_S) is adopted, then in the first stage, both suppliers determine their individually-global wholesale prices concurrently. In the second stage, both retailers choose strategy C_R or NC_R, and the retail prices are determined concurrently (with each price set by either the corresponding retailer herself or the negotiation between the corresponding supplier and retailer). If strategy (NC_S, C_S) is chosen, then in the first stage, retailer R_2 determines her retail pricing strategy (C_R or NC_R). When retailer R_2 uses strategy NC_R, in the second stage supplier S_1 determines wholesale price w_1 individually but supplier S_2 and retailer R_2 negotiate wholesale price w_2. We can obtain the results of w_1 and w_2 in Nash equilibrium. In the third stage, retailer R_1 determines her retail pricing strategy (C_R or NC_R) and retail price p_1 is set, while retailer R_2 determines her retail price individually. When retailer R_2 uses strategy C_R, in the second stage S_1 determines w_1 individually, and in the third stage R_1 determines her retail pricing strategy (C_R or NC_R) and retail price p_1 is set, while S_2 and R_2 negotiate (w_2, p_2). The above applies similarly for strategy (C_S, NC_S).

The equilibrium results given the suppliers' pricing strategies are presented in Theorem 11 in online Appendix B.7. We can note the possibility of the result that a supply chain dissolves but the other reaches an agreement, when one supplier uses strategy C_S and the other supplier uses strategy NC_S. We do not include this result in Theorem 11 mainly because in this possibility, the supplier who breaks up with the retailer has an incentive to change his wholesale pricing strategy and thus, this is not a Nash equilibrium. We also learn from Theorem 11 that the pricing strategies in the two supply chains may involve the negotiations for both the wholesale prices and the retail prices; that is, when the wholesale prices are negotiated, the retailers may also decide to negotiate their retail prices. If any one of the following two conditions does not hold:

$$\begin{cases} \eta_1 \equiv (A-bc)^2 (b+\alpha) \left(2b^2+4\alpha b+\alpha^2\right) (2b+3\alpha) / [(2b+\alpha) \left(4b^2+7\alpha b+\alpha^2\right)^2] \geq d_s, \\ \eta_2 \equiv (A-bc)^2 (b+\alpha)(2b^2+4\alpha b+\alpha^2)^2/[(2b+\alpha)^2 (4b^2+7\alpha b+\alpha^2)^2] \geq d_r, \end{cases}$$

then, when the suppliers choose to negotiate the wholesale prices, the suppliers and the retailers may reach an agreement. However, the suppliers' strategy NC_S can induce all firms to leave the supply chains with no agreement. The result differs from Baron, Berman, and Wu's finding (2016) that supply chain members can always reach an agreement when the suppliers use their non-cooperative strategies. The difference occurs because Baron, Berman, and Wu (2016) assumed zero disagreement points whereas we used general disagreement points for all players.

Prior to turning into numerical simulations, we briefly summarize the equilibrium of the multi-stage game in the duopoly case. For each supply chain, there are three possible equilibria (i.e., (C_S, C_R), (C_S, NC_R), and (NC_S, NC_R)) of the supplier's and the retailer's pricing strategies. This coincides with our results for the monopoly case. For the suppliers, the asymmetric Nash equilibrium (C_S, NC_S) (or (NC_S, C_S)) may appear.

Next, we perform numerical study to discuss the suppliers' wholesale pricing strategies in Nash equilibrium as well as the retailers' best responses. Similar to McGuire and Staelin (1983), we let $A = b = 1$ and $c = 0$. Since Theorem 12 (in online Appendix B.8) indicates the analytic impact of $\bar{\lambda}$ and (d_r, d_s), we mainly focus on the numerical study with α and λ. For many different values of $\bar{\lambda}$ and (d_r, d_s), the results regarding the impact of α and λ are robust. Hence, we plot Figure 5 to show our results when $\bar{\lambda} = 0.6$ and $(d_r, d_s) = (0.01, 0.05)$. Figure 5 exposes that when the competition is higher as the value of α increases from 0 to 2, the retailer who uses strategy C_R changes his strategy to strategy NC_R. Moreover, when each supplier's relative bargaining power in the NC_R case increases from 0 to 1, the retailers change their strategy NC_R to strategy C_R.

Then, we consider the suppliers' wholesale pricing strategies in Nash equilibrium. When both suppliers chose to negotiate their wholesale prices, the retailers may have multiple pure Nash equilibria. As Theorem 12 (in online Appendix B.8) indicates, when multiple Nash equilibria for the retailers appear, there are only two situations: first, the two pure Nash equilibria are (C_R, C_R) and (NC_R, NC_R). For this case, we choose the pure Nash equilibrium that generates higher profits for the retailers. Secondly, the two pure Nash equilibria are (NC_R, C_R) and (C_R, NC_R). Because of the symmetry, we,

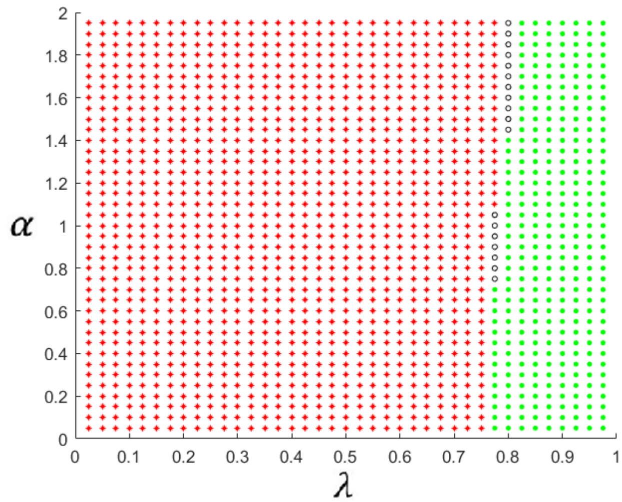

Figure 5: The impact of α and λ on the retail pricing strategies in Nash equilibrium when $\bar{\lambda} = 0.6$, $(d_r, d_s) = (0.01, 0.05)$, and both suppliers choose to negotiate their wholesale prices. Note that, in this figure, the red star symbol "*", the green solid circle"$\bullet$", and the black empty circle "$\circ$" represent the retailers' pricing strategies (NC_R, NC_R), (C_R, C_R), and (C_R, NC_R) or (NC_R, C_R), respectively.

w.l.o.g., choose pure Nash equilibrium (C_R, NC_R). We begin by showing the suppliers' strategies in Nash equilibrium for different values of α and λ. Since our results are basically robust with respect to (d_r, d_s) but are somewhat dependent on $\bar{\lambda}$, we provide Figure 6 in which each figure corresponds to a specific value of $\bar{\lambda}$.

When the value of $\bar{\lambda}$ is sufficiently small (e.g., $\bar{\lambda} \leq 0.2$), the suppliers have only one pure Nash equilibrium (NC_S, NC_S) for all $\lambda \in (0, 1)$ and $\alpha \in (0, 2)$. The reason is that when the value of $\bar{\lambda}$ is low, the retailers have a strong bargaining power under their strategy C_R, and they should prefer to use strategy C_R. This makes the suppliers' profits slightly above their disagreement points. As a result, the suppliers prefer to determine the wholesale price individually for higher profits. As the value of $\bar{\lambda}$ increases, there are two pure Nash equilibria (C_S, NC_S) and (NC_S, C_S). In this case, the retailers still prefer strategy C_R, and the suppliers need to compare their profits under strategies NC_S and C_S. However, if both suppliers choose to negotiate their wholesale prices with the retailers, then the two supply chains have a high competition, which reduces the suppliers' profits dramatically. Hence, only one supplier changes to negotiate his wholesale price whereas the other one still stays with strategy NC_S. That is, only the supplier who negotiates his wholesale price can enjoy the profit increment made by an increase of his bargaining power. When the value of $\bar{\lambda}$ is sufficiently large (e.g., $\bar{\lambda} \geq 0.8$), the suppliers only have a unique pure Nash equilibrium (C_S, C_S) for all $\lambda \in (0, 1)$ and $\alpha \in (0, 2)$. This occurs because the retailers prefer to adopt strategy NC_R when the value of $\bar{\lambda}$ is sufficiently large. Thereby, the suppliers cannot impact the retailers' pricing decisions via the negotiation, which implies that the suppliers may prefer to determine their optimal wholesale price individually.

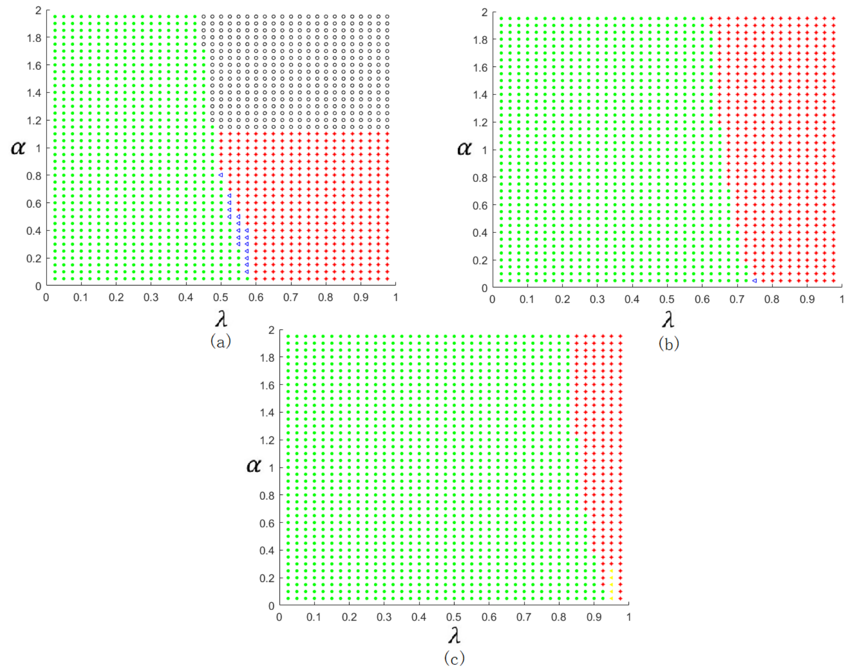

Figure 6: The impact of α and λ on the wholesale pricing strategies in Nash equilibrium when (a) $\bar{\lambda} = 0.5$, $d_r = 0.01$, and $d_s = 0.05$; (b) $\bar{\lambda} = 0.6$, $d_r = 0.01$, and $d_s = 0.05$; (c) $\bar{\lambda} = 0.7$, $d_r = 0.01$, and $d_s = 0.05$. Note that, in this figure, the red star symbol "*", the green solid circle "●", the black empty circle "○", the yellow solid triangle "◀", and the blue empty triangle "◁" represent the wholesale pricing strategies (C_S, C_S), (NC_S, NC_S), $(C_S, C_S)/(NC_S, NC_S)$, (C_S, NC_S), and $(C_S, NC_S)/(NC_S, C_S)$, respectively.

When the value of λ is small, the retailers possess a high bargaining power under their strategy NC_R, which entices both retailers to choose NC_R strategy but allocates a small part of the total profit to the suppliers. Therefore, the suppliers prefer to choose strategy NC_S. When the value of λ is large, since the values of $\bar{\lambda}$ in all three figures in Figure 6 are greater than 0.5, the suppliers hold stronger position in their wholesale price negotiations. Thus, when the competition between the supply chains is not high, the suppliers should negotiate their wholesale prices and gain more profits from supply chain integration. However, when the competition is highly intensive, supply chain integration may lead to a "price war," which may decrease the supply chain-wide profit and thus, it may be another Nash equilibrium when both suppliers choose to determine their wholesale prices individually.

Next, we present the suppliers' strategies in Nash equilibrium for different values of (d_r, d_s). Since the values of $\bar{\lambda}$ and λ have a negligibly small impact on the results but the value of α influences the suppliers' pricing strategies. Accordingly, we plot Figure 7 to indicate the impacts of d_r and d_s for two different values of α. In Figures 7(a) and (b), the blank areas means that both supply chains dissolves due to all firms' high disagreements. Compared with Figure 7(b) (in which the supply chain competition is high), we find that, in Figure 7(a) (in which the competition is low), the firms in both supply chains are more likely to reach an agreement (i.e., the blank area is smaller) and (NC_S, NC_S)

is less likely to be an Nash equilibrium (i.e., the areas of the green solid and black empty circles). Our result coincides with that found by McGuire and Staelin (1983) who exposed that for both suppliers, strategy C_S is the only pure Nash equilibrium, when the supply chain competition is highly intensive.

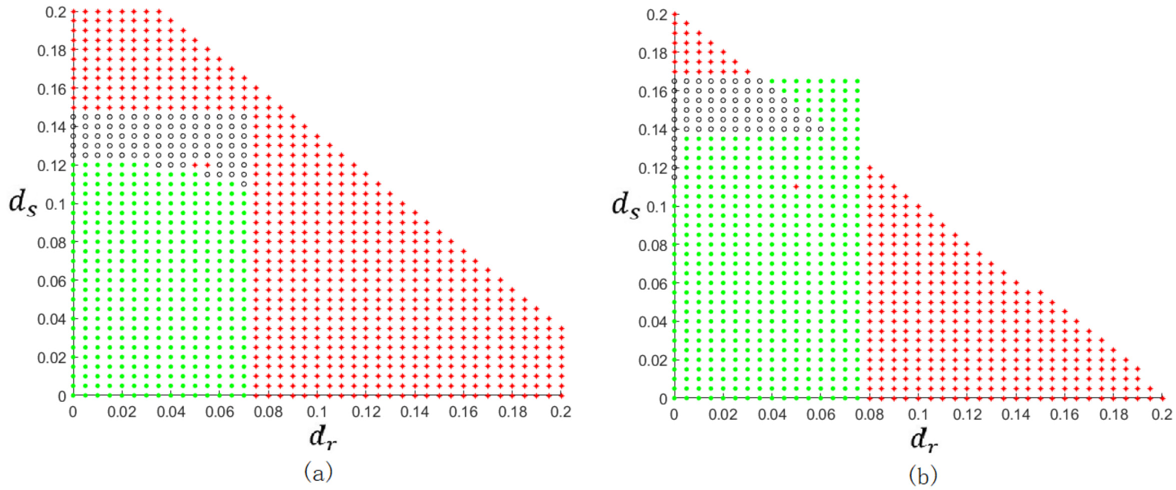

Figure 7: The impact of d_r and d_s on the wholesale pricing strategies in Nash equilibrium when (a) $\bar{\lambda} = 0.6$, $\lambda = 0.4$, and $\alpha = 0.5$; and (b) $\bar{\lambda} = 0.6$, $\lambda = 0.4$, and $\alpha = 1.5$. Note that, in this figure, the red star symbol "*", the green solid circle "•", and the black empty circle "∘" represent the wholesale pricing strategies (C_S, C_S), (NC_S, NC_S), and $(C_S, C_S)/(NC_S, NC_S)$, respectively.

In the monopoly supply chain system analyzed in Section 2, when the supplier's strategy NC_S does not dissolve the supply chain, we find that if the retailer prefers strategy NC_R, then the supplier should adopt strategy NC_S. Nonetheless, this result does not hold in the presence of supply chain competition. Specifically, we can learn from Figure 8 (which is extracted from Figure 7(b), focusing on the results when $d_r \in [0, 0.07]$ and $d_s \in [0.13, 0.15]$) that, in Nash equilibrium for some cases, when the supply chains does not dissolve in case that the suppliers chooses strategy NC_S, both suppliers use strategy C_S whereas both retailers adopt strategy NC_R. The main reason is that, if a supplier changes to strategy NC_S but the other supplier still holds strategy C_S, then the competitiveness of the supply chain involving the supplier who changes his strategy decreases and the supplier is then worse off. For the retailers, an area in Figure 7(b), which corresponds to area (2) in Figure 4, indicates that (NC_R, NC_R) be an equilibrium when λ is sufficiently high.

