Impact of Tax Reduction Policies on Consumer Purchase of New Automobiles: An Analytical Investigation with Real Data-Based Experiments[1]

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Abstract: We investigate and compare the impact of the tax reduction policies implemented in the U.S. and China to stimulate consumer purchase of new automobiles and improve manufacturers' profits. The U.S. policy provides each qualifying consumer with a federal income tax deduction on state and local sales and excise taxes paid on the purchase price (up to a cutoff level), whereas the Chinese policy reduces the vehicle sales tax rate for consumers. We observe that these policy designs are consistent with the tax management system and the economic environment in the respective country. We analytically determine the effects of the two tax reduction policies on the automobile sales and the manufacturer's and the retailer's profits. Numerical examples are then used to provide insights on the importance of certain factors that influence the effects of the two policies. Finally, a numerical experiment with sensitivity analysis based on real data is conducted to compare the merits and characteristics of the two policies under comparable conditions. We find that the U.S. policy is better than the Chinese policy in stimulating the sales of high-end automobiles, whereas the Chinese policy is better than the U.S. policy in improving the sales of low-end automobiles. The U.S. policy is slightly more effective in increasing the profitability of the automobile supply chain; but, in general, the Chinese policy is more cost effective. The methodology developed herein can be used to evaluate other tax reduction policies such as those related to the purchase of energy-saving vehicles and to serve as a decision model to guide the choice of alternative tax reduction policies.

Keywords: Accounting-operations interface; tax reduction policy; supply chain management.

1 Introduction

An automobile industry crisis arose as a result of the recent world-wide financial storm. As an important part of the global economic recession, the crisis first hit American automobile manufacturers in 2008, and then affected European and Asian automobile industries. Many countries had responded to such a crisis by executing stimulus plans to help their automobile industries. A tax reduction policy is a common plan, under which each consumer buying a new qualifying automobile can enjoy a reduction in his or her tax payment.

As two important economies that implemented an automobile tax reduction policy, the U.S. and China have successfully stimulated consumers' purchases of new automobiles. The U.S. government

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proposed an economic stimulus package containing an "Auto Assistance Ownership" amendment, which became effective on February 17, 2009. The amendment stipulates that a consumer will qualify for a special tax reduction if his or her taxable family income is no more than \$260,000 (for joint filers), or if his or her individual taxable income is no more than \$135,000 (for single filers). Each qualifying consumer can enjoy a federal income tax deduction on state and local sales and excise taxes paid on the purchase price (up to \$49,500) of an eligible vehicle that weighs no more than 8,500 pounds. In January 2009, the Chinese government reduced the vehicle sales tax rate from 10% to 5% for consumers who bought fuel-efficient vehicles with a displacement of 1.6 liters or less. Under the Chinese tax reduction policy, a consumer who purchased a qualifying vehicle in 2009 paid a sales tax of only 5% of the retail price. China decided to extend this policy until December 31, 2010 because of its significantly positive impact, but the tax rate was reduced from the *original* 10% to 7.5% (instead of 5%). For more information, see reports by Chang [6], Hong Kong Trade Development Council [19], Liu [24], the State Administration of Taxation of China [30], and `Vietnamnet.vn` [32].

Since the tax reduction policy had been playing an important role in stimulating consumers' automobile purchases, it behooves us to rigorously analyze the impact of such a policy on the automobile industry, and investigate the design of an effective tax policy for public policy makers. The two tax reduction policies in our paper were implemented to help the automobile industry and prevent unemployment in the industry from worsening during the economic recession. Increases in both the sales and the profits are necessary for the survival of the manufacturers. Thus, the effectiveness of the policy in improving the sales and the profit are the policy makers' major concerns. Indeed, we provide empirical evidence from major automobile manufacturers in the U.S. and China, which indicates that their sales and profits are highly and positively correlated in both countries.

We note that the U.S. policy stipulates a cutoff level (\$49,500), whereas the Chinese policy does not have any cutoff level but directly reduces the sales tax rate. Hence, the U.S. tax reduction policy may be regarded as a threshold-based one with the tax reduction limited by the cutoff level on the purchase price; the Chinese policy is a percentage-based one with a reduction on the sales tax rate. Motivated by these differences, we consider the following issues:

1. How effective will the U.S. vs. the Chinese tax reduction policies be in enticing more consumers to buy new automobiles? We will analytically determine the increase in sales volume under each policy. Although there are statistics based on empirical data for the increase of sales, it is still meaningful to provide an analytical methodology to determine the relationship between the tax policy and the increase in sales, and to identify the factors that may affect this relationship. Such a methodology can also be used to evaluate similar tax reduction policies such as those for the purchase of energy-saving vehicles. We will also analytically investigate whether these policies will increase the average retail price of automobiles.
2. How effective will the U.S. and the Chinese tax reduction policies be in improving the profit of the automobile supply chain? A policy is considered worthwhile if it is cost effective, i.e., the after-tax profit increase in the automobile supply chain is no less than the government's "cost" (i.e., reduced tax revenue). We will analytically derive the formulas that can be used to determine the cost effectiveness of the two policies. Moreover, the U.S. public policy makers need to determine a proper cutoff level (which is currently \$49,500) for their policy,

and their Chinese counterparts need to choose a proper tax reduction percentage (which was 50% in 2009, but became 25% in 2010). Each must make a decision that can achieve its targets. Accordingly, for each policy, we provide a methodology to determine the feasible targets (i.e., the additional sales and the additional system-wide profit that can be achieved due to the tax reduction policy) while ensuring that the policy will be cost effective. For a given cutoff level or reduction percentage, we will also identify the government's budget corresponding to the feasible targets.

3. Since the U.S. and China implement their policies in different ways, we will conduct an experiment to compare their merits and perform a real data-based sensitivity analysis to enrich the comparison.

To investigate the above issues, we first consider a two-level automobile supply chain in which a manufacturer sells both a high-end and a low-end qualifying automobiles to a retailer, who then serves the respective market segment of each automobile in a market of a finite size. Hereafter, we shall refer to the high-end automobile and the low-end one as the Type-H and the Type-L automobiles, respectively.

For each tax reduction policy, we begin by deriving the retail price for the consumer of each automobile type, which results from the negotiation between the consumer and the retailer. Next, we determine the impact of the policy on the sales and profitability of the automobile supply chain. We then examine the cost effectiveness of the policy, which is measured by the profit-cost ratio, defined as the profit increase divided by the government's tax expenditure. Accordingly, we derive the sufficient condition for the U.S. policy to be cost effective and the necessary and sufficient condition for the Chinese policy to be cost effective. To provide more insights for public policy makers when choosing an appropriate tax reduction policy during an economic recession, we further compare the two policies. First, we discuss their differences in institutional background. We then perform a sensitivity analysis based on real data to compare their impacts on the expected sales and supply chain-wide profits, and also compare the cost effectiveness (i.e., the profit-cost ratio) of the two policies. We also consider the case with consumers' choices between two types of automobiles and the case with a strategic manufacturer, which are viewed as two extensions to our base model.

We find that the U.S. policy is better in stimulating the sales of the Type-H automobile, whereas the Chinese policy is better in improving the sales of the Type-L automobile. Moreover, the U.S. policy is slightly more effective in increasing the profitability of the automobile supply chain; but, in general, the Chinese policy is more cost effective.

The remainder of the paper is organized as follows: In Section 2, we review relevant literature. In Sections 3 and 4, we discuss the impact of the U.S. and the Chinese policies on the sales and profitability of the automobile supply chain, and investigate the cost effectiveness of each policy. In Section 5, we compare the two policies, and perform a numerical experiment to supplement our comparison. In Section 6, we consider two extensions. We conclude in Section 7. All tables, proofs, as well as numerical examples and results are relegated to online appendices. We also provide a summary of important model assumptions and parameter values with their justifications in online Appendix A.

2 Literature Review

In the economics literature, most of the tax policy-related publications adopted various empirical approaches to examine the impact of tax policies on the sales of automobiles. A few empirical studies in the environmental economics field analyzed the impact of a tax policy on the sales of hybrid vehicles. Their empirical evidences largely support the analytical results derived in our paper. For example, Gallagher and Muehlegger [16] provided empirical evidences that the state sales tax waivers in the United States are associated with more than a ten-fold increase in hybrid vehicle sales relative to income tax credits. Diamond [12] also found that government incentives which provide payments upfront appear to be the most effective in promoting the adoption of hybrid-electric vehicles in the United States. Chandra et al. [5] estimated the effect of tax rebates offered by Canadian provinces on the sales of hybrid electric vehicles, and found that those rebates had led to a huge increase in the market share of hybrid vehicles—that is, 26% of the hybrid vehicles sold during the program period were attributed to the rebates.

It has been well established in the literature that the merit of a tax policy should be evaluated based on the cost-effectiveness of the policy. For example, Fullerton and Gan [15] provided an in-depth analysis on the cost-effectiveness of alternative government policies to reduce vehicle emission. Those policies include a higher tax on gasoline, a tax on distance, and a government subsidy for buying a new car. Similarly, Jenn et al. [22] showed that the Energy Policy Act of 2005 in the U.S. had increased the sales of hybrid electric vehicles from 3% to 20% depending on the vehicle model. They further found that the sales of hybrid vehicles increased by 0.0046% per dollar of government incentive when the incentive was above $1,000.

Very few economics publications analytically studied tax policies for automobile sales. An exception is Chen et al. [7], who focused on a market with two manufacturers selling a homogenous automobile, and numerically examined the impact of durability of the automobile on the effectiveness of sales tax reduction in a dynamic model. Although their model and analysis was motivated by the U.S. sales tax reduction policy, Chen et al. considered a percentage-based policy, which is similar to the Chinese policy discussed in our paper. However, Chen et al. [7] assumed that consumers are price takers, who pay a single price determined by the manufacturers. Thus, all consumers receive an *identical* amount of sales tax reduction, which is *independent* of each consumer's income. Unlike the majority of relevant economics publications, we *analytically* examine the impact of tax reduction policies. We distinguish our paper from Chen et al. [7] by analyzing both the U.S. and the Chinese tax policies in a more realistic way, under which an automobile supply chain sells both a high-end automobile and a low-end one for a negotiated price. We provide generalized analytical results and real data-based numerical experiments and sensitivity analysis. In addition, we use the "generalized Nash bargaining (GNB) scheme" (Nash [26] and Roth [29]) to derive the negotiated retail price for each consumer, which enables us to examine the effect of a retailer's bargaining power on the cost-effectiveness of each policy.

Motivated by the U.S. federal and local governments' subsidy programs for promoting electric vehicles and solar panels, Cohen et al. [9] examined a government subsidy policy for green technology adoption. Our paper differs from [9] in the following four aspects: (i) Cohen et al. considered a fixed subsidy that is independent of the retail price, whereas in our model each consumer's sales tax reduction depends on both the retail price and the consumer's income tax rate (under the U.S. policy) or a percentage reduction in the sales tax rate (under the Chinese policy). (ii) In [9], Cohen et al. assumed that the demand consists of a deterministic component and an additive or multiplicative noise, and the industry sets a single retail price for all consumers. Different from [9], we derive the negotiated retail prices that are different for heterogeneous consumers with utilities drawn from purchasing an automobile, and develop the demand function for the automobile supply chain. Using such negotiated retail prices, we are able to examine the effect of the retailer' bargaining power on the cost effectiveness of each policy. (iii) While Cohen et al. [9] considered a single product, we consider both a high-end automobile and a low-end automobile. (iv) Cohen et al. [9] focused on how the demand uncertainty affects a government's policy design, the industry's decisions of investment in green technology capacity and pricing under the policy, and the consumer surplus. But, we compare the U.S. and the Chinese policies with a focus on their effects on the automobile sales and the manufacturer's and the retailer's profits as well as their cost effectiveness.

Therefore, we can conclude from the above review that our paper differs from previous literature in both methodology and orientation.

3 Impact of the U.S. Automobile Tax Reduction Policy

We first analyze the effectiveness of the U.S. tax reduction policy in the automobile industry, under which the *reduced* federal income tax amount for a consumer is calculated as the federal income tax rate (that applies to the consumer) times the sales tax paid by the consumer based on the lower value of the cutoff level ($49,500) and the consumer's actual purchase (retail) price. Under the U.S. policy, consumers with different taxable income levels will have different tax savings. The federal income tax rate is higher for the consumer with a higher taxable income. To help filers estimate their federal income tax payments, the U.S. Internal Revenue Service (IRS) provided the 2009 tax rate schedules that indicate the *approximate* tax rates applicable to all levels of taxable income. Note that only single filers with a 2009 taxable income no more than $135,000 and joint filers with a 2009 taxable income no more than $260,000 qualify for the U.S. automobile tax reduction. The U.S. tax rate schedule for these single or joint filers (who qualify for the U.S. tax reduction policy) is summarized in Panel A of Table A, which is given in online Appendix B.

The U.S. policy consists of three components: the federal income tax rates, the sales tax rate, and the cutoff level. The federal income tax rates apply to all residents in the United States. Even though the state sales tax rate may vary with states, it is identical for all consumers who reside in the same state. Certain U.S. counties and cities also levy sales tax. Our analysis can be generalized to each sales tax jurisdiction, where the sales tax rate is identical for all consumers.

3.1 Impact of the U.S. Tax Reduction Policy on Each Type of Automobile

We assume that the manufacturer makes the Type-H and the Type-L automobiles at unit production costs c_H and c_L, respectively, which are then sold by the retailer to consumers in each automobile market segment. In practice, the retail price of an automobile is often negotiated by the consumer and the retailer, which is common in the U.S. automobile market, as discussed by Desai and Purohit [11]. In [8], Chen et al. performed an empirical study with transaction data on individual consumer purchases of three popular automobiles in the United States, and found that the means (standard deviations) of the negotiated prices for the three brands are $\$21,200$ ($\$2,400$),

\$20,800 (\$1,600), and \$25,100 (\$1,600). The empirical results in [8] indicate that the range in which the negotiated retail price for an automobile could vary is *not* negligible, which implies that both the retailer's and the consumers' roles in the automobile price negotiation cannot be ignored. Thus, in our paper, the bargaining analysis for the retail price is necessary and important to the practicality of our models and results.

Because the analyses for the retail price and sales of the two automobile types are similar, we use a *generic* automobile to represent each type of automobile and omit the super- or sub-script for the type in the case of no ambiguity. Thus, the unit production cost of the generic automobile is denoted by c.

3.1.1 Negotiated Retail Price

The retail price p_r should naturally be not smaller than the wholesale price w paid by the retailer to the manufacturer. It is thus reasonable to assume that $p_r \geq w$. The U.S. government should determine the cutoff level A such that $A \geq w$ because, otherwise, each consumer's retail price must be greater than the cutoff level, and *no* consumer can enjoy a tax cut on all of his or her expense. This will discourage consumers from purchasing the qualifying automobiles under the policy. In fact, the cutoff level in the current U.S. policy is \$49,500, which is significantly higher than the retail prices and thus the wholesale prices of most qualifying automobiles. Note that the generic automobile can represent any type of automobile with its specific unit production cost c and wholesale price w. A low-end automobile is characterized by small values of c and w, whereas a high-end automobile is characterized by large values of c and w.

