Qualifying for a Government's Scrappage Program to Stimulate Consumers' Trade-In Transactions? Analysis of an Automobile Supply Chain Involving a Manufacturer and a Retailer[1]

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Abstract: We investigate an automobile supply chain where a manufacturer and a retailer serve a market with a fuel-efficient automobile under a scrappage program by the government. The program awards a subsidy to each consumer who trades in his or her used automobile with a new fuel-efficient automobile, if the manufacturer's suggested retail price (MSRP) for the new one does not exceed a cutoff level. We derive the conditions assuring that the manufacturer has an incentive to qualify for the program, and find that when the cutoff level is low, the manufacturer may be unwilling to qualify for the program even if the subsidy is high. We also show that when the manufacturer qualifies for the program, increasing the MSRP cutoff level would raise the manufacturer's expected profit but may decrease the expected sales. A moderate cutoff level can maximize the effectiveness of the program in stimulating the sales of fuel-efficient automobiles, whereas a sufficiently high cutoff level can result in the largest profit for the manufacturer. The retailer's profit always increases when the manufacturer chooses to qualify for the program. Furthermore, we compute the government's optimal MSRP cutoff level and subsidy for a given sales target, and find that as the program budget increases, the government should raise the subsidy but reduce the MSRP cutoff level to maximize sales.

Key words: Supply chain management; government scrappage program; manufacturer; retailer; single period.

1 Introduction

In recent years many scrappage programs have been implemented to encourage the trade-in of old automobiles with more fuel-efficient new automobiles. Such programs generally have

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the dual aim of stimulating the automobile market and removing inefficient, high-emission automobiles from the road for the purpose of environmental protection. Even though some governments introduced similar programs (e.g., tax rebate programs) in the 1990s, the scrappage program was widely adopted in a number of countries only during the global recession that began in 2008. This happened because the 2008 financial storm heavily influenced the world-wide automobile sector, resulting in an unprecedented automobile industry crisis.

In Table 1, we provide a summary of automobile scrappage programs that have been implemented by the governments of eight countries or regions in Asia, Europe, and North America. Under a scrappage program for automobile, a *contingent* subsidy is provided to each consumer who trades in his or her old vehicle for a new, more *fuel-efficient* one that has a combined fuel economy.

Country /Region	Scrappage Program for Automobile Trade-In
China (Chinaautoweb.Com, 2010)	The government provided each consumer with a subsidy of RMB3,000—RMB6,000 for trading in a used, heavy polluting car or truck with a new one since June 2009, and later increased the subsidy to RMB5,000—RMB18,000 in order to implement the program more effectively.
France (IHS Global Insight, 2010)	Each consumer can claim a €1,000 subsidy from the government if he or she trades in a used car that is more than 10 years old with a new car that meets Euro IV emission standards.
Germany (IHS Global Insight, 2010)	The government offers a €2,500 subsidy (with a total subsidy value of €1.5 billion) to consumers who trade in their old vehicles for new ones that have low carbon dioxide emissions.
Greece (IHS Global Insight, 2010)	The government offers the subsidy of €1,900—€4,200 to each buyer who trades in his or her old car with a new one that has ecological credentials.
Japan (Yacobucci and Canis, 2009)	The government provides a subsidy of JPY250,000 for the trade-in of each vehicle first registered 13 years ago or earlier with a new model compliant with 2010 fuel efficiency standards.
Mexico (Niedermeyer, 2009)	The federal government provides a 15,000 pesos subsidy to each consumer who replaces his or her vehicle that is at least 10 years old with a new, fuel-efficient one, if the manufacturer's suggested retail price (MSRP) for the new vehicle is less than or equal to 160,000 pesos.
Slovaks (IHS Global Insight, 2010)	The government provides a €1,000 state subsidy to each buyer who replaces a car that is more than 10 years old with a new one, if the MSRP for the new car is not greater than €25,000.
The United States (Yacobucci and Canis, 2009)	The U.S. president recently signed the "Consumer Assistance to Recycle and Save" (CARS) act. Under this act, if the MSRP for a new automobile is not greater than US$45,000, then the government rewards each consumer with a subsidy of US$3,500 or US$4,500, which depends on the types of both the new and the old automobiles.

Table 1: A summary of scrappage programs that have been implemented by the governments of eight countries or regions in Asia, Europe, and North America.

Some governments (e.g., Hong Kong, the United States, etc.) have reported that the scrappage program is useful for stimulating the sales of fuel-efficient vehicles. For example, the Environmental Protection Department of Hong Kong prepared HK$3.2 billion for its scrappage program, and announced that this amount had been committed within 18 months. As reported by Yacobucci and Canis (2009), the United States appropriated an initial amount of $1 billion for its "Car Allowance Rebate System" (CARS) program to support qualifying transactions. Immediately after the first week of implementing the CARS program, the United States's Department of Transportation announced that this program was embraced by thousands of consumers and by automobile retailers across the country, and nearly all of the funds appropriated for the CARS program were committed. In a response to the high demand resulting from the CARS program, the House of Representatives decided to make available an additional $2 billion to extend the program. In Slovaks, 62.3% of the total sales realized between March and December 2009 were attributed to the government's scrappage program; see the IHS Global Insight (2010).

From our above discussion, we find that the scrappage program can be effective in stimulating sales for fuel-efficient automobiles during the economic recession. Hence, it would be important, and interesting, to consider the following relevant questions. First, we need to investigate if the manufacturer's and the retailer's profits when the manufacturer qualifies for the scrappage program are higher than those when the manufacturer does not qualify. We note that the government's target of implementing the scrappage program is to boost the sales of fuel-efficient automobiles for the purpose of reducing harmful emissions and protecting the environment. Accordingly, we examine the conditions under which the program can effectively stimulate the sales. We find from Table 1 that some governments (e.g., Mexico, Slovaks, and the United States) provide their subsidies only to the buyers who purchase new cars each with a manufacturer's suggested retail price (MSRP) no more than a pre-determined cutoff level, whereas other governments (e.g., China, France, Germany, and Japan) do not set any MSRP cutoff level for their scrappage programs. It thus behooves us to investigate the impact of the MSRP cutoff level and the subsidy amount on the automobile sales. To address our above questions in a *general* setting, we assume in this paper that the scrappage program for automobile trade-in involves a cutoff level and a subsidy.

We consider a two-echelon supply chain where a manufacturer and a retailer serve consumers with a fuel-efficient automobile under a government's scrappage program. The manufacturer makes a wholesale pricing decision and, as in the *real-world* automobile industry (see, e.g., `Leaseguide.com` 2011), determines an MSRP as a markup above the wholesale price. In accordance with the practice, the MSRP can be calculated as the manufacturer's wholesale price plus a markup percentage of the wholesale price. The markup percentage for an automobile *uniquely* corresponds to a gross profit margin, which, in the past two decades, was *usually* between 4% and 13% with an average value in the range $[6\%, 8\%]$, as indicated at `Leaseguide.com` (2011).

The retailer purchases the manufacturer's automobiles at the wholesale price, and then serves heterogenous consumers in a market of a finite size. That is, there are a certain, finite number of consumers each having a net valuation which is the consumer's valuation of a new, fuel-efficient automobile minus the valuation of his or her old automobile. To reflect the heterogeneity, we characterize the consumers' net valuations by a non-negative, finite-valued random variable. As in practice, the retail price for each consumer is determined as a discount of the MSRP, which results from the negotiation between the consumer and the retailer. Accordingly, we analyze the bargaining process to determine the discount of the MSRP and calculate the retail price for each consumer. Since the retailer and the consumer may have different bargaining powers, we apply the generalized Nash bargaining (GNB) scheme—which was developed by Roth (1979)—to analyze the two-player cooperative game and find a unique MSRP discount, which determines the retail price charged to the consumer. For a recent application of GNB in supply chain analysis, see Huang et al. (2013), who performed a numerical study to analyze automobile supply chains under a subsidy scheme, and Luo et al. (2014), who analytically investigated a price-discount scheme for an automobile supply chain.

We then use the negotiated retail prices to develop the manufacturer's profit function, and maximize it to find the manufacturer's unique optimal wholesale price under a scrappage program. We derive the condition under which the manufacturer can benefit from the program and is thus willing to set an MSRP lower than or equal to the cutoff level and qualify for the program. In addition, we show that, when the manufacturer qualifies for the scrappage program, raising the MSRP cutoff level will result in an increase in the manufacturer's expected profit but a decrease in the expected sales. Therefore, if the government intends to increase the expected sales for the fuel-efficient automobile, then it should set a moderate value for the MSRP cutoff level. In addition, we find that a small subsidy may be ineffective in stimulating the sales. The government's optimal subsidy is increasing in its budget, while its optimal MSRP cutoff level is decreasing in the budget.

2 Literature Review

This paper is related to two streams of literature. The first stream explores the trade-in subsidies provided by firms. For example, Levinthal and Purohit (1989) investigated a firm's trade-in scheme for its durable products, and showed that the firm can utilize trade-in rebates to promote an improved product generation by deterring the secondary market. For the trade-in problem by Levinthal and Purohit (1989), van Ackere and Reynolds (1995) found that trade-in rebates can encourage consumers to trade in their used products for new ones. Fudenburg and Tirole (1998) considered the optimal pricing and trade-in rebate decisions for a firm selling its products in a market that involves potential trade-in consumers—who intend to trade in used products for new ones—and potential first-time buyers—who may directly purchase the new products from the firm. Ray et al. (2005) also studied the optimal pricing and trade-in strategies for a firm that satisfies an age-dependent demand with durable and remanufactured products. Bruce et al. (2006) investigated a trade-in problem, and derived the conditions under which the manufacturer of a durable product is willing to offer a cash rebate to each consumer who trades in the used product for a new one. Using prospect theory, Kim et al. (2011) developed an analytical model to explore consumers' choices in their trade-ins. Moreover, some researchers addressed the trade-in issues for general durable goods; see Clerides and Hadjiyiannis (2008), Prince (2009), and Rao et al. (2009).

Our paper differs from this stream of research in that we contribute to the literature by finding (i) the consumers' purchase decisions and the expected sales, (ii) the negotiated retail price, (iii) the optimal wholesale price, and (iv) the government's optimal MSRP cutoff level and subsidy that maximize the expected sales. We can find the differences from both managerial and technical perspectives. From the managerial perspective, the trade-in policy implemented by a government primarily aims at stimulating the sales of new fuel-efficient automobiles to reduce CO_2 emissions, whereas the trade-in policy implemented by a firm mainly aims at increasing the firm's profit. A policy that effectively increases a firm's profit may not be effective in increasing the sales. From the technical perspective, the firm with its trade-in policy needs to make optimal decisions on price, subsidy, and other policy-related variables. However, when the trade-in subsidy is provided by a government, the firm only makes its optimal pricing decision in response to the government's scrappage program, for which the government decides on its optimal cutoff level and subsidy that maximize the expected sales.

The second stream of literature is concerned with the scrappage programs (colloquially known as "cash for clunkers") implemented by governments, under which a subsidy is offered to each consumer who trades in a used automobile for a new one. A number of researchers investigated the environmental and economic consequences of such scrappage programs. For a comprehensive review on the environmental impact of the scrappage programs, see Wee et al. (2011). Some researchers examined the environmental impact of the scrappage programs from the empirical perspective. For instance, by measuring the efficiency of the program in terms of the reduction in greenhouse gas emissions, Morrison et al. (2010) and Zolnik (2012) examined the costs generated by the program. Ryan (2012) estimated the efficiency of the scrappage program in pollution reduction. As an alternative to the subsidy, governments may apply tax policies to influence consumers' trading-in behaviors. Fosgerau and Jensen (2013) considered the effects of the government's tax reform on CO_2 emissions and social welfare. Brand et al. (2013) and Fullerton and West (2010) considered the combinations of tax and subsidy for the control of car pollution.

Besides the above empirical studies, researchers examined the environmental effects of the scrappage program using various analytical models, which include the discrete analysis (e.g., Adda and Cooper, 2000), mass point duration model (e.g., Chen and Niemeier, 2005), integer program model (e.g., Gao and Stasko, 2009), cost-benefit analysis (e.g., Lavee and Becker, 2009), life cycle optimization model (e.g., Kim et al. 2003, Kim et al. 2004, Spitzley et al. 2005, Lenski et al. 2010, and Basbagill et al. 2013), and others (e.g., Chen et al. 2010 and Lorentziadis and Vournas 2011).

Researchers also analytically investigated the economic impact of the scrappage program. For example, Hahn (1995) assessed the likely benefits and costs of the scrappage program, and Alberini et al. (1995) modeled the owner's car tenure and scrappage decision, and forecasted the participation rates in the scrappage program. Incorporating the scrappage cycle length, de Palma and Kilani (2008) presented an economic model to assess the impact of scrap value for old cars and taxes on gasoline. By maximizing the social net benefit, Iwata and Arimura (2009) analyzed the optimal retirement timing of the Japanese program for air pollution regulation. Licanaro and Sampayo (2006) designed a model to quantitatively evaluate Spain's 1997 scrappage program.

In addition, researchers examined the economic effects using the empirical data in some specific programs. Several studies recently investigated the impact of the U.S. 2009 automobile CARS program (see, Yacobucci and Canis 2009, Busse et al. 2012, Mian and Sufi 2012, Copeland and Kahn 2013, Klier and Rubenstein 2013, Li et al. 2013, and Li and Wei 2013). A recent report by IHS Global Insight (2010) assessed the economic, environmental, and safety impact of the scrappage program implemented by countries in European Union. Aldred and Tepe (2011) considered the scrappage programs in Germany and the UK. Huse and Lucinda (2013) investigated the effects of the Swedish "green car rebate" program on CO_2 emission and the program cost.