4 Summary and Concluding Remarks

We investigate a strategic pricing problem in which a supplier can choose to determine his wholesale price individually or negotiate it with the retailer, and a retailer also choose between determining the retail price individually and negotiating the retail price with the supplier. We begin by analyzing a

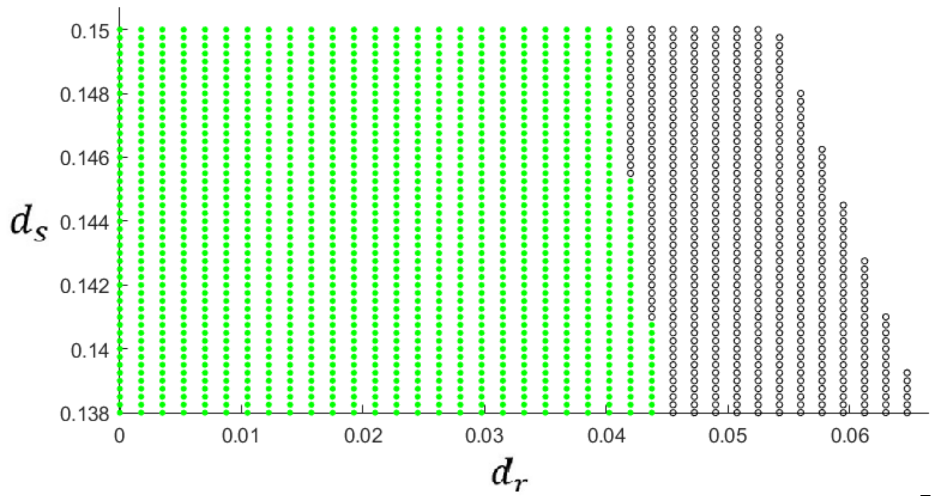

Figure 8: The impact of d_r and d_s on the wholesale pricing strategies in Nash equilibrium when $\bar{\lambda} = 0.6$, $\lambda = 0.4$, and $\alpha = 1.5$. Note that, in this figure, the green solid circle "•" and the black empty circle "∘" represent the suppliers' and retailers' pricing strategies $((C_S, C_S), (C_R, C_R))$ and $((C_S, C_S), (NC_R, NC_R))$, respectively.

single supply chain consisting of a supplier and a retailer. Comparing the two firms' profits in different price setting scenarios, we find that a proper power allocation between the supplier and the retailer is required for negotiating both wholesale price and retail price to benefit the two firms. Moreover, the two firms' willingness to negotiate the retail price increases with the retailer's disagreement point. The results are consistent with the practice in which commercial negotiations between suppliers and retailers may or may not involve a discussion of retail prices.

Although the wholesale pricing negotiations are common in reality, we find that, in the monopoly supply chain, the supplier may not always prefer to negotiate the wholesale price. If the supplier's disagreement point is sufficiently small and his bargaining power in the wholesale price negotiation is sufficiently small, then the supplier should always choose to determine the wholesale price individually. In addition, the wholesale price negotiation is a necessary part of the communications between supply chain members.

We perform our supply chain analysis with a demand function in general form, and find that our results with a linear demand function in the monopoly setting are robust. In addition, to study the impact of supply chain competition on the pricing strategies, we investigate the wholesale and retail pricing strategies when two supply chains compete in a market. We show that, for the duopoly case in which both suppliers choose to negotiate the wholesale prices, most of the insights drawn from the analysis of a monopoly supply chain hold. However, a difference from the monopoly case is that in the monopoly setting, wholesale price and retail price negotiations can always result in supply chain coordination but only the wholesale price negotiation cannot achieve the coordination, whereas in the duopoly setting, the wholesale price and retail price negotiations cannot coordinate both supply chains but only the wholesale price only negotiation can induce the coordination under certain conditions.

We also obtain a number of implications from our duopoly analyses, which cannot be drawn from our monopoly analysis. The wholesale price and retail price negotiations in a supply chain can improve the competitiveness of the supply chain by integrating the supplier and the retailer. As a consequence, the firms who negotiate both the retail price and the wholesale price are more likely to reach an agreement than those do not. However, the competition between the two supply chains may not reduce the profits of both supply chains. When the supply chain competition increases, both the suppliers and the retailers are less likely to negotiate both the retail price and the wholesale price. In addition, if the supplier's disagreement is small and the retailer's disagreement is sufficiently high, then the supply chain-wide profit when each supply chain is coordinated is lower than that when no supply chain can be coordinated.

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Supplemental File

"**Game-Theoretic Analyses of Strategic Pricing Decision Problems in Supply Chains**"

Feimin Zhong, Zhongbao Zhou, Mingming Leng

Appendix A Proofs of Theorems and Propositions in the Main Paper

Proof of Theorem 1. Using (11), we have

$$\frac{1-\lambda}{\pi_r^{nc_R}-d_r}=-\frac{\lambda g'\left(\pi_r^{nc_R}\right)}{g\left(\pi_r^{nc_R}\right)-d_s}\leq\frac{\lambda}{g\left(\pi_r^{nc_R}\right)-d_s}.$$

Thus,

$$\frac{\lambda}{g\left(\pi_r^{nc_R}\right)-d_s}\geq\frac{1-\lambda+\lambda}{\pi_r^{nc_R}-d_r+g\left(\pi_r^{nc_R}\right)-d_s}\geq\frac{1}{\Pi^*\left(c\right)-d_r-d_s}.$$

As a consequence, $\lambda_l=(g(\pi_r^{nc_R})-d_s)/(\Pi^*(c)-d_r-d_s)\leq\lambda$. According to Theorem 7 (in online Appendix B.2), we find that $\pi_r^{nc_R}$ and $\pi_r^{nc_R}+\pi_s^{nc_R}=\pi_r^{nc_R}+g(\pi_r^{nc_R})$ are both decreasing in λ, which means that both λ_u and $\lambda_u-\lambda_l$ are increasing in λ. Since $(\pi_r^{nc_R},g(\pi_r^{nc_R}))$ is on the Pareto frontier of the concave curve $\{(x,g(x))|0\leq x\leq\Pi^*(c)\}$, we have $\partial(g(\pi_r^{nc_R}))/\partial\lambda=g'(\pi_r^{nc_R})\times\partial\pi_r^{nc_R}/\partial\lambda\geq0$, which implies that λ_l is increasing in λ. Since $(1-\lambda)/g'(\pi_r^{nc_R})\leq\partial\pi_r^{nc_R}/\partial d_s\leq0$,

$$\begin{aligned}\frac{\partial\lambda_l}{\partial d_s}&=\frac{g'\left(\pi_r^{nc_R}\right)\frac{\partial\pi_r^{nc_R}}{\partial d_s}-1}{\Pi^*\left(c\right)-d_r-d_s}+\frac{g\left(\pi_r^{nc_R}\right)-d_s}{\left(\Pi^*\left(c\right)-d_r-d_s\right)^2}\\&\leq\frac{g\left(\pi_r^{nc_R}\right)-d_s}{\left(\Pi^*\left(c\right)-d_r-d_s\right)^2}-\frac{\lambda}{\Pi^*\left(c\right)-d_r-d_s},\end{aligned}$$

which is non-positive. This theorem is thus proved. ■

Proof of Theorem 2. We note that the inequality $\lambda\leq\lambda_u$ is equivalent to the inequality $d_r+(1-\lambda)(\Pi^*(c)-d_r-d_s)-\pi_r^{nc_R}\geq0$. Using Theorem 7 (in online Appendix B.2), we have

$$\frac{\partial\left[d_r+(1-\lambda)\left(\Pi^*\left(c\right)-d_r-d_s\right)-\pi_r^{nc_R}\right]}{\partial d_r}=\lambda-\frac{\partial\pi_r^{nc_R}}{\partial d_r}\geq0.$$

Therefore, given the values of λ and d_s, if $(1-\lambda)(\Pi^*(c)-d_s)-\pi_r^{nc_R}|_{d_r=0}\geq0$, then $\lambda\leq\lambda_u$ and we can set $d_t=0$. Otherwise, if $(1-\lambda)(\Pi^*(c)-d_s)-\pi_r^{nc_R}|_{d_r=0}<0$, then we can increase the value of d_r until (d_r,d_s) reaches the Pareto frontier of the concave curve $\{(x,g(x))|0\leq x\leq\Pi^*(c)\}$. It follows that $(\pi_r^{nc_R},\pi_s^{nc_R})=(d_r,d_s)$, which means that $d_r+(1-\lambda)(\Pi^*(c)-d_r-d_s)-\pi_r^{nc_R}=(1-\lambda)(\Pi^*(c)-d_r-d_s)\geq0$. Therefore, there exists a $d_t>0$ such that $d_r+(1-\lambda)(\Pi^*(c)-d_r-d_s)-\pi_r^{nc_R}=0$ when $d_r=d_t$. This theorem is thus proved. ■

Proof of Theorem 3. We begin by calculating the profits of both firms when the supplier chooses strategy NC_S. When the supplier determines a wholesale price w individually, the retailer sets a retail price p to maximize $\pi_r(w,p)=(p-w)(A-bp)$ if the retailer uses strategy NC_R, or the supplier and the retailer jointly determine a retail price p to maximize $[(p-w)(A-bp)-d_r]^{1-\hat{\lambda}}[(w-c)(A-bp)-d_s]^{\hat{\lambda}}$ if the retailer uses strategy C_R. It is easy to note that it is optimal for the retailer to always choose strategy NC_R. As a result, the optimal retail price for the retailer is $p^{nc_S} = (w + A/b)/2$, and the supplier's profit is $(w-c)(A-bp^{nc_S}) = (w-c)(A-wb)/2$. Hence, the supplier's optimal wholesale price is $w^{nc_S} = (c + A/b)/2$. We calculate the supplier's and the retailer's profits as $\pi_s^{nc_S} = (A - cb)^2/(8b)$ and $\pi_r^{nc_S} = (A-cb)^2/(16b)$, respectively. It follows that both the supplier and the retailer should stay in the supply chain if $(A - cb)^2/(8b) > d_s$ and $(A - cb)^2/(16b) > d_r$.

Next, we find the necessary and sufficient condition under which the supplier prefers NC_S–which corresponds to part (i). It is easy to see that the condition in (3) is needed. When C_S is used by the supplier, we find from (1) that the retailer should use C_R (NC_R) if $\bar{\lambda} \leq \lambda_u$ ($\bar{\lambda} \geq \lambda_u$). Thus, if $\bar{\lambda} \geq \lambda_u$, the retailer should use NC_R, and the supplier's profit is $\pi_s^{nc_R}$. It is easy to find that $\pi_s^{nc_S} = g(\pi_r^{nc_S})$ and $g(x)$ reaches its maximum at $x = \pi_r^{nc_S}$. Therefore, $\pi_s^{nc_S} = g(\pi_r^{nc_S}) \geq g(\pi_r^{nc_R}) = \pi_s^{nc_R}$, which means that if $\bar{\lambda} \geq \lambda_u$, then NC_S is always optimal for the supplier. If $\bar{\lambda} \leq \lambda_u$, the retailer should use C_R, and the supplier's profit is $\pi_s^{c_R}$. Therefore, NC_S is optimal for the supplier if $\pi_s^{nc_S} \geq \pi_s^{c_R}$ (i.e., $\bar{\lambda} \leq [(A-cb)^2/(8b) - d_s]/[(A-cb)^2/(4b) - (d_r + d_s)]$). In summary, the supplier prefers strategy NC_S if and only if conditions in (3) and (4) hold. The retailer's optimal response is NC_R.

Finally, although the supplier prefers strategy C_S if any one of the conditions in (3) and (4) is satisfied, the best response of the retailer may differ under different conditions. Specifically, if the condition in (3) does not hold and $\bar{\lambda} \leq \lambda_u$–which corresponds to part (ii), then the supplier uses strategy C_S and the retailer responds by using strategy C_R. If the condition in (3) does not hold and $\bar{\lambda} \geq \lambda_u$–which corresponds to part (iii), then the supplier adopts strategy C_S and the retailer chooses strategy NC_R as a response. If the condition in (3) holds whereas the condition in (4) does not hold, then the supplier uses strategy C_S and $\bar{\lambda} \leq \lambda_u$, and the retailer responds by choosing strategy C_R. ■

Proof of Proposition 1. We first examine the condition under which both the supplier and the retailer prefer C_S. When the condition in (3) holds, according to Theorem 3, the supplier prefers C_S if

$$\bar{\lambda} \in \left[\frac{\Pi^*(c)/2 - d_s}{\Pi^*(c) - (d_r + d_s)}, \lambda_u\right].$$

Then, we discuss the retailer's preference on the supplier's pricing strategy. When $(\Pi^*(c)/2 - d_s)/(\Pi^*(c) - (d_r + d_s)) \leq \bar{\lambda} \leq \lambda_u$, the supplier prefers C_S. If the supplier uses NC_S, then the retailer chooses NC_R and her profit is $\Pi^*(c)/4$. If the supplier uses C_S, then the retailer adopts C_R (since $\bar{\lambda} \leq \lambda_u$) and obtains the profit $\pi_r^{c_R} = d_r + (1 - \bar{\lambda})[\Pi^*(c) - (d_r + d_s)]$. Therefore, the retailer prefers C_S if $\pi_r^{c_R} \geq \Pi^*(c)/4$, i.e.,

$$\bar{\lambda} \leq \frac{3\Pi^*(c)/4 - d_s}{\Pi^*(c) - (d_r + d_s)}. \tag{6}$$

In summary, when the condition in (3) holds, both the supplier and the retailer prefer C_S if

$$\bar{\lambda} \in \left[\frac{\Pi^*(c)/2 - d_s}{\Pi^*(c) - (d_r + d_s)}, \frac{3\Pi^*(c)/4 - d_s}{\Pi^*(c) - (d_r + d_s)}\right].$$

Next, we examine the condition under which both the supplier and the retailer prefer NC_S. When the condition in (3) holds, according to Theorem 3, the supplier prefers NC_S if

$$\bar{\lambda} \in \left[0, \frac{\Pi^*(c)/2 - d_s}{\Pi^*(c) - (d_r + d_s)}\right] \cup [\lambda_u, 1]. \tag{7}$$

For the retailer, we should consider two cases: $\bar{\lambda} \leq \lambda_u$ and $\bar{\lambda} \geq \lambda_u$.

1. Case $\bar{\lambda} \leq \lambda_u$. Similar to our above analysis for the condition under which both the supplier and the retailer prefer C_S, we learn that the retailer prefers NC_S if the condition in (6) is not satisfied. Noting that $\bar{\lambda} \geq (3\Pi^*(c)/4 - d_s)/(\Pi^*(c) - d_r - d_s)$ and the condition in (7) cannot hold simultaneously in this case, we conclude that both the supplier and the retailer cannot prefer NC_S concurrently.
2. Case $\bar{\lambda} \geq \lambda_u$. Using similar arguments, we find that the retailer prefers NC_S if λ is sufficiently large such that $\pi_r^{nc_R} \leq \Pi^*(c)/4$. Also using the conditions in (7), we find that both the supplier and the retailer prefer NC_S if $\bar{\lambda} \geq \lambda_u$ and λ is sufficiently large.

■

Proof of Proposition 2. Under the condition in (3), $\lambda_u \leq (\Pi^*(c)/2 - d_s)/[\Pi^*(c) - (d_r + d_s)]$ is equivalent to $\Pi^*(c) \leq 2\pi_r^{nc_R}$. As the value of λ increases from 0 to 1, the negotiation can lead the result under the retailer's non-cooperative strategy to stay along the Pareto frontier of curve $g(x)$ from the bottom (i.e., $(\pi_r^{nc_R}, g(\pi_r^{nc_R})) = (g^{-1}(d_s), d_s)$) to the top (i.e., $(\pi_r^{nc_R}, g(\pi_r^{nc_R})) = (\Pi^*(c)/4, \Pi^*(c)/2)$). Therefore, if $d_s \leq g(\Pi^*(c)/2)$, the retailer's profit $\pi_r^{nc_R}$ at the bottom of the curve $g(x)$ is larger than or equal to $\Pi^*(c)/2$, i.e., $\lambda_u \leq (\Pi^*(c)/2 - d_s)/[\Pi^*(c) - (d_r + d_s)]$. As a result, if the supplier's bargaining power λ is sufficiently small such that $(\pi_r^{nc_R}, g(\pi_r^{nc_R}))$ is sufficiently close to the bottom point $(g^{-1}(d_s), d_s)$, then $\Pi^*(c) \leq 2\pi_r^{nc_R}$, i.e., $\lambda_u \leq (\Pi^*(c)/2 - d_s)/[\Pi^*(c) - (d_r + d_s)]$. ■

Proof of Proposition 3.

The system-wide profit is $\Pi(p) = (p - c)D(p)$. The globally-optimal retail price $p^*(c)$ satisfies the first order condition $D(p^*(c)) - cD'(p^*(c)) = 0$, and the maximum system-wide profit is $\Pi^*(c) \equiv \Pi(p^*(c))$. Next, we show some important results which are used to prove this proposition.