The retailer sells the automobile to a market segment of a finite base B, which is defined as the total number of potential consumers (single or joint filers) who are qualified for tax reduction. We learn from Panel A of online Table A that, corresponding to each federal income tax rate, the income bracket for the single filer differs from that for the joint filer. In fact, we only use a consumer's income tax rate rather than the specific income to calculate the tax reduction amount for the consumer. To simplify our analysis, we only need the joint distribution of the federal income tax rates for both the single and joint filers in the U.S. rather than two separate distributions for the single and the joint filers. Hence, we assume that the income tax rate γ of consumers in a market segment is randomly distributed with the probability mass function (p.m.f.) $\Pr\{\gamma = \gamma_i\} = \delta_i$ (for $i = 1, \ldots, n+1$). For the purpose of generality, we assume that there are n federal income tax rates $\gamma_i \in [0, 1)$ ($i = 1, \ldots, n$) for eligible consumers, which are set such that $\gamma_i < \gamma_{i+1}$, for $i = 1, \ldots, n-1$. Under the U.S. policy, n is equal to 5. If a consumer's federal income tax rate is γ_i, then the reduced tax amount is computed as $\gamma_i \times t \times \min(A, p_r)$, where t denotes the sales tax rate and $t \in [0, 1]$. The value of δ_{n+1} represents the proportion of consumers who are not eligible for tax reduction (because their taxable incomes exceed the threshold); thus, we let $\gamma_{n+1} \equiv 0$ for the purpose of calculating the amount of tax reduction, which is 0.

To derive the negotiated retail price, we begin by calculating a consumer's net gain and the retailer's profit obtained from their transaction. We assume heterogeneity of consumers' valuations of an automobile. These valuations represent the after-tax prices that the consumers are willing to pay for the automobile, and are characterized by a non-negative independent and identically distributed (i.i.d.) random parameter θ with the p.d.f. $f(\theta)$ and the c.d.f. $F(\theta)$ on the interval $[0, \bar{\theta}]$. Breidert [3] discussed major approaches that can be used to estimate the probability distribution of the parameter θ. It then follows that the consumer's net gain, denoted by $u(p_r;\theta,\gamma_i)$, can be computed as his or her valuation θ *minus* the purchase-related expense $(1+t)\times p_r$ *plus* the tax reduction $\gamma_i \times t \times \min(A, p_r)$; that is, $u(p_r;\theta,\gamma_i) = \theta - (1+t)\times p_r + \gamma_i \times t \times \min(A, p_r)$. From the transaction with the consumer, the retailer obtains the sale revenue p_r. Since the retailer spends the wholesale price w to purchase the automobile, the retailer's after-tax profit is calculated as $\pi_R(p_r) = (1-T)(p_r - w)$, where $T \in [0,1]$ denotes the corporate tax rate for a firm in the tax jurisdiction.

The above utility model for consumers is called a "willingness-to-pay" model, which has been widely used in the marketing and operations management areas to calculate consumers' utilities from purchasing a product when the consumers are heterogenous in their valuations for the product. For the applications of such utility model in the analysis of the automobile market, see, e.g., Chen et al. [8], Desai and Purohit [10], Desai and Purohit [11], Huang et al. [20], Iyer [21], and Purohit [27].

We analyze the bargaining problem of the consumer and the retailer using the cooperative-game solution concept of "generalized Nash bargaining (GNB) scheme." For an application of the concept in accounting, see Arya and Mittendorf [2] who study a transfer pricing-related problem. To find the retail price for a consumer with the valuation θ and income tax rate γ_i, we solve the following constrained maximization problem:

$$\begin{array}{rl} \max_{p_r} & \Lambda \equiv [\theta - (1+t)p_r + \gamma_i t \min(A, p_r)]^{1-\kappa}[(1-T)(p_r - w)]^{\kappa} \\ \text{s.t.} & \theta - (1+t)p_r + \gamma_i t \min(A, p_r) \geq 0 \quad \text{and} \quad (1-T)(p_r - w) \geq 0, \end{array} \tag{1}$$

where the parameters $1-\kappa \in [0,1]$ and $\kappa \in [0,1]$ denote the consumer's and the retailer's *relative* bargaining powers, respectively. Chen et al. [8] also adopted a GNB scheme to model the negotiated retail price. Using the data of individual consumer purchases of three popular automobiles in the U.S., they estimated that the bargaining powers of dealers (retailers) for three automobiles are between 0.384 and 0.453. Since the retailer may have a lower bargaining power during the economic recession because of the pressure of dealing with excess inventory, we can reasonably assume that the retailer's bargaining power is $\kappa = 0.4$ for our numerical experiments.

Theorem 1 Given that the cutoff level in the U.S. tax reduction policy is A and the manufacturer's wholesale price is w, the retailer and a consumer with a valuation $\theta \in [0, \bar{\theta}]$ and a federal income tax rate $\gamma_i \in [0,1]$ bargain over the retail price. If $0 \leq \theta < \tilde{\theta}_i \equiv [1+(1-\gamma_i)t]w$, then the retailer and the consumer cannot reach an agreement on the retail price, and do not complete any transaction. If $\theta \geq \tilde{\theta}_i$, then the consumer buys from the retailer at the negotiated retail price p_r^* determined as

$$p_r^* = \begin{cases} \tilde{p}_r \equiv \dfrac{\kappa\theta}{1+(1-\gamma_i)t} + (1-\kappa)w, & \text{if } \tilde{\theta}_i \leq \theta \leq \hat{\theta}_i, \\ A, & \text{if } \hat{\theta}_i \leq \theta \leq \bar{\theta}_i, \\ \hat{p}_r \equiv \dfrac{\kappa(\theta + \gamma_i t A)}{1+t} + (1-\kappa)w, & \text{if } \bar{\theta}_i \leq \theta \leq \bar{\theta}, \end{cases} \tag{2}$$

where $\hat{\theta}_i \equiv [A-(1-\kappa)w][1+(1-\gamma_i)t]/\kappa$ and $\bar{\theta}_i \equiv [A-(1-\kappa)w](1+t)/\kappa - \gamma_i t A$. ■

In (2), $\tilde{\theta}_i$ represents the minimum valuation of the consumers who are willing to make a purchase; $\hat{\theta}_i$ is the minimum valuation of the consumers who pay a price equal to the cutoff level A

and thus enjoy the tax reduction in amount of $\gamma_i tA$; $\bar{\theta}_i$ is the minimum valuation of the consumers who incur purchase costs higher than A but only receive the tax reduction $\gamma_i tA$.

We find from Theorem 1 that the negotiated retail price is increasing in the consumer's valuation θ such that $\tilde{p}_r \leq A \leq \hat{p}_r$, because consumers with higher valuations are willing to pay more. For the consumer with a relatively low valuation $\theta \in [\tilde{\theta}_i, \hat{\theta}_i)$, the negotiated price $\tilde{p}_r$ is below the cutoff level A and is thus independent of A. It is easy to show that $\tilde{p}_r$ is increasing in the wholesale price w and the retailer's relative bargaining power κ, which means that the negotiated retail price is higher when the manufacturer charges a larger wholesale price or when the retailer has a stronger bargaining power. When the consumer has a valuation $\theta = \hat{\theta}_i$, the negotiated retail price is A, and the consumer can obtain the maximum tax reduction amount $\gamma_i tA$. For the consumer with a higher valuation in the range $(\hat{\theta}_i, \bar{\theta}_i]$, the consumer cannot gain a tax reduction more than $\gamma_i tA$ if he or she pays a retail price higher than A. Thus, the retailer fixes the retail price at A to benefit the consumer. When the consumer's valuation is greater than $\bar{\theta}_i$, the consumer is willing to pay the price $\hat{p}_r$, which is higher than A and is increasing in the consumer's valuation. But, because the consumer's tax reduction amount is capped at $\gamma_i tA$, the negotiated price $\hat{p}_r$ is dependent on A and also increasing in A. In addition, $\hat{p}_r$ is increasing in w and κ.

We learn from Theorem 1 that the probability for a successful transaction is $\int_{\tilde{\theta}_i}^{\bar{\theta}} f(\theta)d\theta = 1 - F(\tilde{\theta}_i)$. The expected retail price is calculated as

$$\bar{\mu} = \sum_{i=1}^{n+1} \left(\delta_i \times \mu(\gamma_i)\right), \tag{3}$$

where δ_i $(i = 1, \ldots, n+1)$ represents the federal income tax rate distribution of consumers in the market segment; $\mu(\gamma_i)$ is the expected retail price for a consumer with the federal income tax rate γ_i, and is computed as

$$\mu(\gamma_i) = E[p_r^* | \theta \geq \tilde{\theta}_i] = \left\{ \int_{\tilde{\theta}_i}^{\hat{\theta}_i} \tilde{p}_r f(\theta)d\theta + \left(F(\bar{\theta}_i) - F(\hat{\theta}_i)\right) A + \int_{\bar{\theta}_i}^{\bar{\theta}} \hat{p}_r f(\theta)d\theta \right\} \Big/ [1 - F(\tilde{\theta}_i)]. \tag{4}$$

We use empirical data to illustrate the above analytical result in Example 1 (in online Appendix E). We find the best-fitting distribution for the taxable incomes of the U.S. single filers and that of the joint filers, which can be used to obtain the joint distribution of the federal income tax rate (i.e., the value of δ_i) for both single and joint filers.

3.1.2 Impact of the U.S. Policy on the Sales and the Profitability of Each Type of Automobile

According to our analysis in Section 3.1.1, we can compute the retailer's expected after-tax profit in an automobile market segment as

$$\Pi_R = B(1-T) \times \sum_{i=1}^{n+1} \left[\delta_i \times \left(1 - F(\tilde{\theta}_i)\right) \times (\mu(\gamma_i) - w)\right]. \tag{5}$$

Moreover, the expected sales of this automobile type can be calculated as

$$D = B \times \sum_{i=1}^{n+1} \left[\delta_i \times \left(1 - F(\tilde{\theta}_i)\right)\right], \tag{6}$$

and the manufacturer's expected after-tax profit in this market segment is thus computed as

$$\Pi_M = (1-T)(w-c)D = B(1-T)(w-c)\sum_{i=1}^{n+1}\left[\delta_i \times \left(1-F(\tilde{\theta}_i)\right)\right]. \tag{7}$$

We assume the wholesale price w is exogenously determined. This assumption is reasonable because, during an economic recession, a manufacturer has to rely heavily on cash incentives and price cuts to boost sales and help deal with excess inventory. Hence, the manufacturer is unlikely to increase its wholesale price under the tax reduction policy and mitigate the effect of the policy in stimulating the sales (see `Businessweek.com` [4] and `ResearchInChina.com` [28]).

Remark 1 Given the U.S. tax reduction policy with a cutoff level A, the expected sales D, the retailer's expected profit Π_R, and the manufacturer's expected profit Π_M can be determined by using (6), (5), and (7), respectively.

The values of D, Π_R, and Π_M are all higher than those before the policy is implemented. The degrees of improvement in D, Π_R, and Π_M increase in the federal income tax rates γ_i $(i = 1, \ldots, n)$. The improvements are also dependent on the income tax rate distribution. That is, if two income tax rate distributions $\{\delta_i\}$ and $\{\delta_i'\}$ $(i = 1, \ldots, n+1)$ are such that their cumulative probabilities satisfy $\sum_{i=1}^{m}\delta_i \geq \sum_{i=1}^{m}\delta_i'$ for any $m < n+1$, then the improvements in D, Π_R, and Π_M under the distribution $\{\delta_i'\}$ would be greater than those under $\{\delta_i\}$. ■

For an analytical justification for the above remark, see online Appendix D. The above remark indicates that the U.S. policy can improve the sales and the profits of the retailer and the manufacturer. The improvements are higher when more consumers have relatively high incomes (i.e., the income tax rate distribution indicates a higher probability for larger income tax rates). We provide a real data-based Example 2 in online Appendix E, which indicates that the sales and the profits of the retailer and the manufacturer can increase after the U.S. policy is executed.

Theorem 2 For the U.S. policy with a cutoff level A, we have the following results.

1. When the price elasticity of demand $\tilde{\theta}_i f(\tilde{\theta}_i)/\left(1-F(\tilde{\theta}_i)\right)$ satisfies the following condition for all income tax rates γ_i of eligible consumers $(i = 1, \ldots, n)$, the expected retail price $\bar{\mu}$ is lower than that before the policy is implemented:

$$\frac{\tilde{\theta}_i f(\tilde{\theta}_i)}{1-F(\tilde{\theta}_i)} > \frac{\frac{1}{1+(1-\gamma_i)t}\int_{\tilde{\theta}_i}^{\hat{\theta}_i}\theta f(\theta)d\theta + \frac{A[1+(1-\gamma_i)t]}{1+t}\left(1-F(\bar{\theta}_i)\right)}{\frac{1}{1+(1-\gamma_i)t}\int_{\tilde{\theta}_i}^{\hat{\theta}_i}\left(\theta-\tilde{\theta}_i\right)f(\theta)d\theta + \frac{1}{1+t}\int_{\bar{\theta}_i}^{\bar{\theta}}\left(\theta-\bar{\theta}_i\right)f(\theta)d\theta + \left(1-F(\hat{\theta}_i)\right)\frac{A-w}{\kappa}}. \tag{8}$$

2. When the cutoff level $A \geq \bar{A} \equiv \kappa\bar{\theta}/[1+(1-\gamma_n)t] + (1-\kappa)w$, increasing A will not affect $\bar{\mu}$ and Π_R; but, when $A < \bar{A}$, increasing A will raise both $\bar{\mu}$ and Π_R. Moreover, when $A \geq w$, increasing A will not affect D and Π_M. ■

Using the data in online Example 1 with the retailer's bargaining power $\kappa = 0.4$, we can calculate the threshold for the price elasticity of demand as 2.94 for the lowest income tax rate (10%) and as 2.89 for the highest income tax rate (33%). Our estimates are roughly consistent with the relevant evidence from Goodwin et al. [18] who showed that the price elasticity of the demand for Ford compact automobile is 2.8. During the economic recession, the price elasticity of demand could be higher and satisfy the condition in (8). That is, the expected retail price could decrease as a result of implementing the tax reduction policy. This occurs because, if the demand is sufficiently price elastic, then a tax reduction can lead to a large increase in the demand of *low-valuation* consumers. These consumers would enjoy low negotiated prices, which result in a reduction in the average retail price.

In addition, using the data in online Example 1, we can find the threshold for the cutoff level as $\bar{A} = \$37,487$, which is lower than the cutoff level $\$49,500$ in the U.S. policy. Then, according to Theorem 2, both the expected retail price and the retailer's profit should be constant for any cutoff level above $\bar{A} = \$37,487$. This happens mainly because, when the cutoff level A is sufficiently large, each consumer (even the consumer with the highest valuation $\bar{\theta}$) will pay a negotiated retail price lower than A. Therefore, a large cutoff level may not be effective in influencing the retail price and the retailer's profit.

3.2 Cost Effectiveness of the U.S. Tax Reduction Policy on the Automobile Supply Chain

Recall from Section 1 that a tax reduction policy is implemented mainly to stimulate consumer purchase of new automobiles and increase the automobile supply chain-wide profit. We empirically verify that, in the U.S. automobile industry, sales increase results in profit increase. As shown in online Table B, for both GM and DaimlerChrysler, the correlation coefficients between sales and profits from 2001 to 2009 are 0.85 and 0.62, respectively. This implies that the U.S. government actually aims at both sales and profit increases to help its automobile industry survive and to prevent worsening unemployment during the economic recession.