Most publications were concerned with empirical study for the scrappage program. Though some researchers developed analytical models for such a program, our analytical work still significantly differs from them in several aspects.

1. The government's scrappage program under our study involves an MSRP cutoff level (in addition to a subsidy). To the best of our knowledge, *no* existent publication has analytically considered such a cutoff level for any scrappage scheme, which is actually important to the program's effectiveness in stimulating the sales because of the following fact. The cutoff level could lead consumers to buy qualifying automobiles—i.e., those with MSRPs lower than or equal to the cutoff level—and obtain the subsidy from the government. Manufacturers may respond to consumers' purchase behaviors by reducing their wholesale prices and MSRPs to qualify for the scrappage program. Thus, the choice of the cutoff level affects the effectiveness of the scrappage program. It behooves us to consider the scrappage program including the MSRP cutoff level and the subsidy. This saliently distinguishes our paper from extant publications.
2. The modeling approach in our paper differs from that in any existent publication. None of the publications regarding the government's scrappage program analyzed consumers' trade-in decisions, which is, however, important because the program aims at encouraging consumers to buy qualifying automobiles. In this paper, we consider each consumer's purchase decision by using the generalized Nash bargaining (GNB) scheme to compute the retail price that is negotiated by the retailer and the consumer. This differs from most extant publications which (i) ignored consumers' behaviors in their automobile transactions, (ii) allowed the retailer to unilaterally decide on an optimal retail price maximizing the retailer's individual profit, and (iii) presumed that consumers in all successful transactions accept the retail price. Moreover, we allow heterogeneity in consumers' valuations, which is important because it enables us to investigate how the scrappage program entices low-valuation consumers to trade in their used automobiles. Thus, compared with relevant publications, the modeling approach in our paper is more realistic. Using our analytical results characterizing consumers' purchase behaviors, we derive a demand function, which is expected to analytically describe automobile sales in an accurate manner.
3. Our realistic model and analysis for the scrappage program that has appeared in real life generate a number of new managerial insights that have not been found in extant publications. For a summary of our new insights, see Section 7.

3 Preliminaries

We consider a two-level automobile supply chain in the presence of a scrappage program, where a manufacturer produces a fuel-efficient automobile—that meets relevant requirements in the scrappage program—at the unit acquisition cost c. The manufacturer sells its products to a retailer in a market at the wholesale price w.

In practice, an automobile manufacturer usually recommends to its retailers a suggested retail price p_s which is commonly calculated as the manufacturer's wholesale price plus a markup percentage ($\rho \geq 0$) of the wholesale price—i.e., $p_s = (1+\rho)w$—as indicated by a large number of facts in the automobile industry, see, e.g., Martí (2000), `Biz.Yahoo.Com` (2011), and `Leaseguide.com` (2011). Note that some firms may use the concept of "gross margin" instead of the markup percentage to calculate the MSRP. In fact, the gross margin and the markup percentage uniquely correspond to each other, such that the gross margin is equal to $\rho/(1+\rho)$ and the value of ρ can be found as (profit margin)/(1−profit margin). Therefore, using the markup percentage to determine the MSRP does not result in any loss of generality of our model and analysis.

The above discussion indicates that, in the automobile industry, the MSRP actually reflects a markup ceiling that a manufacturer allows its retailer to take; for details, see, e.g., a case report by Martí (2000). Since the MSRP of the automobile is known to consumers, the actual retail prices are usually not higher than the MSRP, because the retail price charged to a consumer is commonly determined as the result of negotiation between the retailer and the consumer over a discount from the MSRP. Therefore, we can calculate the retail price p_r for a consumer as $p_r = (1-\alpha)p_s = (1-\alpha)(1+\rho)w$, where $\alpha \in [0,1]$ represents the discount from MSRP p_s. Because the retail price is greater than or equal to the wholesale price w, i.e., $p_r \geq w$, we should determine the value of α such that $(1-\alpha)(1+\rho) \geq 1$, or, $\alpha \leq \rho/(1+\rho)$.

In our paper, the market that the retailer serves has a finite consumer base B consisting of all potential consumers who may trade in their old automobiles for new ones. Moreover, each consumer in the market only buys one unit of automobile in a transaction. Consumers on the base B may have different valuations over their used automobiles; they may also draw different valuations from the new automobile. To reflect the fact, we assume the heterogeneity of consumers' *net* trade-in valuations. We define a consumer's net trade-in valuation as the consumer's valuation of the new automobile minus his or her valuation of the used automobile. The heterogeneous net valuations of consumers are characterized by a non-negative random parameter θ with p.d.f. $f(\theta)$ and c.d.f. $F(\theta)$ on support $[0, \bar{\theta}]$, where $\bar{\theta}$ is the maximum net valuation of all consumers in the market.

Let A and s denote the cutoff level and subsidy for the scrappage program, respectively. We find that each consumer can gain at most $\bar{\theta} + s$ from trade-in when $p_s \leq A$, or, $w \leq A/(1+\rho)$, but can obtain at most $\bar{\theta}$ when $w > A/(1+\rho)$. Thus, to assure the effectiveness of the MSRP p_s, the manufacturer should determine its wholesale price such that $w \leq p_s \leq \bar{\theta} + \mathbf{1}_{\{w \leq A/(1+\rho)\}} \times s$, where $\mathbf{1}_{\{w \leq A/(1+\rho)\}} \equiv 1$, if $w \leq A/(1+\rho)$; $\mathbf{1}_{\{w \leq A/(1+\rho)\}} \equiv 0$, if $w > A/(1+\rho)$. Under this condition, we find that if $A < w$, then the manufacturer certainly cannot qualify for the program; but, if $A \geq \bar{\theta} + \mathbf{1}_{\{w \leq A/(1+\rho)\}} \times s$, then the manufacturer can always qualify for the program. Taking the above into account, the government should determine its cutoff level A such that $w \leq A \leq \bar{\theta} + \mathbf{1}_{\{w \leq A/(1+\rho)\}} \times s$. In addition, we assume that the manufacturer's wholesale price is greater than the subsidy s, i.e., $w > s$. This assumption is consistent with practice. For example, under the U.S. CARS program, the automobile type-dependent subsidy for each qualified transaction is \$3,500 or \$4,500, which is significantly lower than most manufacturers' wholesale prices.

Next, we develop a net surplus function for a consumer who possesses a specific net trade-in valuation θ. Note that the consumer's net surplus is computed as his or her net trade-in valuation θ minus the purchase cost [i.e., the retail price $p_r = (1-\alpha)(1+\rho)w$] *possibly* plus the subsidy s. Since whether or not the consumer can get the subsidy s depends on the comparison between the cutoff level A and the MSRP p_s (which is dependent on the wholesale price w), we can write the consumer's net surplus function $u(\alpha, w)$ as,

$$u(\alpha, w) = \theta - (1-\alpha)(1+\rho)w + \mathbf{1}_{\{w \leq A/(1+\rho)\}} \times s. \tag{1}$$

According to the above discussion, we find that, given the government's scrappage program (A, s), the manufacturer first determines its wholesale price w and releases it to the retailer, who then bargains with each consumer over a retail price. The MSRP p_s is visible to all consumers in their price negotiations. Such a decision process is depicted by Figure 1, where the arrow between the manufacturer and retailer denotes that the wholesale price w is unilaterally determined by the manufacturer, and the double-head arrow between the retailer and a consumer represents that the retail price p is determined as a result of the bargaining between the retailer and the consumer.

Figure 1: The decision process in the two-level automobile supply chain.

To help readers easily follow our modeling and analysis, we summarize the notations used in this paper in Table 2.

4 Negotiated Retail Price

We investigate the price negotiation between the retailer and a consumer with a specific net valuation θ, who bargain over the discount α of the MSRP p_s, given the scrappage program (A, s) and the manufacturer's wholesale price w. The bargaining issue is important to our paper mainly because of the following fact. If we do not consider the bargaining issue to find the negotiated retail price but determine the optimal retail price by maximizing the retailer's expected profit, then there will be a unique retail price for all consumers, which prevents the consumers with low valuations from buying. This results in underestimation of the expected sales and the total carbon emission reduction, because as a result of price negotiation, those

A :	MSRP cutoff level in the scrappage program;	α :	negotiated discount from the MSRP p_s;
B :	finite consumer base;	β :	consumer's bargaining power relative to
C_s :	government's total subsidy cost;		the retailer, which is a random variable
$\bar{C}_s$:	government's budget for the program;		with p.d.f. $g(\beta)$ and c.d.f. $G(\beta)$;
c :	manufacturer's unit acquisition cost;	θ :	consumer's net trade-in valuation,
p_r :	unit retail price;		which is a random variable with
s :	subsidy in the scrappage program;		p.d.f. $f(\theta)$ and c.d.f. $F(\theta)$;
u :	consumer's net surplus;	$\bar{\theta}$:	maximum net valuation of all consumers;
w :	unit wholesale price;	ρ :	markup percentage of the MSRP.

Table 2: List of notations that are used in this paper.

low-valuation consumers may purchase automobiles at lower retail prices. That is, ignoring the bargaining issue can undervalue the impact of the government's scrappage program on the sales and the automobile supply chain.

Since, in practice, the retailer and each consumer may have different bargaining powers, we use the generalized Nash bargaining (GNB) scheme—developed by Roth (1979)—to find the negotiated discount and retail price for the two players. In our bargaining problem, without loss of generality, we assume that the consumer is player 1 with the bargaining power β and the retailer is player 2 with the bargaining power $1-\beta$. Consumers may possess different bargaining powers when negotiating with the retailer. To incorporate the heterogeneity of consumers in their bargaining powers, we assume that β is a random variable with the p.d.f. $g(\beta)$ and the c.d.f. $G(\beta)$ on the support $[0,1]$.

We next compute these two players' profits (y_1, y_2) and security levels (y_1^0, y_2^0). The consumer's net surplus $u(\alpha, w)$ in (1) can be regarded as his or her "profit;" that is, $y_1 = u(\alpha, w)$. The retailer's profit from the transaction is calculated as its sales revenue (i.e., the retail price p_r) minus wholesale price w that the retailer pays to the manufacturer, that is, $y_2 = p_r - w = [\rho - \alpha(1+\rho)]w$. Moreover, for our problem, the retailer's and the consumer's security levels (also known as *status quo* points) are defined as these two players' guaranteed profits when they do not complete any transaction. Since neither the retailer nor the consumer will gain any profit if no transaction occurs between them, the status quo points are $(y_1^0, y_2^0) = (0,0)$ and the set of Pareto optimal solutions is $\mathcal{P} = \{(y_1, y_2) | u(\alpha, w) \geq 0$ and $[\rho - \alpha(1+\rho)]w \geq 0\}$. Using the above, we write the GNB model for our bargaining problem as,

$$\begin{array}{rl} \max_\alpha & \Lambda \equiv [\theta - (1-\alpha)(1+\rho)w + \mathbf{1}_{\{w \leq A/(1+\rho)\}} \times s]^\beta [\rho w - \alpha(1+\rho)w]^{1-\beta} \\ \text{s.t.} & \theta - (1-\alpha)(1+\rho)w + \mathbf{1}_{\{w \leq A/(1+\rho)\}} \times s \geq 0 \quad \text{and} \quad \rho w - \alpha(1+\rho)w \geq 0. \end{array} \tag{2}$$

The negotiated retail price obtained from the above Nash bargaining model cannot exceed the MSRP p_s. We derive the negotiated discount and retail price as follows.

Theorem 1 If $\theta < \theta_1 \equiv w - 1_{\{w \leq A/(1+\rho)\}} \times s$, then the consumer does not buy any automobile from the retailer. Otherwise, if $\theta_1 \leq \theta \leq \bar{\theta}$, then the consumer and the retailer complete the transaction with the following GNB-characterized discount:

$$\alpha^* = \begin{cases} \dfrac{(1+\rho-\beta)w - (1-\beta)(\theta + \mathbf{1}_{\{w \leq A/(1+\rho)\}} \times s)}{(1+\rho)w}, & \text{if } \theta_1 \leq \theta \leq \theta_2, \\ 0, & \text{if } \theta_2 \leq \theta \leq \bar{\theta}, \end{cases} \tag{3}$$

where $\theta_2 \equiv (1+\rho-\beta)w/(1-\beta) - 1_{\{w \leq A/(1+\rho)\}} \times s$, and the retail price charged to the consumer is thus computed as

$$p_r(w|\theta) = \begin{cases} \beta w + (1-\beta)(\theta + \mathbf{1}_{\{w \leq A/(1+\rho)\}} \times s), & \text{if } \theta_1 \leq \theta \leq \theta_2, \\ (1+\rho)w, & \text{if } \theta_2 \leq \theta \leq \bar{\theta}. \end{cases}$$

Proof. For a proof of this theorem and the proofs for all subsequent theorems, see Appendix A. ■

We learn from the above theorem that only consumers with sufficiently large net valuations are willing to trade in their used automobiles for new ones under the scrappage program.