Result 1 *When the retailer adopts the NC_R to determine her retail price individually, we find that, if $d_r < \Pi^*(c)$ and $d_s < \bar{d}_s \equiv \max\{g(x)|d_r \leq x \leq \Pi^*(c)\}$, where $g(x)$ is a concave function on domain $[0, \Pi^*(c)]$ and represents the upper boundary of*

$$\begin{aligned} BS \equiv \quad & \{\delta(\Pi^*(w_1), (w_1 - c)D(p^*(w_1))) + (1 - \delta)(\Pi^*(w_2), \\ & (w_2 - c)D(p^*(w_2)))|c \leq w_1, w_2 \leq T, 0 \leq \delta \leq 1\}, \end{aligned} \tag{8}$$

then the supplier and the retailer can reach an agreement on their wholesale price negotiation. The negotiated wholesale price w^{nc_R} equals $w_1^{nc_R}$ with probability δ^{nc_R} and $w_2^{nc_R}$ with probability $1 - \delta^{nc_R}$. In addition, $(w_1^{nc_R}, w_2^{nc_R}; \delta^{nc_R}, 1 - \delta^{nc_R})$ satisfies

$$\begin{aligned} (\pi_r^{nc_R}, g(\pi_r^{nc_R})) \quad = \quad & \delta^{nc_R}(\Pi^*(w_1^{nc_R}), (w_1^{nc_R} - c)D(p^*(w_1^{nc_R}))) \\ & + (1 - \delta^{nc_R})(\Pi^*(w_2^{nc_R}), (w_2^{nc_R} - c)D(p^*(w_2^{nc_R}))), \end{aligned}$$

where $\pi_r^{nc_R}$ can be uniquely obtained by solving the equation $(1-\lambda)/(\pi_r^{nc_R}-d_r)+\lambda g'(\pi_r^{nc_R})/(g(\pi_r^{nc_R})-d_s)=0$. The optimal retail price is $p^{nc_R}=p^*(w^{nc_R})$. The supplier's and the retailer's expected profits are $\pi_s^{nc_R}=g(\pi_r^{nc_R})$ and $\pi_r^{nc_R}$, respectively. We also find that $-(\pi_r^{nc_R}-d_r)/(1-\lambda)\leq \partial\pi_r^{nc_R}/\partial\lambda\leq 0$, $\lambda\geq\partial\pi_r^{nc_R}/\partial d_r\geq 0$, and $(1-\lambda)/g'(\pi_r^{nc_R})\leq\partial\pi_r^{nc_R}/\partial d_s\leq 0$. Moreover, the supply chain-wide profit (i.e., $\pi_r^{nc_R}+g(\pi_r^{nc_R})$) is decreasing in λ and d_s but increasing in d_r.

Proof of Result 1: For any given value of wholesale price w, the retailer maximizes $\pi_r(w,p)=(p-w)D(p)$ to obtain his optimal retail price as $p^{nc_R}=p^*(w)$. Thus, the retailer's and the supplier's profits are $\Pi^*(w)$ and $(w-c)D(p^*(w))$, respectively. Note that, when w changes from c to T, $(\Pi^*(w),(w-c)D(p^*(w)))$ draws a curve $\{(\Pi^*(w),(w-c)D(p^*(w)))|c\leq w\leq T\}$, which may not be concave. Hence, the bargaining set of the supplier and the retailer is the convex hull of such curve, i.e., BS, as given in (8).

The negotiated wholesale price could be a probabilistic decision. That is, it is w_1 with probability δ and w_2 with probability $1-\delta$. Since $\Pi^*(w)+(w-c)D(p^*(w))\leq\Pi^*(c)$, $(\Pi^*(c),(c-c)D(p^*(c)))=(\Pi^*(c),0)$, and $(\Pi^*(T),(T-c)D(p^*(T)))=(0,0)$, BS is contained in the triangle $\{(\pi_r,\pi_s)|\pi_r\geq 0,\pi_s\geq 0,\pi_r+\pi_s\leq\Pi^*(c)\}$, and upper boundary of BS can be represent by a concave function $g(x)$ with domain $[0,\Pi^*(c)]$ and $g(\Pi^*(c))=0$. It then follows that the negotiation between the retailer and the supplier can end up with an agreement, if $d_r<\Pi^*(c)$ and $d_s<\bar{d}_s\equiv\max\{g(x)|d_r\leq x\leq\Pi^*(c)\}$ hold, i.e., (d_r,d_s) is in areas (1) and (2) in Figure 1. We can obtain Nash bargaining solution by solving the following problem: $\max_{d_r\leq\pi_r,d_s\leq g(\pi_r)}(\pi_r-d_r)^{1-\lambda}(g(\pi_r)-d_s)^{\lambda}$, in which π_r and $g(\pi_r)$ represent the expected profits of the retailer and the supplier, respectively. Letting $\Lambda(x)\equiv(1-\lambda)\ln(x-d_r)+\lambda\ln(g(x)-d_s)$, we have

$$\Lambda''(x)=-\frac{1-\lambda}{(x-d_r)^2}+\frac{\lambda g''(x)(g(x)-d_s)-\lambda g'(x)^2}{(g(x)-d_s)^2}\leq 0,$$

which implies that $\Lambda(x)$ has a unique maximizer ξ that satisfies $d_r\leq\xi$ and $d_s\leq g(\xi)$. That is, ξ is the solution of the following first order condition: $(1-\lambda)/(\xi-d_r)+\lambda g'(\xi)/(g(\xi)-d_s)=0$. As a consequence, $(\pi_r^{nc_R},g(\pi_r^{nc_R}))$, where $\pi_r^{nc_R}=\xi$, is the Nash bargaining solution that characterizes the negotiation outcome of the retailer and the supplier. The negotiated wholesale price is a probabilistic combination of the two wholesale prices $(w_1^{nc_R},w_2^{nc_R};\delta^{nc_R},1-\delta^{nc_R})$ that satisfies

$$\delta^{nc_R}(\Pi^*(w_1^{nc_R}),(w_1^{nc_R}-c)D(p^*(w_1^{nc_R})))+(1-\delta^{nc_R})(\Pi^*(w_2^{nc_R}),(w_2^{nc_R}-c)D(p^*(w_2^{nc_R})))=(\pi_r^{nc_R},g(\pi_r^{nc_R})).$$

Similar to the proof of Theorem 7 (in online Appendix B.2), we can show that $-(\pi_r^{nc_R}-d_r)/(1-\lambda)\leq\partial\pi_r^{nc_R}/\partial\lambda\leq 0$, $\lambda\geq\partial\pi_r^{nc_R}/\partial d_r\geq 0$, and $(1-\lambda)/g'(\pi_r^{nc_R})\leq\partial\pi_r^{nc_R}/\partial d_s\leq 0$. Moreover, the supply chain-wide profit (i.e., $\pi_r^{nc_R}+g(\pi_r^{nc_R})$) is decreasing in λ and d_s but increasing in d_r. Thus, Result 1 is proved. ◀

We can find that the results in Result 1 are similar to those in Theorem 7 (in online Appendix B.2). That is, when the demand function is given in general form, these results in the monopoly setting are robust.

Result 2 *When the retailer adopts strategy C_R to negotiate the wholesale and retail prices with* *the supplier, the negotiation cannot succeed if* $d_r + d_s \geq \Pi^*(c)$*. Otherwise, if* $d_r + d_s < \Pi^*(c)$*, then the retailer and the supplier can successfully reach an agreement with* $p^{c_R} = p^*(c)$ *and* $w^{c_R} = c + \{d_s + \bar{\lambda}[\Pi^*(c) - d_r - d_s]\}/D(p^*(c))$*. The retailer's and the suppler's profits are* $\pi_r^{c_R} = d_r + (1-\bar{\lambda})[\Pi^*(c) - d_r - d_s]$ *and* $\pi_s^{c_R} = d_s + \bar{\lambda}[\Pi^*(c) - d_r - d_s]$*, respectively.*

Proof of Result 2: When the supplier and the retailer bargain over wholesale price w and retail price p, their profits are $\pi_s(w,p) = (w-c)\Pi(p)$ and $\pi_r(w,p) = (p-w)\Pi(p)$. Hence, the bargaining set is the convex hull of $\{(\pi_r(w,p), \pi_s(w,p)) | T \geq p \geq w \geq c\}$. Since the maximum supply chain profit is $\Pi^*(c)$, the bargaining set is contained in the triangle $\{(\pi_r, \pi_s) | \pi_r \geq 0, \pi_s \geq 0, \pi_r + \pi_s \leq \Pi^*(c)\}$, which is identical to the bargaining set because of the following facts. If $p = w = c$, then $(0,0)$ is in the bargaining set. If $w = c$ and $p = p^*(c)$, then $(\Pi^*(c), 0)$ is in the bargaining set. However, if $p = p^*(c)$ and $w = c + \Pi^*(c)/p^*$, then $(0, \Pi^*(c))$ is in the bargaining set. Therefore, the bargaining set is identical to the triangle. As a consequence, the negotiation cannot succeed if $d_r + d_s \geq \Pi^*(c)$. Otherwise, if $d_r + d_s < \Pi^*(c)$, then the generalized Nash bargaining solution is obtained as $\pi_r^{c_R} = d_r + (1-\bar{\lambda})[\Pi^*(c) - (d_r + d_s)]$ and $\pi_s^{c_R} = d_s + \bar{\lambda}[\Pi^*(c) - (d_r + d_s)]$. We thus prove Result 2. ◀

We learn from Result 2 that the condition for successful negotiation in the case of a general demand function is similar to the result in Theorem 6 (in online Appendix B.1). In addition, similar to our discussion in Section 2.3, we compare results 1 and 2 to investigate the conditions under which the retailer prefers to use strategy C_R. We find that, similar to our results with the linear demand function, when (d_r, d_s) does not belong to areas (1) and (2) in Figure 1, the retailer and the supplier benefit from retail price negotiation. However, when (d_r, d_s) is in areas (1) and (2), the condition for the retailer to adopt strategy C_R is obtained as in (1), and the condition for the supplier to benefit from the retailer's cooperative strategy is attained as in (2). In addition, both firms can benefit from strategy C_R if $\bar{\lambda} \in [\lambda_l, \lambda_u]$. We can thus conclude that our results about two firms' preferences on the retailer's pricing strategy are robust.

Because the proofs of Theorems 1 and 2 are independent of the forms of the demand function and the function g, only the concavity property of g affect the results in these two theorems, which thus hold when the demand function is given in general form.

Result 3 *In Stackelberg equilibrium, the supplier's optimal choice and the retailer's response are summarized below.*

(i) *If the conditions in (3) and (4) hold, then the supplier's optimal pricing strategy is* NC_S*, to which the retailer's best response is strategy* NC_R*.*

(ii) *If the condition in (3) does not hold and* $\bar{\lambda} \leq \lambda_u$*, then the supplier's optimal pricing strategy is* C_S*, to which the retailer's best response is strategy* C_R*.*

(iii) *If the condition in (3) does not hold and* $\bar{\lambda} \geq \lambda_u$*, then the supplier's optimal pricing strategy is* C_S*, to which the retailer's best response is strategy* NC_R*.*

(iv) *If the condition in (3) holds whereas the condition in (4) does not hold, then the supplier's optimal pricing strategy is* C_S*, to which the retailer's best response is strategy* C_R*.*

Proof of Result 3: We first find the profits of the supplier and the retailer if the supplier adopts strategy NC_S. For a given wholesale price w, the retailer maximizes $\pi_r(w,p) = (p-w)D(p)$ to obtain an optimal retail price as $p^{nc_S} = p^*(w)$. Thus, the supplier's profit is $(w-c)D(p^*(w))$.

Hence, the supplier determines his optimal wholesale price as $w^{nc_S} = \arg\max_w (w - c)D(p^*(w))$. As a result, the supplier's and the retailer's profits are $\pi_s^{nc_S} = (w^{nc_S} - c)D(p^*(w^{nc_S}))$ and $\pi_r^{nc_S} = (p - w^{nc_S})D(p^*(w^{nc_S}))$, respectively. It follows that both the supplier and the retailer are willing to transact in the supply chain if $(w^{nc_S} - c)D(p^*(w^{nc_S})) > d_s$ and $(p - w^{nc_S})D(p^*(w^{nc_S})) > d_r$.

Next, we find the necessary and sufficient condition under which the supplier prefers NC_S. As the condition in (4) always holds, the supplier's and the retailer's profits are $\pi_s^{nc_S} = (w^{nc_S} - c)D(p^*(w^{nc_S}))$ and $\pi_r^{nc_S} = (p - w^{nc_S})D(p^*(w^{nc_S}))$, respectively, if the supplier chooses strategy NC_S. When the supplier uses strategy C_S, the retailer should choose strategy NC_R (C_R) if $\bar{\lambda} \geq \lambda_u$ ($\bar{\lambda} \leq \lambda_u$). If $\bar{\lambda} \geq \lambda_u$, then the retailer adopts strategy NC_R and the profit of the supplier is $\pi_s^{nc_R}$. Noting that $\pi_s^{nc_S} = \max_w (w - c)D(p^*(w)) = \max_{0 \leq x \leq \Pi^*(c)} g(x)$, we find $\pi_s^{nc_S} \geq g(\pi_r^{nc_R}) = \pi_s^{nc_R}$. This means that if $\bar{\lambda} \geq \lambda_u$, strategy NC_S is optimal for the supplier. If $\bar{\lambda} \leq \lambda_u$, then strategy C_R is used and the profit of the supplier is $\pi_s^{c_R}$. Hence, strategy NC_S is optimal for the supplier if $\pi_s^{nc_S} \geq \pi_s^{c_R}$ (i.e., $\bar{\lambda} \leq [\Pi^*(c)/2 - d_s]/[\Pi^*(c) - (d_r + d_s)]$). In summary, the supplier's optimal choice is strategy NC_S if the conditions in (3) and (4) hold, to which the retailer's best response is strategy NC_R.

Finally, the supplier's optimal choice is strategy C_S if any one of the conditions in (3) and (4) does not hold, and we can find the retailer's best response accordingly. If the condition in (3) does not hold, then the supplier uses strategy C_S. We note that the retailer uses strategy NC_R (C_R) if $\bar{\lambda} \geq \lambda_u$ ($\bar{\lambda} \leq \lambda_u$). If the condition in (3) holds whereas the condition in (4) does not hold, the supplier chooses strategy C_S and $\bar{\lambda} \leq \lambda_u$. Hence, the retailer uses strategy C_S. ◀

We find that the supplier's wholesale pricing strategy for the case of a general demand function is similar to that in Theorem 3. ■

Proof of Proposition 4. When both retailers use strategy C_R, $(b + \alpha)[(A - bc)/(2b + \alpha)]^2$ is decreasing in α. Hence, agreements are less likely to be achieved if the supply chain competition increases.

If one retailer (e.g., R_1) uses strategy C_R while the other one uses strategy NC_R, then $g^{c_R, nc_R}(x)$ reaches its maximum when $x = (b + \alpha)(A - bc)^2/[4(2b + \alpha)^2]$ and

$$\begin{aligned} \frac{\partial}{\partial \alpha}\left[g^{c_R,nc_R}\left(\frac{(b+\alpha)(A-bc)^2}{4(2b+\alpha)^2}\right)\right] &= \frac{\partial}{\partial \alpha}\left[\frac{2b^2+3\alpha b}{2b^2+4\alpha b+\alpha^2} \times \frac{(b+\alpha)\,(A-bc)^2}{4\,(2b+\alpha)\,b}\right] \\ &= \frac{(A-bc)^2}{4} \times \frac{2b^4-9\alpha^2 b^2-10\alpha^3 b-3\alpha^4}{(2b+\alpha)^2\,(2b^2+4\alpha b+\alpha^2)^2}. \end{aligned}$$

Thus, the maximum value of $g^{c_R,nc_R}(x)$ is decreasing in α, if α is sufficiently large (for example

$\alpha \geq b/3$). Moreover, when $x \geq (b+\alpha)(A-bc)^2/[4(2b+\alpha)^2]$, we have

$$\begin{aligned}\frac{\partial g^{c_R,nc_R}(x)}{\partial \alpha} &= \frac{\left(\alpha b^2 - 1.5\alpha^3\right)(A-bc)\sqrt{x}}{(2b^2+4\alpha b+\alpha^2)^2\sqrt{b+\alpha}} - \frac{4\alpha b(\alpha+b)x}{(2b^2+4\alpha b+\alpha^2)^2} \\ &= \frac{(A-bc)\sqrt{x}}{(2b^2+4\alpha b+\alpha^2)^2\sqrt{b+\alpha}}\left[\alpha b^2 - 1.5\alpha^3 - \frac{4\alpha b(\alpha+b)\sqrt{(b+\alpha)x}}{A-bc}\right] \\ &\leq \frac{(A-bc)\sqrt{x}}{(2b^2+4\alpha b+\alpha^2)^2\sqrt{b+\alpha}}\left[\alpha b^2 - 1.5\alpha^3 - \frac{2\alpha b(b+\alpha)^2}{(2b+\alpha)}\right] \\ &\leq 0;\end{aligned}$$

thus, the union of areas (1) and (2) in Figure 3 shrinks if α increases when it is sufficiently large (for example, $\alpha \geq b/3$). In addition,

$$\frac{\partial}{\partial \alpha}\left[(b+\alpha)\left(\frac{A-(b+\alpha)c}{2b+2\alpha}\right)^2\right] = -\frac{A^2}{4(b+\alpha)^2} + \frac{c^2}{4} < 0,$$

i.e., the upper boundary of area (3) in Figure 3 moves downward, if α increases.