As mentioned in Theorem 2, when the cutoff level A is not very large, the retailer's profit can be increased by raising A. Therefore, in theory, the U.S. government could decide a higher cutoff level to achieve a larger system-wide profit increase. It thus behooves public policy makers to determine a cost-effective tax reduction policy, under which the system-wide after-tax profit increase induced by the policy (i.e., "output") would be at least equal to the government's "cost" for implementing the policy (i.e., "input"). Because the tax reduction policy aims at helping the automobile industry, if the government cuts a large amount of tax to stimulate the demand, but generates a small amount of profit increase for the industry, then the policy should not be considered as a successful one. We define "*profit-cost ratio*" as the ratio of the additional system-wide after-tax profit generated by a policy to the government's cost (tax reduction amount) of implementing the policy.

Definition 1 A tax reduction policy is cost effective if and only if the profit-cost ratio is greater than or equal to 1. ■

If the cutoff level A cannot be found for the tax reduction policy to meet the condition in Definition 1, then from an economic perspective, such a policy should not be viewed as a stimulus plan for the automobile industry. On the other hand, if there exists at least one cost-effective policy, then it would be worthwhile for policy makers to consider such a plan. As discussed previously, the U.S. policy is implemented to increase the automobile sales and the profit of the automobile supply chain. In order to adopt a policy that meets the cost-effectiveness condition given in Definition 1, the policy makers need to consider the following question: What are the maximum increases in the system-wide after-tax profit and in sales that could be induced by a cost-effective tax reduction policy? The solution to the question is important because it indicates the upper bounds for the government's target profit and sales increases.

Remark 2 We can use the following two-step approach to determine a proper tax reduction policy.

Stage 1: Sufficient condition for a cost-effective policy. Derive the sufficient condition under which a policy is cost effective. Public policy makers can then determine whether or not there exists at least one tax reduction policy that meets the cost effectiveness condition.

Stage 2: Feasible targets and corresponding budget for the policy. The sales increase should be maximized under the cost effectiveness constraint to find the maximum target sales increase that can be achieved. Similarly, the system-wide profit increase should also be maximized under the cost effectiveness constraint to obtain the maximum target profit increase. If the government has a budget limit, then the limit can also be considered as a constraint. The policy makers can choose *feasible* target sales increase and profit increase which do not exceed their maximum values, and then determine the corresponding budget. ■

If there is no cost-effectiveness policy, then we would not need Stage 1 in the above remark and the government may still intend to stimulate sales with a feasible target sales increase. This is especially true in the occurrence of an economic recession, during which automakers have to operate with a low capacity utilization and high inventories. It is thus of interest for the government to help stimulate sales and improve the automakers' cash turnover ratios.

3.2.1 Sufficient Conditions for the Cost Effectiveness of the U.S. Policy

We first find the system-wide after-tax profit increase generated by the policy. For the supply chain involving both the Type-H and Type-L automobiles, the system-wide after-tax profit is given as $\sum_{j\in\{H,L\}}(\Pi_R^j + \Pi_M^j)$, where Π_R^j and Π_M^j are defined in (5) and (7), respectively, with $B = B_j$, $w = w_j$, $c = c_j$, $\delta_i = \delta_i^j$, $\tilde{\theta}_i = \tilde{\theta}_i^j$, $\mu(\gamma_i) = \mu_j(\gamma_i)$, $\kappa = \kappa_j$, $F(\cdot) = F_j(\cdot)$, and $f(\cdot) = f_j(\cdot)$ for the Type-j automobile ($j \in \{H, L\}$). Note that the retailer's (the manufacturer's) after-tax profit increase is calculated as the difference between the after-tax profit when the U.S. government implements its policy and the after-tax profit without the policy. Using the results in Section 3.1.2, we calculate the system-wide after-tax profit under the policy as

$$\begin{aligned}\Pi = \sum_{j\in\{H,L\}} \Bigg\{ B_j(1-T)\sum\nolimits_{i=1}^{n+1} \Bigg[\delta_i^j \Bigg(\int_{\tilde{\theta}_i^j}^{\hat{\theta}_i^j} \tilde{p}_r^j f_j(\theta)d\theta + \left(F_j(\bar{\theta}_i^j) - F_j(\hat{\theta}_i^j)\right) A \\ + \int_{\bar{\theta}_i^j}^{\bar{\theta}_j} \hat{p}_r^j f_j(\theta)d\theta - \left(1 - F_j(\tilde{\theta}_i^j)\right) c_j \Bigg) \Bigg] \Bigg\},\end{aligned} \tag{9}$$

where $\hat{\theta}_i^j$, $\bar{\theta}_i^j$, $\bar{\theta}_j$, $\hat{p}_r^j$, $f_j(\theta)$, and $F_j(\theta)$ are defined similar to $\hat{\theta}_i$, $\bar{\theta}_i$, $\bar{\theta}$, $\hat{p}_r$, $f(\theta)$, and $F(\theta)$, respectively.

Let θ_0^j denote the minimum valuation of a consumer in the Type-j automobile market segment above which the consumer will buy a Type-j automobile when the U.S. policy is not implemented. Similar to our analysis in Section 3.1, we can find that, before the policy is implemented, the system-wide expected profit Π_0 is computed as

$$\Pi_0 = \sum\nolimits_{j\in\{H,L\}} \left[B_j(1-T)\left(1 - F_j(\theta_0^j)\right)(\mu_0^j - c_j) \right], \tag{10}$$

where

$$\mu_0^j \equiv \int_{\theta_0^j}^{\bar{\theta}_j} p_{r0}^j f_j(\theta)d\theta \Big/ \left(1 - F_j(\theta_0^j)\right), \text{ and } \theta_0^j = (1+t)w_j; \tag{11}$$

$p_{r0}^j \equiv \kappa_j\theta/(1+t) + (1-\kappa_j)w_j$ is the negotiated retail price of a Type-j automobile. In (10), μ_0^j represents the expected pre-tax retail price for the Type-j automobile, $(1-T)(\mu_0^j - c_j)$ is the expected after-tax unit profit for the Type-j automobile, and $B_j\left(1 - F_j(\theta_0^j)\right)$ is the expected sales for the Type-j automobile. We note from Remark 1 that $\Pi > \Pi_0$ because the manufacturer's and the retailer's expected profits are higher after the policy is implemented. Let S represent the system-wide after-tax profit increase generated by the U.S. policy. We calculate S as Π in (9) minus Π_0 in (10), i.e.,

$$S = \sum\nolimits_{j\in\{H,L\}} \left\{B_j(1-T)\sum\nolimits_{i=1}^{n}\left\{\delta_i^j\left[\mu_j(\gamma_i)\left(1-F_j(\tilde{\theta}_i^j)\right) - \mu_0^j\left(1-F_j(\theta_0^j)\right) + c_j\left(F_j(\tilde{\theta}_i^j) - F_j(\theta_0^j)\right)\right]\right\}\right\}, \tag{12}$$

which must be positive because $\Pi > \Pi_0$.

Next, we determine the government's expense (or cost) for its policy. We learn from Theorem 1 that the consumer in the Type-j automobile market segment with the valuation $\theta \in [\tilde{\theta}_i^j, \bar{\theta}_j]$ and the income tax rate γ_i purchases an automobile at the price p_r^{j*} as given in (2), and thus reduces the tax payment by the amount of $\gamma_i \times t \times \min(A, p_r^{j*})$. Using (2), we compute the expected tax reduction amount for a consumer in the Type-j automobile market segment as

$$x_j \equiv t\sum\nolimits_{i=1}^{n}\left\{\delta_i^j\gamma_i\left[\int_{\tilde{\theta}_i^j}^{\hat{\theta}_i^j}\tilde{p}_r^j f_j(\theta)d\theta + A\left(1-F_j(\hat{\theta}_i^j)\right)\right]\right\}, \tag{13}$$

which is regarded as the expected reimbursement to the consumer for the automobile purchase. The government's expected total amount of tax reduction is $\sum_{j\in\{H,L\}}(B_jx_j)$ because there are B_j consumers in the Type-j automobile market segment. Noting that the manufacturer and the retailer should pay to the government their corporate tax $TS/(1-T)$, we compute the government's net expense as $X \equiv \sum_{j\in\{H,L\}}(B_jx_j) - TS/(1-T)$.

Then, the profit-cost ratio can be calculated by the following formula:

$$\phi \equiv S/X = (1-T)S \Big/ \left[(1-T)\sum\nolimits_{j\in\{H,L\}}(B_jx_j) - TS\right]. \tag{14}$$

As Definition 1 indicates, for a cost-effective policy, ϕ should be greater than or equal to 1. One may note that consumer surplus is not involved in (14). We do not consider consumer surplus, mainly because it would have no impact on our major results. Specifically, our previous analytical results indicate that introducing a consumer surplus term into our model does not change our analytical approach and most of the results, including the negotiated retail price, the sales, the manufacturer's profit, the retailer's profit, and the government's expense for its policy. If consumer surplus is included in the analysis, then when we evaluate the cost-effectiveness of the policy, we need to consider the consumer surplus increase together with the system-wide profit increase. However, because consumers and the retailer share the tax reduction, the government's gain from the policy is very likely to exceed the government's expense and thus the policy would always be cost effective.

Theorem 3 The U.S. policy is cost effective if the retailer's bargaining power κ_j for the Type-j automobile ($j = H, L$) satisfies

$$\sum\nolimits_{j\in\{H,L\}} \left(B_j\kappa_j\varphi_j^l(\gamma)\right) \geq \sum\nolimits_{j\in\{H,L\}} \left(B_j\lambda_j^l(\gamma)\right) \text{ for } l = 1, 2, \tag{15}$$

where $\varphi_j^l(\gamma)$ and $\lambda_j^l(\gamma)$ (for $l = 1, 2$ and $j \in \{H, L\}$) are defined as,

$$\begin{aligned}\varphi_j^1(\gamma) \equiv& \sum\nolimits_{i=1}^n \left\{\frac{\delta_i^j}{1+(1-\gamma_i)t}\left[(1-\gamma_i t)\int_{\tilde\theta_i^j}^{\theta_0^j}(\theta-\tilde\theta_i^j)f_j(\theta)d\theta\right.\right.\\ &\left.\left.-\gamma_i t\int_{\theta_0^j}^{\bar\theta_j}\left(\frac{t}{1+t}\theta-\tilde\theta_i^j\right)f_j(\theta)d\theta\right]\right\}, \end{aligned}\tag{16}$$

$$\lambda_j^1(\gamma) \equiv \sum\nolimits_{i=1}^n \left\{\delta_i^j\left[\gamma_i t w_j\left(1-F_j(\tilde\theta_i^j)\right)-(w_j-c_j)\left(F_j(\theta_0^j)-F_j(\tilde\theta_i^j)\right)\right]\right\}, \tag{17}$$

$$\varphi_H^2(\gamma) \equiv \frac{1}{1+t}\sum\nolimits_{i=1}^n \left\{\delta_i^H\left[\int_{\tilde\theta_i^H}^{\bar\theta_H}(\theta-\tilde\theta_i^H)f_H(\theta)d\theta-\int_{\theta_0^H}^{\bar\theta_H}(\theta-\theta_0^H)f_H(\theta)d\theta\right]\right\}, \tag{18}$$

$$\lambda_H^2(\gamma) \equiv \sum\nolimits_{i=1}^n \left\{\delta_i^H\left[\gamma_i t w_H\left(1-F_H(\tilde\theta_i^H)\right)-(w_H-c_H)\left(F_H(\theta_0^H)-F_H(\tilde\theta_i^H)\right)\right]\right\}, \tag{19}$$

$$\begin{aligned}\varphi_L^2(\gamma) \equiv& \frac{1}{1+t}\sum\nolimits_{i=1}^n \left\{\delta_i^L\left[\frac{(1-\gamma_i t)(1+t)}{1+(1-\gamma_i)t}\int_{\tilde\theta_i^L}^{\hat\theta_i^L}\left(\theta-\tilde\theta_i^L\right)f_L(\theta)d\theta\right.\right.\\ &\left.\left.+\int_{\bar\theta_i^L}^{\bar\theta_L}\left[\theta-\tilde\theta_i^L+\gamma_i t(w_H-w_L)\right]f_L(\theta)d\theta-\int_{\theta_0^L}^{\bar\theta_L}(\theta-\theta_0^L)f_L(\theta)d\theta\right]\right\}, \end{aligned}\tag{20}$$

$$\begin{aligned}\lambda_L^2(\gamma) \equiv& \sum\nolimits_{i=1}^n \delta_i^L\left[\left\{(w_H-w_L)\left[(1-\gamma_i t)F_L(\hat\theta_i^L)+\gamma_i t-F_L(\bar\theta_i^L)\right]\right.\right.\\ &\left.\left.+\gamma_i t w_L\left(1-F_L(\tilde\theta_i^L)\right)-(w_L-c_L)\left(F_L(\theta_0^L)-F_L(\tilde\theta_i^L)\right)\right\}\right]. \end{aligned}\tag{21}$$

If $\lambda_j^l(\gamma) < 0$ for $j = H, L$ and $l = 1, 2$, then a cost-effective policy always exists. A cost-effective policy exists if $\kappa_j \geq \lambda_j^l(\gamma)/\varphi_j^l(\gamma)$ for $j = H, L$ and $l = 1, 2$. If $\lambda_j^l(\gamma) > \varphi_j^l(\gamma)$, then there may not exist a cost-effective policy. ∎

The above theorem indicates that the U.S. tax reduction policy may or may not be cost effective, depending on the retailer's bargaining power κ_j, which reflects the retailer's ability to negotiate retail prices with consumers in the Type-j automobile market segment. The conditions that $\sum_{j\in\{H,L\}}\left(B_j\kappa_j\varphi_j^l(\gamma)\right) \geq \sum_{j\in\{H,L\}}\left(B_j\lambda_j^l(\gamma)\right)$ ($l = 1, 2$) imply that a cost-effective U.S. policy is *more likely* to exist if the automobile supply chain includes a retailer with a *stronger* bargaining power. This is because if the retailer's bargaining power κ_j is sufficiently high, then the retail price negotiated between the retailer and each consumer would be high, resulting in a large profit for the retailer. As a consequence, the system-wide profit would increase and the profit-cost ratio would be high. The cost-effectiveness of the policy also depends on (i) the sales tax rate t; (ii) the federal income tax rate distribution (i.e., δ_i^j, for $i = 1, \ldots, n+1$) of consumers in each automobile market segment; (iii) the values of n federal income tax rates of eligible consumers (i.e., γ_i, for $i = 1, \ldots, n$); and (iv) the size B_j of each segment. Since it is difficult to interpret the sufficient condition for the cost-effectiveness of the U.S. policy in Theorem 3, we provide a simpler sufficient condition and discuss its insight below.

Remark 3 For the smaller value of $t\sum_{i=1}^{n}(\delta_i^j \times \gamma_i)$, a cost-effective tax reduction policy involving a cutoff level is more likely to exist. In addition, if B_L is greater than B_H, then a cost-effective policy is more likely to exist. ■

For an analytical justification for the above remark, see online Appendix D. As discussed in Section 3.1.1, δ_i^j is the percentage of Type-j ($j = H, L$) automobile consumers who are eligible for the tax reduction based on the federal income tax rate γ_i ($i = 1, \ldots, n$). Hence, the term $\sum_{i=1}^{n}(\delta_i^j \times \gamma_i)$ is the expected federal income tax rate for the eligible Type-j automobile consumers. Because the tax reduction amount for an eligible consumer who pays his or her income tax at the rate γ_i is calculated as the tax reduction rate $\gamma_i t$ times $\min(A, p_r^*)$, $t\sum_{i=1}^{n}(\delta_i^j \times \gamma_i)$ can be regarded as the expected tax reduction rate for the Type-j automobile consumers.