According to Theorem 1, given the wholesale price w, we can calculate the *expected* retail price for each transaction as follows:

$$\begin{aligned} \mu_r(w) &= E[p_r(w|\theta)|\theta_1 \leq \theta \leq \bar{\theta}] \\ &= \frac{\int_0^1 \left[\int_{\theta_1}^{\theta_2} [\beta w + (1-\beta)(\theta + \mathbf{1}_{\{w \leq A/(1+\rho)\}} \times s)] f(\theta) d\theta\right] g(\beta) d\beta}{1 - F(\theta_1)} \\ &+ \frac{\int_0^1 \left[(1+\rho)w \int_{\theta_2}^{\bar{\theta}} f(\theta) d\theta\right] g(\beta) d\beta}{1 - F(\theta_1)}, \end{aligned} \tag{4}$$

which must be greater than the wholesale price w because $w < \theta + \mathbf{1}_{\{w \leq A/(1+\rho)\}} \times s$. We then develop the manufacturer's and the retailer's expected profit functions as

$$\begin{cases} \Pi_M(w) = B(w-c)[1 - F(\theta_1)], \\ \Pi_R(w) = B \int_0^1 [(1-\beta) \int_{\theta_1}^{\theta_2} (\theta + \mathbf{1}_{\{w \leq A/(1+\rho)\}} \times s - w) f(\theta) d\theta + \rho w \int_{\theta_2}^{\bar{\theta}} f(\theta) d\theta] g(\beta) d\beta. \end{cases} \tag{5}$$

Moreover, we can compute the expected sales (i.e., the total number of automobiles that consumers on the base B buy) as $D(w) = B[1 - F(\theta_1)]$.

Remark 1 Theorem 1 also indicates consumers' purchase decisions when the government does not implement a scrappage program, which corresponds to the case that $s = 0$. In the case of no scrappage program, a consumer with $\theta < \theta_1|_{s=0} = w$ does not buy from the retailer; but if $\theta_1|_{s=0} \leq \theta \leq \bar{\theta}$, then the consumer is willing to buy at the following retail price:

$$p_r(w|\theta)|_{s=0} = \begin{cases} \beta w + (1-\beta)\theta, & \text{if } \theta_1|_{s=0} \leq \theta \leq \theta_2|_{s=0} = w[1 + \rho/(1-\beta)], \\ (1+\rho)w, & \text{if } \theta_2|_{s=0} \leq \theta \leq \bar{\theta}. \end{cases} \tag{6}$$

Noting that $\theta_1|_{s=0} > \theta_1|_{s>0}$ when $w \leq A/(1+\rho)$, we find that the scrappage program can entice more consumers to buy from the retailer. ■

5 Manufacturer's Optimal Wholesale Price

We maximize the manufacturer's expected profit $\Pi_M(w)$ in (5) to find its optimal wholesale pricing decision under a given scrappage program (A, s). As Theorem 1 indicates, the value of θ_1 in (5) depends on the condition that $w \leq A/(1+\rho)$. Therefore, in order to find the optimal wholesale price for the manufacturer, we need to consider the following two scenarios: $w \leq A/(1+\rho)$ and $w > A/(1+\rho)$, and maximize $\Pi_M(w)$ for each scenario. Then, we compare the maximum profits obtained for the above two scenarios to find the manufacturer's *globally* optimal wholesale price.

5.1 Profit Maximization under the Constraint that $w \leq A/(1+\rho)$

When $w \leq A/(1+\rho)$, the manufacturer's optimization problem can be described as,

$$\max_{w} \Pi_{M1}(w) \equiv B(w-c)[1 - F(w-s)], \quad \text{s.t. } w \leq A/(1+\rho). \tag{7}$$

Lemma 1 If $f(\theta)$ is continuously differentiable and *log-concave* on $[0, \overline{\theta}]$, then the manufacturer's expected profit $\Pi_{M1}(w)$ is a *unimodal* function of the wholesale price w.

Proof. For our proof, see Appendix B. ■

The above lemma indicates that the unimodality of function $\Pi_{M1}(w)$ depends on the condition that $f(\theta)$ is continuously differentiable and log-concave. Such a condition is acceptable, because, as Bagnoli and Bergstrom (2005) showed, many commonly-used distributions—e.g., uniform, normal, exponential, logistic, Laplace (double exponential), Weibull, Gamma, etc.—satisfy the condition. To that end, we assume that the p.d.f. $f(\theta)$ possesses the properties of continuous differentiability and log-concavity.

Theorem 2 Given the scrappage program (A, s), the manufacturer's optimal wholesale price w_1^* when $w \leq A/(1+\rho)$ can be *uniquely* determined as

$$w_1^* = \begin{cases} \hat{w}_1, & \text{if } \hat{w}_1 \leq A/(1+\rho), \\ A/(1+\rho), & \text{if } \hat{w}_1 > A/(1+\rho), \end{cases} \tag{8}$$

where $\hat{w}_1$ is a *unique* solution to the following equation:

$$\hat{w}_1 = [1 - F(\hat{w}_1 - s)]/f(\hat{w}_1 - s) + c. \quad ■ \tag{9}$$

Then, we can compute the expected sales as $D(w_1^*) = B[1 - F(w_1^* - s)]$, and calculate the total subsidy cost as $C_s(s) = sD(w_1^*) = Bs[1 - F(w_1^* - s)]$. Using w_1^* in (8), we rewrite the manufacturer's expected profit as $\Pi_{M1}(w_1^*) = B(w_1^* - c)[1 - F(w_1^* - s)]$.

Corollary 1 The value of $\hat{w}_1$ is increasing in the subsidy s, i.e., $\partial\hat{w}_1/\partial s > 0$; but the increment in $\hat{w}_1$ is smaller than that in the subsidy, i.e., $\partial\hat{w}_1/\partial s < 1$.

Proof. For a proof of this corollary and that for the next corollary, see Appendix C. ■

The above corollary shows that, if $\hat{w}_1 \leq A/(1+\rho)$, then increasing the subsidy s leads the manufacturer to raise its wholesale price with an increment lower than the rise in s. Thus, the MSRP $p_s = (1+\rho)w$ is also increasing in the subsidy s.

Theorem 3 If $\hat{w}_1 \leq A/(1+\rho)$, then the expected sales, and the manufacturer's and the retailer's expected profits are strictly increasing in the subsidy s. In addition, the total subsidy cost is increasing in s. ■

The above theorem implies that both the manufacturer and the retailer can benefit from an increase in the subsidy s, when the government chooses a sufficiently high cutoff level such that $A \geq (1+\rho)\hat{w}_1$. Though, the government may not need to choose a very large subsidy because, otherwise, it will incur a high "expense" $C_s(s)$, which is considered as an "input" for implementing the scrappage program. Recall from Section 1 that some governments (e.g., China, France, Germany, Greece, and Japan) do not involve any cutoff level into their scrappage programs, so that each consumer can obtain a subsidy no matter what retail price the manufacturer suggests. That is, for those governments' scrappage programs, the cutoff level in our model is set as the maximum value $\bar{\theta} + s$. Even though other governments (e.g., Mexico, Slovaks, and the United States) explicitly apply cutoff levels to their programs, we find that those cutoff levels are *significantly* higher than the suggested prices of qualifying automobiles. For example, the U.S. government's cutoff level for its scrappage program is \$45,000, as indicated by Table 1. According to the report by IHS (2010), which is a leading global source of critical information and insight, the average MSRP for new vehicles under the U.S. scrappage program was \$22,450, which is significantly lower than the cutoff level \$45,000.

5.2 Profit Maximization under the Constraint that $w > A/(1+\rho)$

When $w > A/(1+\rho)$, all consumers who buy the manufacturer's automobiles cannot obtain the subsidy s, and, $\theta_1 = w$. Therefore, under the constraint that $w > A/(1+\rho)$, the manufacturer maximizes the following expected profit:

$$\Pi_{M2}(w) \equiv B(w-c)[1-F(w)]. \tag{10}$$

Theorem 4 Given the scrappage program (A, s), we maximize the manufacturer's expected profit $\Pi_{M2}(w)$ in (10) subject to $w > A/(1+\rho)$, and find that the manufacturer's optimal wholesale price w_2^* can be *uniquely* determined as follows:

$$w_2^* = \begin{cases} \hat{w}_2, & \text{if } A/(1+\rho) < \hat{w}_2 \leq \overline{\theta}/(1+\rho), \\ \overline{\theta}/(1+\rho), & \text{if } A/(1+\rho) < \overline{\theta}/(1+\rho) < \hat{w}_2, \\ A/(1+\rho)+\varepsilon, & \text{if } \hat{w}_2 \leq A/(1+\rho), \end{cases} \tag{11}$$

where ε is a positive infinitesimal, and $\hat{w}_2$ is a unique solution to the equation that $\hat{w}_2 = [1 - F(\hat{w}_2)]/f(\hat{w}_2) + c$. ■

Using w_2^* in (11), we can rewrite the manufacturer's expected profit function in (10) as $\Pi_{M2}(w_2^*) = B(w_2^* - c)[1 - F(w_2^*)]$, and compute the maximum expected sales as $D(w_2^*) = B[1 - F(w_2^*)]$.

Corollary 2 If $s > 0$, then $\hat{w}_1 > \hat{w}_2$. ■

The above corollary means that, if no cutoff level is involved into a scrappage program implemented by, e.g., China, France, Germany, etc., then the resulting wholesale price is greater than that in the absence of the program. That is, the scrappage program may result in an increase in the wholesale price.

5.3 The Optimal Wholesale Price

Using our analytic results in Sections 5.1 and 5.2, we now derive the manufacturer's globally optimal wholesale price maximizing its expected profit under *no* constraint, in the presence of the government's scrappage program (A, s). To do so, we need to compare the manufacturer's maximum profit when $w \leq A/(1+\rho)$ (the manufacturer qualifies for the scrappage program) and that when $w > A/(1+\rho)$ (the manufacturer does not qualify for the program).

Theorem 5 Given the government's program (A, s) for automobile scrappage, the manufacturer's globally optimal wholesale price w^* can be *uniquely* determined as follows:

$$w^* = \begin{cases} \hat{w}_1 & \text{if } A/(1+\rho) \geq \hat{w}_1, \\ A/(1+\rho) & \text{if } \hat{w}_2 \leq A/(1+\rho) < \hat{w}_1, \\ \tilde{w}_1 & \text{if } A/(1+\rho) < \hat{w}_2 \leq \overline{\theta}/(1+\rho), \\ \tilde{w}_2 & \text{if } A/(1+\rho) < \overline{\theta}/(1+\rho) < \hat{w}_2, \end{cases}$$

where $\tilde{w}_1$ and $\tilde{w}_2$ are given as

$$\tilde{w}_1 \equiv \begin{cases} A/(1+\rho), & \text{if } \Pi_{M1}(A/(1+\rho)) \geq \Pi_{M2}(\hat{w}_2), \\ \hat{w}_2, & \text{if } \Pi_{M1}(A/(1+\rho)) < \Pi_{M2}(\hat{w}_2); \end{cases}$$

$$\tilde{w}_2 \equiv \begin{cases} A/(1+\rho), & \text{if } \Pi_{M1}(A/(1+\rho)) \geq \Pi_{M2}(\bar{\theta}/(1+\rho)), \\ \bar{\theta}/(1+\rho), & \text{if } \Pi_{M1}(A/(1+\rho)) < \Pi_{M2}(\bar{\theta}/(1+\rho)). \end{cases}$$ ■

The above theorem suggests that the scrappage program significantly affects the manufacturer's optimal wholesale pricing decision. Specifically, if the manufacturer's optimal wholesale price w^* is $\hat{w}_1$ or $A/(1+\rho)$, then the manufacturer can qualify for the program and achieve the expected sales $D(w^*)$. As a result, the government needs to spend the total subsidy $C_s(s) = sD(w^*)$. Otherwise, if the manufacturer does not qualify for the program, then each consumer buying the manufacturer's product cannot obtain any subsidy from the government, which thus incurs no subsidy cost. We next provide a numerical example to illustrate our above analysis.

Example 1 We assume that each consumer's net valuation θ is normally distributed on the support $[0, \bar{\theta}]$ with mean $E(\theta) = \$30,000$ and standard deviation $\sigma = \$4,000$. The average value of consumers' net valuations roughly approximates \$30,000 because, as Markiewicz (2012) reported, the average transaction price for new fuel cars in April 2012 is \$30,748. We also note from `Jiskha.Com` (2010) that, in 2010, the transaction prices of new fuel vehicles roughly followed a normal distribution with mean \$23,000 and standard deviation \$3,500. Accordingly, in this numerical example, it should be reasonable to suppose that the consumers' net consumption gains are normally distributed with $E(\theta) = \$30,000$ and $\sigma = \$4,000$.

The maximum net valuation is set as $\bar{\theta} = \$60,000$. We truncate the normal distribution function at zero and assume that the probability of negative values is added to that of zero; we also truncate the distribution at $\theta = \bar{\theta}$, assuming that the probability of $\theta > \bar{\theta}$ is added to that of $\bar{\theta}$. We learn from `Leaseguide.com` (2011) that an automobile retailer's profit margin is usually between 4% and 13% with an average value falling in range [6%,8%]. Thus, it is reasonable to assume that $\rho = 0.07$. According to the discussions from Cuenca et al. (2000) and Chen et al. (2010), we assume that the manufacturer's unit production cost is $c = \$25,000$, and the retailer serves a market with the size $B = 1,000,000$. In addition, as indicated by Chen et al. (2008), it is reasonable to assume that the retailer's bargaining power follows a normal distribution with mean 0.4 and standard deviation 0.1.