When both retailers use strategy NC_R,

$$\begin{aligned} & g^{nc_R,nc_R}\left((b+\alpha)\left\{\frac{A-bc}{2b+\alpha} \times \frac{2(b+\alpha)^2-\alpha^2}{(2b+3\alpha)b+\left[2(b+\alpha)^2-\alpha^2\right]}\right\}^2\right) \\ = \; & (A-bc)^2\,\frac{b+\alpha}{2b+\alpha}\,\frac{(2b+3\alpha)\left(2b^2+4\alpha b+\alpha^2\right)}{(4b^2+7\alpha b+\alpha^2)^2},\end{aligned}$$

which is decreasing in α, if α is sufficiently large. Moreover, for all values of x such that

$$x \geq (b+\alpha)\left\{\frac{A-bc}{2b+\alpha} \times \frac{2(b+\alpha)^2-\alpha^2}{(2b+3\alpha)b+\left[2(b+\alpha)^2-\alpha^2\right]}\right\}^2,$$

we have

$$\begin{aligned}\frac{\partial g^{nc_R,nc_R}(x)}{\partial \alpha} &= \sqrt{\frac{x}{b+\alpha}}\left[\frac{A-bc}{2b} - \frac{\sqrt{x(b+\alpha)}}{b}\right] \\ &\leq \sqrt{\frac{x}{b+\alpha}}\frac{A-bc}{b}\left(\frac{1}{2} - \frac{b+\alpha}{2b+\alpha} \times \frac{2b^2+4\alpha b+\alpha^2}{4b^2+7\alpha b+\alpha^2}\right),\end{aligned}$$

which is negative for a sufficiently large value of α. Therefore, when the supply chain competition is above a certain level (i.e., the value of α is sufficiently large), a higher competition (i.e., a higher value of α) makes both supply chains more likely to dissolve. ■

Proof of Theorem 4. When (d_r, d_s) is in area (1) in Figure 4, by replacing $(g(x), \Pi^*(c))$ with $(g^{c_R,nc_R}(x), (b+\alpha)(A-bc)^2/(2b+\alpha)^2)$ in the proof of Theorem 1, we can find that $\lambda_u^{c_R,c_R} \geq \lambda_l^{c_R,c_R}$, $\lambda \geq \lambda_l^{c_R,c_R}$, $\partial\lambda_u^{c_R,c_R}/\partial\lambda \geq 0$, $\partial\lambda_l^{c_R,c_R}/\partial\lambda \geq 0$, $\partial(\lambda_u^{c_R,c_R} - \lambda_l^{c_R,c_R})/\partial\lambda \geq 0$, and $\partial\lambda_l^{c_R,c_R}/\partial d_s \leq 0$. ■

Proof of Theorem 5. The proof of this theorem is similar to that of Theorem 2. ■

Appendix B Supplementary Analytic Results

B.1 Theorem 6 and Its Proof

Theorem 6 *When the supplier and the retailer adopt strategy (C_S, C_R), the negotiation cannot succeed if $d_r + d_s \geq \Pi^*(c) = b(A/b - c)^2/4$. Otherwise, if $d_r + d_s < \Pi^*(c)$, then the retailer and the supplier can successfully reach an agreement with*

$$p^{c_R} = \frac{1}{2}\left(\frac{A}{b} + c\right) \text{ and } w^{c_R} = c + \frac{d_s + \bar{\lambda}[b(A/b-c)^2/4 - d_r - d_s]}{A - bc}. \tag{9}$$

The retailer's and the suppler's profits are $\pi_r^{c_R} = d_r + (1-\bar{\lambda})[b(A/b-c)^2/4 - d_r - d_s]$ and $\pi_s^{c_R} = d_s + \bar{\lambda}[b(A/b-c)^2/4 - d_r - d_s]$, respectively.

Proof. When the supplier and the retailer bargain over wholesale price w and retail price p, the profits of the retailer and the supplier are $\pi_r(w,p) = (p-w)(A-bp)$ and $\pi_s(w,p) = (w-c)(A-bp)$, respectively. Hence, the bargaining set is the convex hull of $\{(\pi_r(w,p), \pi_s(w,p)) | A/b \geq p \geq w \geq c\}$. Since the maximum supply chain profit is $\Pi^*(c)$, the bargaining set is contained in the triangle $\{(\pi_r, \pi_s) | \pi_r \geq 0, \pi_s \geq 0, \pi_r + \pi_s \leq \Pi^*(c)\}$, which is identical to the bargaining set as shown below: if $p = w = c$, then $(0,0)$ is in the bargaining set. If $w = c$ and $p = p^*(c)$, then $(\Pi^*(c), 0)$ is in the bargaining set. However, if $p = p^*(c)$ and $w = c + \Pi^*(c)/p^*$, then $(0, \Pi^*(c))$ is in the bargaining set. Therefore, the bargaining set is identical to the triangle. As a consequence, the negotiation is not successful if $d_r + d_s \geq \Pi^*(c)$.

Otherwise, if $d_r + d_s < \Pi^*(c)$, then the generalized Nash bargaining solution is obtained as $\pi_r^{c_R} = d_r + (1-\bar{\lambda})[\Pi^*(c) - (d_r + d_s)]$ and $\pi_s^{c_R} = d_s + \bar{\lambda}[\Pi^*(c) - (d_r + d_s)]$. Moreover, the pricing decisions can be found as given in (9). We thus prove this theorem. ■

B.2 Theorem 7 and Its Proof

Theorem 7 *When the supplier and the retailer adopt strategy (C_S, NC_R), we find that, if and only if*

$$d_r < \frac{b}{4}\left(\frac{A}{b} - c\right)^2 \text{ and } d_s < \bar{d}_s \equiv \max\left\{ g(x) \,\middle|\, d_r \leq x \leq \frac{b}{4}\left(\frac{A}{b} - c\right)^2 \right\}, \tag{10}$$

where $g(x) \equiv -2x + A\sqrt{x/b} - c\sqrt{bx}$, then the supplier and the retailer can reach an agreement on their wholesale pricing negotiation. The negotiated wholesale price is $w^{nc} = (A - 2\sqrt{b\xi})/b$, where $\xi \in [d_r, b(A/b-c)^2/4]$ with $g(\xi) > d_s$ can be uniquely obtained by solving the following equation for x,

$$(1-\lambda)/(x - d_r) + \lambda g'(x) / (g(x) - d_s) = 0. \tag{11}$$

The optimal retail price is $p^{nc_R} = A/b - \sqrt{\xi/b}$. The supplier's and the retailer's profits are $\pi_s^{nc_R} = g(\xi)$ and $\pi_r^{nc_R} = \xi$, respectively. We also find that $-(\pi_r^{nc_R} - d_r)/(1-\lambda) \leq \partial \pi_r^{nc_R}/\partial \lambda \leq 0$, $\lambda \geq \partial \pi_r^{nc_R}/\partial d_r \geq$

0, and $(1-\lambda)/g'(\pi_r^{nc_R}) \leq \partial \pi_r^{nc_R}/\partial d_s \leq 0$. Moreover, the supply chain-wide profit (i.e., $\xi + g(\xi)$) is decreasing in λ and d_s but increasing in d_r.

Proof. Under strategy NC_R, for any given value of wholesale price w, the retailer maximizes $\pi_r(w,p) = (p-w)(A-bp)$ to obtain an optimal retail price as $p^{nc_R} = 0.5\,(w + A/b)$. Thus, the retailer's and the supplier's profits are $\Pi^*(w)$ and $(w-c)(A-bp^{nc_R})$, respectively. Note that, when w changes from c to A/b, $(\Pi^*(w), (w-c)(A-bp^{nc_R}))$ draws a concave curve $\{(x, g(x)) | 0 \leq x \leq \Pi^*(c)\}$. It then follows that the negotiation between the retailer and the supplier can end up with an agreement, if the condition in (10) holds, i.e., (d_r, d_s) is in areas (1) and (2) in Figure 1. We can obtain Nash bargaining solution by solving the following problem: $\max_{d_r \leq \pi_r, d_s \leq g(\pi_r)} (\pi_r - d_r)^{1-\lambda}(g(\pi_r) - d_s)^{\lambda}$. Letting $\Lambda(x) \equiv (1-\lambda)\ln(x-d_r) + \lambda \ln(g(x) - d_s)$, we have

$$\Lambda''(x) = -\frac{1-\lambda}{(x-d_r)^2} + \frac{\lambda g''(x)\,(g(x)-d_s) - \lambda g'(x)^2}{(g(x)-d_s)^2} \leq 0,$$

which implies that $\Lambda(x)$ has a unique maximizer ξ that satisfies $d_r \leq \xi$ and $d_s \leq g(\xi)$. That is, ξ is the solution of the first order condition in (11). As a consequence, $(\pi_r^{nc_R}, g(\pi_r^{nc_R}))$, where $\pi_r^{nc_R} = \xi$, is the Nash bargaining solution that characterizes the negotiation outcome of the retailer and the supplier. The negotiated wholesale price is $w^{nc_R} = (A - \sqrt{4b\pi_r^{nc_R}})/b$.

Since $\Lambda'(\pi_r^{nc_R}) \equiv 0$, we find that

$$\begin{aligned}
0 &= \frac{\partial}{\partial \lambda}\left[\Lambda'(\pi_r^{nc_R})\right] \\
&= -\frac{1}{\pi_r^{nc_R} - d_r} + \frac{g'(\pi_r^{nc_R})}{g(\pi_r^{nc_R}) - d_s} + \Lambda''(\pi_r^{nc_R})\frac{\partial \pi_r^{nc_R}}{\partial \lambda} \\
&= -\frac{1}{\lambda(\pi_r^{nc_R} - d_r)} + \Lambda''(\pi_r^{nc_R})\frac{\partial \pi_r^{nc_R}}{\partial \lambda} \\
&\leq \Lambda''(\pi_r^{nc_R})\frac{\partial \pi_r^{nc_R}}{\partial \lambda},
\end{aligned}$$

$$0 = \frac{\partial}{\partial d_r}\left[\Lambda'(\pi_r^{nc_R})\right] = \frac{1-\lambda}{(\pi_r^{nc_R} - d_r)^2} + \Lambda''(\pi_r^{nc_R})\frac{\partial \pi_r^{nc_R}}{\partial d_r} \geq \Lambda''(\pi_r^{nc_R})\frac{\partial \pi_r^{nc_R}}{\partial d_r},$$

and

$$\begin{aligned}
0 &= \frac{\partial}{\partial d_s}\left[\Lambda'(\pi_r^{nc_R})\right] \\
&= \frac{\lambda g'(\pi_r^{nc_R})}{(g(\pi_r^{nc_R}) - d_s)^2} + \Lambda''(\pi_r^{nc_R})\frac{\partial \pi_r^{nc_R}}{\partial d_s} \\
&= -\frac{1-\lambda}{(\pi_r^{nc_R} - d_r)(g(\pi_r^{nc_R}) - d_s)} + \Lambda''(\pi_r^{nc_R})\frac{\partial \pi_r^{nc_R}}{\partial d_s} \\
&\leq \Lambda''(\pi_r^{nc_R})\frac{\partial \pi_r^{nc_R}}{\partial d_s}.
\end{aligned}$$

As $\Lambda''(\pi_r^{nc_R}) \leq 0$, we find that $\partial \pi_r^{nc_R}/\partial \lambda \leq 0$, $\partial \pi_r^{nc_R}/\partial d_r \geq 0$, and $\partial \pi_r^{nc}/\partial d_s \leq 0$. In addition,

$$\begin{aligned}
\frac{\partial \pi_r^{nc_R}}{\partial \lambda} &= \frac{1}{\lambda\left(\pi_r^{nc_R} - d_r\right)}\left[\Lambda''\left(\pi_r^{nc_R}\right)\right]^{-1} \\
&\geq \frac{1}{\lambda\left(\pi_r^{nc_R} - d_r\right)}\left[-\frac{1-\lambda}{\left(\pi_r^{nc_R} - d_r\right)^2} - \frac{\lambda g'\left(\pi_r^{nc_R}\right)^2}{\left(g\left(\pi_r^{nc_R}\right) - d_s\right)^2}\right]^{-1} \\
&= -\frac{\pi_r^{nc_R} - d_r}{1-\lambda},
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \pi_r^{nc_R}}{\partial d_r} &= -\frac{1-\lambda}{\left(\pi_r^{nc_R} - d_r\right)^2}\left[\Lambda''\left(\pi_r^{nc_R}\right)\right]^{-1} \\
&\leq -\frac{1-\lambda}{\left(\pi_r^{nc_R} - d_r\right)^2}\left[-\frac{1-\lambda}{\left(\pi_r^{nc_R} - d_r\right)^2} - \frac{\lambda g'\left(\pi_r^{nc_R}\right)^2}{\left(g\left(\pi_r^{nc_R}\right) - d_s\right)^2}\right]^{-1} \\
&= \lambda,
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial \pi_r^{nc_R}}{\partial d_s} &= \frac{1-\lambda}{\left(\pi_r^{nc_R} - d_r\right)\left(g\left(\pi_r^{nc_R}\right) - d_s\right)}\left[\Lambda''\left(\pi_r^{nc_R}\right)\right]^{-1} \\
&\geq \frac{1-\lambda}{\left(\pi_r^{nc_R} - d_r\right)\left(g\left(\pi_r^{nc_R}\right) - d_s\right)}\left[-\frac{1-\lambda}{\left(\pi_r^{nc_R} - d_r\right)^2} - \frac{\lambda g'\left(\pi_r^{nc_R}\right)^2}{\left(g\left(\pi_r^{nc_R}\right) - d_s\right)^2}\right]^{-1} \\
&= \frac{1-\lambda}{g'\left(\pi_r^{nc_R}\right)}.
\end{aligned}$$

We next show that $\pi_r^{nc_R} + \pi_s^{nc_R} = \pi_r^{nc_R} + g(\pi_r^{nc_R})$ is decreasing in λ and d_s but increasing in d_r. It is easy to see that

$$x + g\left(x\right) = -x + A\sqrt{x/b} - c\sqrt{bx} = b\sqrt{\frac{x}{b}}\left(\frac{A}{b} - c - \sqrt{\frac{x}{b}}\right) \leq \Pi^*\left(c\right);$$

hence, $g\left(x\right)$ is a concave curve in the triangle $\{(\pi_r, \pi_s) | \pi_r \geq 0, \pi_s \geq 0, \pi_r + \pi_s \leq \Pi^*(c)\}$ for all $x \in [0, \Pi^*(c)]$. Therefore, $g'(\pi_r^{nc_R}) \geq -1$ and

$$\begin{cases}
\dfrac{\partial\left[\pi_r^{nc_R} + g\left(\pi_r^{nc_R}\right)\right]}{\partial \lambda} = \left(1 + g'\left(\pi_r^{nc_R}\right)\right)\dfrac{\partial \pi_r^{nc_R}}{\partial \lambda} \leq 0, \\
\dfrac{\partial\left[\pi_r^{nc_R} + g\left(\pi_r^{nc_R}\right)\right]}{\partial d_s} = \left(1 + g'\left(\pi_r^{nc_R}\right)\right)\dfrac{\partial \pi_r^{nc_R}}{\partial d_s} \leq 0, \\
\dfrac{\partial\left[\pi_r^{nc_R} + g\left(\pi_r^{nc_R}\right)\right]}{\partial d_r} = \left(1 + g'\left(\pi_r^{nc_R}\right)\right)\dfrac{\partial \pi_r^{nc_R}}{\partial d_r} \geq 0.
\end{cases}$$

Thus, the theorem is proved. ■

B.3 Theorem 8 and Its Proof

Theorem 8 *When both suppliers adopt strategy C_S and both retailers adopt strategy C_R, we find that, if $d_r + d_s < (b+\alpha)[(A-bc)/(2b+\alpha)]^2$, then, in both supply chains, the firms can reach agreements with the pricing decisions as: $p_1^{c_R,c_R} = p_2^{c_R,c_R} = [A+(b+\alpha)c]/(2b+\alpha)$ and $w_1^{c_R,c_R} = w_2^{c_R,c_R} = c+\bar{\lambda}(A-bc)/(2b+\alpha)+(2b+\alpha)[(1-\bar{\lambda})d_s-\bar{\lambda}d_r]/[(b+\alpha)(A-bc)]$. Otherwise, the firms in each supply chain cannot reach any agreement.*

Proof. If all firms in both supply chains can reach agreements with $(p_1^{c_R,c_R}, w_1^{c_R,c_R})$ and $(p_2^{c_R,c_R}, w_2^{c_R,c_R})$ and there exists a Nash equilibrium, then, for $i = 1, 2$,

$$\begin{aligned}(p_i^{c_R,c_R}, w_i^{c_R,c_R}) \;&=\; \arg\max_{p_i,w_i}\{[A-bp_i+\alpha(p_{3-i}^{c_R,c_R}-p_i)](p_i-w_i)-d_r\}^{1-\bar{\lambda}} \\ &\quad\times\{[A-bp_i+\alpha(p_{3-i}^{c_R,c_R}-p_i)](w_i-c)-d_s\}^{\bar{\lambda}}.\end{aligned}$$