Consider the joint federal income tax rate distribution P_i ($i = 1, \ldots, n+1$) for single and joint filers in the U.S., where $P_i \equiv (B_L\delta_i^L + B_H\delta_i^H)/(B_L + B_H)$ represents the proportion of consumers in the automobile supply chain who are eligible for tax reduction at the income tax rate γ_i (note that $\gamma_{n+1} = 0$). Remark 3 implies that the U.S. policy will more likely be cost effective when consumers' income levels decline so that the expected tax reduction rate [i.e., $\sum_{j\in\{H,L\}}[B_j/(B_L + B_H)]\left(t\sum_{i=1}^{n}\delta_i^j \times \gamma_i\right) = t\sum_{i=1}^{n}(P_i \times \gamma_i)$] for all consumers is reduced, that is, a lower cost for the government. This means that it would be appropriate for the U.S. government to implement the tax reduction policy during the global economic recession.

As reported by the Federation of Tax Administrators (FTA), as of January 1, 2010, the highest sales tax rate is 9.75% in Los Angeles (Federation of Tax Administrators [14]). That is, in Section 3, $t \leq 9.75\%$. Using Example 1 and online Table A, we can find that, in the U.S., $\sum_{i=1}^{n}(P_i \times \gamma_i) = 18.64\%$, and $t\sum_{i=1}^{n}(P_i \times \gamma_i) \leq 1.82\%$, which is a small percentage of consumers' income being deducted for tax purposes. Therefore, according to Remark 3, we conclude that for the current sales rate and federal income tax rate, the U.S. policy is likely to be cost effective.

To illustrate Remark 3, we provide online Example 3, from which we learn that the market share for each automobile type can affect the cost effectiveness of the U.S. tax reduction policy, and the degree of consensus among consumers can also be a key factor that affects the cost effectiveness of the U.S. policy.

3.2.2 Feasible Targets and the Corresponding Budget for the U.S. Policy

We find from Section 3.1.2 that the U.S. policy with $A \geq w_H$ can generate the additional sales of the Type-j automobile ($j = H, L$) as

$$\varsigma_j \equiv D_j - D_0^j = B_j \times \left[F_j((1+t)w_j) - \sum_{i=1}^{n+1}\left(\delta_i^j \times F_j(\tilde{\theta}_i^j)\right)\right], \quad (22)$$

where $D_0^j = B_j \times (1 - F_j((1+t)w_j))$ is the expected sales of the Type-j automobile when the policy is not implemented. We learn from (22) that the additional sales are independent of the cutoff level A. This happens because as Theorem 1 indicates, the cutoff level does not influence the value of $\tilde{\theta}_i^j$, which is the minimum valuation of the consumers who are willing to buy the Type-j automobile.

The policy makers need to set the feasible target sales increase for the Type-j automobile as ς_j in (22). To find the feasible target system-wide profit increase, the policy makers should first investigate the maximum profit increase S^* that can be achieved for the supply chain, and then determine the feasible target profit increase as a specific value that does not exceed the maximum value. We can find S^* by solving the following problem:

$$\max_A S, \quad \text{s.t.} \quad \phi \geq 1 \quad \text{and} \quad A \geq w_H. \tag{23}$$

Theorem 4 When the sufficient condition in (15) for the cost-effectiveness of a tax reduction policy holds, the optimal cutoff level A^* for the problem in (23) is any value such that $A^* \geq \bar{A}_H \equiv \kappa_H \bar{\theta}_H / [1 + (1-\gamma_n)t] + (1-\kappa_H) w_H$, where $\bar{A}_H$ is the retail price of the Type-H automobile paid by a consumer with the highest valuation $\bar{\theta}_H$ and the largest income tax rate γ_n. ■

The above theorem implies that, in order to maximize the system-wide profit increase, the U.S. government may set a sufficiently high cutoff level or no cutoff level. In this way, each consumer can enjoy a tax reduction on the full retail price of a qualifying automobile that he or she buys. In fact, in the current U.S. policy, the cutoff level is $49,500, which is significantly higher than the retail prices of most qualifying automobiles. However, considering the "expense" for the policy, the government may need to choose a *feasible target* $\tilde{S}$ that is equal to or less than the maximum value S^* resulting from the adoption of the optimal cutoff level A^*, and choose a finite cutoff level to induce the target value $\tilde{S}$.

Because the cutoff level does not affect the additional sales, the U.S. government can determine its cutoff level and budget by only considering the target profit increase $\tilde{S}$. Accordingly, we can find the government's budget by following two steps: (i) solve the equation that $S = \tilde{S}$ to find the cutoff level $\tilde{A}$ [note that S is given as in (12)], and (ii) substitute $\tilde{A}$ into the government's "expense" function $X = Bx - TS/(1-T)$ and determine the budget $\tilde{X}$. We illustrate the above in online Example 4.

4 Impact of the Chinese Automobile Tax Reduction Policy

As explained in Section 1, the Chinese tax reduction policy is percentage based, under which the sales tax rate is reduced by α $(0 \leq \alpha \leq 1)$ from t to $(1-\alpha)t$, where $t \in [0,1]$ is the *original* sales tax rate (10% in China). Note that $\alpha = 50\%$ in 2009 and $\alpha = 25\%$ in 2010. This means that the Chinese government's decision is the *reduction percentage* α in its tax reduction policy. In order to distinguish the notations of the variables for the Chinese policy from those for the U.S. policy, we, hereafter, use the hat symbol ($\hat{}$) to indicate the Chinese policy.

4.1 Impact of the Chinese Tax Reduction Policy on Each Type of Automobile

Similar to Section 3.1, we consider a generic automobile in this section. Using the same methodology explained in Section 3.1.1, we can determine a consumer's net gain as his or her valuation θ minus the after-tax expense $(1+t) \times \hat{p}_r$ plus the consumer's tax reduction $\alpha \times t \times \hat{p}_r$; that is,

$$\hat{u}(\hat{p}_r; \theta) = \theta - [1 + (1-\alpha) \times t] \times \hat{p}_r, \tag{24}$$

where $\hat{p}_r$ is the retail price. Note that, similar to the term $t \sum_{i=1}^{n} (\delta_i \times \gamma_i)$ in the analysis of the U.S. policy, αt is considered as the *tax reduction rate* (that applies to the retail price paid by each consumer) for the Chinese policy. Moreover, the retailer's marginal after-tax profit is calculated as

$\hat{\pi}_R(\hat{p}_r) = (1-T)(\hat{p}_r - w)$. Using the GNB approach specified in Section 3.1.1, we can determine the negotiated retail price.

Theorem 5 Given that the sales tax reduction percentage in the Chinese policy is α and the manufacturer's wholesale price is w, the retailer and a consumer with the valuation $\theta \in [0, \bar{\theta}]$ negotiate the retail price as follows: If the consumer's valuation θ is smaller than his or her purchase expense in terms of the wholesale price w (the minimum retail price that the retailer can offer), i.e., $0 \leq \theta < \hat{\theta} \equiv [1 + (1-\alpha)t]w$, then the retailer and the consumer cannot reach an agreement on the retail price and they do not complete any transaction. If the consumer's valuation is above $\hat{\theta}$, i.e., $\theta \geq \hat{\theta}$, then the consumer buys from the retailer at the negotiated retail price $\hat{p}_r$ as

$$\hat{p}_r = \frac{\kappa\theta}{1 + (1-\alpha)t} + (1-\kappa)w. \quad \blacksquare \tag{25}$$

The above theorem indicates that each consumer and the retailer complete a transaction with the probability $1 - F(\hat{\theta})$. Recalling that the consumer base is B, we can calculate the expected sales $\hat{D}$ as

$$\hat{D} = B\left(1 - F(\hat{\theta})\right). \tag{26}$$

Since $\hat{\theta}$ is decreasing in α, the Chinese government can raise the sales by increasing the reduction percentage. The retail price depends on the consumer's valuation; thus, we compute the *expected* retail price $\hat{\mu}$ as

$$\hat{\mu} = E[\hat{p}_r | \theta \geq \hat{\theta}] = \frac{\int_{\hat{\theta}}^{\bar{\theta}} \hat{p}_r f(\theta) d\theta}{1 - F(\hat{\theta})} = \frac{\kappa \int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta) d\theta}{[1 + (1-\alpha)t]\left(1 - F(\hat{\theta})\right)} + (1-\kappa)w. \tag{27}$$

Moreover, we can compute the retailer's and the manufacturer's expected profits as

$$\hat{\Pi}_R = B(1-T)\int_{\hat{\theta}}^{\bar{\theta}} (\hat{p}_r - w) f(\theta) d\theta = B\kappa(1-T)\left[\frac{\int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta) d\theta}{1 + (1-\alpha)t} - \left(1 - F(\hat{\theta})\right) w\right], \tag{28}$$

$$\hat{\Pi}_M = B(1-T)(w-c)\left(1 - F(\hat{\theta})\right). \tag{29}$$

Note that $\hat{D}$, $\hat{\Pi}_R$, and $\hat{\Pi}_M$ are independent of consumers' income tax rate distribution.

Remark 4 The negotiated retail price $\hat{p}_r$, the expected sales $\hat{D}$, the manufacturer's expected profit $\hat{\Pi}_M$, and the retailer's expected profit $\hat{\Pi}_R$ are higher than those before the policy is implemented. The magnitude of each increase is measured by the difference between each of the equations (25), (26), (28), and (29) and the corresponding value under no tax reduction, and mainly depends on the sales tax reduction percentage α. $\blacksquare$

For an analytical justification for the above remark, see online Appendix D. The above remark indicates that the Chinese tax reduction policy should be helpful in stimulating the automobile market and improving the performance of the automobile industry. In fact, compared with the annual sales in 2008, the annual sales in China in 2009 increased by 46.15% to 13.64 million units (see, e.g., Hong Kong Trade Development Council [19] and Liu [24]).

Theorem 6 If the consumer demand is price inelastic—that is, the generalized failure rate $\hat{\theta} f(\hat{\theta}) \Big/ \Big(1 - F(\hat{\theta})\Big)$ is smaller than 1 (see, e.g., Lariviere [23]), then the expected retail price $\hat{\mu}$ increases in α. Moreover, $\hat{D}$, $\hat{\Pi}_R$, and $\hat{\Pi}_M$ are increasing in α. ■

Theorem 6 presents an interesting result that, even though each consumer's retail price $\hat{p}_r$ increases in α, the expected (average) retail price $\hat{\mu}$ may *not* increase when α is raised because it depends on the price elasticity of demand. If the demand is price inelastic, then the retail price increases. But, this may not occur when the demand is price elastic, because a small increase in α (i.e., a small reduction in the sales tax rate) can stimulate a large number of *low-valuation* consumers to purchase the automobile, who are only willing to pay low retail prices. Although the retail prices paid by high-valuation consumers (who intend to buy even without the tax reduction policy) would *slightly* increase (in α), the proportion of low retail prices greatly increases. Therefore, the expected (average) retail price for all consumers may decrease in α.

To illustrate the above results, we provide online Example 5, which shows that, as α is increased from 25% to 50%, the expected retail price, the sales, and the retailer's and the manufacturer's profits would be increased by 0.51%, 10.91%, 22.99%, and 10.91%, respectively. Moreover, when the demand is significantly price elastic, the expected retail price will decrease in α.

4.2 Cost Effectiveness of the Chinese Tax Reduction Policy on the Automobile Supply Chain

To examine the cost effectiveness of the Chinese policy, we need to consider both the Type-H and Type-L automobiles to calculate the total expense for implementing the policy. We follow the two steps specified in Remark 2 to find the feasible target sales increase and system-wide profit increase and determine the corresponding budget for the targets. In online Table B, we show that the sales and profits of two major Chinese automobile firms—Shanghai Automobile Corporation and Dongfeng Automobile Corporation—are positively correlated, with the correlation coefficients being 0.89 and 0.85, respectively. This evidence suggests that, similar to the U.S., China also aims at increasing both the sales and the profitability of its automobile industry.

4.2.1 The Necessary and Sufficient Condition for the Cost Effectiveness of the Chinese Policy

Using $\hat{\Pi}_R$ in (28) and $\hat{\Pi}_M$ in (29), we calculate the system-wide (after-tax) profit when the policy is implemented, denoted by $\hat{\Pi}$, as

$$\hat{\Pi} = \sum_{j \in \{H,L\}} \left\{ B_j(1-T) \left(\frac{\kappa_j}{1+(1-\alpha)t} \int_{\hat{\theta}_j}^{\bar{\theta}_j} \theta f_j(\theta) d\theta + \Big(1 - F_j(\hat{\theta}_j)\Big) [(1-\kappa_j)w_j - c_j] \right) \right\}, \quad (30)$$

which is increasing in the tax reduction percentage α. The system-wide profit increase generated by the Chinese policy is calculated as $\hat{S} = \hat{\Pi} - \Pi_0$, where Π_0 as given in (10) is the system-wide expected profit when the policy is not implemented. Thus,

$$\hat{S} = \sum_{j\in\{H,L\}} \left\{ B_j(1-T) \left(\kappa_j \left[\frac{\int_{\hat{\theta}_j}^{\bar{\theta}_j} \theta f_j(\theta)d\theta}{1+(1-\alpha)t} - \frac{\int_{\theta_0^j}^{\bar{\theta}_j} \theta f_j(\theta)d\theta}{1+t} \right] + \left(F_j(\theta_0^j) - F_j(\hat{\theta}_j)\right) [(1-\kappa_j)w_j - c_j] \right) \right\}, \tag{31}$$

which must be positive because according to Remark 4, the manufacturer's and the retailer's expected profits are higher than those before the policy is implemented. Note that $\theta_0^j = (1+t)w_j$ and $\hat{\theta}_j = [1+(1-\alpha)t]w_j$.