Suppose that the scrappage program involves the MSRP cutoff level $A = \$35,000$ and the subsidy $s = \$4,000$. The manufacturer determines its optimal wholesale price as $w^* = \hat{w}_1 = \$32,297.56$. As a result, the MSRP is $p_s = (1+\rho)w^* = \$34,558.39$, which is less than the cutoff level A. Thus, the manufacturer qualifies for the scrappage program. We can also find the manufacturer's and the retailer's expected profits as $\Pi_M(w^*) = \$4.85 \times 10^9$ and $\Pi_R(w^*) = \$1.23 \times 10^9$, respectively, the resulting expected sales are $D(w^*) = 634,803$, and the government's total subsidy cost is $C_s(s) = sD(w^*) = \$2.54 \times 10^9$.

When the cutoff level A is reduced from \$35,000 to \$32,000, the manufacturer still qualifies for the scrappage program, choosing its optimal wholesale price as $w^* = A/(1+\rho) = \$29,906.5$ and its MSRP as $p_s = A = \$32,000$. The resulting sales are given as, $D(w^*) = 677,932$; the manufacturer's and retailer's expected profits are $\Pi_M(w^*) = \$4.16 \times 10^9$ and $\Pi_R(w^*) = \$1.7 \times 10^9$, respectively; the government's total subsidy cost is $C_s(s) = \$2.71 \times 10^9$.

When the cutoff level A is further reduced from \$32,000 to \$28,000, the manufacturer's optimal wholesale price and MSRP are $w^* = \hat{w}_2 = \$30,006.63$ and $p_s = \$32,107.06 > A = \$28,000$, which means that the manufacturer does not qualify for the scrappage program. The resulting expected sales are computed as $D(w^*) = 499,338$; the manufacturer's and retailer's expected profits are determined as $\Pi_M(w^*) = \$2.50 \times 10^9$ and $\Pi_R(w^*) = \$1.06 \times 10^9$, respectively. Because no consumer can obtain the subsidy, the government's total subsidy cost is zero.

From the above three scenarios, we observe that the manufacturer's expected profit when $A = \$35,000$ is significantly larger than those when $A = \$28,000$ and $A = \$32,000$. This means that increasing the MSRP cutoff level may increase the manufacturer's expected profit. However, the sales when $A = \$35,000$ are greater than those when $A = \$28,000$ but less than those when $A = \$32,000$. $\blacksquare$

6 Sensitivity Analysis and Managerial Implications

In this section, we perform analytical and numerical sensitivity analysis to explore the impact of the MSRP cutoff level A, the subsidy s, and the MSRP markup percentage ρ on the manufacturer's optimal wholesale price, the expected sales, and the manufacturer's and the retailer's maximum expected profits. Moreover, we find the government's optimal cutoff level and subsidy decisions that maximize the expected sales.

6.1 The Scrappage Programs

We first investigate the impact of A and s, and then derive the government's optimal decisions.

6.1.1 The Impact of the MSRP Cutoff Level A

We learn from Theorem 5 that if A is sufficiently low, then the manufacturer may be unwilling to reduce its wholesale price and MSRP to qualify for the scrappage program. This occurs possibly because qualifying for the program with a low cutoff level may deteriorate the manufacturer's profit margin. As a result, all consumers who buy the manufacturer's automobiles cannot get the subsidy, which means that the scrappage program is ineffective in increasing the sales and reducing air pollution. Therefore, it is important to execute a proper trade-in scheme for the automobile scrappage. Next, we analytically derive the minimum MSRP cutoff level that induces the manufacturer to qualify for the scrappage program.

Theorem 6 Given a subsidy s, the minimum MSRP cutoff level $\tilde{A}(s)$ for the manufacturer to willingly qualify for the scrappage program is obtained as follows:

1. If $(1+\rho)\hat{w}_2 \leq \bar{\theta}$, then $\tilde{A}(s)$ is the unique solution that satisfies $\Pi_{M1}(A/(1+\rho)) = \Pi_{M2}(\hat{w}_2)$.
2. If $(1+\rho)\hat{w}_2 > \bar{\theta}$, then $\tilde{A}(s)$ is the unique solution that satisfies $\Pi_{M1}(A/(1+\rho)) = \Pi_{M2}(\bar{\theta}/(1+\rho))$.

The minimum cutoff level $\tilde{A}(s)$ is decreasing in the subsidy s. $\blacksquare$

If $A \geq \tilde{A}(s)$, the manufacturer determines a wholesale price such that the MSRP is lower than or equal to the cutoff level A and thus qualifies for the scrappage program. Otherwise, the manufacturer does not qualify for the program. Next, using the parameter values in Example 1 but increasing A from \$24,000 to \$38,000 in increments of \$1,000, we plot Figure 2 to show the effect of the cutoff level A on the manufacturer's optimal wholesale price w^* [in Figure 2(a)], the expected sales $D(w^*)$ [in Figure 2(b)], the manufacturer's expected profit $\Pi_M(w^*)$ [in Figure 2(c)], and the retailer's expected profit $\Pi_R(w^*)$ [in Figure 2(d)]. We find that $\tilde{A}(s) = \$29,584.19$. Figure 2 indicates that if A is less than $\tilde{A}(s)$ (Zone 1), then the manufacturer has no intention to qualify for the scrappage program; but if A is greater than $\tilde{A}(s)$ (Zone 2), then the manufacturer decides to qualify for the program.

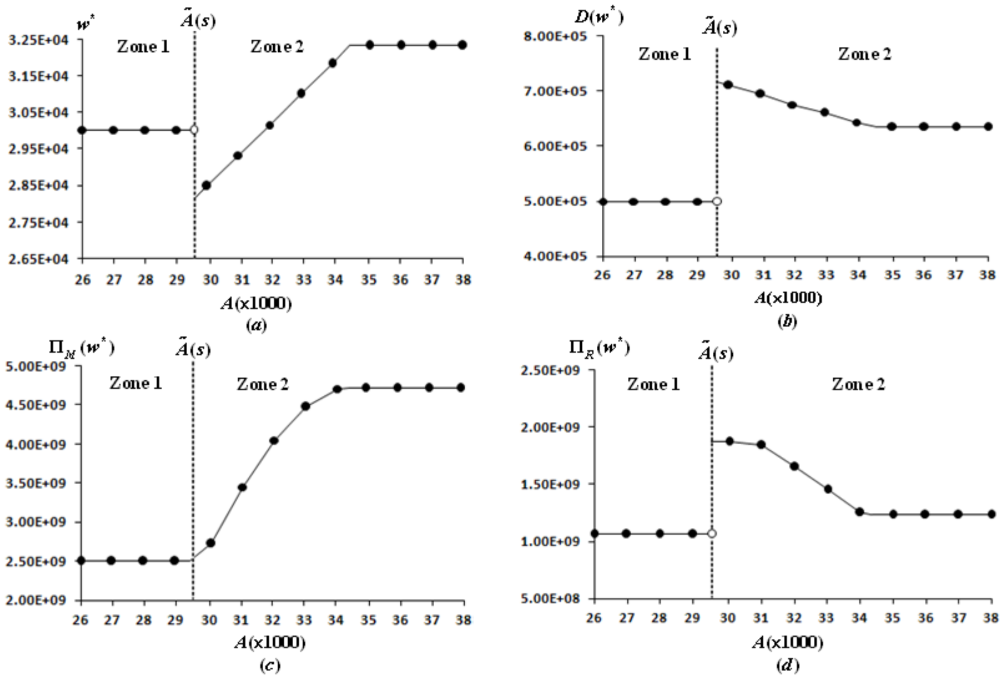

Figure 2: The impact of the cutoff level A on the manufacturer's optimal wholesale price w^*, the expected sales $D(w^*)$, the manufacturer's expected profit $\Pi_M(w^*)$, and the retailer's expected profit $\Pi_R(w^*)$. Note that, in Zone 1 (where $A < A(\tilde{s})$), the manufacturer's optimal wholesale price w^* is greater than $A/(1+\rho)$, and the manufacturer does not qualify for the scrappage program, which is thus ineffective. In Zone 2 (where $A \geq A(\tilde{s})$), $w^* \leq A/(1+\rho)$ and the manufacturer qualifies for the program, which is thus effective.

When A is in Zone 1, the manufacturer determines its wholesale price as $\hat{w}_2 = \$30,006.63$ and is unwilling to qualify for the scrappage program. This happens because of the following fact: If the manufacturer reduces its wholesale price, then more consumers are willing to buy and the expected sales are increased. But, when the cutoff level A is small, reducing the wholesale price to satisfy $(1+\rho)w \leq A$—i.e., to qualify for the scrappage program—will greatly decrease the manufacturer's profit margin but may not significantly increase the expected sales. As a result, when A is in Zone 1, the manufacturer is worse off from reducing its wholesale price, thereby keeping the wholesale price as $\hat{w}_2$, which is independent of the cutoff level A. Therefore, we observe that in Figure 2, when $A < \tilde{A}(s)$, the curves of the optimal wholesale price, the expected sales, the manufacturer's and retailer's expected profits are horizontal lines.

When A is increased from a value in Zone 1 (where $A < \tilde{A}(s)$) to $A = \tilde{A}(s)$ (which belongs to Zone 2), the manufacturer can enjoy a higher profit by reducing its wholesale price to $A/(1+\rho)$ and thus qualifying for the scrappage program. The manufacturer's price reduction across two zones appears to be a downward jump in Figure 2(a). Moreover, such a price reduction generates a significant increase in the expected sales [i.e., an upward jump in Figure 2(b)], which causes an increase in the retailer's expected profit [i.e., an upward jump in Figure 2(d)]. However, there is no jump in the manufacturer's expected profit at the boundary between two zones, as indicated in Figure 2(c). This happens because, as Theorem 6 shows, at the boundary (i.e., $A = \tilde{A}(s)$), the manufacturer is indifferent between qualifying and not qualifying for the program.

We learn from Theorem 5 that when $\tilde{A}(s) \leq A < (1+\rho)\hat{w}_1 = \$34,558.39$, the manufacturer's optimal wholesale price is $w^* = A/(1+\rho)$, which is increasing in A, as shown in Figure 2(a). Hence, the expected sales, $D(w^*) = 1 - F(A/(1+\rho) - s)$, are decreasing in A; see Figure 2(b). Such a result follows the fact that if the cutoff level is higher than the minimum level (for the manufacturer's program qualification) but lower than a sufficiently large value (that does not influence the manufacturer's wholesale pricing decision), then the manufacturer intends to raise its wholesale price to attain a higher expected profit while qualifying for the scrappage program. Though, an increase in the wholesale price does not significantly reduce the expected sales, as indicated by Figure 2(b), because the government's subsidy s awarded to each consumer helps alleviate the negative impact of the price increase on the sales.

As discussed above, when $\tilde{A}(s) \leq A < (1+\rho)\hat{w}_1$, the manufacturer's expected profit, $\Pi_{M1}(A/(1+\rho)) = B[A/(1+\rho) - c][1 - F(A/(1+\rho) - s)]$, is increasing in A (indicated by Figure 2(c)), because $\Pi_{M1}(w)$ is a unimodal function with a unique maximizing value $\hat{w}_1$. In addition, we find from Figure 2(d) that the retailer's expected profit $\Pi_R(A/(1+\rho))$ is decreasing in A, which is attributed to the following fact. An increase in A leads to a reduction in the expected sales but does not significantly raise the retail price to increase the retailer's profit margin, because the retail price results from the negotiation between the retailer and each consumer.

When $A \geq (1+\rho)\hat{w}_1$, we find from Theorems 3 and 5 that the manufacturer determines its optimal wholesale price as $\hat{w}_1 = \$32,297.56$, and the optimal sales are $D(w^*) = 1 - F(\hat{w}_1 - s)$. Accordingly, any further increase in A has no impact on the optimal wholesale price, the expected sales, and the manufacturer's and retailer's expected profits. The reason is given as follows: if the manufacturer further increases its wholesale price, then the subsidy for a consumer cannot help reduce the consumer's purchase cost and the sales are thus significantly reduced. Therefore, the manufacturer does not change its wholesale price; and the expected sales, and the manufacturer's and the retailer's expected profits are unchanged. The corresponding curves in Figure 2 are thus horizontal lines.

The effect of A on the expected sales in Zone 2 also implies that, compared with a scrappage program with a subsidy only (i.e., the MSRP cutoff level A is infinitely large), a program with a moderate value of the cutoff level is *more* effective in reducing the wholesale price and thus boosting the sales of fuel efficient automobiles. For example, comparing the expected sales when $A = \tilde{A}(s)$ with those when $A = (1+\rho)\hat{w}_1$ in our numerical experiments, we find that a scrappage program with the MSRP cutoff level $\tilde{A}(s)$ can effectively improve the expected sales by 21.3% than a program without a cutoff level.

The manufacturer's profit reaches its maximum value when $A \geq (1+\rho)\hat{w}_1$. Any further increase in A cannot improve its profit any more. This means that the manufacturer's expectation on the value of the cutoff level significantly differs from the government's optimal cutoff level that maximizes the expected sales. Since the scrappage program is implemented mainly to control carbon emissions and protect the environment, we next discuss the impact of the sales of the fuel efficient automobile on the reduction in CO_2 emissions. According to Sachs (2009) and Zolnik (2012), we can roughly estimate the reduction in CO_2 emissions when more consumers trade in their used automobiles for new, more fuel-efficient automobiles. Specifically, new fuel-efficient automobiles can run 24.9 miles per gallon (mpg) on average, whereas the average mpg for trade-in automobiles is 15.8. This means that the average difference in fuel efficiency between new and used automobiles is 9.1 mpg. Assuming that an automobile can run for $12,000$ miles per year, we find that the new and used automobiles need 482 and 759 gallons of gasoline, respectively. That is, such a trade-in can reduce the gasoline usage by 277 gallons. Because around 8.8 kilograms of CO_2 is generated by burning one gallon of gasoline, the trade-in can decrease CO_2 emissions by 2.44 metric tons.