Since the total profit of R_i and S_i is $[A-bp_i+\alpha(p_{3-i}^{c_R,c_R}-p_i)](p_i-c)$, which is independent of w_i, an agreement is achievable if $\max_{p_i}[A-bp_i+\alpha(p_{3-i}^{c_R,c_R}-p_i)](p_i-c) \geq d_r+d_s$. When both supply chains reach agreements,

$$p_i^{c_R,c_R} = \arg\max_{p_i}\left(A-bp_i+\alpha\left(p_{3-i}^{c_R,c_R}-p_i\right)\right)(p_i-c) = \frac{1}{2}\left(\frac{A+\alpha p_{3-i}^{c_R,c_R}}{b+\alpha}+c\right);$$

thus, $p_i^{c_R,c_R}$ $(i = 1, 2)$ is given as in this theorem. Then,

$$\begin{aligned}w_i^{c_R,c_R} \;&=\; \arg\max_{w_i}\left[\left(A-b\frac{A+(b+\alpha)c}{2b+\alpha}\right)\left(\frac{A+(b+\alpha)c}{2b+\alpha}-w_i\right)-d_r\right]^{1-\bar{\lambda}} \\ &\quad\times\left[\left(A-b\frac{A+(b+\alpha)c}{2b+\alpha}\right)(w_i-c)-d_s\right]^{\bar{\lambda}} \\ &=\; c+\frac{\bar{\lambda}(A-bc)}{2b+\alpha}+\frac{(2b+\alpha)\left[\left(1-\bar{\lambda}\right)d_s-\bar{\lambda}d_r\right]}{(b+\alpha)(A-bc)};\end{aligned}$$

and supply chain agreements are achievable if

$$d_r+d_s < \left(A-b\frac{A+(b+\alpha)c}{2b+\alpha}\right)\left(\frac{A+(b+\alpha)c}{2b+\alpha}-c\right) = (b+\alpha)\left(\frac{A-bc}{2b+\alpha}\right)^2.$$

Otherwise, if the above inequality cannot be satisfied, then no agreement can be reached in any supply chain, according to our previous discussions. We examine if one supply chain can reach an agreement but the other cannot. Without loss of generality, we assume that S_1 and R_1 in the first supply chain reach an agreement $(\bar{p}_1, \bar{w}_1)$ whereas the second supply chain dissolves. Then, the demand faced by R_1 is $A-\bar{p}_1(b+\alpha)$, and

$$\begin{aligned}(\bar{p}_1, \bar{w}_1) \;&=\; \arg\max_{p_1,w_1}\left[(A-(b+\alpha)p_1)(p_1-w_1)-d_r\right]^{1-\bar{\lambda}} \\ &\quad\times\left[(A-(b+\alpha)p_1)(w_1-c)-d_s\right]^{\bar{\lambda}}.\end{aligned}$$

Thus, a necessary condition for achieving the agreement is

$$d_r + d_s < \max_{p_1} \left(A - (b+\alpha) p_1\right)(p_1 - c) = \frac{(A-(b+\alpha)c)^2}{4b+4\alpha}.$$

However,

$$\frac{A-bc}{2b+\alpha} - \frac{A-(b+\alpha)c}{2b+2\alpha} = \frac{\alpha\left(A+(b+\alpha)c\right)}{(2b+\alpha)(2b+2\alpha)} \geq 0,$$

which means that the necessary condition cannot hold if $d_r + d_s \geq (b+\alpha)[(A-bc)/(2b+\alpha)]^2$. ■

B.4 Theorem 9 and Its Proof

Theorem 9 *When both suppliers choose strategy C_S and both retailers use strategy NC_R, we find that, if $0 \leq d_r < (b+\alpha)(A-bc)^2/(2b+\alpha)^2$ and*

$$d_s < g^{nc_R,nc_R}\left(\max\left\{d_r, (b+\alpha)\left[\frac{A-bc}{2b+\alpha} \times \frac{2(b+\alpha)^2-\alpha^2}{(2b+3\alpha)b+2(b+\alpha)^2-\alpha^2}\right]^2\right\}\right), \tag{12}$$

where $g^{nc_R,nc_R}(x) \equiv (A/b-c)\sqrt{(b+\alpha)x} - x(2+\alpha/b)$, i.e., (d_r, d_s) is in areas (1) and (2) in Figure 2, then a symmetric equilibrium exists with the negotiated wholesale prices and optimal retail prices as $w_1^{nc_R,nc_R} = w_2^{nc_R,nc_R} = \hat{w}$ and $p_1^{nc_R,nc_R} = p_2^{nc_R,nc_R} = [A+\hat{w}(b+\alpha)]/(2b+\alpha)$, where $\hat{w}$ can be uniquely obtained by solving the following equation for w:

$$\frac{2(1-\lambda)\dfrac{\left[2(b+\alpha)^2-\alpha^2\right]}{(2b+\alpha)(2b+3\alpha)}\left(\dfrac{A-bw}{2b+\alpha}\right)}{(b+\alpha)\left(\dfrac{A-bw}{2b+\alpha}\right)^2 - d_r} = \frac{\lambda\left[-\dfrac{\left[2(b+\alpha)^2-\alpha^2\right]}{(2b+\alpha)(2b+3\alpha)}(w-c) + \dfrac{A-bw}{2b+\alpha}\right]}{(b+\alpha)\left(\dfrac{A-bw}{2b+\alpha}\right)(w-c) - d_s}. \tag{13}$$

Otherwise, if (d_r, d_s) is outside areas (1) and (2) in Figure 2, i.e., the condition in (12) does not hold, then the negotiations in both supply chains cannot end up with an agreement.

Proof. If the two supply chains reach agreements with negotiated wholesale prices $w_1^{nc_R,nc_R}$ and $w_2^{nc_R,nc_R}$, then there is a Nash equilibrium for the retailers' pricing decisions $(p_1^{nc_R,nc_R}, p_2^{nc_R,nc_R})$, i.e.,

$$\begin{aligned} p_i^{nc_R,nc_R} &= \arg\max_{p_i}[A - bp_i + \alpha\left(p_{3-i}^{nc_R,nc_R} - p_i\right)](p_i - w_i^{nc_R,nc_R}) \\ &= [w_i^{nc_R,nc_R} + (A + \alpha p_{3-i}^{nc_R,nc_R})/(b+\alpha)]/2. \end{aligned}$$

Therefore, for $i = 1, 2$,

$$p_i^{nc_R,nc_R} = \frac{A}{2b+\alpha} + \frac{2(b+\alpha)^2 w_i^{nc_R,nc_R}}{(2b+\alpha)(2b+3\alpha)} + \frac{(b+\alpha)\alpha w_{3-i}^{nc_R,nc_R}}{(2b+\alpha)(2b+3\alpha)};$$

and the retailers' and the suppliers' profits are

$$\begin{aligned}\pi_{r_i} &= (b+\alpha)\left(\frac{A}{2b+\alpha} - \frac{\left[2\,(b+\alpha)^2-\alpha^2\right] w_i^{nc_R,nc_R}}{(2b+\alpha)\,(2b+3\alpha)} + \frac{\alpha\,(b+\alpha)\,w_{3-i}^{nc_R,nc_R}}{(2b+\alpha)\,(2b+3\alpha)}\right)^2,\\ \pi_{s_i} &= (b+\alpha)\left(\frac{A}{2b+\alpha} - \frac{\left[2\,(b+\alpha)^2-\alpha^2\right] w_i^{nc_R,nc_R}}{(2b+\alpha)\,(2b+3\alpha)} + \frac{\alpha\,(b+\alpha)\,w_{3-i}^{nc_R,nc_R}}{(2b+\alpha)\,(2b+3\alpha)}\right)\left(w_i^{nc_R,nc_R}-c\right).\end{aligned}$$

Letting

$$\begin{aligned}f_1(x,y) &\equiv (b+\alpha)\left(\frac{A}{2b+\alpha} - \frac{2\,(b+\alpha)^2-\alpha^2}{(2b+\alpha)\,(2b+3\alpha)}x + \frac{\alpha\,(b+\alpha)}{(2b+\alpha)\,(2b+3\alpha)}y\right)^2,\\ f_2(x,y) &\equiv (b+\alpha)\left(\frac{A}{2b+\alpha} - \frac{2\,(b+\alpha)^2-\alpha^2}{(2b+\alpha)\,(2b+3\alpha)}x + \frac{\alpha\,(b+\alpha)}{(2b+\alpha)\,(2b+3\alpha)}y\right)(x-c)\,,\end{aligned}$$

we find that $\pi_{r_i} = f_1(w_i^{nc_R,nc_R}, w_{3-i}^{nc_R,nc_R})$ and $\pi_{s_i} = f_2(w_i^{nc_R,nc_R}, w_{3-i}^{nc_R,nc_R})$. Noting that a Nash equilibrium exists for the two negotiated results, we have $w_i^{nc_R,nc_R} = \arg\max_{w_i}(1-\lambda)\ln[f_1(w_i, w_{3-i}^{nc_R,nc_R}) - d_r] + \lambda\ln[f_2(w_i, w_{3-i}^{nc_R,nc_R}) - d_s]$. Therefore, $w_i^{nc_R,nc_R}$ satisfies the first order condition of the above function. For the symmetric equilibrium (i.e., $w^{nc_R,nc_R} \equiv w_1^{nc_R,nc_R} = w_2^{nc_R,nc_R}$), the first order condition is the equation in (13). Therefore, the symmetric agreements can be achieved if the following conditions are satisfied,

$$\begin{cases} d_s < (b+\alpha)\left(\dfrac{A-bw^{nc_R,nc_R}}{2b+\alpha}\right)\left(w^{nc_R,nc_R}-c\right),\\ d_r < (b+\alpha)\left(\dfrac{A-bw^{nc_R,nc_R}}{2b+\alpha}\right)^2,\\ w^{nc_R,nc_R} \le \dfrac{(2b+3\alpha)\,A + c\left[2\,(b+\alpha)^2-\alpha^2\right]}{(2b+3\alpha)\,b+\left[2\,(b+\alpha)^2-\alpha^2\right]}. \end{cases}$$

That is, (d_r, d_s) belongs to areas (1) and (2) in Figure 2; or equivalently, the condition in (12) is satisfied.

Next, we show that the condition is sufficient. If (d_r, d_s) belongs to areas (1) and (2) in Figure 2, then the expression

$$-\frac{2\,(1-\lambda)\,[2(b+\alpha)^2-\alpha^2][(A-bx)/(2b+\alpha)]}{(2b+\alpha)\,(2b+3\alpha)\,\{(b+\alpha)[(A-bx)/(2b+\alpha)]^2-d_r\}} \tag{14}$$

is decreasing in x when $bx < A-(2b+\alpha)\sqrt{d_r/(b+\alpha)}$, and it approaches $-\infty$ when $bx \to A-(2b+\alpha)\sqrt{d_r/(b+\alpha)}$. Moreover, the expression

$$\frac{\lambda\{-[2(b+\alpha)^2-\alpha^2](x-c)/[(2b+\alpha)\,(2b+3\alpha)] + (A-bx)/(2b+\alpha)\}}{(b+\alpha)[(A-bx)/(2b+\alpha)](x-c)-d_s} \tag{15}$$

is decreasing in x when $r_0 < x \leq \{A(2b+3\alpha)+c[2(b+\alpha)^2-\alpha^2]\}/\{b(2b+3\alpha)+2(b+\alpha)^2-\alpha^2\}$, where r_0 is the smaller solution of $(b+\alpha)(A-bx)(x-c)/(2b+\alpha)-d_s=0$. Since (d_r,d_s) belongs to areas (1) and (2) in Figure 2, we find that $r_0 < A-(2b+\alpha)\sqrt{d_r/(b+\alpha)}$; thus, the sum of expressions in (14) and (15), denoted by ζ, is decreasing in x when

$$r_0 < x \leq \min\left\{A-(2b+\alpha)\sqrt{d_r/(b+\alpha)}, \frac{(2b+3\alpha)A+\left[2(b+\alpha)^2-\alpha^2\right]c}{(2b+3\alpha)b+\left[2(b+\alpha)^2-\alpha^2\right]}\right\}.$$

Noting that

$$\begin{cases} \zeta \to +\infty, & \text{if } x \to r_0, \\ \zeta \to -\infty, & \text{if } x \to A-(2b+\alpha)\sqrt{d_r/(b+\alpha)}, \\ \zeta < 0, & \text{if } x \to \dfrac{(2b+3\alpha)A+\left[2(b+\alpha)^2-\alpha^2\right]c}{(2b+3\alpha)b+\left[2(b+\alpha)^2-\alpha^2\right]}, \end{cases}$$

we conclude that the equation in (13) has a unique solution, which induces a symmetric equilibrium.

Then, we show that if the condition in (12) does not hold, both supply chains dissolve. We prove it by contradiction. Since the equation in (12) does not hold, the firms in each supply chain cannot reach any agreement. If an agreement is reached in one supply chain whereas the other breaks up, without loss of generality, we assume that S_1 and R_1 can reach an agreement with wholesale price $\bar{w}_1$ whereas S_2 and R_2 cannot. For any given w_1, R_1's optimal retail price is $\bar{p}_1 = \arg\max_{p_1}[A-(b+\alpha)p_1](p_1-w_1) = [w_1+A/(b+\alpha)]/2$. Hence, S_1's and R_1's profits are

$$\pi_{r_1} = (b+\alpha)\left(\frac{A}{2b+2\alpha}-\frac{w_1}{2}\right)^2 \text{ and } \pi_{s_1} = \left(\frac{A}{b+\alpha}-c\right)\sqrt{\pi_{r_1}(b+\alpha)}-2\pi_{r_1}.$$

Since S_1 and R_1 reach an agreement with wholesale price $\bar{w}_1$, the following inequalities must hold: $\pi_{r_1} > d_r$ and $[A/(b+\alpha)-c]\sqrt{\pi_{r_1}(b+\alpha)}-2\pi_{r_1} > d_s$. Therefore, we have

$$\begin{cases} d_r < \dfrac{b+\alpha}{4}\left(\dfrac{A}{b+\alpha}-c\right)^2 \leq \dfrac{(b+\alpha)(A-bc)^2}{(2b+\alpha)^2}, \\ d_s < \left(\dfrac{A}{b+\alpha}-c\right)\sqrt{x(b+\alpha)}-2x\Big|_{x=\max\left\{d_r, \frac{b+\alpha}{16}\left(\frac{A}{b+\alpha}-c\right)^2\right\}}. \end{cases}$$

Since the condition in (12) does not hold,

$$d_s \geq g^{nc_R,nc_R}\left(\max\left\{d_r, (b+\alpha)\left(\frac{A-bc}{2b+\alpha}\times\frac{2(b+\alpha)^2-\alpha^2}{(2b+3\alpha)b+\left[2(b+\alpha)^2-\alpha^2\right]}\right)^2\right\}\right).$$

If

$$d_r \geq (b+\alpha)\left(\frac{A-bc}{2b+\alpha}\times\frac{2(b+\alpha)^2-\alpha^2}{(2b+3\alpha)b+\left[2(b+\alpha)^2-\alpha^2\right]}\right)^2,$$

then

$$d_r \geq \frac{(b+\alpha)}{4}\left(\frac{A-bc}{2b+\alpha}\right)^2 \geq \frac{(b+\alpha)}{16}\left(\frac{A}{b+\alpha}-c\right)^2,$$

and

$$\begin{aligned}
d_s &\geq g^{nc_R,nc_R}(d_r) \\
&= \left(\frac{A}{b+\alpha}-c\right)\sqrt{x(b+\alpha)}-2x\bigg|_{x=d_r}+\frac{\alpha}{b}\sqrt{d_r}\left(\frac{A}{\sqrt{(b+\alpha)}}-\sqrt{d_r}\right) \\
&> d_s+\frac{\alpha}{b}\sqrt{d_r}\left(\frac{A}{\sqrt{(b+\alpha)}}-\sqrt{d_r}\right) \\
&> d_s+\frac{\alpha}{b}\sqrt{d_r}\left(\frac{A}{\sqrt{(b+\alpha)}}-\sqrt{\frac{(b+\alpha)}{4}\left(\frac{A}{b+\alpha}-c\right)^2}\right) \\
&\geq d_s,
\end{aligned}$$

which is a contradiction. Hence, the next inequality must hold

$$d_r < (b+\alpha)\left(\frac{A-bc}{2b+\alpha}\times\frac{2(b+\alpha)^2-\alpha^2}{(2b+3\alpha)b+\left[2(b+\alpha)^2-\alpha^2\right]}\right)^2.$$