Similar to our discussion in Section 3.2.1, we compute the government's expense as $\hat{X} = \sum_{j\in\{H,L\}}(B_j\hat{x}_j) - T\hat{S}/(1-T)$, where $\hat{x}_j$ denotes the expected tax reduction amount for a consumer in the Type-j automobile market segment, i.e.,

$$\hat{x}_j = \alpha \times t \times E[\hat{p}_r^j] = \alpha \times t \times \left[\frac{\kappa_j}{1+(1-\alpha)t} \int_{\hat{\theta}_j}^{\bar{\theta}_j} \theta f_j(\theta)d\theta + w_j(1-\kappa_j)\left(1 - F_j(\hat{\theta}_j)\right) \right]. \tag{32}$$

Then, we compute the profit-cost ratio of the policy as

$$\hat{\phi} = \hat{S}/\hat{X} = (1-T)\hat{S} \Bigg/ \left[\sum_{j\in\{H,L\}} (B_j\hat{x}_j)(1-T) - T\hat{S} \right]. \tag{33}$$

Theorem 7 The Chinese policy is cost effective (i.e., $\hat{\phi} \geq 1$) if and only if $\sum_{j\in\{H,L\}}[B_j \times (\kappa_j\tau_j^1 + \tau_j^2)] \geq 0$, where

$$\tau_j^1 \equiv \frac{1-\alpha t}{1+(1-\alpha)t} \int_{\hat{\theta}_j}^{\bar{\theta}_j} (\theta - \hat{\theta}_j) f_j(\theta)d\theta - \frac{1}{1+t} \int_{\theta_0^j}^{\bar{\theta}_j} (\theta - \theta_0^j) f_j(\theta)d\theta, \tag{34}$$

$$\tau_j^2 \equiv [(1-\alpha t)w_j - c_j]\left(1 - F_j(\hat{\theta}_j)\right) - (w_j - c_j)\left(1 - F_j(\theta_0^j)\right). \tag{35}$$

In general, $\tau_j^1 > 0$, $\partial\tau_j^1/\partial\alpha > 0$, and $\tau_j^2 < 0$, $\partial\tau_j^2/\partial\alpha < 0$. Therefore, a sufficient condition for the Chinese policy to be cost effective is $\kappa_j \geq -\tau_j^2/\tau_j^1$ for $j = H, L$. ■

As the above theorem indicates, the cost effectiveness of the Chinese policy depends on the values of κ_j, τ_j^1, τ_j^2, and B_j. According to our discussion in online Appendix G, we find that τ_j^1 can be viewed as the expected increase in the extra before-tax valuation of a Type-j automobile consumer that is generated by the Chinese policy, and τ_j^2 can be explained as the manufacturer's tax reduction-exclusive profit increase from a Type-j automobile under the Chinese policy. From (34) and (35), we find that the Chinese government can choose α to affect the values of τ_j^1 and τ_j^2. Therefore, we cannot conclude from Theorem 7 that κ_j must be large to assure the cost effectiveness of the Chinese policy. That is, the Chinese policy may still be cost effective when the retailer is in a weak position (i.e., κ_j is small) to negotiate the retail price with consumers. We illustrate our analysis on the cost effectiveness of the Chinese policy in online Example 6, which shows that whether or not the Chinese policy is cost effective significantly depends on the retailer's bargaining power.

4.2.2 Feasible Targets and the Corresponding Budget for the Chinese Policy

According to Section 4.1, the expected sales increase of the Type-j automobile (for $j = H, L$) generated by the Chinese policy with $\alpha \in [0, 1]$ are computed as

$$\hat{\varsigma}_j \equiv B_j \left(F(\theta_0^j) - F(\hat{\theta}_j) \right), \tag{36}$$

which is positive because $\theta_0^j = (1+t)w_j > \hat{\theta}_j = [1 + (1-\alpha)t]w_j$. Thus, a feasible target sales increase of the Type-j automobile can be calculated using the formula in (36). In addition, we can obtain the maximum system-wide profit increase by solving the constrained maximization problem: $\max_\alpha \hat{S}$, s.t. $\hat{\phi} \geq 1$ and $0 \leq \alpha \leq 1$, and determine a feasible target profit increase that is no more than the maximum value. Because Theorem 6 has shown that the expected sales and the manufacturer's and the retailer's expected profits from each type of automobile are increasing in α, we obtain the following corollary.

Corollary 1 For a cost-effective Chinese policy, the optimal tax reduction percentage α^* maximizing the system-wide profit $\hat{S}$ can be uniquely determined as $\alpha^* = \max \Theta$, where $\Theta \equiv \{\alpha | \hat{\phi} \geq 1$ and $0 \leq \alpha \leq 1\}$. ■

Therefore, the optimal reduction percentage must exist if the condition for the cost effectiveness of policy in Theorem 7 is satisfied. If the condition is not satisfied, then $\Theta = \varnothing$ and the Chinese policy may not be worth implementing; otherwise, $\Theta \neq \varnothing$, and the policy makers can choose a *feasible target* sales increase $\hat{\varsigma}_j^C$ for the Type-j automobile and a *feasible target* system-wide profit increase $\hat{S}^C$, which are capped by the maximum sales increase and profit increase (generated by adopting the optimal reduction percentage α^*), respectively.

Next, to achieve the targets $\hat{\varsigma}_j^C$ and $\hat{S}^C$, we find the budget using the following three steps: (i) For the Type-j automobile ($j = H, L$), we solve the equation that $\hat{\varsigma}_j = \hat{\varsigma}_j^C$, where $\hat{\varsigma}_j$ is given as in (36), and find the tax reduction percentage $\hat{\alpha}_j^1$. Substituting $\hat{\alpha}_1 = \max(\hat{\alpha}_H^1, \hat{\alpha}_L^1)$ into the Chinese government's expense function $\hat{X} = \sum_{j \in \{H,L\}} (B_j \hat{x}_j) - T\hat{S}/(1-T)$ gives the corresponding budget X_1^C for achieving the sales increase $\hat{\varsigma}_j^C$. (ii) Similarly, we solve the equation that $\hat{S} = \hat{S}^C$, where $\hat{S}$ is given as in (31), and find the tax reduction percentage $\hat{\alpha}_2$. Substituting $\hat{\alpha}_2$ into the expense function $\hat{X}$ gives the corresponding budget X_2^C for achieving the profit increase $\hat{S}^C$. (iii) We then determine the government's budget as the higher value of X_1^C and X_2^C because both the demand and profit increase targets should be met. This approach is illustrated by online Example 7, where the optimal tax reduction percentage is $\alpha = 47.96\%$, resulting in a profit-cost ratio $\hat{\phi} = 1.0654$.

5 Comparison Between the U.S. and the Chinese Policies

We note from Sections 3 and 4 that the U.S. stipulates a finite cutoff level in its tax reduction policy with a tax reduction percentage determined by the federal income tax rate. In contrast, China does not consider any cutoff level but only adjusts the sales tax reduction percentage to influence consumers' purchase of new automobiles. Theorems 2 and 6 analytically determine the effectiveness of the U.S. and the Chinese policies in stimulating the automobile sales and improving the profitability of the automobile industry for a type of automobile, respectively. However, we cannot *immediately* determine which policy is more effective under comparable conditions while assuring the cost effectiveness of the policy. Next, we discuss the institutional differences between the two policies. We then conduct a numerical experiment with sensitivity analysis based on real parameters to compare their impact and cost effectiveness.

5.1 Background Comparison between the U.S. and the Chinese Policies

We analyze the background of these two policies by discussing the sales tax management systems, the consumers' taxable income levels, the retail prices of qualifying automobiles, and the retailers' bargaining powers in their automobile markets.

5.1.1 Sales Tax Management Systems

The U.S. and China use different ways to assist their automobile industries. Specifically, the U.S. consumers cannot get a tax cut immediately after purchasing a new automobile, but can enjoy a reduction in their federal income tax when they report their tax liability. In China, each consumer can immediately benefit from the policy by paying less sales tax. That is, the U.S. consumers receive "delayed" benefits from the U.S. policy, and those who do not pay or pay little income tax have no or little benefit. The Chinese consumers receive immediate benefits from the Chinese policy, and each consumer who buys an automobile can enjoy the same percentage of sales tax reduction.

One reason for the above difference in the two tax policies is the fact that sales tax is a local tax in the U.S. but a federal tax in China. Therefore, in terms of tax administration, it is easier for China's central government to reduce the sales tax.

5.1.2 Taxable Incomes of Consumers and Retail Prices of Qualifying Automobiles

We learn from Sections 3 and 4 that the U.S. and Chinese policies have different impact on consumers' benefits and the automobile industry. We first discuss a consumer's tax reduction amount under each policy. Note from Section 3.1.1 that an eligible U.S. consumer with the federal income tax rate $\gamma_i \in (0,1)$ can enjoy the tax reduction amount calculated as $\gamma_i t$ of the minimum of the cutoff level A and the retail price. In Section 3.2.1, we calculate the average federal income tax rate for the U.S. consumers as $\sum_{i=1}^{n}(P_i \times \gamma_i) = 18.64\%$. For each U.S. consumer, the average ratio of a reduced tax to the cost of the automobile purchased (up to \$49,500) is $t\sum_{i=1}^{n}(P_i \times \gamma_i) \leq 1.82\%$. We also find from Section 4.1 that for each Chinese consumer, the ratio of a tax reduction amount to the retail price is $\alpha t = 5\%$ in 2009 and $\alpha t = 2.5\%$ in 2010.

Comparing $t\sum_{i=1}^{n}(P_i \times \gamma_i)$ in the U.S. policy with αt in the Chinese policy, we find that the percentage of tax reduction amount in the retail price of a qualifying automobile for the Chinese consumers is significantly greater than that for the U.S. consumers. This happens possibly because of two reasons: First, the average income of consumers in China is lower than that in the United States. Second, the automobile retail prices in China (especially for imported automobiles) are higher than those in the U.S. markets for the same model. To effectively stimulate the automobile market, the Chinese government needs to reduce the consumer's tax burden by a larger amount so as to ensure that more Chinese consumers are able to afford the purchase of new automobiles.

According to the above, if the taxable incomes of consumers in a country are low and/or the retail prices of new automobiles are high, then the government may consider the Chinese policy to directly reduce sales tax by a significant percentage and thus immediately help consumers afford their purchases. Compared with the Chinese policy, the U.S. policy provides consumers with less tax reductions and thus needs a smaller budget, which may be desirable when the government has a large deficit.

5.1.3 Retailers' Bargaining Powers in Automobile Markets

Retail prices are negotiated under both the U.S. and Chinese policies, which mainly differ in that under the U.S. policy, the consumers with different income tax rates will obtain different sales tax reduction rate, whereas under the Chinese policy, all consumers enjoy an identical reduction in the sales tax rate. To focus our comparison between the two policies on their key differences and obtain analytical results and managerial insights, we now assume that the retailer's bargaining power in the U.S. is identical to that in China. This assumption also enables us to investigate the impact of the bargaining power on the two policies; otherwise, we cannot reach any comparable results.

We find from Theorem 3 that the U.S. tax reduction policy is likely to be cost effective if the retailer is in a sufficiently strong position to negotiate retail prices with consumers. This is because a strong retailer bargaining power means a high negotiated retail price and profit. As discussed in Section 3.2.1, only a small percentage of consumers' income is deducted, and thus the increase in the government's cost of implementing the policy is likely to be less than the profit increase of the retailer and the supply chain.

On the other hand, we note from Theorem 7 that the Chinese tax reduction policy is also likely to be cost effective if the retailer's bargaining power κ_j is greater than $-\tau_j^2/\tau_j^1$ for $j = H, L$. In contrast to the U.S. policy, the Chinese policy imposes a deduction directly on the sales tax, and the government can easily adjust the tax reduction percentage α to ensure the cost-effectiveness of the policy. As demonstrated in online Example 6, even when the bargaining power is small (e.g., $\kappa_j = 0.2$ for $j = H, L$), the government can choose a small α to achieve this purpose.

The above findings should be helpful in guiding public policy makers to choose a proper tax reduction policy. That is, if the automobile retailer in a country is in a weak position to bargain the retail price with consumers (because of overstocking, weak demand, or intense competition), then the policy makers may consider the Chinese policy with a tax reduction percentage α that is set such that the policy is cost effective. On the other hand, if the retailer is strong in negotiating the retail prices, then the policy makers may consider either the U.S. policy or the Chinese policy with a proper value of α.

5.2 A Numerical Experiment with Sensitivity Analysis Comparing the U.S. and the Chinese Policies

We perform a numerical experiment with sensitivity analysis based on real data to compare the impact and the cost effectiveness of the U.S. and the Chinese policies. Such an analysis together with the analytical results obtained in this paper can serve as a decision model on the choice of alternative tax reduction policies by public policy makers (see Macnaughton [25]). We now consider a *generic* government (say, a third-world country) which can implement either the U.S. policy with a cutoff level or the Chinese policy with a tax reduction percentage. We vary the values of decision variables and parameters in the two policies, and compare their impact and cost effectiveness to find condition(s) under which the U.S. or the Chinese policy is more appropriate for the generic government to use for stimulating the sales and improving the profitability of the automobile industry.

In Sections 3 and 4, we provided numerical examples to illustrate our analytical results for the U.S. and the Chinese policies, respectively. We note that the U.S. policy is dependent on the distribution of consumers' income tax rates (i.e., P_i, $i = 1, \ldots, n+1$) and the federal income tax rates (i.e., γ_i, $i = 1, \ldots, n$), whereas the Chinese policy does not deal with the income-related issues but depends on the tax reduction percentage α. The models that we developed are applicable for all sales tax rates.

To compare the two policies *in the same setting*, we consider the parameter values that are used for the U.S. policy in Section 3, and *"simulate"* the implementation of the Chinese policy in the U.S. setting. Our results for both policies are then compared to draw insights about the comparison between their impact (on the expected sales as well as the expected profits of the retailer, the manufacturer, and the supply chain) and cost effectiveness (i.e., the profit-cost ratio).

To simulate the Chinese policy in the U.S. setting, we assume that the sales tax rate is $t = 5\%$ (a typical rate) and $\alpha = \sum_{i=1}^{n}(P_i \times \gamma_i) = 18.64\%$, because $\sum_{i=1}^{n}(P_i \times \gamma_i)$ is the expected federal income tax rate in the U.S. policy that is equivalent in meaning to the "tax reduction percentage" in the Chinese policy. Using $t = 5\%$ and $\alpha = 18.64\%$, we apply other parameter values in Example 3 to identify the expected sales for the Type-j automobile, the expected profits of the retailer, the manufacturer, and the supply chain, as well as the profit-cost ratio. The results for both policies are given in Panel A of online Table C.

We find from the results that, although the expected federal income tax rate in the U.S. policy is equal to the tax reduction percentage in the Chinese policy, the impact and cost effectiveness of two policies are different. In the current U.S. setting, the Chinese policy is more effective in stimulating the sales of the Type-L automobile, but is less effective in stimulating the sales of the Type-H automobile. In comparison, the Chinese policy is more cost effective because the profit-cost ratio is higher.

Even though we can compare the U.S. policy with the Chinese policy in the current U.S. setting, we still need to examine which policy is better in different settings. Accordingly, as discussed below, we perform a sensitivity analysis by changing the values of decision variables and parameters in the two policies around their *base* values in the current U.S. setting.

5.2.1 Impact of the Decision Variables on the U.S. and the Chinese Policies

We first consider the impact of the cutoff level A in the U.S. policy by changing its value around the base value \$49,500. The value of A is increased from \$34,500 to \$57,000 in increments of \$2,500. Our sensitivity analysis results are given in Panel B of online Table C, where we calculate: (i) the expected sales of each type of automobile, (ii) the retailer's, the manufacturer's, and the system-wide expected profits, and (iii) the profit-cost ratio. The results here are only for the U.S. policy because the Chinese policy does not involve the cutoff level. We find that the cutoff level A does not influence the sales and the manufacturer's profit as long as $A > w_H = \$34,000$, as discussed in Section 3.1.2. Moreover, when $w_H < A \leq \$39,500$, increasing the cutoff level will raise the retailer's profit. However, when A is sufficiently high ($A > \$39,500$), any further increase of A

will not affect the profits in the supply chain and the profit-cost ratio. In this case, the government may choose a cutoff level lower than \$49,500 and have the same market effects.