When $A = \tilde{A}(s)$, the expected sales increase to the maximum level of $713,973$, the reduction in the gasoline consumption approximates 19.78 million gallons, and the corresponding reduction in CO_2 emissions is calculated as 1.74 million metric tons. But, when $A \geq (1+\rho)\hat{w}_1$, the expected sales decrease to $634,803$ and thus, the reduction in the gasoline consumption and that in CO_2 emissions are 17.65 million gallons and 1.55 million metric tons, respectively. As the above results indicate, the effectiveness of the scrappage program in reducing CO_2 emissions is dependent on the sales of the fuel-efficient automobile. As Figure 2(a) suggests, the government should adopt a moderate cutoff level (e.g., $A = \tilde{A}(s)$) rather than implementing a high cutoff level $A > \tilde{A}(s)$, in order to effectively increase the sales of new automobiles.

When the government does not implement any scrappage program ($s = 0$), the consumer's purchase decision is given as in Remark 1, and the manufacturer determines the wholesale price as $\hat{w}_2$. The resulting expected sales are $D(\hat{w}_2) = 1 - F(\hat{w}_2)$, and the manufacturer's expected profit is $\Pi(\hat{w}_2)$. As indicated by Figure 2(b), $D(\hat{w}_2)$ (the expected sales in Zone 1) is much lower than the expected sales when the manufacturer qualifies for the program (in Zone 2), which are $D(A/(1+\rho))$ when $\tilde{A}(s) \leq A < (1+\rho)\hat{w}_1$ and $D(\hat{w}_1)$ when $A \geq (1+\rho)\hat{w}_1$. This suggests that a scrappage program with a sufficiently large MSRP cutoff level A can significantly increase the sales of new fuel-efficient automobiles.

6.1.2 The Impact of the Subsidy s

Using Theorem 5, we can investigate the impact of subsidy s on the manufacturer's optimal wholesale price and maximum expected profit, and the expected sales. Specifically, if $A \geq (1+\rho)\hat{w}_1$, then the manufacturer determines its optimal wholesale price as $\hat{w}_1$, and its maximum expected profit and the expected sales are increasing in s, according to Theorem 3. If $(1+\rho)\hat{w}_2 \leq A < (1+\rho)\hat{w}_1$, then the manufacturer decides to qualify for the scrappage program and determines its optimal wholesale price as $A/(1+\rho)$, which is independent of the subsidy s. However, the expected sales $D(A/(1+\rho))$ and the manufacturer's expected profit $\Pi_{M1}(A/(1+\rho))$ are both increasing in s. If $A < (1+\rho)\hat{w}_2$, then the manufacturer may or may not desire to qualify for the scrappage program. When $\Pi_{M1}(A/(1+\rho)) \geq \Pi_{M2}(\hat{w}_2)$, the manufacturer chooses to qualify for the scrappage program by setting its optimal wholesale price as $A/(1+\rho)$. Since $\Pi_{M1}(A/(1+\rho))$ is increasing in the subsidy s, we find that if the government increases its subsidy s to a sufficiently high level, then the manufacturer certainly has an incentive to qualify for the scrappage program and obtain a greater profit. A critical question then arises as follows: What is the minimum subsidy assuring that the manufacturer chooses to qualify for the scrappage program?

Theorem 7 Given an MSRP cutoff level A, the minimum subsidy $\tilde{s}(A)$ that leads the manufacturer to qualify for the scrappage program can be given as follows:

1. If $A/(1+\rho) < \hat{w}_2 \leq \bar{\theta}/(1+\rho)$, then $\tilde{s}(A)$ is the unique solution that satisfies $\Pi_{M1}(A/(1+\rho)) = \Pi_{M2}(\hat{w}_2)$.
2. If $A/(1+\rho) \leq \bar{\theta}/(1+\rho) < \hat{w}_2$, then $\tilde{s}(A)$ is the unique solution that satisfies $\Pi_{M1}(A/(1+\rho)) = \Pi_{M2}(\bar{\theta}/(1+\rho))$.

Moreover, $\tilde{s}(A)$ is decreasing in A. ■

Recall from Theorem 5 that the manufacturer always decides to qualify for the scrappage program when $A \geq \hat{w}_2(1+\rho)$. We learn from Theorem 7 that, when $A < \hat{w}_2(1+\rho)$, the subsidy s should be greater than or equal to $\tilde{s}(A)$, so as to assure that the manufacturer decides to qualify for the program. Accordingly, if both the MSRP cutoff level and the subsidy are sufficiently low, then the scrappage program may be ineffective, because the manufacturer is unwilling to reduce its wholesale price and MSRP to qualify for the program. It follows that a government's scrappage program with a sufficiently high value of A [e.g., $A \geq \hat{w}_2(1+\rho)$] is likely to be effective even if the subsidy s is not large. This result is in agreement with the practice that some governments (e.g., Mexico, Slovaks, and the United States) implement a significantly high cutoff level, and other governments (e.g., China, France, Germany, and Japan) do not set any cutoff level. Moreover, another important reason why the governments are willing to increase the cutoff level rather than the subsidy is that awarding a large subsidy to consumers may bring a heavy financial burden to the governments.

However, even if the cutoff level A is so high that the manufacturer decides to qualify for the scrappage program, the expected sales and the manufacturer's expected profit cannot be significantly improved when the subsidy is very low. According to Theorem 3, we find that when $A \geq \tilde{A}(s)$, both the manufacturer's expected profit and the expected sales are increasing in s. This means that, keeping A unchanged, the government can raise its subsidy to increase the manufacturer's expected profit and the expected sales, which can entice the manufacturer to qualify for the scrappage program. In fact, we find from Theorem 6 that $\tilde{A}(s)$ is decreasing in s, which implies that, to encourage the manufacturer to qualify for the scrappage program, the government should increase s to a sufficiently high level $\hat{s}$ such that $A \geq \tilde{A}(\hat{s})$. Since a very large subsidy may cause a heavy financial burden, the government should determine a proper value of subsidy subject to its budget constraint. This can be demonstrated by the practice in China: In June 2009, a nationwide scrappage program was implemented to offer subsidies of RMB3,000—RMB6,000 (equivalently, US\$450—US\$900) to consumers who trade in used heavy polluting automobiles or trucks for new ones. However, this program could only result in a little success in the first several months; hence, the Chinese government raised its subsidy to RMB5,000—RMB18,000 (equivalently, US\$750—US\$2,700). The new scrappage program was proved to be effective, because, until October 2010, the number of replaced automobiles had been 2.84×10^5.

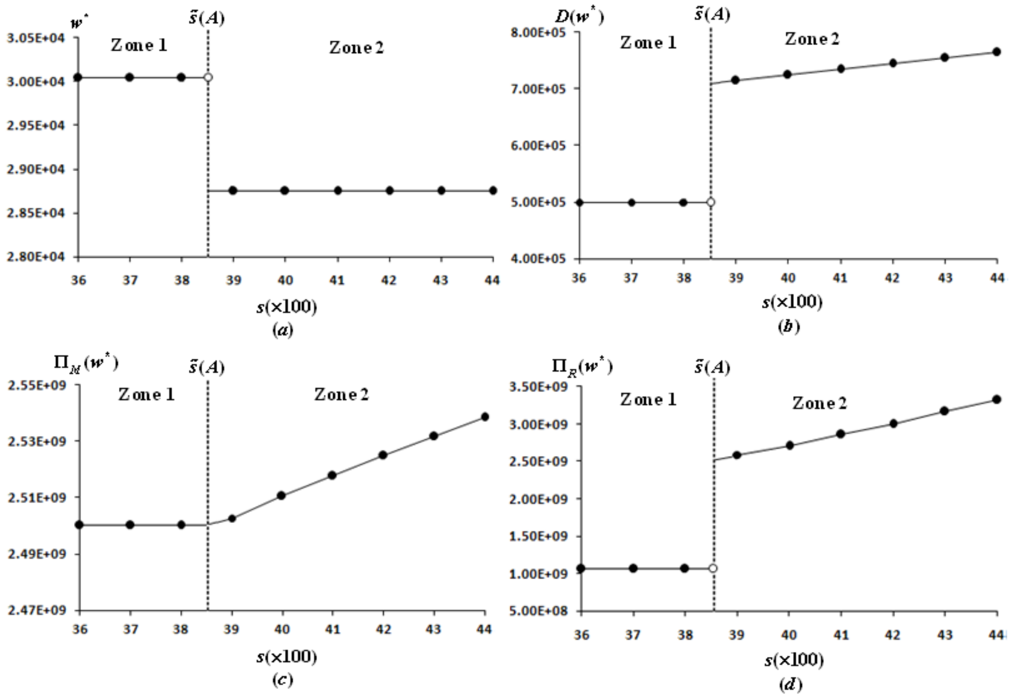

Figure 3: The impact of the subsidy s on the manufacturer's optimal wholesale price w^*, the expected sales $D(w^*)$, the manufacturer's expected profit $\Pi_M(w^*)$, and the retailer's expected profit $\Pi_R(w^*)$. Note that, in Zone 1 (where $s < \tilde{s}(A)$), the manufacturer's optimal wholesale price w^* is greater than $A/(1+\rho)$, and the manufacturer does not qualify for the scrappage program, which is thus ineffective. In Zone 2 (where $s \geq \tilde{s}(A)$), $w^* \leq A/(1+\rho)$ and the manufacturer qualifies for the program, which is thus effective.

We now perform numerical experiments to examine the effect of subsidy s on the manufacturer's optimal wholesale price w^*, expected sales $D(w^*)$, and the manufacturer's and retailer's expected profits $\Pi_M(w^*)$ and $\Pi_R(w^*)$, assuming $A = \$30,000$ but using the values of other parameters as in Example 1. We increase the value of s from $\$3,600$ to $\$4,400$ in increments of \$100, and plot Figures 3(a)-(d) to show the impact of s on w^*, $D(w^*)$, $\Pi_M(w^*)$, and $\Pi_R(w^*)$. We find that $\tilde{s}(A) = \$3,835.98$.

When s is small such that $s < \tilde{s}(A)$ (Zone 1), the manufacturer determines its wholesale price as $\hat{w}_2$ and does not qualify for the scrappage program. As a result, consumers cannot obtain the subsidy from the government, and the expected sales $D(\hat{w}_2)$ are thus low, as shown in Figure 3(b); the government's total subsidy cost is zero. It follows that the optimal wholesale price $\hat{w}_2$, the expected sales $D(\hat{w}_2)$, the manufacturer's and retailer's expected profits are independent of s; therefore, all the corresponding curves are horizontal lines in Zone 1, as indicated by Figure 3.

When s is increased from $s < \tilde{s}(A)$ (Zone 1) to $s \geq \tilde{s}(A)$ (Zone 2), the manufacturer reduces its wholesale price to $A/(1+\rho)$ in order to qualify for the program, and each consumer can enjoy the subsidy. This generates a large increase in the expected sales, and the retailer enjoys a great increase in its expected profit. Therefore, we observe jumps at the boundary between Zone 1 and Zone 2 in Figures 3(a), (b), and (d). We note from Theorem 7 that at $s = \tilde{s}(A)$, the manufacturer is indifferent in his expected profit between qualifying and not qualifying for the program; hence, we do not observe a jump in the manufacturer's profit at the boundary between Zone 1 and Zone 2 in Figure 3(c).

In Zone 2, the expected sales are $D(A/(1+\rho)) = 1 - F(A/(1+\rho) - s)$, and the government's total subsidy cost is $C_s(s) = s[1 - F(A/(1+\rho) - s)]$. As s increases, the manufacturer has no incentive to reduce its wholesale price because the manufacturer intends to assure its profit margin while each consumer enjoys a greater net gain. In addition, when the government increases s, the manufacturer does not increase its wholesale price because, otherwise, the manufacturer will not qualify for the program. Therefore, the manufacturer's optimal wholesale price is independent of s when $s \geq s(\tilde{A})$, as indicated by Figure 3(a). As a result, increasing the value of s can entice more low-valuation consumers to trade in their used automobiles, which implies that the expected sales are increasing in s; see Figure 3(b). Since the manufacturer's profit margin is not reduced (as argued above), its expected profit is increasing in s, as shown in Figure 3(c). In addition, a higher subsidy helps raise the retail price and thus improve the retailer's profit margin. It follows that the retailer's expected profit increases as the value of s rises, as depicted in Figure 3(d).

From the above, we find that even when the government sets a sufficiently high cutoff level A, the subsidy is still helpful to increase the sales of fuel-efficient automobiles for reducing CO_2 emissions and to raise the manufacturer's and the retailer's expected profits for assuring the two firms' incentives. However, we note from Figures 2 and 3 that when $s \geq \tilde{s}(A)$, increasing s does not significantly raise the sales and two firms' expected profits, compared with the impact of A. In fact, a larger subsidy benefits the high-valuation consumers but is not significantly effective in attracting low-valuation consumers to buy. Therefore, the government incurs a high subsidy cost without saliently improving the sales. This suggests that the government needs to implement an appropriate MSRP cutoff level but may not need to apply a very large subsidy.

We also learn from Figure 3(b) that the expected sales $D(\hat{w}_2)$ when the government does not implement any program are significantly lower than the expected sales $D(w^*)$ in the presence of an effective scrappage program (i.e., $s \geq \tilde{s}(A)$). This implies that when the subsidy is sufficiently large, the scrappage program can significantly stimulate more consumers to buy fuel-efficient automobiles.