Therefore,

$$\begin{aligned}
d_s &\geq g^{nc_R,nc_R}\left((b+\alpha)\left(\frac{A-bc}{2b+\alpha}\times\frac{2(b+\alpha)^2-\alpha^2}{(2b+3\alpha)b+\left[2(b+\alpha)^2-\alpha^2\right]}\right)^2\right) \\
&= (b+\alpha)\frac{(A-bc)^2}{2b+\alpha}\frac{\left(2(b+\alpha)^2-\alpha^2\right)(2b+3\alpha)}{\left[(2b+3\alpha)b+2(b+\alpha)^2-\alpha^2\right]^2} \\
&\geq (b+\alpha)\frac{(A-(b+\alpha)c)^2}{2b+\alpha}\frac{\left(2(b+\alpha)^2-\alpha^2\right)(2b+3\alpha)}{\left[(2b+3\alpha)b+2(b+\alpha)^2-\alpha^2\right]^2} \\
&\geq (b+\alpha)(A-(b+\alpha)c)^2\frac{\left(2(b+\alpha)^2-\alpha^2\right)}{\left[(2b+3\alpha)b+2(b+\alpha)^2-\alpha^2\right]^2} \\
&\geq (b+\alpha)\left(\frac{A-(b+\alpha)c}{2b+2\alpha}\right)^2\frac{\left(2(b+\alpha)^2-\alpha^2\right)}{(2b+3\alpha)b+2(b+\alpha)^2-\alpha^2} \\
&\geq \frac{b+\alpha}{8}\left(\frac{A-(b+\alpha)c}{b+\alpha}\right)^2 \\
&= \max_x\left(\frac{A}{b+\alpha}-c\right)\sqrt{(b+\alpha)x}-2x \\
&> d_s,
\end{aligned}$$

which is also a contradiction. Thus, both supply chains dissolve if the equation in (12) does not hold. ■

B.5 Lemma 1 and Its Proof

Lemma 1 *Letting* $g^{c_R,nc_R}(x) \equiv g^{nc_R,nc_R}(x)(2b^2+3\alpha b)/(2b^2+4\alpha b+\alpha^2)$, *we find that if*

$$d_r < (b+\alpha)\left[(A-bc)/(2b+\alpha)\right]^2 \text{ and } d_s < \max_{x\geq d_r} g^{c_R,nc_R}(x), \tag{16}$$

i.e., (d_r, d_s) *belongs to areas (1) and (2) in Figure 3, then solving the following equation for* y *yields a unique solution, which is* $\pi_{r_2}^{c_R,nc_R}$,

$$(1-\lambda)/(y-d_r) + \lambda(\partial g^{c_R,nc_R}(y)/\partial y)/(g^{c_R,nc_R}(y)-d_s) = 0. \tag{17}$$

We note that $\pi_{r_2}^{c_R,nc_R}$ *depends on* (λ, d_r, d_s), *and* $-(\pi_{r_2}^{c_R,nc_R}-d_r)/(1-\lambda) \leq \partial\pi_{r_2}^{c_R,nc_R}/\partial\lambda \leq 0$, $\lambda \geq \partial\pi_{r_2}^{c_R,nc_R}/\partial d_r \geq 0$, *and* $(1-\lambda)/(g^{c_R,nc_R})'(\pi_{r_2}^{c_R,nc_R}) \leq \partial\pi_{r_2}^{c_R,nc_R}/\partial d_s \leq 0$.

Proof. We learn from the proof of Theorem 7 (in online Appendix B.2) that the concavity property of g is sufficient to show that $-(\pi_r^{nc_R}-d_r)/(1-\lambda) \leq \partial\pi_r^{nc_R}/\partial\lambda \leq 0$, $\lambda \geq \partial\pi_r^{nc_R}/\partial d_r \geq 0$, $(1-\lambda)/g'(\pi_r^{nc_R}) \leq \partial\pi_r^{nc_R}/\partial d_s \leq 0$, and $\pi_r^{nc_R} + g(\pi_r^{nc_R})$ is decreasing in λ. Similarly, since g^{c_R,nc_R} is a concave function, by comparing the equations in (11) and (17), we find that $-(\pi_{r_2}^{c_R,nc_R}-d_r)/(1-\lambda) \leq \partial\pi_{r_2}^{c_R,nc_R}/\partial\lambda \leq 0$, $\lambda \geq \partial\pi_{r_2}^{c_R,nc_R}/\partial d_r \geq 0$, $(1-\lambda)/(g^{c_R,nc_R})'(\pi_{r_2}^{c_R,nc_R}) \leq \partial\pi_{r_2}^{c_R,nc_R}/\partial d_s \leq 0$, and $\pi_{r_2}^{c_R,nc_R} + g^{c_R,nc_R}(\pi_{r_2}^{c_R,nc_R})$ is decreasing in λ. ■

B.6 Theorem 10 and Its Proof

Theorem 10 *When both suppliers adopt strategy* C_S, *retailer* R_1 *uses strategy* C_R *but retailer* R_2 *adopts strategy* NC_R, *the firms in each supply chain can reach an agreement if the condition in (16) holds, i.e.,* (d_r, d_s) *belongs to areas (1) and (2) in Figure 3. The pricing decisions* $(w_1^{c_R,nc_R}, p_1^{c_R,nc_R}; w_2^{c_R,nc_R}, p_2^{c_R,nc_R})$ *in both supply chains are given as in Table 2.*

If $d_r + d_s < [A-c(b+\alpha)]^2/(4b+4\alpha)$ *and the condition in (16) does not hold, i.e.,* (d_r, d_s) *is in area (3) in Figure 3, then retailer* R_1 *and supplier* S_1 *can reach an agreement with negotiated prices as* $\bar{p}_1^{c_R,nc_R} = [A/(b+\alpha)+c]/2$ *and* $\bar{w}_1^{c_R,nc_R} = c+\bar{\lambda}[A-c(b+\alpha)]/[4(b+\alpha)]+[(1-\bar{\lambda})d_s-\bar{\lambda}d_r]/[A-c(b+\alpha)]$, *whereas retailer* R_2 *and supplier* S_2 *cannot complete their transaction. Otherwise, if* $d_r + d_s \geq [A-c(b+\alpha)]^2/(4b+4\alpha)$ *and the condition in (16) does not hold, then no supply chain has a successful transaction.*

Proof. We first examine the equilibrium that both supply chains can reach agreements. For any negotiated wholesale price w_2 by R_2 and S_2, there is a Nash equilibrium for the negotiation results $(w_1^{c_R,nc_R}, p_1^{c_R,nc_R})$ and R_2's optimal decision $p_2^{c_R,nc_R}$, i.e.,

$$p_2^{c_R,nc_R} = \arg\max_{p_2}(p_2-w_2)\left(A-bp_2+\alpha\left(p_1^{c_R,nc_R}-p_2\right)\right) = \frac{1}{2}\left(w_2+\frac{A+\alpha p_1^{c_R,nc_R}}{b+\alpha}\right),$$

and

$$\begin{aligned}\left(w_1^{c_R,nc_R}, p_1^{c_R,nc_R}\right) &= \arg\max_{w_1,p_1} \left[(p_1 - w_1)\left(A - bp_1 + \alpha\left(p_2^{c_R,nc_R} - p_1\right)\right) - d_r\right]^{1-\bar{\lambda}} \\ &\quad \times \left[(w_1 - c)\left(A - bp_1 + \alpha\left(p_2^{c_R,nc_R} - p_1\right)\right) - d_s\right]^{\bar{\lambda}}.\end{aligned}$$

It is easy to see that

$$\begin{cases} p_1^{c_R,nc_R} = \dfrac{1}{2}\left(c + \dfrac{A + \alpha p_2^{c_R,nc_R}}{b+\alpha}\right), \\ w_1^{c_R,nc_R} = c + \dfrac{\bar{\lambda}}{2}\left(\dfrac{A + \alpha p_2^{c_R,nc_R}}{b+\alpha} - c\right) + \dfrac{2\left[\left(1-\bar{\lambda}\right)d_s - \bar{\lambda} d_r\right]}{A + \alpha p_2^{c_R,nc_R} - (b+\alpha)c}, \end{cases}$$

and R_1's and S_1's profits are

$$\begin{cases} \pi_{r_1}^{c_R,nc_R} = d_r + \left(1-\bar{\lambda}\right)\left[\dfrac{b+\alpha}{4}\left(\dfrac{A + \alpha p_2^{c_R,nc_R}}{b+\alpha} - c\right)^2 - d_r - d_s\right], \\ \pi_{s_1}^{c_R,nc_R} = d_s + \bar{\lambda}\left[\dfrac{b+\alpha}{4}\left(\dfrac{A + \alpha p_2^{c_R,nc_R}}{b+\alpha} - c\right)^2 - d_r - d_s\right]. \end{cases}$$

Hence, the agreement can be achieved, if $d_r + d_s < (b+\alpha)[(A + \alpha p_2^{c_R,nc_R})/(b+\alpha) - c]^2/4$; and, we find that

$$\begin{cases} p_1^{c_R,nc_R} = \dfrac{A}{2b+\alpha} + \dfrac{2(b+\alpha)^2 c}{(2b+\alpha)(2b+3\alpha)} + \dfrac{(b+\alpha)\alpha w_2}{(2b+\alpha)(2b+3\alpha)}, \\ p_2^{c_R,nc_R} = \dfrac{A}{2b+\alpha} + \dfrac{2(b+\alpha)^2 w_2}{(2b+\alpha)(2b+3\alpha)} + \dfrac{(b+\alpha)\alpha c}{(2b+\alpha)(2b+3\alpha)}. \end{cases}$$

Therefore, we calculate the profits of R_2 and S_2 as

$$\pi_{r_2}^{c_R,nc_R} = (b+\alpha)\left(\frac{A - bw_2}{2b+\alpha} - \frac{(b+\alpha)\alpha(w_2 - c)}{(2b+\alpha)(2b+3\alpha)}\right)^2 \text{ and } \pi_{s_2}^{c_R,nc_R} = g^{c_R,nc_R}\left(\pi_{r_2}^{c_R,nc_R}\right).$$

Hence, R_2 and S_2 can reach an agreement if the condition in (16) is satisfied. The negotiated wholesale price $w_2^{c_R,nc_R}$ is

$$\begin{aligned} w_2^{c_R,nc_R} &= \arg\max_{w_2}\left[(b+\alpha)\left(\frac{A - bw_2}{2b+\alpha} - \frac{(b+\alpha)\alpha(w_2-c)}{(2b+\alpha)(2b+3\alpha)}\right)^2 - d_r\right]^{1-\lambda} \\ &\quad \times \left[(b+\alpha)(w_2 - c)\left(\frac{A - bw_2}{2b+\alpha} - \frac{(b+\alpha)\alpha(w_2-c)}{(2b+\alpha)(2b+3\alpha)}\right) - d_s\right]^{\lambda}, \end{aligned}$$

or equivalently, according to Lemma 1 (in online Appendix B.5), the profit of R_2 is $\pi_{r_2}^{c_R,nc_R} = \arg\max_x (x - d_r)^{1-\lambda}(g^{c_R,nc_R}(x) - d_s)^{\lambda}$. Thus, we obtain $w_2^{c_R,nc_R}$ as in this theorem.

Noting that

$$\begin{aligned}
& \frac{1}{4}\left(\frac{A+\alpha p_2^{c_R,nc_R}}{b+\alpha}-c\right)^2-\frac{\left(\pi_{r2}^{c_R,nc_R}+g^{c_R,nc_R}\left(\pi_{r2}^{c_R,nc_R}\right)\right)}{b+\alpha} \\
= & \frac{\left(w_2^{c_R,nc_R}-c\right)\left[\alpha A+(b+\alpha)^2 w_2^{c_R,nc_R}-\left(b^2+3\alpha b+\alpha^2\right)c\right]}{(2b+\alpha)(2b+3\alpha)} \\
\geq & \frac{\left(w_2^{c_R,nc_R}-c\right)}{(2b+\alpha)(2b+3\alpha)}\left[\alpha bc+(b+\alpha)^2 c-\left(b^2+3\alpha b+\alpha^2\right)c\right] \\
= & 0,
\end{aligned}$$

we find that $d_r+d_s<(b+\alpha)[(A+\alpha p_2^{c_R,nc_R})/(b+\alpha)-c]^2/4$ is always satisfied if the condition in (16) holds. Therefore, both supply chains can reach agreements if the condition in (16) is satisfied.

Next, we examine if there is an equilibrium when only one supply chain reaches an agreement when the condition in (16) does not hold. There are two possible equilibria: only R_1 and S_1 reach an agreement, or only R_2 and S_2 reach an agreement.

1. If R_1 and S_1 reach an agreement whereas R_2 and S_2 do not, then $p_2=0$ and the demand faced by R_1 is $A-(b+\alpha)p_1$. Assuming that $(\bar{p}_1^{c_R,nc_R},\bar{w}_1^{c_R,nc_R})$ is the negotiated result by R_1 and S_1, we find that

$$\begin{aligned}
\left(\bar{p}_1^{c_R,nc_R},\bar{w}_1^{c_R,nc_R}\right) = & \arg\max_{p_1,w_1}\left[\left(p_1-w_1\right)\left(A-(b+\alpha)p_1\right)-d_r\right]^{1-\bar{\lambda}} \\
& \times\left[\left(w_1-c\right)\left(A-(b+\alpha)p_1\right)-d_s\right]^{\bar{\lambda}}.
\end{aligned}$$

   Hence, an agreement is achievable if $d_r+d_s<(b+\alpha)\{[A-c(b+\alpha)]/(2b+2\alpha)\}^2$. When an agreement is achievable, the negotiated prices are $\bar{p}_1^{c_R,nc_R}$ and $\bar{w}_1^{c_R,nc_R}$, as given in this theorem. The profits of R_1 and S_1 are

$$\begin{cases}
\bar{\pi}_{r1}^{c_R,nc_R}=d_r+\left(1-\bar{\lambda}\right)\left[(b+\alpha)\left(\dfrac{A-(b+\alpha)c}{2b+2\alpha}\right)^2-d_r-d_s\right], \\
\bar{\pi}_{s1}^{c_R,nc_R}=d_s+\bar{\lambda}\left[(b+\alpha)\left(\dfrac{A-(b+\alpha)c}{2b+2\alpha}\right)^2-d_r-d_s\right].
\end{cases}$$

   Therefore, if $d_r+d_s<[A-c(b+\alpha)]^2/(4b+4\alpha)$ and the condition in (16) does not hold, then R_1 and S_1 can reach an agreement whereas R_2 and S_2 break up.

2. If R_2 and S_2 reach an agreement whereas R_1 and S_1 break up, then $p_1=0$ and the demand faced by R_2 is $A-p_2(b+\alpha)$. For any given negotiated result $\bar{w}_2$ by R_2 and S_2, $\bar{p}_2^{c_R,nc_R}$ is the optimal retail price for R_2. Then,

$$\bar{p}_2^{c_R,nc_R}=\arg\max_{p_2}\left(A-(b+\alpha)p_2\right)\left(p_2-\bar{w}_2\right)=\frac{1}{2}\left(\frac{A}{b+\alpha}+\bar{w}_2\right),$$

and the profits of R_2 and S_2 are

$$\bar{\pi}_{r_2}^{c_R,nc_R} = (b+\alpha)\left(\frac{A-(b+\alpha)\,\bar{w}_2}{2b+2\alpha}\right)^2 \text{ and } \bar{\pi}_{s_2}^{c_R,nc_R} = \frac{1}{2}\left(\bar{w}_2 - c\right)\left(A-(b+\alpha)\,\bar{w}_2\right).$$

Thus, the negotiated wholesale price $\bar{w}_2^{c_R,nc_R}$ is

$$\bar{w}_2^{c_R,nc_R} = \arg\max_{w_2}\left\{\left[(b+\alpha)\left(\frac{A-(b+\alpha)\,w_2}{2b+2\alpha}\right)^2 - d_r\right]^{1-\lambda}\left[\frac{1}{2}\left(w_2-c\right)\left(A-(b+\alpha)\,w_2\right)-d_s\right]^{\lambda}\right\}.$$

Defining $\bar{g}^{c_R,nc_R}(x) \equiv \sqrt{(b+\alpha)\,x}[A/(b+\alpha)-c]-2x$, we find that $\bar{g}^{c_R,nc_R}(x)$ is a concave function and $\bar{\pi}_{s_2}^{c_R,nc_R} = \bar{g}^{c_R,nc_R}(\bar{\pi}_{r_2}^{c_R,nc_R})$. Therefore, an agreement is achievable only if $d_r < (b+\alpha)\{[A-c(b+\alpha)]/(2b+2\alpha)\}^2$ and $d_s < \max_{x\geq d_r} \bar{g}^{c_R,nc_R}(x)$. Note that, when $x < [A-c(b+\alpha)]^2/(4b+4\alpha)$,

$$\left(2b^2+4\alpha b+\alpha^2\right)\left(g^{c_R,nc_R}(x) - \bar{g}^{c_R,nc_R}(x)\right) = \sqrt{(b+\alpha)\,x}\left(\frac{\left(\alpha b+2\alpha^2\right)A}{(b+\alpha)} + \left(\alpha b+\alpha^2\right)c\right) - \alpha^2 x \geq 0.$$

Therefore, the condition in (16) always holds when R_2 and S_2 can reach an agreement. Thus, R_1 and S_1 also reach an agreement, which means that there does not exist an equilibrium when R_2 and S_2 reach an agreement whereas R_1 and S_1 break up.