We then consider the impact of the tax reduction percentage α in the Chinese policy by increasing its value from 0.1 to 0.74 in increments of 0.08. The results are given in Panel C of online Table C, where we also provide the results when $\alpha = 18.64\%$ and 25%. Because the U.S. policy does not consider the reduction percentage, we only investigate the impact here for the Chinese policy. Panel C of Table C indicates that, when the government increases α, the expected sales of each type of automobile and the expected profits of the manufacturer, the retailer, and the supply chain all increase, but the profit-cost ratio decreases. Moreover, if $\alpha \geq 42\%$, the profit-cost ratio is smaller than 1 and thus the Chinese policy is not cost effective. The maximum value of α (that is smaller than 42%) for the policy cost effectiveness is obtained for our numerical experiment based on the data of the Chinese policy *in the U.S. setting* (where the sales tax rate is 5%), whereas the maximum value of $\alpha = 60.3\%$ found in Example 7 is for the analysis of the Chinese policy *in the Chinese setting* (where the sales tax rate is 10%). Comparing Panels B and C of online Table C, we find that, when α is chosen to be higher than the expected federal income tax rate 18.64% (e.g., $\alpha \geq 25\%$), both the sales and the system-wide profit are higher under the Chinese policy than those under the U.S. policy (for all values of the cutoff level A).

Remark 5 According to our above discussion, we find that if a government intends to stimulate the sales and increase the profitability of its automobile industry more effectively, then it may choose the Chinese policy with a tax reduction percentage that is greater than the expected federal income tax rate 18.64%, but cannot be too large (e.g., $\alpha < 42\%$) to render the policy cost-ineffective. ■

5.2.2 Impact of the Sales Tax Rate

In online Table D, we vary the sales tax rate t from 1% to 10% in increments of 1%, and calculate the expected sales and profits under the U.S. and the Chinese policies and the percentage increases in the sales and profits compared with those before each policy is implemented. From Table D, we find that, when the sales tax rate t is larger, the percentage increases in the sales and profits under both policies are higher, thus both policies are more effective in improving the sales of each type of automobile and the profit of the automobile supply chain. Given each sales tax rate, both policies are more effective in improving the sales of the Type-H automobile than that of the Type-L automobile; the U.S. policy is better in improving the sales of the Type-H automobile whereas the Chinese policy is better in improving those of the Type-L automobile. The U.S. policy is more effective in increasing the profitability of the automobile supply chain, mainly because the manufacturer's profit margin from the Type-H automobile is higher than that from the Type-L automobile. We also note that the U.S. policy has a smaller profit-cost ratio, indicating that the government's expense under the U.S. policy would be higher than that under the Chinese policy. In addition, both policies are more cost effective in the case of a larger value of t; when t is small (e.g., $t \leq 3\%$), both policies are not cost effective in terms of the profit-cost ratio.

Remark 6 From our sensitivity analysis of the sales tax rate t, we find that, if a government aims at increasing the sales of the Type-L automobile, then it may choose the Chinese policy under any sales tax rate. Otherwise, if a government's main purpose is to improve the sales of the Type-H automobile or the profitability of the automobile industry, then the government may adopt the U.S. policy. If the government just aims at implementing a cost-effective policy, then it would be better to consider the Chinese policy. ■

5.2.3 Impact of the Market Share of Each Type of Automobile

To investigate the impact of market share on the effectiveness of each policy, we fix the total market size as $10,000$, and change the market share of the Type-L automobile from 10% to 90% in steps of 10%. Hence the size of the Type-L automobile market segment varies from $1,000$ to $9,000$ in increments of $1,000$. We also fix the income tax rate distribution of consumers in the Type-L automobile market segment as that in online Example 3. As discussed in Example 3, because the overall income tax rate distribution of consumers in both market segments follows the U.S. income tax rate distribution P_i $(i = 1, \dots, 6)$, the income tax rate distribution of consumers in the Type-H automobile market segment will vary when we change the market share of the Type-L automobile. As online Table E indicates, our results regarding the comparison between the U.S. and the Chinese policies are similar to those for the impact of the sales tax rate.

In addition, we find that the effectiveness of the U.S. policy in improving the sales of the Type-H automobile depends on the market share of the Type-L automobile, which, however, does not affect the effectiveness of the U.S. policy in improving the sales of the Type-L automobile. This occurs because we have fixed the income tax rate distribution of consumers in the Type-L automobile market segment so that the income tax rate distribution of consumers in the Type-H automobile market segment varies with the market share. Therefore, if the income tax rate distribution of consumers of an automobile type is independent of the market share, then the effectiveness of the U.S. policy in improving the sales of this automobile does not vary with the market share. We also find that the effectiveness of the Chinese policy in improving the sales of each automobile type is independent of the market share of the Type-L automobile. The effectiveness of both policies in improving the profit of the automobile supply chain is decreasing in the market share of the Type-L automobile because the Type-L automobile has a smaller profit margin than the Type-H automobile.

5.2.4 Impact of the Mean and Standard Deviation of Consumers' Valuations

A consumer's valuation reflects the after-tax price that the consumer is willing to pay for an automobile. The mean value of consumers' valuations on two types of automobiles is given as $E(\theta) = \sum_{j\in\{H,L\}}[B_j/(B_L + B_H)]E(\theta_j)$. To investigate the effect of $E(\theta)$, we increase $E(\theta_H)$ from 35,000 to 39,500 and $E(\theta_L)$ from 24,000 to 28,500, both in increments of 500; hence, $E(\theta)$ is changed from 27,300 to 31,800 in steps of 500. This range contains the estimated value of $E(\theta)$ equal to 29,916.6 that we provided in Example 1. From online Table F, we find that, both the U.S. and the Chinese policies are not cost effective (i.e., $\phi < 1$) when consumers' mean valuations on the two automobiles are sufficiently high [i.e., $E(\theta_H) \geq 37,500$ and $E(\theta_L) \geq 26,500$], whereas they become cost effective when consumers' mean valuations are not large [i.e., $E(\theta_H) < 37,500$ and $E(\theta_L) < 26,500$]. The cost effectiveness of each policy is decreasing in consumers' mean valuations on two automobile types; however, the Chinese policy is more cost effective in improving the profitability of the automobile supply chain. Moreover, both policies are less effective in improving the sales and the supply chain profit when consumers' mean valuations on two automobiles increase.

For each pair of values of $E(\theta_H)$ and $E(\theta_L)$, we can find that the comparison between the U.S. and the Chinese policies generates results similar to those for the impact of the sales tax rate and the market share of the Type-L automobile.

We then examine the impact of the dispersion of consumers' valuations by increasing the standard deviation σ_θ for both types of automobiles from 2,000 to 6,500 in steps of 500. From online Table G, we find that the dispersion of consumers' valuations (σ_θ) has a similar effect as the mean values of consumers' valuations. The cost effectiveness of each policy decreases with σ_θ. When σ_θ is small (i.e., $\sigma_\theta < 4,500$), both the U.S. and the Chinese policies are cost effective; but when consumers' valuations on an automobile are highly diverged (i.e., $\sigma_\theta \geq 4,500$), both policies will not be cost effective (i.e., $\phi < 1$). The result can be interpreted as follows. When the standard deviation of consumers' valuations increases, consumers' valuations are more diverged. As a result, it would be more difficult for the government to implement its tax policy for enticing low-valuation consumers to buy the automobile. This would result in a less significant increase in the manufacturer's and the retailer's profits in the supply chain. Thus, the profit-cost ratio becomes smaller and the policy is less cost effective.

For the Chinese policy and normally-distributed consumers' valuations, we can analytically derive the above result; see online Appendix H. For the U.S. policy, however, it is intractable to show the above result. Moreover, we find that, for each value of σ_θ, the comparison between the U.S. and the Chinese policies shows similar results as those for the impact of the sales tax rate and the market share of the Type-L automobile.

Remark 7 If the standard deviation of the consumers' valuations is larger (i.e., consumers' valuations become more diverged), then both policies are less effective in improving the sales and the supply chain profit, which means that the policies is of less significance to benefit the manufacturer and the retailer. But, if the standard deviation is within a *moderate* range (e.g., $\sigma_\theta < 4,500$ in our sensitivity analysis), then the government may choose the Chinese policy when aiming at a cost-effective policy. ■

5.2.5 Robustness of the Sensitivity Analysis Results

For our numerical experiments, we have used normal distributions for consumers' valuations. To test the robustness of our numerical results and managerial insights, we consider the case in which consumers' valuations satisfy a gamma distribution Gamma(k_1, k_2). We let the shape parameters k_1 of the Type-L and the Type-H automobiles be $k_L = 49$ and $k_H = 69$, and the scale parameter $k_2 = 535$ for both automobiles. Correspondingly, the consumers' valuations have the mean $E(\theta_L) = \$26,215$ and standard deviation $\sigma_L = \$3,745$ for the Type-L automobile and $E(\theta_H) = \$36,915$ and $\sigma_H = \$4,444$ for the Type-H automobile. Other parameters are kept the same as those in the above numerical analysis. The results for the impact of the decision variables of the U.S. and the Chinese policies, the sales tax rate, and the market share of each automobile are presented in online Tables H, I, and J (see Appendix B), respectively, which indicate that all the managerial insights that we obtained under the normal distribution still hold under the gamma distribution. In addition, we investigate the impact of the shape parameter by increasing k_H from 65 to 74 and k_L from 45 to 54, both in increments of 1; we also examine the impact of the scale parameter by increasing k_2 for both types of automobiles from 503 to 575 in steps of 8. We present the results in online Tables K and L, which indicate managerial implications similar to those for the impacts of the mean and standard deviation of normally-distributed consumers' valuations. Therefore, our managerial insights still hold when consumers' valuations satisfy a gamma distribution.

5.2.6 Summary of Major Managerial Insights

We can draw the following major insights from the above sensitivity analysis.

1. When the U.S. tax reduction policy is adopted, a government can effectively influence automobile sales by setting the cutoff level up to a certain limit. Under the Chinese policy, the government can directly reduce the sales tax rate and generate a greater increase in sales and supply chain profit than those under the U.S. policy.
2. Both policies are more effective in improving the sales of high-end automobile than those of low-end automobile. But, the U.S. policy is better in improving the sales of high-end automobile, whereas the Chinese policy is better in improving those of the low-end automobile. Moreover, the U.S. policy is more effective in increasing the supply chain profit.
3. The effectiveness of both policies in improving the sales and the supply chain profit increases with the sales tax rate, but it decreases when consumers' valuations for automobiles are higher or become more diverged.
4. Both policies are less effective in improving the supply chain profit as the Type-L automobile's market share increases.

6 Extensions

We consider two important extensions, which are (i) consumers' choices between two automobiles and (ii) the analysis of the problem with a strategic manufacturer.

6.1 Consumers' Choices Between Two Automobiles

We have considered the case where two automobiles have their individual market segments, and consumers in each segment would only choose the automobile in that segment. The underlying assumption is that consumers are divided into two segments *ex ante* according to their choices between two automobiles. That is, a consumer in the Type-H (or Type-L) automobile segment has a higher net gain from purchasing the Type-H (or Type-L) automobile than that from the Type-L (or Type-H) automobile. We now consider an extension by explicitly modeling consumers' choices between the two automobiles.

6.1.1 Negotiated Retail Price, Sales, and Profit under Each Policy

In practice, consumers' valuations for the two types of automobiles (i.e., θ_j for $j = H, L$) can be correlated with each other. We thus assume that θ_H and θ_L are jointly distributed with the bivariate probability density function $f(\theta_H, \theta_L)$ as

$$f(\theta_H, \theta_L) = \frac{1}{2\pi\sqrt{|\mathbf{V}|}} \exp\left(-\frac{1}{2}(\theta - \mu)^T \mathbf{V}^{-1}(\theta - \mu)\right),$$

where $\theta \equiv [\theta_H, \theta_L]^T$ denotes a random vector; $\mu \equiv [\mu_H, \mu_L]^T \equiv [E(\theta_H), E(\theta_L)]^T$ is the vector of the consumers' mean valuations on the Type-H and the Type-L automobiles; $|\mathbf{V}|$ is the determinant of the 2×2 covariance matrix $\mathbf{V}$. The matrix $\mathbf{V}$ is written as,

$$\mathbf{V} \equiv \left[\begin{array}{cc} \sigma_H^2 & \rho\sigma_H\sigma_L \\ \rho\sigma_H\sigma_L & \sigma_L^2 \end{array} \right],$$

where σ_j $(j = H, L)$ denote the standard deviations of θ_j and ρ is the correlation coefficient between θ_H and θ_L. The bivariate cumulative distribution function is $F(\theta_H, \theta_L)$.

For the U.S. policy, using the result in Section 3.1.1, we find that when a consumer buys a Type-j automobile $(j = H, L)$, the consumer receives a net gain $u(p_j; \theta_j, \gamma_i) = \theta_j - (1 + t) \times p_j + \gamma_i \times t \times \min(A, p_j)$, and the retailer obtains the after-tax profit $\pi_R(p_j) = (1 - T)(p_j - w_j)$, where p_j and w_j represent the retail and the wholesale prices of the Type-j automobile. To simplify our analysis, we assume the retailer's bargaining powers for the two automobiles are equal, i.e., $\kappa_H = \kappa_L$. The GNB model for the bargaining of the Type-j automobile retail price in (1) becomes:

$$\begin{array}{rl} \max_{p_j} & \Lambda_j \equiv [\theta_j - (1+t)p_j + \gamma_i t \min(A, p_j) - u_j^0]^{1-\kappa}[(1-T)(p_j - w_j) - v_j^0]^{\kappa} \\ \text{s.t.} & \theta_j - (1+t)p_j + \gamma_i t \min(A, p_j) \geq u_j^0 \quad \text{and} \quad (1-T)(p_j - w_j) \geq v_j^0, \end{array} \tag{37}$$

where $u_j^0 \equiv \max\{u(p_k; \theta_k, \gamma_i), 0\}$ and $v_j^0 \equiv \max\{(1 - T)(p_k - w_k), 0\}$ $(k = H, L; k \neq j)$ are the consumer's and the retailer's disagreement payoffs for the Type-j automobile, respectively. Then we can find the negotiated retail price p_j^* when the consumer chooses to buy a Type-j automobile as in online Appendix I.1.

Similarly, for the Chinese policy, we derive the negotiated retail price $\hat{p}_j^*$ when the consumer chooses to buy a Type-j automobile as in online Appendix I.2. Then, for each policy, we can use the negotiated retail price to derive the expected sales for each automobile and the expected profits of the manufacturer, the retailer, and the supply chain, and calculate the profit-cost ratio.

6.1.2 Sensitivity Analysis Comparing the U.S. and the Chinese Policies

Similar to Section 5.2, we perform a sensitivity analysis to compare the impact and cost effectiveness of the U.S. and the Chinese policies based on the consumer choice assumption. We consider a market with $B = 10,000$ potential consumers, who may purchase one of the two types of automobiles. For the case of correlated valuations, we do not impose a market size for each type of automobile; we use similar parameter values as those in Section 5.2 and assume $\rho = 0.5$.

The numerical results are presented in online Tables U–X, from which we can draw similar conclusions and insights as those in Section 5.2. In addition, we find that, no matter whether the U.S. or the Chinese policy is implemented, the increase in the sales of the Type-H automobile is always higher than that of the Type-L automobile. This is attributed to the fact that under a tax reduction policy, each consumer can gain a greater tax reduction amount from buying the Type-H automobile than the Type-L automobile.