6.1.3 The Government's Optimal Decisions

In practice, the government usually has a budget (denoted by $\bar{C}_s$) for its scrappage program. For example, the budget for the US government's "CARS" scrappage program was \$3.0 billion, and the German government afforded its scrappage program with a total amount of about \$7 billion (€5 billion); see Aldred and Tepe (2011) and Lenski et al. (2010). It thus behooves us to consider the following important question. Given a budget for the government's scrappage program, what are the government's optimal MSRP cutoff level and subsidy that maximize the expected sales?

We recall from Section 6.1.1 that given a subsidy s, in order to maximize the sales, the government should set the MSRP cutoff level at $A = \tilde{A}(s)$, and the manufacturer determines its wholesale price as $w^* = \tilde{A}(s)/(1+\rho)$. The corresponding expected sales are $D(\tilde{A}(s)) = 1 - F(\tilde{A}(s)/(1+\rho) - s)$. The scrappage program include both an MSRP cutoff level and a subsidy. Thus, besides the cutoff level, the government should also determine the optimal subsidy for the given budget $\bar{C}_s$. In order to effectively improve the sales, the total subsidy cost—i.e., $C_s = s \times D(\tilde{A}(s))$—should be equal to $\bar{C}_s$; that is,

$$s \times D(\tilde{A}(s)) = \bar{C}_s. \tag{12}$$

As Theorem 6 indicates that $\tilde{A}(s)$ is decreasing in s, and the expected sales $D(\tilde{A}(s))$ is increasing in s. The total subsidy cost $C_s(s) = s \times D(\tilde{A}(s))$ is a monotone function. Consequently, equation (12) has a unique optimal solution s^*.

Remark 2 In order to maximize the expected sales, the government should determine its optimal MSRP cutoff level as $A^* = \tilde{A}(s)$, and choose its optimal subsidy as $s^* = \{s | s \times D(\tilde{A}(s)) = \bar{C}_s\}$, where $D(\tilde{A}(s)) = 1 - F(\tilde{A}(s)/(1+\rho) - s)$. ■

Next, using the parameter values in Example 1 but increasing the budget $\bar{C}_s$ from $\$1.5 \times 10^9$ to $\$8.0 \times 10^9$ in increments of $\$5 \times 10^8$, we plot Figure 4 to show the influences of $\bar{C}_s$ on (i) the government's optimal decisions A^* and s^* [in Figures 4(a) and (b), respectively], (ii) the expected sales $D(w^*)$ [in Figure 4(c)], and (iii) the manufacturer's optimal wholesale price w^* [in Figure 4(d)].

Using Figure 4, we can address the following two questions. Given the target sales of fuel-efficient automobiles, how much should the government prepare for its scrappage program? and what are the optimal MSRP cutoff level and subsidy? For example, if the government intends to attain the expected sales of 8.2×10^5, then we can learn from Figure 4(c) that

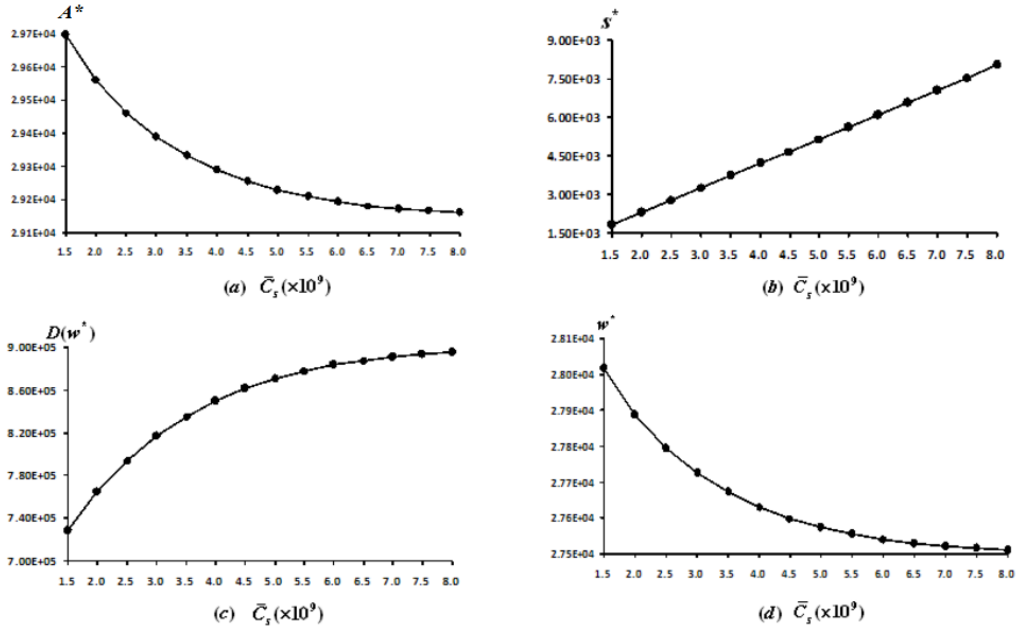

Figure 4: The impact of the government's budget on its optimal decisions, the expected sales, and the manufacturer's optimal wholesale price.

the government should offer a total budget of $\$3.0 \times 10^9$, and we can use Figures 4(a) and (b) to find the optimal cutoff level A^* and subsidy s^* as $\$29,400$ and $\$3,270$, respectively. For a given budget for the scrappage program, we can also use Figure 4 to make the optimal decisions for the government and estimate the resulting expected sales. For example, if the government has a budget of $\$5.0 \times 10^9$ for the program, then the optimal MSRP cutoff level and subsidy should be $\$29,230$ and $\$5,150$, respectively, and the resulting expected sales approximate 8.7×10^5.

We also observe from Figures 4(a) and (b) that the optimal subsidy is increasing in the government's budget $\bar{C}_s$, whereas the optimal MSRP cutoff level is decreasing in $\bar{C}_s$. This result may be surprising because the government could respond to a higher budget by decreasing its cutoff level to encourage the manufacturer to reduce its wholesale price—see Figure 4(d)—and qualify for the program, thus improving sales. We also observe from Figure 4(c) that the expected sales are increasing in $\bar{C}_s$ but at a decreasing rate, which implies that a large budget may not help the scrappage program saliently increase the sales.

6.2 The Impact of the MSRP Markup Percentage ρ

As discussed in Section 3, the MSRP p_s is calculated as a percentage markup of the wholesale price w, i.e., $p_s = (1+\rho)w$, where ρ is the markup percentage. Such an approach for the calculation of MSRP has been widely used in the real-world automobile industry. Using Theorem 5, we can find that, given the scrappage program (A, s), if ρ is sufficiently small such that $A/(1+\rho) \geq \hat{w}_1$, then the manufacturer determines its wholesale price as $\hat{w}_1$ (which is independent of ρ) and qualifies for the scrappage program, achieving the profit $\Pi_{M1}(\hat{w}_1)$. As a result, the expected sales are $D(\hat{w}_1) = 1 - F(\hat{w}_1 - s)$. If ρ is given such that $\hat{w}_2 \leq A/(1+\rho) < \hat{w}_1$, then the manufacturer still decides to qualify for the scrappage program, determining its wholesale price as $A/(1+\rho)$. The resulting expected sales are $D(w^*) = 1 - F(A/(1+\rho) - s)$, and the manufacturer's expected profit is $\Pi_{M1}(A/(1+\rho))$. Note that $\Pi_{M1}(A/(1+\rho))$ is lower than $\Pi_{M1}(\hat{w}_1)$.

If the markup percentage ρ is given such that $A/(1+\rho) < \hat{w}_2 \leq \bar{\theta}/(1+\rho)$, then the manufacturer may or may not decide to qualify for the scrappage program. Specifically, when $\rho \leq \tilde{\rho}$, where $\tilde{\rho}$ is the unique solution to the equation that $\Pi_{M1}(A/(1+\rho)) = \Pi_{M2}(\hat{w}_2)$, the manufacturer will choose the optimal wholesale price as $A/(1+\rho)$ to qualify for the program. But, when $\rho > \tilde{\rho}$, the manufacturer will not qualify for the program. We also find that the manufacturer's expected profit when $A/(1+\rho) < \hat{w}_2 \leq \bar{\theta}/(1+\rho)$ is larger than that when $A/(1+\rho) \leq \bar{\theta}/(1+\rho) < \hat{w}_2$.

From the above, we conclude that the markup percentage ρ greatly influences the manufacturer's decision on whether or not to qualify for the scrappage program. That is, the manufacturer with a larger value of ρ has less incentive to qualify for the program, which means that the program is less likely to be effective.

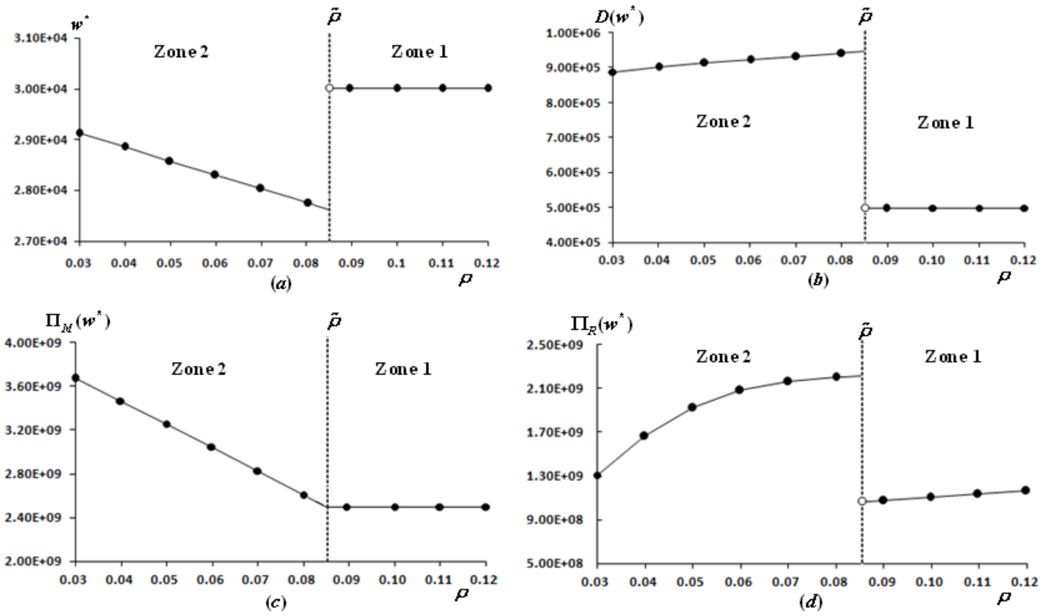

Figure 5: The impact of the markup percentage ρ on the manufacturer's optimal wholesale price w^*, the expected sales $D(w^*)$, the manufacturer's expected profit $\Pi_M(w^*)$, and the retailer's expected profit $\Pi_R(w^*)$. Note that, in Zone 1 (where $\rho \geq \tilde{\rho}$), the manufacturer's optimal wholesale price w^* is greater than $A/(1+\rho)$, and the manufacturer does not qualify for the scrappage program, which is thus ineffective. In Zone 2 (where $\rho < \tilde{\rho}$), $w^* \leq A/(1+\rho)$, and the manufacturer qualifies for the program, which is thus effective.

To demonstrate our above results, we use the parameter values as in Example 1 but increase ρ from 0.03 to 0.12 in increments of 0.01, and plot Figure 5 to show the impact of ρ on the manufacturer's optimal wholesale price, the expected sales, and two firms' expected profits. When ρ falls in Zone 2 (i.e., $\rho \leq \tilde{\rho}$), the manufacturer determines its optimal wholesale price as $A/(1+\rho)$, so as to qualify for the scrappage program. Accordingly, each consumer can enjoy a subsidy from the government. Moreover, the wholesale price $A/(1+\rho)$ is decreasing in ρ; thus, a larger value of ρ results in higher expected sales, as shown in Figure 5(b). As a consequence, the retailer benefits from a larger value of ρ by achieving higher expected sales and a higher profit margin, as indicated by Figure 5(d). But, the manufacturer is worse off, because the manufacturer's profit margin is greatly reduced due to a decrease in its wholesale price, as shown in Figure 5(c). The expected sales are $D(w^*) = 1 - F(A/(1+\rho) - s)$, and the government's total subsidy cost is $C_s(s) = sD(A/(1+\rho))$, which are both increasing in ρ.

We also learn from Figure 5(a) that, when the value of ρ is increased from $\rho \leq \tilde{\rho}$ (Zone 2) to $\rho > \tilde{\rho}$ (Zone 1), the manufacturer loses its incentive to qualify for the scrappage program, because the manufacturer needs to determine a significantly low wholesale price to qualify for the program. To assure the profit margin, the manufacturer keeps its wholesale price at a high level as $\hat{w}_2$. As a result, no consumer enjoys a subsidy from the government, which entails a large decrease in the number of consumers trading in their used automobiles. The decrease in the expected sales generates a large decrease in the retailer's expected profit. Therefore, there appear the jumps at the boundary between Zone 1 and Zone 2 in Figures 5(a), (b), and (d). Moreover, the manufacturer's and retailer's expected profits in Zone 1 are lower than those in Zone 2, as shown in Figures 5(c) and (d). Moreover, Figure 5(d) indicates that the retailer's profit reaches its maximum value when $\rho = \tilde{\rho}$. In Zone 1, the optimal wholesale price $\hat{w}_2$ is independent of ρ, and thus the expected sales and the manufacturer's expected profit do not depend on ρ, which are represented by the horizontal lines in Figures 5(a), (b), and (c). However, the retailer's profit is increasing in ρ because an increase in ρ generates a higher value of the MSRP and then increases the retailer's profit margin that is dependent on the MSRP.