In summary, if the condition in (16) holds, then both of the two supply chains can reach agreements. If $d_r+d_s < [A-c(b+\alpha)]^2/(4b+4\alpha)$ and the condition in (16) does not hold, then R_1 and S_1 can reach an agreement whereas R_2 and S_2 break up. Otherwise, there is no agreement in any supply chain. ∎

B.7 Theorem 11 and Its Proof

Theorem 11 *If a supplier adopts strategy NC_S, then his retailer also chooses strategy NC_R regardless of what strategy is used in the other supply chain. Moreover, we can derive the condition under which the supplier and the retailer in each supply chain can reach an agreement, and also compute all firms' profits, as presented in Table 3 in which*

$$\frac{\lambda\left[\left(g^{c_R,nc_R}\right)'(h_1(x)) + h_2(x)/(2\sqrt{h_1(x)})\right]}{g^{c_R,nc_R}(h_1(x)) + h_2(x)\sqrt{h_1(x)} - d_s} + \frac{1-\lambda}{h_1(x)-d_r} = 0, \tag{18}$$

where $w_1^{nc_S,nc_R} \equiv \{c+[A(2b+3\alpha)+w_2^{nc_S,nc_R}(\alpha b+\alpha^2)]/(2b^2+4\alpha b+\alpha^2)\}/2$, and the functions $h_1(x)$, $h_2(x)$, $g^{nc_S,nc_R}(x,y)$, and the coefficients $(\eta_1,\eta_2,\eta_3,\eta_4,\eta_5,\eta_6)$ are defined as in (19) and (20) in the proof of this theorem.

Proof. When both suppliers use strategy NC_S, we find that for retailer R_1, strategy NC_R dominates strategy C_R. Given wholesale prices w_1 and w_2, if retailer R_1 adopts strategy NC_R, then she sets a retail price p_1 to maximize $\pi_{r_1} = (A-bp_1+\alpha\,(p_2-p_1))\,(p_1-w_1)$. If retailer R_1 uses strategy NC_R, then p_1 is set to maximize $[(A-bp_1+\alpha\,(p_2-p_1))\,(p_1-w_1)-d_r]^{1-\hat{\lambda}}[(A-bp_1+\alpha\,(p_2-p_1))\,(w_1-c)-d_s]^{\hat{\lambda}}$. Since retail prices p_1 and p_2 are determined concurrently, retailer R_1 is always better off from

Pricing strategy	Condition for successful transactions	Profits
$(S_1, S_2) = (NC_S, NC_S)$	$\eta_1 \geq d_s$ and $\eta_2 \geq d_r$	$\begin{cases} \pi_{s_1}^{nc_S,nc_S} = \pi_{s_2}^{nc_S,nc_S} = \eta_1, \\ \pi_{r_1}^{nc_S,nc_S} = \pi_{r_2}^{nc_S,nc_S} = \eta_2. \end{cases}$
$(S_1, S_2) = (NC_S, C_S)$ $(R_2) = (C_R)$	$\eta_3 \geq d_s$ and $\eta_4 \geq d_r$	$\begin{cases} (\pi_{s_1}^{nc_S,c_R}, \pi_{r_1}^{nc_S,c_R}) = (\eta_3, \eta_4), \\ (\pi_{s_2}^{nc_S,c_R}, \pi_{r_2}^{nc_S,c_R}) = (d_s + \bar{\lambda}\eta_5, d_r + (1-\bar{\lambda})\,\eta_5). \end{cases}$
$(S_1, S_2) = (NC_S, C_S)$ $(R_2) = (NC_R)$	The equation in (18) has a solution $x = w_2^{nc_S,nc_R}$, $\pi_{s_1}^{nc_S,nc_R} \geq d_s$ and $\pi_{r_i}^{nc_S,nc_R} \geq d_r$	$\begin{cases} (\pi_{r_1}^{nc_S,nc_R}, \pi_{s_1}^{nc_S,nc_R}) \\ = (\eta_6^2(b+\alpha), \eta_6(b+\alpha)(w_1^{nc_S,nc_R} - c)), \\ (\pi_{r_2}^{nc_S,nc_R}, \pi_{s_2}^{nc_S,nc_R}) \\ = (h(w_2^{nc_S,nc_R}), g^{nc_S,nc_R}(h(w_2^{nc_S,nc_R}), w_1^{nc_S,nc_R})); \end{cases}$

Table 3: A summary of our results when suppliers make their wholesale prcing strategies.

using strategy NC_R to determine retail price p_1. Similarly, for retailer R_2, strategy NC_R dominates strategy C_R. Hence, both retailers should choose strategy NC_R. For any given wholesale prices w_1 and w_2, the retailers' pricing decisions in Nash equilibrium ($p_i^{nc_S,nc_S}(w_1, w_2)$, $i = 1, 2$) satisfy

$$\begin{aligned} p_i^{nc_S,nc_S}(w_1, w_2) &= \arg\max_{p_i} \left(A - bp_i + \alpha\left(p_{3-i}^{nc_S,nc_S}(w_1, w_2) - p_i\right)\right)(p_i - w_i) \\ &= \frac{1}{2}\left(w_i + \frac{A + \alpha p_{3-i}^{nc_S,nc_S}(w_1, w_2)}{b + \alpha}\right). \end{aligned}$$

Therefore,

$$p_i^{nc_S,nc_S}(w_1, w_2) = \frac{A}{2b + \alpha} + \frac{2\,(b+\alpha)^2\, w_i}{(2b+\alpha)\,(2b+3\alpha)} + \frac{(b+\alpha)\,\alpha w_{3-i}}{(2b+\alpha)\,(2b+3\alpha)};$$

and the suppliers' profits are

$$\pi_{s_i} = \frac{b+\alpha}{2b+\alpha}\left(A - \frac{\left[2\,(b+\alpha)^2 - \alpha^2\right] w_i}{2b+3\alpha} + \frac{\alpha\,(b+\alpha)\, w_{3-i}}{2b+3\alpha}\right)(w_i - c)\,.$$

Hence, the Nash equilibrium for the suppliers' pricing decisions ($w_i^{nc_S,nc_S}$, $i = 1, 2$) satisfy

$$\begin{aligned} w_i^{nc_S,nc_S} &= \arg\max_{w_i}\left(A - \frac{\left[2\,(b+\alpha)^2 - \alpha^2\right] w_i}{2b+3\alpha} + \frac{\alpha\,(b+\alpha)\, w_{3-i}^{nc_S,nc_S}}{2b+3\alpha}\right)(w_i - c) \\ &= \frac{2b+3\alpha}{4b^2 + 7\alpha b + \alpha^2} A + \frac{2b^2 + 4\alpha b + \alpha^2}{4b^2 + 7\alpha b + \alpha^2} c. \end{aligned}$$

Therefore, the profits of the suppliers and the retailers are

$$\begin{cases} \pi_{s_i}^{nc_S,nc_S} = \eta_1 \equiv (A - bc)^2\,(b+\alpha)\left(2b^2 + 4\alpha b + \alpha^2\right)(2b+3\alpha)\,/[(2b+\alpha)\left(4b^2 + 7\alpha b + \alpha^2\right)^2], \\ \pi_{r_i}^{nc_S,nc_S} = \eta_2 \equiv (A - bc)^2\,(b+\alpha)(2b^2 + 4\alpha b + \alpha^2)^2/[(2b+\alpha)^2\,(4b^2 + 7\alpha b + \alpha^2)^2], \end{cases}$$

and the condition for all firms to stay in supply chains are given in Table 3.

When supplier S_1 determines the wholesale price individually whereas supplier S_2 chooses to negotiate the wholesale price, and R_2 uses strategy C_R, similar to the case of (NC_S, NC_S), we can show that retailer R_1 always chooses strategy NC_R. Then, for any given value of w_1, retailer R_1's optimal retail price $p_1^{nc_S,c_R}(w_1)$ and the negotiation result $(w_2^{nc_S,c_R}(w_1), p_2^{nc_S,c_R}(w_1))$ is a Nash equilibrium as follows

$$\begin{aligned}
p_1^{nc_S,c_R}(w_1) &= \arg\max_{p_1}(p_1 - w_1)(A - bp_1 + \alpha(p_2^{nc_S,c_R}(w_1) - p_1)) \\
&= \frac{1}{2}\left(w_1 + \frac{A + \alpha p_2^{nc_S,c_R}(w_1)}{b+\alpha}\right), \\
(w_2^{nc_S,c_R}(w_1), p_2^{nc_S,c_R}(w_1)) &= \arg\max_{(w_2,p_2)} \left[(w_2 - c)\left(A - bp_2 + \alpha(p_1^{nc_S,c_R}(w_1) - p_2)\right) - d_s\right]^{\bar{\lambda}} \\
&\quad \times \left[(p_2 - w_2)(A - bp_2 + \alpha(p_1^{nc_S,c_R}(w_1) - p_2)) - d_r\right]^{1-\bar{\lambda}}.
\end{aligned}$$

The negotiation between S_2 and R_2 is successful if

$$\begin{aligned}
d_s + d_r &\leq \max_{p_2} (p_2 - c)\left(A - bp_2 + \alpha(p_1^{nc_S,c_R}(w_1) - p_2)\right) \\
&= \frac{b+\alpha}{4}\left(\frac{A + \alpha p_1^{nc_S,c_R}(w_1)}{b+\alpha} - c\right)^2,
\end{aligned}$$

the negotiation result $p_2^{nc_S,c_R}(w_1)$ and the optimal retail price $p_1^{nc_S,c_R}(w_1)$ are

$$\begin{cases}
p_1^{nc_S,c_R}(w_1) = \dfrac{A}{2b+\alpha} + \dfrac{2\,(b+\alpha)^2\, w_1}{(2b+\alpha)\,(2b+3\alpha)} + \dfrac{(b+\alpha)\,\alpha c}{(2b+\alpha)\,(2b+3\alpha)}, \\
p_2^{nc_S,c_R}(w_1) = \dfrac{A}{2b+\alpha} + \dfrac{2\,(b+\alpha)^2\, c}{(2b+\alpha)\,(2b+3\alpha)} + \dfrac{(b+\alpha)\,\alpha w_1}{(2b+\alpha)\,(2b+3\alpha)}.
\end{cases}$$

The profit of S_1 is $(b+\alpha)(w_1-c)\{(A-bw_1)/(2b+\alpha) - [\alpha(b+\alpha)(w_1-c)]/[(2b+\alpha)(2b+3\alpha)]\}$; hence, S_1's optimal wholesale price is

$$w_1^{nc_S,c_R} = \frac{1}{2}\left(\frac{2b+3\alpha}{2b^2+4\alpha b+\alpha^2}A + \frac{2b^2+5\alpha b+2\alpha^2}{2b^2+4\alpha b+\alpha^2}c\right);$$

the profits of S_1 and R_1 are

$$\begin{cases}
\pi_{s_1}^{nc_S,c_R} = \eta_3 \equiv b(b+\alpha)\,(2b+3\alpha)\,\Pi^*(c)/[(2b+\alpha)(2b^2+4\alpha b+\alpha^2)], \\
\pi_{r_1}^{nc_S,c_R} = \eta_4 \equiv b\,(b+\alpha)\,\Pi^*(c)/\,(2b+\alpha)^2;
\end{cases}$$

and the condition for S_1 and R_1 to stay in the supply chain is given in Table 3. Further, when the condition under S_1 and R_1 to complete their transaction is satisfied, we have

$$\begin{aligned}
&\frac{b+\alpha}{4}\left(\frac{A + \alpha p_1^{nc_S,c_R}(w_1^{nc_S,c_R})}{b+\alpha} - c\right)^2 \\
= \;& \frac{(b+\alpha)\,b}{(2b+\alpha)^2}\left(\frac{4b^2+9\alpha b+3\alpha^2}{2b^2+4\alpha b+\alpha^2}\right)^2 \Pi^*(c) \\
\geq \;& \frac{b+\alpha}{2b+\alpha}\,\frac{(2b+3\alpha)\,b}{2b^2+4\alpha b+\alpha^2}\Pi^*(c) + \frac{(b+\alpha)\,b}{(2b+\alpha)^2}\Pi^*(c) \\
\geq \;& d_s + d_r,
\end{aligned}$$

i.e., S_2 and R_2 can reach an agreement if both S_1 and R_1 stay in the supply chain. In summary, if the condition for scenario $[(S_1, S_2) = (NC_S, C_S); (R_2) = (C_R)]$ is satisfied, then S_1 and R_1 stay in the supply chain, and S_2 and R_2 can reach an agreement; and, all firms' profits $(\pi_{s_1}^{nc_S,c_R}, \pi_{r_1}^{nc_S,c_R}; \pi_{s_2}^{nc_S,c_R}, \pi_{r_2}^{nc_S,c_R})$ are obtained as in Table 3, in which

$$\begin{aligned} \eta_5 &\equiv \frac{(b+\alpha)\,b}{(2b+\alpha)^2}\left(\frac{4b^2+9\alpha b+3\alpha^2}{2b^2+4\alpha b+\alpha^2}\right)^2 \Pi^*(c) - d_s - d_r \\ &= \eta_4 \left(\frac{4b^2+9\alpha b+3\alpha^2}{2b^2+4\alpha b+\alpha^2}\right)^2 - d_s - d_r. \end{aligned}$$

When S_1 determines the wholesale price individually whereas S_2 chooses to negotiate the wholesale price, and R_2 uses strategy NC_R, similar to the case of (NC_S, NC_S), we find that retailer R_1 always uses strategy NC_R. Then, for any given w_1 and w_2, similar to the proof of Theorem 9 (in online Appendix B.4), the Nash equilibrium for the retailers' pricing decisions are

$$p_i^{nc_S,nc_R}(w_1, w_2) = \frac{A}{2b+\alpha} + \frac{2\,(b+\alpha)^2\, w_i}{(2b+\alpha)\,(2b+3\alpha)} + \frac{(b+\alpha)\,\alpha w_{3-i}}{(2b+\alpha)\,(2b+3\alpha)},$$

and all firms' profits are $\pi_{r_i}(w_1, w_2) = f_1(w_1, w_2)$, $\pi_{s_i}(w_1, w_2) = f_2(w_1, w_2)$. The negotiation result $w_2^{nc_S,nc_R}$ and S_1's optimal wholesale price $w_1^{nc_S,nc_R}$ are in Nash equilibrium as follows:

$$\begin{aligned} w_1^{nc_S,nc_R} &= \frac{1}{2}\left(c + \frac{2b+3\alpha}{2b^2+4\alpha b+\alpha^2}A + \frac{\alpha b+\alpha^2}{2b^2+4\alpha b+\alpha^2} w_2^{nc_S,nc_R}\right), \\ w_2^{nc_S,nc_R} &= \arg\max_{w_2} \left[\pi_{s_2}(w_1^{nc_S,nc_R}, w_2) - d_s\right]^{\lambda} \left[\pi_{r_2}(w_1^{nc_S,nc_R}, w_2) - d_r\right]^{1-\lambda}. \end{aligned}$$