We investigate the impact of the correlation coefficient ρ of consumers' valuations on two automobile types by changing ρ from -0.8 to 0.8 in steps of 0.2. Most of the insights we obtained for positive values of ρ remain unchanged. We also find that when the value of ρ is very small, a consumer's valuations on the two automobile types are strongly and negatively associated, and the consumer has a strong preference for one automobile and makes a purchase accordingly. Thus, the sales of both automobiles are high. As ρ increases, the preference becomes weaker and more consumers may not prefer to buy any automobile type, resulting in a reduction in the total sales. When a positive value of ρ becomes higher, a consumer with a higher (or lower) valuation on the Type-H automobile also has a higher (or lower) valuation on the Type-L automobile, which implies that an increase in the consumer's valuation on the Type-H automobile may not induce the consumer to buy the Type-H automobile, because the wholesale price of the Type-L automobile is lower. It thus follows that when the value of ρ increases, the sales of the Type-H automobile become significantly smaller than those of the Type-L automobile, as indicated by our numerical results in Table Y in online Appendix I.3.

When a tax reduction policy is implemented, a consumer can enjoy a higher tax reduction from buying the Type-H automobile than that from buying the Type-L automobile, thus having a greater incentive to change the purchase choice to the Type-H automobile. This is more likely to happen when the consumer's valuations for the two automobiles are less negatively or more positively correlated, and as a result, both the U.S. and the Chinese policies are more effective in improving the sales of the Type-H automobile, as shown in online Table Y. Moreover, we find that both policies are the most effective in improving the sales of the Type-L automobile when ρ takes a moderate positive value. This result could be interpreted as follows: When the value of ρ is sufficiently small, a consumer's valuation drawn from buying the Type-H automobile has no significant positive relationship with that from buying the Type-L automobile. Any tax reduction policy does not significantly influence each consumer's purchase choice between the two automobiles. Thus, as the value of ρ increases, each consumer with a high valuation on the Type-L automobile is more likely to buy the Type-L automobile rather than the Type-H automobile. However, when the value of ρ is sufficiently high (close to 1), the consumer's valuations on two automobile types are highly interdependent, which reduces the impact of a tax reduction policy on the sales of the Type-L automobile because the tax reduction is smaller for the purchase of the Type-L automobile than for the purchase of the Type-H automobile.

We also learn that, when ρ becomes larger, both the U.S. and the Chinese policies can more significantly improve the sales of the Type-H automobile, and are thus more effective in improving the profit of the automobile supply chain. Moreover, for any value of ρ, the Chinese policy is more cost effective than the U.S. policy, whereas the U.S. policy can induce a slightly larger (smaller) increase in the supply chain profit when ρ is positive (negative).

One may note that most of our results generated by the analysis of separate market segments are preserved after we introduce the consumer choice. This occurs because incorporating the correlation of consumer's valuations on two automobile types in our analysis will only result in consumer's choices between two automobiles, and the different impact of a tax reduction policy on consumer's valuations for different automobiles will only affect the choice.

6.2 A Procedure for Numerical Investigation with a Strategic Manufacturer

In Section 3.1.2, we assume that the manufacturer's wholesale price is exogenously given. Even though relaxing this assumption will make the analysis of most issues intractable, we can still develop a procedure for numerical investigation. That is, if we consider a strategic manufacturer, then we have to perform numerical experiments using the following five steps.

1. Given a manufacturer's wholesale price w and a tax reduction policy, we find the price negotiated between a consumer and the retailer. Then, using the negotiated price, we derive the demand function and the manufacturer's expected profit function. In this step, we can find analytical results as shown in Section 3.1.
2. We maximize the manufacturer's expected profit to find the optimal wholesale price under the tax reduction policy. For the U.S. policy, we can find from the manufacturer's profit function in (7) that the optimal wholesale price is a unique solution to the equation

$$(w-c)\sum_{i=1}^{n+1}\{\delta_i f(\tilde{\theta}_i)[1+(1-\gamma_i)t]\} = \sum_{i=1}^{n+1}\left[\delta_i\left(1-F(\tilde{\theta}_i)\right)\right],$$

where $\tilde{\theta}_i = [1+(1-\gamma_i)t]w$.
For the Chinese policy, we can find from the manufacturer's profit function in (29) that the optimal wholesale price is a unique solution to the equation

$$(w-c)[1+(1-\alpha)t]f(\hat{\theta}) = 1-F(\hat{\theta}),$$

where $\hat{\theta} = [1+(1-\alpha)t]w$.
Since we *cannot* find a closed-form solution for w from each of the above two equations, all subsequent analysis needs to be conducted numerically.
3. Using the optimal wholesale price, we calculate the expected sales, the manufacturer's and the retailer's expected profits, and the system-wide expected profit under the tax reduction policy.
4. For the case when the policy is not implemented, we repeat the analysis in Steps 1 to 3 but let all $\gamma_i = 0$ for the U.S. policy and $\alpha = 0$ for the Chinese policy. We compute the manufacturer's optimal wholesale price, and find the corresponding manufacturer's and retailer's expected profits, and system-wide expected profit.
5. Comparing the results in Steps 3 and 4, we can calculate the sales increase, system-wide profit increase, and the government's expense under the policy, which are then used to evaluate the cost effectiveness of the policy.

To gain insights into how the effectiveness of a tax reduction policy is affected by the presence of a strategic manufacturer, we provide Examples 8 and 9 in online Appendix E for the U.S. and the Chinese policies, respectively. The two examples show that a strategic manufacturer responds to the implementation of each policy by raising its wholesale price and gains a higher profit. Although a tax reduction policy can improve the sales and the profits of the retailer and the supply chain, the policy's impact on the sales and profits is less than that when the manufacturer fixes its wholesale price. Because a strategic manufacturer only changes the wholesale price under a tax reduction policy, our findings and managerial insights for the case of an exogenous wholesale price still hold.

7 Summary and Concluding Remarks

In this paper, we investigate the impact and cost effectiveness of the tax reduction policies implemented by the U.S. and China to stimulate consumer purchase of new automobiles in their markets. For our analysis, we consider a two-level automobile supply chain involving a manufacturer and a retailer who serves heterogenous consumers in a market. We use the GNB concept to characterize the negotiated retail price, and then compute the expected sales, and the manufacturer's and the retailer's expected profits. We analytically determine the effectiveness of both the U.S. and the Chinese policies in increasing the sales and improving the profitability of the automobile supply chain.

We then propose a two-stage approach to (i) find the (necessary and) sufficient condition for a cost effective tax reduction policy, and (ii) determine the maximum target values (i.e., the sales increase and the system-wide after-tax profit increase) that the policy can achieve, and find the corresponding budget for the feasible targets that do not exceed the maximum target values. We show that the U.S. policy is more likely to be cost effective if the retailer has a strong position to bargain with consumers over the retail prices. The Chinese policy could be cost effective even when the retailer has a weak bargaining position. According to the retailer's bargaining power in the automobile market, the Chinese government can choose a proper tax reduction percentage to assure the cost effectiveness of its policy.

Our sensitivity analysis based on real data generates the following results. Both the U.S. and the Chinese policies are more effective in improving the sales of high-end automobile than the sales of low-end automobile. The U.S. policy is better than the Chinese policy in increasing the sales of high-end automobiles whereas the Chinese policy is better than the U.S. policy in increasing the sales of low-end automobiles. The U.S. policy results in a greater increase in the profit of the automobile supply chain, whereas overall the Chinese policy is more cost effective in terms of the profit-cost ratio. A high sales tax rate can greatly increase the effectiveness of both policies in increasing the sales and the supply chain profit and improve the cost effectiveness of both policies. Moreover, a large mean value or a large dispersion of the consumers' valuations could significantly reduce the impact and cost effectiveness of both policies.

Both the U.S. and the Chinese tax reduction policies are useful for enticing consumers to purchase new automobiles and increasing the profitability of the automobile supply chain. We provide a methodology to determine this impact and the cost effectiveness of these policies. Although our discussions on the policies are motivated by the practices of the U.S. and China (who responded to the recent global economic recession), our analytic approach and the real data-based sensitivity analysis should be useful for any government who intends to choose a proper tax reduction policy to stimulate a market and improve the profitability of an industry.

We note that the Chinese policy was implemented in a weaning away process (i.e., reducing the sales tax rate from 10% to 5% in 2009 and to 7.5% in 2010). By implementing a tax reduction policy in a weaning away process, a government can gradually reduce the dependency of an industry on its policy and mitigate the shock to the industry when the policy is no longer available. Moreover, the government's weaning-away process is also ascribed to the uncertainties of the outcome of the policy. For the sales stimulation, a government usually sets a target (e.g., the sales increase) in order to determine its budget, and decides on an implementation period. But, the outcome may be different from what the government desires due to the following two facts. First, when the government designs a policy, it could be difficult for the government to accurately estimate certain parameter values (e.g. consumers' valuations on the automobile), because those parameters are affected by, e.g., the economic conditions and consumer confidence, and thus vary over time. Second, the uncertainties such as the changing market conditions also exist during the implementation progress. It thus follows that the government may take more than one phase to implement its policy. At the first phase, if the policy turns out to be more effective than expected, then the government may want to continue but provide less incentive to reduce the cost incurred for the policy. Otherwise, if the policy is not as effective as expected, the government may enhance the incentive to meet the overall target. In the situation with a weaning-away process, we can use our model and methodology in this paper to analyze each phase of the process during the implementation period.

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Online Appendices

"Impact of Tax Reduction Policies on Consumer Purchase of New Automobiles: An Analytical Investigation with Real Data-Based Experiments"

K. Hung Chan, Mingming Leng, Liping Liang

Appendix A References for Major Model Assumptions

Model	**Reference**
Consumer utility model (Willingness-to-pay model)	Chen et al. [8], Desai and Purohit [10], Desai and Purohit [11], Huang et al. [20], Iyer [21], Purohit [27]
GNB scheme for the negotiated retail price	Chen et al. [8], Desai and Purohit [11]
Model Parameter	**Reference**
Average valuation of consumers, $E(\theta) = \$29,916.6$	As reported by `Edmunds.com` [13], the average retail price of a car or truck in the U.S. in 2009 was $\$28,492$, we can reasonably assume the average valuation $E(\theta)$ of consumers is the average retail price $\$28,492$ plus the sales tax $5\% \times \$28,492 = \$1,424.6$. That is, $E(\theta) = \$29,916.6$.
Standard deviation of consumers' valuations $\sigma_\theta = \$4,000$	Using the data of individual consumer purchases of three popular automobiles in the U.S., Chen et al. [8] found that the standard deviations of consumers' valuations for three popular automobiles are between $\$2,100$ and $\$6,600$. Thus, $\sigma_\theta = \$4,000$.
Retailer's bargaining power $\kappa = 0.4$	Using the data of individual consumer purchases of three popular automobiles in the U.S., Chen et al. [8] estimated that the bargaining powers of dealers for three automobiles are between 0.384 and 0.453. Since the retailer may have a lower bargaining power during the economic recession because of the pressure of disposing excess inventory, we assume that $\kappa = 0.4$.
Wholesale price $w = \$27,000$	`Edmunds.com` [13] reported that the average retail price of a car or truck in the U.S. in 2009 was $\$28,492$. Albuquerque and Bronnenberg [1] found that retailers' average gross margin is approximately $\$1,600$. Thus, we assume that $w = \$27,000$.

Appendix B Data and Sensitivity Analysis Results

Table A: Panel A indicates the 2009 federal tax rate schedule for qualifying single and joint filers under the U.S. automobile tax reduction policy. Panel B presents the percentage distribution of U.S. single filers' taxable incomes and that of joint filers' taxable incomes in 2009. Let I denote the taxable income.

Panel A

	2009 Taxable Income Bracket ($)	
Estimated Federal Tax Rate	Single Filers	Joint Filers
10%	$0 \leq I < 8,350$	$0 \leq I < 16,700$
15%	$8,350 \leq I < 33,950$	$16,700 \leq I < 67,900$
25%	$33,950 \leq I < 82,250$	$67,900 \leq I < 137,050$
28%	$82,250 \leq I \leq 135,000$	$137,050 \leq I < 208,850$
33%	–	$208,850 \leq I \leq 260,000$

Panel B

	2009 Taxable Income Distribution	
Income Range	Single Filers	Joint Filers
$b_0 = 0 \leq I \leq b_1 = \$10,000$	$Perc_1^S = 27.76\%$	$Perc_1^J = 17.77\%$
$b_1 < I \leq b_2 = \$25,000$	$Perc_2^S = 20.15\%$	$Perc_2^J = 0.69\%$
$b_2 < I \leq b_3 = \$40,000$	$Perc_3^S = 19.50\%$	$Perc_3^J = 5.65\%$
$b_3 < I \leq b_4 = \$75,000$	$Perc_4^S = 23.35\%$	$Perc_4^J = 24.51\%$
$b_4 < I \leq b_5 = \$100,000$	$Perc_5^S = 4.78\%$	$Perc_5^J = 18.36\%$
$b_5 < I \leq b_6 = \$200,000$	$Perc_6^S = 3.59\%$	$Perc_6^J = 25.38\%$
$b_6 < I \leq b_7 = \$500,000$	$Perc_7^S = 0.70\%$	$Perc_7^J = 6.25\%$
$b_7 < I \leq b_8 = \$1,000,000$	$Perc_8^S = 0.11\%$	$Perc_8^J = 0.95\%$
$b_8 < I < b_9 = \infty$	$Perc_9^S = 0.07\%$	$Perc_9^J = 0.44\%$
Percentage of Total Filers	57.57%	42.23%

Table B: The correlation coefficient r between the sales and the profit of the U.S. and the Chinese automobile manufacturers. Note that the empirical data for the two U.S. manufacturers are obtained from the firms' annual reports, and those for the two Chinese manufacturers are retrieved online from http://stockdata.stock.hexun.com.

Year	The U.S. Automobile Manufacturers				The Chinese Automobile Manufacturers			
	GM		DaimlerChrysler		Shanghai Auto Co.		Dongfeng Auto Co.	
	Sales ($\times 10^3$)	Profit ($\times 10^6$ in U.S.$)	Sales ($\times 10^3$)	Profit ($\times 10^6$ in €)	Sales ($\times 10^3$)	Profit ($\times 10^6$ in ¥)	Sales ($\times 10^3$)	Profit ($\times 10^6$ in ¥)
2001	8,037	445	2,756	22,338	320	965	28	-20
2002	8,411	1,988	2,823	25,225	577	1,149	64	-70
2003	8,098	553	2,638	23,460	797	1,649	66	186
2004	8,984	11,226	2,780	24,362	848	1,923	61	1,123
2005	9,179	1,341	2,813	17,127	1,056	1,179	107	1,315
2006	9,093	7,328	2,655	18,901	1,344	5,636	126	1,146
2007	9,370	12,576	2,100	22,520	1,691	14,555	150	1,410
2008	8,362	-1,525	2,100	20,109	1,826	12,170	171	1,523
2009	7,477	-8,079	1,600	12,660	—	—	208	1,724
r	0.85		0.62		0.89		0.85	

Table C: Normal Distribution for Consumers' Valuations: In Panel A, we calculate the expected sales of each type of automobile (D_H and D_L), the retailer's and the manufacturer's expected profits (Π_R and Π_M), the system-wide profit (Π), and the profit-cost ratio (ϕ) for the U.S. and the Chinese policies in the current U.S. setting. In Panel B, we provide the results for the sensitivity analysis on the cutoff level A (for the U.S. policy only). Note that $A > w_H$. In Panel C, we provide the results for the sensitivity analysis on the tax reduction percentage α (for the Chinese policy only).