From the above, we conclude that even when the government chooses a sufficiently high cutoff level A (as discussed in Section 6.1.1) and a proper subsidy s (as discussed in Section 6.1.2), the manufacturer may not benefit from the scrappage program, because, for the case of a large MSRP markup percentage ρ, the manufacturer does not intend to reduce the wholesale price to qualify for the scrappage program.

7 Summary and Concluding Remarks

In this paper, we considered a two-level automobile supply chain, where a manufacturer and a retailer serve heterogeneous consumers in a fuel-efficient automobile market under a government's scrappage program involving a cutoff level and a subsidy. We examined the impact of the program on the government's optimal decisions, the manufacturer's optimal wholesale price, the negotiated retail price, and consumers' purchase decisions. Since the retail price charged to a consumer is determined as a result of the negotiation between the consumer and the retailer, who may have different bargaining powers, we used the GNB concept to characterize the negotiated retail price. We then developed the manufacturer's expected profit function, which was maximized to find the optimal wholesale price for the manufacturer. In addition, we compared the optimal wholesale price, the expected sales, and the manufacturer's and retailer's expected profits under the scrappage program and those in the absence of such a program. We also investigated the impact of the MSRP cutoff level, the subsidy, the government's budget, and the markup percentage (that was used to calculate the MSRP) on the manufacturer's optimal pricing decision, the expected sales, the manufacturer's and retailer's expected profits, and the government's optimal decisions.

We summarize our major managerial insights as follows:

1. Given a subsidy s, there exists a minimum cutoff level $\tilde{A}(s)$ for the manufacturer to qualify for the program, which is decreasing in s. When the cutoff level A is increased from a value lower than $\tilde{A}(s)$ to a value higher than $\tilde{A}(s)$, the manufacturer's optimal wholesale price is decreased, whereas both the expected sales and the manufacturer's expected profit are increased.

   If A is sufficiently large such that $A \geq (1+\rho)\hat{w}_1 \geq \tilde{A}(s)$, then the manufacturer's optimal wholesale price and the expected sales are independent of A but increasing in s; as a result, both the manufacturer and the retailer can benefit from the scrappage program. If $(1+\rho)\hat{w}_1 \geq A \geq \tilde{A}(s)$, then the optimal wholesale price and the manufacturer's profit are increasing in A, but the expected sales and the retailer's expected profit are decreasing in A. When A is equal to $\tilde{A}(s)$, the expected sales reach the maximum value and the effectiveness of the scrappage program is thus maximized.
2. The manufacturer may not prefer a moderate MSRP cutoff level (i.e., $A = \tilde{A}(s)$), because its expected profit is the highest when the cutoff level takes a sufficiently high value (i.e., $A \geq (1+\rho)\hat{w}_1$). This means that the manufacturer's preferred cutoff level significantly differs from the government's optimal cutoff level that maximizes the expected sales. Therefore, the government's optimal decision may influence the manufacturer's motivation for qualifying for the scrappage program.
3. Given a cutoff level A, if $s \geq \tilde{s}(A)$, then the manufacturer desires to qualify for the scrappage program, and the expected sales and the manufacturer's and the retailer's profits are increasing in s. That is, the effectiveness of a scrappage program requires that the subsidy should be sufficiently high such that $s \geq \tilde{s}(A)$. Any increase in the subsidy in the range $[0, \tilde{s}(A)]$ cannot help increase the sales. Although a larger subsidy when $s \geq \tilde{s}(A)$ can result in higher sales, it also increases the government's cost. Thus, the optimal subsidy should be determined according to the target sales that the government expects the manufacturer to achieve.
4. Given the government's sales target, we can uniquely compute the optimal MSRP cutoff level and subsidy, and find the corresponding budget for the government. The government's optimal subsidy is increasing in the government's budget, while its optimal MSRP cutoff level is decreasing in the budget. This may be a surprising insight because the government could respond to a higher budget by raising its cutoff level to encourage the manufacturer to qualify for the program and thus improving sales. Moreover, the expected sales are increasing in the budget but at a decreasing rate, which implies that a larger budget may not help the scrappage program saliently increase the sales.
5. Given a scrappage program, there exists a maximum markup percentage $\tilde{\rho}$ for the manufacturer to qualify for the program. Moreover, when $\rho \leq \tilde{\rho}$, the manufacturer's

expected profit is decreasing in ρ, while the expected sales and the retailer's profit are increasing in ρ. The manufacturer with a smaller markup percentage ρ is more likely to reduce its wholesale price and qualify for the scrappage program. The insight is important since the government needs to understand which manufacturers are more likely to qualify for its scrappage program.

In conclusion, we find that the scrappage program with a sufficiently large MSRP cutoff level and a proper subsidy is useful to stimulating the sales of fuel-efficient automobiles, and also increasing the profitability of the automobile manufacturer, who thus has an incentive to qualify for the program. Moreover, for the scrappage program, the government's optimal cutoff level and subsidy that maximize the expected sales depend on the budget for such a program.

In future, some research directions may be worth considering. First, we may investigate the scrappage program in a dynamic, temporal setting, focusing on its impact on the manufacturer's future pricing decisions, its long-run average profit, and the expected sales. Secondly, it could be important for the manufacturer to consider the investment in its product design. Since the government implements a program to encourage the use of fuel-efficient automobiles, we may need to investigate if and how the program can induce the manufacturer to invest in designing a green automobile for environment protection. Thirdly, in practice a consumer may be willing to sell his or her used automobile in a secondary market instead of trading it in. In another research direction, we may consider the supply chain when a government's scrappage program and a secondary market jointly exist, and examine if and how the existence of the secondary market impacts the manufacturer's decisions and the effectiveness of the government's scrappage program in stimulating consumers' trade-ins.

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Online Supplements

"Qualifying for a Government's Scrappage Program to Stimulate Consumers' Trade-In Transactions? Analysis of an Automobile Supply Chain Involving a Manufacturer and a Retailer"

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Appendix A Proofs of Theorems

Proof of Theorem 1. To find the GNB discount α^*, we need to solve the constrained maximization problem in (2). First, temporarily ignoring the constraints that $u(\alpha, w) \geq 0$ and $\rho w - \alpha(1+\rho)w \geq 0$, we maximize Λ in (2) to find the optimal discount for the unconstrained problem.

The first- and second-order derivatives of Λ in (2) w.r.t. α are calculated as,

$$\frac{d\Lambda}{d\alpha} = \left[\frac{\beta}{\theta - (1-\alpha)(1+\rho)w + \mathbf{1}_{\{w \leq A/(1+\rho)\}} \times s} - \frac{1-\beta}{\rho w - \alpha(1+\rho)w}\right](1+\rho)w\Lambda, \tag{13}$$

and

$$\begin{aligned}\frac{d^2\Lambda}{d\alpha^2} &= -\left[\frac{\beta}{[\theta - (1-\alpha)(1+\rho)w + \mathbf{1}_{\{w \leq A/(1+\rho)\}} \times s]^2} + \frac{1-\beta}{(\rho w - \alpha(1+\rho)w)^2}\right](1+\rho)^2 w^2 \Lambda \\ &\quad + \left[\frac{\beta}{\theta - (1-\alpha)(1+\rho)w + \mathbf{1}_{\{w \leq A/(1+\rho)\}} \times s} - \frac{1-\beta}{\rho w - \alpha(1+\rho)w}\right]^2 (1+\rho)^2 w^2 \Lambda \\ &= -\left[\frac{\beta}{[\theta - (1-\alpha)(1+\rho)w + \mathbf{1}_{\{w \leq A/(1+\rho)\}} \times s]^2} + \frac{1-\beta}{(\rho w - \alpha(1+\rho)w)^2}\right](1+\rho)^2 w^2 \Lambda \\ &\quad + \frac{d\Lambda}{d\alpha}\left[\frac{\beta}{\theta - (1-\alpha)(1+\rho)w + \mathbf{1}_{\{w \leq A/(1+\rho)\}} \times s} - \frac{1-\beta}{\rho w - \alpha(1+\rho)w}\right](1+\rho)w.\end{aligned}$$

We can find that at any point (the value of α) satisfying $d\Lambda/d\alpha = 0$, the second-order derivative $d^2\Lambda/d\alpha^2$ is negative, i.e., $d^2\Lambda/d\alpha^2 < 0$. It thus follows that Λ is a unimodal function of the discount α with a unique maximizing value α^*.

We equate $d\Lambda/d\alpha$ in (13) to zero, solve the resulting equation for α, and find the GNB discount α^* as follows:

$$\alpha^* = \frac{(1+\rho-\beta)w - (1-\beta)[\theta + \mathbf{1}_{\{w \leq A/(1+\rho)\}} \times s]}{(1+\rho)w}.$$

However, we should note that the negotiated retail price p_r should be less than or equal to the MSRP, i.e., $p_r = (1-\alpha^*)(1+\rho)w \leq p_s = (1+\rho)w$, which means that $\alpha^* \geq 0$. We thus have the following discussions:

1. When $(1-\beta)[\theta + 1_{\{w \leq A/(1+\rho)\}} \times s] \leq (1+\rho-\beta)w$, or $\theta \leq \theta_2 \equiv [(1+\rho-\beta)w]/(1-\beta) - 1_{\{w \leq A/(1+\rho)\}} \times s$, the optimal $\alpha^* \geq 0$. We substitute the value of α^* into the consumer's

and the retailer's objective functions, and obtain

$$u(\alpha^*, w) = \beta(\theta+\mathbf{1}_{\{w\le A/(1+\rho)\}}\times s-w) \quad \text{and} \quad \rho w-\alpha^*(1+\rho)w = (1-\beta)(\theta+\mathbf{1}_{\{w\le A/(1+\rho)\}}\times s-w),$$

which must be nonnegative when $\theta \ge \theta_1 \equiv w - 1_{\{w\le A/(1+\rho)\}} \times s$. Accordingly, when $\theta_1 \le \theta \le \theta_2$, the negotiated retail price is

$$\begin{aligned}
p_r(w|\theta) &= (1-\alpha^*)(1+\rho)w \\
&= \left[1 - \frac{(1+\rho-\beta)w - (1-\beta)[\theta + \mathbf{1}_{\{w\le A/(1+\rho)\}} \times s]}{(1+\rho)w}\right](1+\rho)w \\
&= (1+\rho)w - (1+\rho-\beta)w + (1-\beta)[\theta + 1_{\{w\le A/(1+\rho)\}} \times s] \\
&= \beta w + (1-\beta)[\theta + 1_{\{w\le A/(1+\rho)\}} \times s].
\end{aligned}$$

2. When $\theta_2 < \theta \le \bar{\theta}$, $\alpha^* = 0$, and the negotiated retail price is $p_r(w|\theta) = p_s = (1+\rho)w$.

■

Proof of Theorem 2. First, we ignore the constraint that $c < w \le A/(1+\rho)$. As Lemma 1 indicates, the manufacturer's expected profit $\Pi_{M1}(w)$ is a log-concave function of wholesale price w with a unique optimal solution as $\hat{w}_1$. Next, we determine the manufacturer's optimal wholesale price under the constraint that $c < w \le A/(1+\rho)$. There exists two possible scenarios:

1. If $\hat{w}_1 \le A/(1+\rho)$, then the optimal wholesale price is $w_1^* = \hat{w}_1$.
2. If $\hat{w}_1 > A/(1+\rho)$, then the optimal wholesale price is $w_1^* = A/(1+\rho)$, since profit function $\Pi_{M1}(w)$ is a log-concave function of wholesale price w, and thus it is increasing in w when $w < \hat{w}_1$.

This theorem is thus proved. ■

Proof of Theorem 3. To prove this theorem, we need to consider the following three steps:

1. The sales realized by the retailer are calculated as $D(\hat{w}_1) = B[1 - F(\hat{w}_1 - s)]$, which is only dependent on subsidy s. Recall from Corollary 1 that $0 < \partial\hat{w}_1/\partial s < 1$. We thus obtain that, $\partial D(\hat{w}_1)/\partial s = Bf(\hat{w}_1 - s)(1 - \partial\hat{w}_1/\partial s) > 0$, which means that sales $D(\hat{w}_1)$ are strictly increasing in s.
2. The manufacturer's and the retailer's expected profits are rewritten as

$$\begin{aligned}
&\Pi_M(\hat{w}_1) = B(\hat{w}_1 - c)[1 - F(\hat{w}_1 - s)], \\
&\Pi_R(\hat{w}_1) = B\int_0^1 [(1-\beta)\int_{\hat{w}_1-s}^{\theta_2(\hat{w}_1)} (\theta + s - \hat{w}_1) f(\theta)d\theta + \rho\hat{w}_1 \int_{\theta_2(\hat{w}_1)}^{\bar{\theta}} f(\theta)d\theta] g(\beta)d\beta,
\end{aligned}$$

where $\theta_2(\hat{w}_1) \equiv (1+\rho-\beta)\hat{w}_1/(1-\beta) - s$. First-order derivative of $\Pi_M(\hat{w}_1)$ w.r.t. s and that of $\Pi_R(\hat{w}_1)$ w.r.t. s are calculated as

$$\begin{aligned}
\frac{\partial\Pi_M(\hat{w}_1)}{\partial s} &= B\frac{\partial\hat{w}_1}{\partial s} + Bf(\hat{w}_1 - s)\left(1 - \frac{\partial\hat{w}_1}{\partial s}\right) > 0, \\
\frac{\partial\Pi_R(\hat{w}_1)}{\partial s} &= B\left[\int_{\hat{w}_1-s}^{\theta_2(\hat{w}_1)} \left(1 - \frac{\partial\hat{w}_1}{\partial s}\right) f(\theta)d\theta + \rho\frac{\partial\hat{w}_1}{\partial s}\int_{\theta_2(\hat{w}_1)}^{\bar{\theta}} f(\theta)d\theta\right]\int_0^1 g(\beta)d\beta > 0.
\end{aligned}$$

That is, both $\Pi_M(\hat{w}_1)$ and $\Pi_R(\hat{w}_1)$ are increasing in s.