Note that $\pi_{s_2}(w_1^{nc_S,nc_R}, w_2) = g^{nc_S,nc_R}(\pi_{r_2}(w_1^{nc_S,nc_R}, w_2), w_1^{nc_S,nc_R})$, in which $g^{nc_S,nc_R}(x,y) = g^{c_R,nc_R}(x) + \sqrt{(b+\alpha)\,x}[\alpha(b+\alpha)(y-c)]/[(2b+\alpha)(2b+3\alpha)]$, S_1 and R_1 can reach an agreement if

$$d_r < (b+\alpha)\left(\frac{A-bc}{2b+\alpha} + \frac{\alpha\,(b+\alpha)}{(2b+\alpha)\,(2b+3\alpha)}\,(w_1^{nc_S,nc_R} - c)\right)^2 \text{ and } d_s < \max_{x \geq d_r} g^{nc_S,nc_R}(x, w_1^{nc_S,nc_R}),$$

and R_2's profit is $\pi_{r_2}^{nc_S,nc_R} = \arg\max_x [g^{nc_S,nc_R}(x, w_1^{nc_S,nc_R}) - d_s]^{\lambda}(x - d_r)^{1-\lambda}$, i.e., $\pi_{r_2}^{nc_S,nc_R}$ satisfies the first order condition:

$$\frac{\lambda(\partial g^{nc_S,nc_R}/\partial x)(\pi_{r_2}^{nc_S,nc_R}, w_1^{nc_S,nc_R})}{g^{nc_S,nc_R}(\pi_{r_2}^{nc_S,nc_R}, w_1^{nc_S,nc_R}) - d_s} + \frac{1-\lambda}{\pi_{r_2}^{nc_S,nc_R} - d_r} = 0$$

or,

$$\frac{\lambda\left[(g^{c_R,nc_R})'(\pi_{r_2}^{nc_S,nc_R}) + \dfrac{\alpha\,(b+\alpha)\,(w_1^{nc_S,nc_R} - c)}{2\,(2b+\alpha)\,(2b+3\alpha)}\sqrt{\dfrac{b+\alpha}{\pi_{r_2}^{nc_S,nc_R}}}\right]}{g^{c_R,nc_R}(\pi_{r_2}^{nc_S,nc_R}) + \dfrac{\alpha\,(b+\alpha)\,(w_1^{nc_S,nc_R} - c)}{(2b+\alpha)\,(2b+3\alpha)}\sqrt{(b+\alpha)\pi_{r_2}^{nc_S,nc_R}} - d_s} + \frac{1-\lambda}{\pi_{r_2}^{nc_S,nc_R} - d_r} = 0.$$

Using

$$w_1^{nc_S,nc_R} = \frac{1}{2}\left(c + \frac{2b+3\alpha}{2b^2+4\alpha b+\alpha^2}A + \frac{\alpha b+\alpha^2}{2b^2+4\alpha b+\alpha^2}w_2^{nc_S,nc_R}\right),$$

we have

$$\pi_{r_2}^{nc_S,nc_R} = (b+\alpha)\left(\frac{A}{2b+\alpha} - \frac{\left[2\left(b+\alpha\right)^2-\alpha^2\right]w_2^{nc_S,nc_R}}{\left(2b+\alpha\right)\left(2b+3\alpha\right)} + \frac{\alpha\left(b+\alpha\right)w_1^{nc_S,nc_R}}{\left(2b+\alpha\right)\left(2b+3\alpha\right)}\right)^2 = h_1(w_2^{nc_S,nc_R}),$$

where $h_1(x) \equiv (b+\alpha)[(A-bx)(4b^2+9\alpha b+3\alpha^2)/(2b^2+4\alpha b+\alpha^2)-\alpha(b+\alpha)(x-c)/(2b+3\alpha)]^2/[4(2b+\alpha)^2]$. Hence, the negotiated wholesale price $w_2^{nc_S,nc_R}$ satisfies

$$\frac{\lambda\left[(g^{c_R,nc_R})'(h_1(w_2^{nc_S,nc_R}))+h_2(w_2^{nc_S,nc_R})/\left(2\sqrt{h_1(w_2^{nc_S,nc_R})}\right)\right]}{g^{c_R,nc_R}(h_1(w_2^{nc_S,nc_R}))+h_2(w_2^{nc_S,nc_R})\sqrt{h_1(w_2^{nc_S,nc_R})}-d_s} + \frac{1-\lambda}{h_1(w_2^{\mathrm{seq},nc})-d_r} = 0,$$

where $h_2(x) \equiv \alpha(b+\alpha)\sqrt{b+\alpha}[(2b+3\alpha)(A-bc)+\alpha(b+\alpha)(x-c)]/[2(2b+\alpha)(2b+3\alpha)(2b^2+4\alpha b+\alpha^2)]$. The profits of the suppliers and the retailers are $\pi_{r_1}^{nc_S,nc_R} = (b+\alpha)\eta_6^2$, $\pi_{s_1}^{nc_S,nc_R} = (b+\alpha)\eta_6(w_1^{nc_S,nc_R}-c)$, $\pi_{r_2}^{nc_S,nc_R} = h_1(w_2^{nc_S,nc_R})$, $\pi_{s_2}^{nc_S,nc_R} = g^{nc_S,nc_R}(h_1(w_2^{nc_S,nc_R}), w_1^{nc_S,nc_R})$, in which

$$\begin{cases} w_1^{nc_S,nc_R} = \{c+[A(2b+3\alpha)+w_2^{nc_S,nc_R}(\alpha b+\alpha^2)]/(2b^2+4\alpha b+\alpha^2)\}/2, \\ \eta_6 \equiv \{A\left(2b+3\alpha\right)-\left[2\left(b+\alpha\right)^2-\alpha^2\right]w_1^{nc_S,nc_R}+\alpha\left(b+\alpha\right)w_2^{nc_S,nc_R}\}/[(2b+\alpha)\left(2b+3\alpha\right)]. \end{cases}$$

as given in Table 3. The firms in each supply chain can reach an agreement if $\pi_{r_i}^{nc_S,nc_R} > d_r$ and $\pi_{s_i}^{nc_S,nc_R} > d_s$.

According to the above analysis, we obtain the functions and coefficients in the theorem as

$$\begin{cases} h_1(x) \equiv \dfrac{b+\alpha}{4(2b+\alpha)^2}\left[\dfrac{(A-bx)(4b^2+9\alpha b+3\alpha^2)}{2b^2+4\alpha b+\alpha^2} - \dfrac{\alpha(b+\alpha)(x-c)}{2b+3\alpha}\right]^2, \\ h_2(x) \equiv \dfrac{\alpha(b+\alpha)\sqrt{b+\alpha}[(2b+3\alpha)(A-bc)+\alpha(b+\alpha)(x-c)]}{2(2b+\alpha)(2b+3\alpha)(2b^2+4\alpha b+\alpha^2)}, \\ g^{nc_S,nc_R}(x,y) = g^{c_R,nc_R}(x) + \dfrac{\alpha(b+\alpha)(y-c)\sqrt{(b+\alpha)\,x}}{(2b+\alpha)(2b+3\alpha)}; \end{cases} \tag{19}$$

and,

$$\begin{cases} \eta_1 \equiv (A-bc)^2\left(b+\alpha\right)\left(2b^2+4\alpha b+\alpha^2\right)\left(2b+3\alpha\right)/[(2b+\alpha)\left(4b^2+7\alpha b+\alpha^2\right)^2], \\ \eta_2 \equiv (A-bc)^2\left(b+\alpha\right)(2b^2+4\alpha b+\alpha^2)^2/[(2b+\alpha)^2\left(4b^2+7\alpha b+\alpha^2\right)^2], \\ \eta_3 \equiv b(b+\alpha)\left(2b+3\alpha\right)\Pi^*(c)/[(2b+\alpha)(2b^2+4\alpha b+\alpha^2)], \\ \eta_4 \equiv b\left(b+\alpha\right)\Pi^*(c)/\left(2b+\alpha\right)^2, \\ \eta_5 \equiv \{\eta_4[(4b^2+9\alpha b+3\alpha^2)/(2b^2+4\alpha b+\alpha^2)]^2-d_s-d_r\} \\ \eta_6 \equiv \{A\left(2b+3\alpha\right)-[2\left(b+\alpha\right)^2-\alpha^2]w_1^{nc_S,nc_R}+\alpha\left(b+\alpha\right)w_2^{nc_S,nc_R}\}/[(2b+\alpha)\left(2b+3\alpha\right)]. \end{cases} \tag{20}$$

■

B.8 Theorem 12 and Its Proof

Theorem 12 *The retailers' pricing strategy in equilibrium depends on parameters (d_r, d_s), λ, $\bar{\lambda}$, α, A, and b. When (d_r, d_s) is in area (3) in Figure 4, (C_R, C_R) is the unique pure Nash equilibrium. When (d_r, d_s) is in area (2) of Figure 4 and*

$$\lambda_l^{nc_R,nc_R} \equiv 1 - \frac{(b+\alpha)\left[(A - bw^{nc_R,nc_R})/(2b+\alpha)\right]^2 - d_r}{(b+\alpha)\left[(A+\alpha p_2^{c_R,nc_R})/(b+\alpha) - c\right]^2/4 - d_r - d_s},$$

the retailers' pure Nash equilibria are shown in Figure 9(a). When (d_r, d_s) is in area (1) of Figure 4 and

$$\lambda_u^{c_R,c_R} \equiv 1 - \frac{\pi_{r2}^{c_R,nc_R} - d_r}{(b+\alpha)\left[(A-bc)/(2b+\alpha)\right]^2 - d_r - d_s},$$

the retailers' pure Nash equilibria are presented in Figure 9(b). When (d_r, d_s) is in area (5) of Figure 4, then the retailers' pure Nash equilibria are shown in Figure 9(c). When (d_r, d_s) is in area (4) of Figure 4, (NC_R, NC_R) is the unique pure Nash equilibrium.

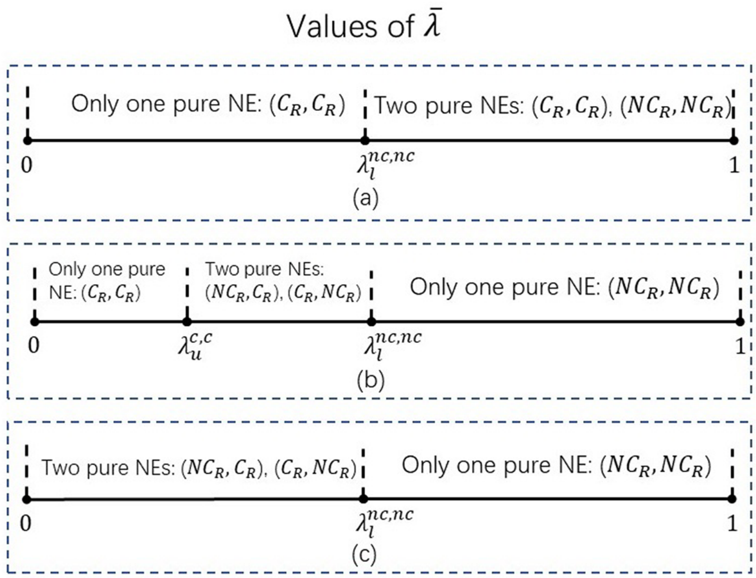

Figure 9: The retailers' pricing strategies when (d_r, d_s) is in: (a) area (2) of Figure 4, (b) area (1) of Figure 4, and (c) area (5) of Figure 4.

Proof. In order to find all possible pure Nash equilibria, we need to combine the results of the retailers' all possible pricing strategies. We begin by exploring the relation between Figures 2 and 3. When (d_r, d_s) is in area (3) in Figure 3, we first show that there is no Nash equilibrium when one retailer (e.g., R_1) uses strategy C_R whereas the other one uses strategy NC_R. This result is justified as follows: R_2 actually has an incentive to change strategy NC_R to strategy C_R, because, as a result of the change, his profit increases from his disagreement d_r to $d_r + (1-\bar{\lambda})\{(b+\alpha)[(A-bc)/(2b+\alpha)]^2 - d_r - d_s\}$. Hence, area (3) in Figure 3 can disappear when we consider the pure Nash equilibrium of the retailers' pricing strategies. It is also easy to see that $g^{c_R,nc_R}(x) = g^{nc_R,nc_R}(x)(2b^2+3\alpha b)/(2b^2+4\alpha b+\alpha^2) \leq g^{nc_R,nc_R}(x)$

and

$$\begin{aligned}
& \frac{g^{nc_R,nc_R}\left((b+\alpha)\left[\frac{A-bc}{2b+\alpha}\times\frac{2\,(b+\alpha)^2-\alpha^2}{(2b+3\alpha)\,b+2\,(b+\alpha)^2-\alpha^2}\right]^2\right)}{\max_x g^{c_R,nc_R}(x)} \\
= \; & \frac{(4b^2+8\alpha b+2\alpha^2)^2}{(4b^2+7\alpha b+\alpha^2)^2} \\
\geq \; & 1.
\end{aligned}$$

Hence, areas (1) and (2) in Figure 3 are contained in areas (1) and (2) in Figure 2.

When (d_r, d_s) is in area (3) in Figure 4, we find that if both retailers use strategy C_R, both supply chains can reach agreements. Hence, (C_R, C_R) is the only pure Nash equilibrium. When (d_r, d_s) is in area (2) in Figure 4, if one retailer uses strategy C_R whereas the other uses strategy NC_R, then either the non-cooperative supply chain breaks up or both supply chains break up. As a consequence, the retailer with strategy NC_R has an incentive to change strategy NC_R to strategy C_R. This means that (NC_R, C_R) and (C_R, NC_R) cannot be pure Nash equilibria. If both retailers use strategy C_R, then none of the retailers has any incentive to choose strategy NC_R, which may result in the disagreement payoff. Hence, (C_R, C_R) is a pure Nash equilibrium. If one retailer keeps strategy NC_R and one retailer changes to strategy C_R, then the payoff of the retailer with strategy C_R changes from $\pi_{r_1}^{nc_R,nc_R}$ to $\pi_{r_1}^{c_R,nc_R}$. Hence, (NC_R, NC_R) is a Nash equilibrium if $\pi_{r_1}^{c_R,nc_R} \leq \pi_{r_1}^{nc_R,nc_R}$ and $\pi_{r_2}^{nc_R,c_R} \leq \pi_{r_2}^{nc_R,nc_R}$, i.e., $\bar{\lambda} \geq \lambda_l^{nc_R,nc_R}$, as $\pi_{r_1}^{c_R,nc_R} = \pi_{r_2}^{nc_R,c_R}$ and $\pi_{r_1}^{nc_R,nc_R} = \pi_{r_2}^{nc_R,nc_R}$.

When (d_r, d_s) is in area (1) in Figure 4, the retailers are facing the following 2×2 symmetric Nash game:

$$\begin{array}{ccc|c|c|}
 & & \multicolumn{2}{c}{R_2} \\
 & & \multicolumn{1}{c}{C} & \multicolumn{1}{c}{NC} \\
\cline{3-4}
R_1 & C & (\pi_{r_1}^{c_R,c_R}, \pi_{r_2}^{c_R,c_R}) & (\pi_{r_1}^{c_R,nc_R}, \pi_{r_2}^{c_R,nc_R}) \\
\cline{3-4}
 & NC & (\pi_{r_1}^{nc_R,c_R}, \pi_{r_2}^{nc_R,c_R}) & (\pi_{r_1}^{nc_R,nc_R}, \pi_{r_2}^{nc_R,nc_R}) \\
\cline{3-4}
\end{array}$$

in which $\pi_{r_1}^{c_R,c_R} = \pi_{r_2}^{c_R,c_R}$, $\pi_{r_1}^{nc_R,nc_R} = \pi_{r_2}^{nc_R,nc_R}$, $\pi_{r_1}^{c_R,nc_R} = \pi_{r_2}^{nc_R,c_R}$, and $\pi_{r_2}^{c_R,nc_R} = \pi_{r_1}^{nc_R,c_R}$. Therefore, (C_R, C_R) is a pure Nash equilibrium, if $\pi_{r_2}^{c_R,nc_R} \leq \pi_{r_2}^{c_R,c_R}$ and $\pi_{r_1}^{nc_R,c_R} \leq \pi_{r_1}^{c_R,c_R}$, i.e., $\bar{\lambda} \leq \lambda_u^{c_R,c_R}$. In addition, (NC_R, NC_R) is a pure Nash equilibrium, if $\bar{\lambda} \geq \lambda_l^{nc_R,nc_R}$. Moreover, the two retailers play a Hawk-Dove game if $\lambda_l^{nc_R,nc_R} \leq \bar{\lambda} \leq \lambda_u^{c_R,c_R}$.

When (d_r, d_s) is in area (4) in Figure 4, both negotiations can reach agreements only with the (NC_R, NC_R) strategy; hence, (NC_R, NC_R) is the only Nash equilibrium. When (d_r, d_s) is in area (5) in Figure 4, both negotiations cannot end up with an agreement on the (C_R, C_R) strategy; thus, $\pi_{r_1}^{c_R,c_R} = \pi_{r_2}^{c_R,c_R} = d_r \leq \pi_{r_1}^{nc_R,c_R} = \pi_{r_2}^{c_R,nc_R}$. Hence, (C_R, C_R) can never be a Nash equilibrium. In addition, (NC_R, NC_R) is a Nash equilibrium if $\pi_{r_1}^{c_R,nc_R} \leq \pi_{r_1}^{nc_R,nc_R}$ and $\pi_{r_2}^{nc_R,c_R} \leq \pi_{r_2}^{nc_R,nc_R}$ i.e., $\bar{\lambda} \geq \lambda_l^{nc_R,nc_R}$; and, (NC_R, C_R) (or (C_R, NC_R)) is a Nash equilibrium if $\pi_{r_1}^{c_R,nc_R} \geq \pi_{r_1}^{nc_R,nc_R}$ and $\pi_{r_2}^{nc_R,c_R} \geq \pi_{r_2}^{nc_R,nc_R}$ i.e., $\bar{\lambda} \leq \lambda_l^{nc_R,nc_R}$. ■