3. Differentiating total subsidy $C_s(s) = sB[1 - F(\hat{w}_1 - s)]$ once w.r.t. s gives,

$$\partial(C_s(s))/\partial s = B\{[1 - F(\hat{w}_1 - s)] - sf(\hat{w}_1 - s)(\partial\hat{w}_1/\partial s - 1)\} > 0,$$

because $\partial\hat{w}_1/\partial s < 1$, according to Corollary 1.

We thus complete the proof of this theorem. ■

Proof of Theorem 4. Similar to the proof of Lemma 1, we can show that without the constraint that $A/(1+\rho) < w \le \overline{\theta}/(1+\rho)$, the manufacturer's profit function $\Pi_{M2}(w) = B(w-c)[1-F(w)]$ is log-concave in w, and we can use the first-order condition of $\Pi_{M2}(w)$ to obtain the optimal solution as $\hat{w}_2 = [1 - F(\hat{w}_2)]/f(\hat{w}_2) + c$. Considering the constraint that $A/(1+\rho) < w \le \overline{\theta}/(1+\rho)$, we find:

1. If $A/(1+\rho) < \hat{w}_2 \le \overline{\theta}/(1+\rho)$, then the optimal wholesale price is $\hat{w}_2$, i.e., $w_2^* = \hat{w}_2$.
2. If $\hat{w}_2 \le A/(1+\rho)$, then the optimal wholesale price is determined as $w_2^* = A/(1+\rho)+\varepsilon$, where ε is an infinitesimally positive number.
3. If $A/(1+\rho) < \overline{\theta}/(1+\rho) < \hat{w}_2$, then the optimal wholesale price is $w_2^* = \overline{\theta}/(1+\rho)$.

This theorem is thus proved. ■

Proof of Theorem 5. We learn from Corollary 2 that $\hat{w}_2 < \hat{w}_1$ given the MSRP cutoff level $A \le \bar{\theta}$ and subsidy $s > 0$, and note from (7) and (10) that $\Pi_{M1}(w) > \Pi_{M2}(w)$ for any $w > c$. Thus, to find the manufacturer's optimal wholesale price, we need to consider the following four cases:

1. If $A/(1+\rho) \ge \hat{w}_1$, then $w_1^* = \hat{w}_1$ and $w_2^* = A/(1+\rho) + \varepsilon$. Since

$$\Pi_{M2}(A/(1+\rho)+\varepsilon) < \Pi_{M2}(A/(1+\rho)) < \Pi_{M1}(A/(1+\rho)) < \Pi_{M1}(\hat{w}_1),$$

the manufacturer should determine its optimal wholesale price as $w^* = \hat{w}_1$.

2. If $\hat{w}_2 \le A/(1+\rho) < \hat{w}_1$, then $w_1^* = A/(1+\rho)$ and $w_2^* = A/(1+\rho) + \varepsilon$. Since $\Pi_{M2}(A/(1+\rho)+\varepsilon) < \Pi_{M2}(A/(1+\rho)) < \Pi_{M1}(A/(1+\rho))$, the manufacturer should choose its optimal wholesale price as $w^* = A/(1+\rho)$.
3. If $A/(1+\rho) < \hat{w}_2 \le \bar{\theta}/(1+\rho)$, then $w_1^* = A/(1+\rho)$ and $w_2^* = \hat{w}_2$. Note that $\Pi_{M1}(A/(1+\rho))$ may or may not be larger than $\Pi_{M2}(\hat{w}_2)$, which depends on the values of $A/(1+\rho)$ and s. Therefore, the manufacturer should compare $\Pi_{M1}(A/(1+\rho))$ and $\Pi_{M2}(\hat{w}_2)$ to find the optimal wholesale price; that is, w^* can be obtained as shown in this theorem.
4. If $A/(1+\rho) < \bar{\theta}/(1+\rho) < \hat{w}_2$, then $w_1^* = A/(1+\rho)$ and $w_2^* = \bar{\theta}/(1+\rho)$. Noting that $\Pi_{M1}(\bar{\theta}/(1+\rho))$ may or may not be larger than $\Pi_{M2}(\hat{w}_2)$, which depends on the values of $\bar{\theta}/(1+\rho)$ and s, we can find the manufacturer's optimal wholesale price w^* as shown in this theorem.

We thus complete the proof of this theorem. ■

Proof of Theorem 6. When $A \ge (1+\rho)\hat{w}_2$, the manufacturer qualifies for the scrappage program; but, when $A < (1+\rho)\hat{w}_2$, the manufacturer may or may not qualify for the scheme. To prove this theorem, we need to consider the scenario that $A < (1+\rho)\hat{w}_2$, and derive the minimum cutoff level $\tilde{A}(s)$ that assures the willingness of the manufacturer to qualify for the scheme.

We find from Theorem 5 that, if $A/(1+\rho) < \hat{w}_2 \leq \bar{\theta}/(1+\rho)$, $\Pi_{M1}(A/(1+\rho))$ may or may not be larger than $\Pi_{M2}(\hat{w}_2)$, which depends on the values of $A/(1+\rho)$ and s. Note that, when $w < \hat{w}_2$, both $\Pi_{M1}(w)$ and $\Pi_{M2}(w)$ are increasing in w, because of the following two facts: (i) $\Pi_{M1}(w) > \Pi_{M2}(w)$ for any $w > c$, as indicated by (7) and (10); and (ii) $\hat{w}_1 > \hat{w}_2$ if $s > 0$, as indicated by Corollary 2. It thus follows that, given a subsidy s, we can find a *unique* MSRP cutoff level $\tilde{A}(s)$ that is the solution of the equation that $\Pi_{M1}(A/(1+\rho)) = \Pi_{M2}(\hat{w}_2)$. Because $\Pi_{M1}(w)$ is increasing in w when $w < \hat{w}_2$, $\Pi_{M1}(A/(1+\rho))$ is increasing in A when $A/(1+\rho) < \hat{w}_2$. Noting that $\hat{w}_2$ is independent of A, as shown in Theorem 4, we find that, when $A > \tilde{A}(s)$, $\Pi_{M1}(A/(1+\rho)) > \Pi_{M2}(\hat{w}_2)$; when $A < \tilde{A}(s)$, $\Pi_{M1}(A/(1+\rho)) < \Pi_{M2}(\hat{w}_2)$. Similarly, we can find a unique MSRP cutoff level $\tilde{A}(s)$ when $\hat{w}_2 > \bar{\theta}/(1+\rho)$.

Next, we show that $\tilde{A}(s)$ is a decreasing function of s. Using (7), we have,

$$\Pi_{M1}(A/(1+\rho)) \equiv B(A/(1+\rho) - c)[1 - F(A/(1+\rho) - s)],$$

which is increasing in both A and s. Thus, to assure one of the equations that $\Pi_{M1}(A/(1+\rho)) = \Pi_{M2}(\hat{w}_2)$ and $\Pi_{M1}(A/(1+\rho)) = \Pi_{M2}(\bar{\theta}/(1+\rho))$—where both $\Pi_{M2}(\hat{w}_2)$ and $\Pi_{M2}(\bar{\theta}/(1+\rho))$ are independent of A and s, we need to decrease the cutoff level A when s increases. This theorem is thus proved. ■

Proof of Theorem 7. Similar to the proof of Theorem 6, we focus on the case that $A < (1+\rho)\hat{w}_2$. When $A/(1+\rho) < \hat{w}_2 \leq \bar{\theta}/(1+\rho)$, we find that, given the MSRP cutoff level A, a unique $\tilde{s}(A)$ is determined such that $\Pi_{M1}(A/(1+\rho)) = \Pi_{M2}(\hat{w}_2)$, because, as indicated by the proof of Theorem 6, the manufacturer's expected profit $\Pi_{M1}(A/(1+\rho))$ is increasing in the subsidy s. Similarly, we can calculate the minimum subsidy $\tilde{s}(A)$ when $A/(1+\rho) < \bar{\theta}/(1+\rho) < \hat{w}_2$.

We learn from the proof of Theorem 6 that $\Pi_{M1}(A/(1+\rho))$ is increasing in both the cutoff level A and the subsidy s. It thus follows that increasing A should result in a decrease in the subsidy s, so that one of the following equations holds: $\Pi_{M1}(A/(1+\rho)) = \Pi_{M2}(\hat{w}_2)$ and $\Pi_{M1}(A/(1+\rho)) = \Pi_{M2}(\bar{\theta}/(1+\rho))$. That is, the minimum subsidy $\tilde{s}(A)$ is decreasing in the cutoff level A. We thus complete the proof of this theorem. ■

Appendix B Proof of Lemma 1

If $w \leq A/(1+\rho)$, then differentiating $\Pi_{M1}(w)$ in (7) once w.r.t. w yields,

$$\frac{\partial \Pi_{M1}(w)}{\partial w} = B[1 - F(\hat{w}_1 - s)] - B(w - c)f(\hat{w}_1 - s).$$

Solving the first-order condition that $\partial\Pi_{M1}(w)/\partial w = 0$, we find that $\hat{w}_1 = [1 - F(\hat{w}_1 - s)]/f(\hat{w}_1 - s) + c$. Then, we compute the second-order derivative of $\Pi_{M1}(w)$ w.r.t. w, and have

$$\frac{\partial^2\Pi_{M1}(w)}{\partial w^2} = -2Bf(\hat{w}_1 - s) - B(w - c)f^{'}(\hat{w}_1 - s).$$

Substituting $\hat{w}_1$ into the second-order derivative gives

$$\left.\frac{\partial^2\Pi_{M1}(w)}{\partial w^2}\right|_{w=\hat{w}_1} = -\frac{2B[f(\hat{w}_1 - s)]^2 + B[1 - F(\hat{w}_1 - s)]f'(\hat{w}_1 - s)}{f(\hat{w}_1 - s)}.$$

If $f(\theta)$ is continuously differentiable and log-concave on $[0, \overline{\theta}]$, then we can find that $[f(\hat{w}_1 - s)]^2 + [1 - F(\hat{w}_1 - s)]f'(\hat{w}_1 - s) > 0$, and $\partial^2\Pi_{M1}(w)/\partial w^2|_{w=\hat{w}_1} < 0$; thus, the manufacturer's profit $\Pi_{M1}(w)$ is unimodal in w.

Appendix C Proof of Corollaries

Proof of Corollary 1. Recall from Theorem 2 that $\hat{w}_1 = [1 - F(\hat{w}_1 - s)]/f(\hat{w}_1 - s) + c$. We differentiate both sides of this equation once w.r.t. s, and have

$$\frac{d\hat{w}_1}{ds} = -\frac{[f(\hat{w}_1 - s)]^2 + [1 - F(\hat{w}_1 - s)]f'(\hat{w}_1 - s)}{[f(\hat{w}_1 - s)]^2}\left(\frac{d\hat{w}_1}{ds} - 1\right). \quad (14)$$

Because $f(\theta)$ is continuously differentiable and log-concave, $[1 - F(\theta)]$ must be also log-concave, as shown by Bagnoli and Bergstrom (Bagnoli & Bergstrom 2005). It then follows that $[f(\hat{w}_1 - s)]^2 + [1 - F(\hat{w}_1 - s)]f(\hat{w}_1 - s) > 0$. We thus find from (14) that $d\hat{w}_1/ds$ and $(d\hat{w}_1/ds - 1)$ must have different signs; this means that $0 < d\hat{w}_1/ds < 1$.

We thus complete this proof. ■

Proof of Corollary 2. From Theorem 2, we find that the optimal wholesale price $\hat{w}_1$ satisfies the equation that $\hat{w}_1 = [1 - F(\hat{w}_1 - s)]/f(\hat{w}_1 - s) + c$. Differentiating the manufacturer's expected profit $\Pi_{M2}(w)$ w.r.t. w yields

$$\frac{\partial\Pi_{M2}(w)}{\partial w} = B[1 - F(w)] - B(w - c)f(w).$$

Since the p.d.f. $f(\theta)$ is continuously differentiable and log-concave, θ must have an increasing failure rate—that is, $f(\theta)/[1 - F(\theta)]$ is an increasing function of θ. Substituting $\hat{w}_1$ into $\partial\Pi_{M2}(w)/\partial w$ gives

$$\begin{aligned}\left.\frac{\partial\Pi_{M2}(w)}{\partial w}\right|_{w=\hat{w}_1} &= B[1 - F(\hat{w}_1)] - B\frac{[1 - F(\hat{w}_1 - s)]f(\hat{w}_1)}{f(\hat{w}_1 - s)} \\ &< B[1 - F(\hat{w}_1)] - B\frac{[1 - F(\hat{w}_1)]f(\hat{w}_1)}{f(\hat{w}_1)} \\ &= 0.\end{aligned}$$

Noting that $\hat{w}_2$ satisfies the equation that $\Pi_{M2}(w)/\partial w = 0$, we conclude that, if $s > 0$, then $\hat{w}_2 < \hat{w}_1$. ■