Panel A

Policy	Sales of Type-H Automobile	Sales of Type-L Automobile	Retailer's Profit ($\times 10^7$)	Manufacturer's Profit ($\times 10^7$)	System-wide Profit ($\times 10^7$)	Profit-Cost Ratio
U.S.	1982.59	4198.03	0.6559	2.0029	2.6588	1.0504
Chinese	1970.92	4206.96	0.6548	2.0012	2.6560	1.0552

Panel B

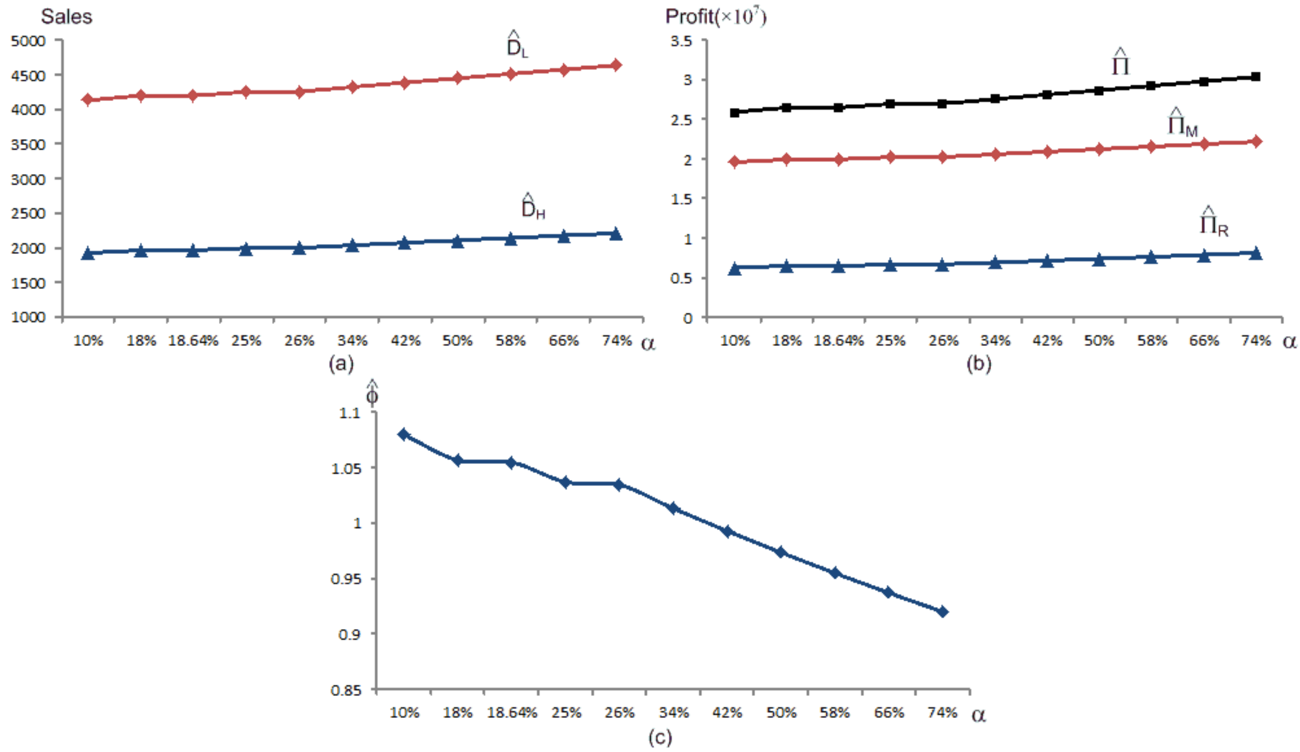
	Impact of the U.S. Policy					
A	D_H	D_L	Π_R ($\times 10^7$)	Π_M ($\times 10^7$)	Π ($\times 10^7$)	ϕ
34500	1982.59	4198.03	0.6539	2.0029	2.6568	1.0478
37000	1982.59	4198.03	0.6555	2.0029	2.6584	1.0480
39500	1982.59	4198.03	0.6559	2.0029	2.6588	1.0504
42000	1982.59	4198.03	0.6559	2.0029	2.6588	1.0504
44500	1982.59	4198.03	0.6559	2.0029	2.6588	1.0504
47000	1982.59	4198.03	0.6559	2.0029	2.6588	1.0504
49500	1982.59	4198.03	0.6559	2.0029	2.6588	1.0504
52000	1982.59	4198.03	0.6559	2.0029	2.6588	1.0504
54500	1982.59	4198.03	0.6559	2.0029	2.6588	1.0504
57000	1982.59	4198.03	0.6559	2.0029	2.6588	1.0504

Panel C

	Impact of the Chinese Policy					
α	$\hat{D}_H$	$\hat{D}_L$	$\hat{\Pi}_R$ ($\times 10^7$)	$\hat{\Pi}_M$ ($\times 10^7$)	$\hat{\Pi}$ ($\times 10^7$)	$\hat{\phi}$
10%	1930.13	4136.68	0.6315	1.9648	2.5963	1.0802
18%	1967.92	4201.77	0.6531	1.9985	2.6516	1.0570
18.64%	1970.92	4206.96	0.6548	2.0012	2.6560	1.0552
25%	2000.57	4258.39	0.6724	2.0277	2.7001	1.0375
26%	2005.20	4266.46	0.6752	2.0319	2.7071	1.0348
34%	2041.93	4330.72	0.6978	2.0649	2.7627	1.0135
42%	2078.08	4394.51	0.7209	2.0976	2.8185	0.9932
50%	2113.62	4457.81	0.7444	2.1300	2.8744	0.9738
58%	2148.51	4520.58	0.7685	2.1619	2.9304	0.9552
66%	2182.73	4582.79	0.7931	2.1934	2.9865	0.9373
74%	2216.25	4644.41	0.8183	2.2244	3.0427	0.9203

For the U.S. policy, the results in Panel B of Table C indicate that, when the cutoff level A is sufficiently large and falls in the range $[34,500, 57,000]$, an increase in A will have little impact on the expected sales of the Type-H and Type-L automobiles, the expected profits of the retailer, the manufacturer, and the supply chain, as well as the cost effectiveness of the policy.

For the Chinese policy, we plot the results in Panel C of Table C to illustrate the impact of the tax reduction percentage α.

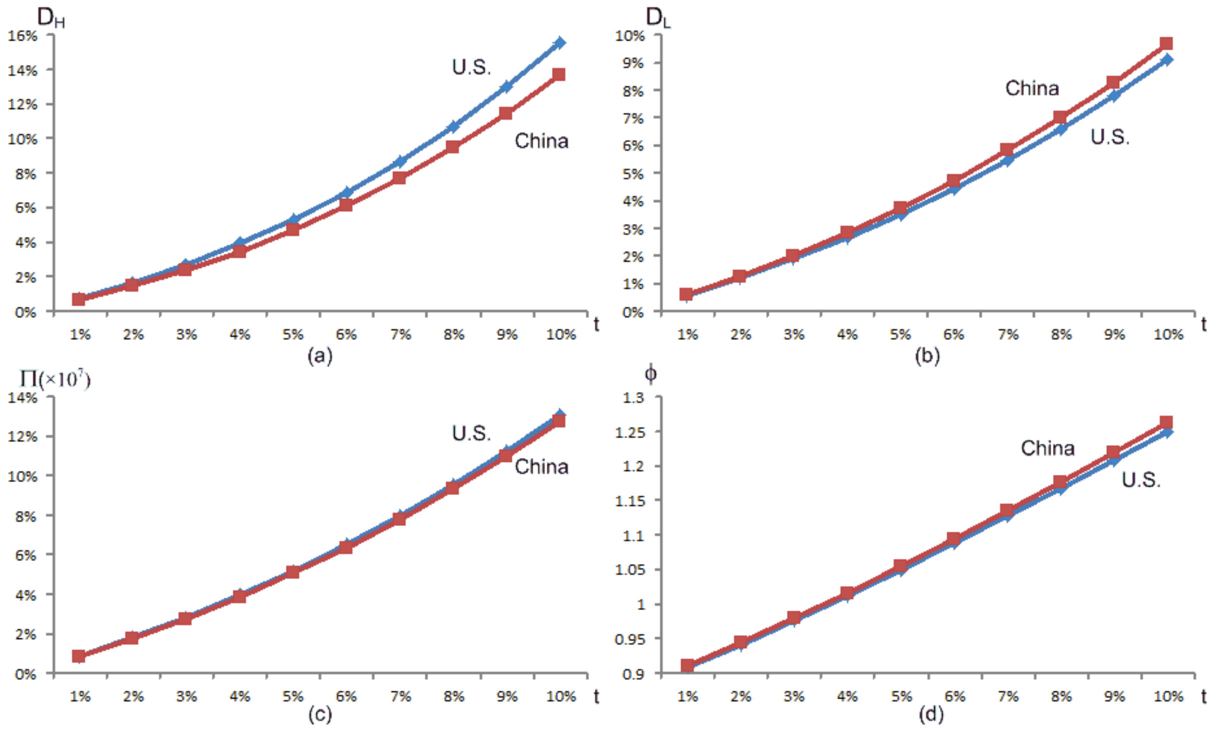


Chinese Policy: The impact of the tax reduction percentage α on: (a) the expected sales $\hat{D}_H$ and $\hat{D}_L$, (b) the retailer's expected profit $\hat{\Pi}_R$, the manufacturer's expected profit $\hat{\Pi}_M$, and the system-wide expected profit $\hat{\Pi}$, and (c) the profit-cost ratio $\hat{\phi}$.

Table D: Normal Distribution for Consumers' Valuations: A sensitivity analysis on the sales tax rate t. Each number inside the brackets is the percentage change in the value (which is given outside the brackets) after a tax reduction policy is implemented.

	Impact of the U.S. Policy					
t	D_H	D_L	Π_R $(\times 10^7)$	Π_M $(\times 10^7)$	Π $(\times 10^7)$	ϕ
1%	2258.07 (0.765%)	4716.85 (0.568%)	0.8497 (1.467%)	2.2618 (0.642%)	3.1115 (0.866%)	0.9088
2%	2193.11 (1.668%)	4590.67 (1.201%)	0.7981 (3.032%)	2.1996 (1.375%)	2.9978 (1.811%)	0.9421
3%	2125.40 (2.721%)	4461.96 (1.903%)	0.7487 (4.702%)	2.1356 (2.207%)	2.8843 (2.843%)	0.9769
4%	2055.14 (3.937%)	4330.98 (2.679%)	0.7013 (6.484%)	2.0700 (3.144%)	2.7712 (3.969%)	1.0131
5%	1982.59 (5.332%)	4198.03 (3.532%)	0.6559 (8.384%)	2.0029 (4.193%)	2.6588 (5.196%)	1.0504
6%	1907.99 (6.920%)	4063.43 (4.467%)	0.6126 (10.409%)	1.9345 (5.361%)	2.5471 (6.532%)	1.0889
7%	1831.65 (8.718%)	3927.48 (5.487%)	0.5713 (12.569%)	1.8651 (6.655%)	2.4364 (7.985%)	1.1283
8%	1753.89 (10.746%)	3790.51 (6.599%)	0.5320 (14.870%)	1.7949 (8.082%)	2.3268 (9.562%)	1.1684
9%	1675.04 (13.022%)	3652.87 (7.807%)	0.4946 (17.321%)	1.7240 (9.651%)	2.2186 (11.272%)	1.2090
10%	1595.44 (15.568%)	3514.89 (9.117%)	0.4591 (19.930%)	1.6528 (11.368%)	2.1119 (13.124%)	1.2498

	Impact of the Chinese Policy					
t	$\hat{D}_H$	$\hat{D}_L$	$\hat{\Pi}_R$ $(\times 10^7)$	$\hat{\Pi}_M$ $(\times 10^7)$	$\hat{\Pi}$ $(\times 10^7)$	$\hat{\phi}$
1%	2256.04 (0.675%)	4718.50 (0.603%)	0.8495 (1.439%)	2.2616 (0.630%)	3.1111 (0.850%)	0.9105
2%	2188.88 (1.472%)	4594.05 (1.276%)	0.7977 (2.973%)	2.1990 (1.349%)	2.9967 (1.776%)	0.9443
3%	2118.82 (2.403%)	4467.14 (2.022%)	0.7480 (4.608%)	2.1347 (2.163%)	2.8827 (2.787%)	0.9798
4%	2046.08 (3.479%)	4338.01 (2.846%)	0.7004 (6.350%)	2.0687 (3.080%)	2.7691 (3.888%)	1.0168
5%	1970.92 (4.712%)	4206.96 (3.752%)	0.6548 (8.206%)	2.0012 (4.105%)	2.6560 (5.087%)	1.0552
6%	1893.63 (6.115%)	4074.25 (4.745%)	0.6113 (10.182%)	1.9324 (5.244%)	2.5437 (6.390%)	1.0948
7%	1814.53 (7.702%)	3940.20 (5.829%)	0.5699 (12.287%)	1.8625 (6.506%)	2.4324 (7.806%)	1.1356
8%	1733.97 (9.488%)	3805.10 (7.010%)	0.5304 (14.528%)	1.7918 (7.896%)	2.3221 (9.342%)	1.1774
9%	1652.30 (11.487%)	3669.28 (8.292%)	0.4929 (16.911%)	1.7204 (9.421%)	2.2133 (11.005%)	1.2200
10%	1569.90 (13.719%)	3533.06 (9.681%)	0.4573 (19.446%)	1.6486 (11.090%)	2.1059 (12.803%)	1.2631

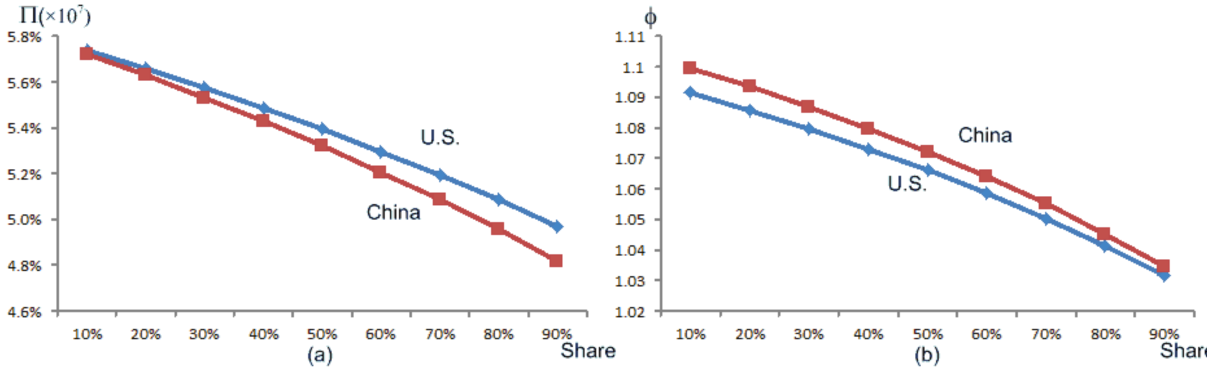


The impact of the sales tax rate t on: (a) the percentage change in D_H, (b) the percentage change in D_L, (c) the percentage change in Π, and (d) the profit-cost ratio ϕ.

Table E: Normal Distribution for Consumers' Valuations: A sensitivity analysis on the market share of the Type-L automobile. Each number inside the brackets is the percentage change in the value (which is given outside the brackets) after a tax reduction policy is implemented.

	Impact of the U.S. Policy					
Market Share of Type-L Auto	D_H	D_L	Π_R $(\times 10^7)$	Π_M $(\times 10^7)$	Π $(\times 10^7)$	ϕ
10%	5913.92 (4.733%)	599.72 (3.532%)	0.7198 (9.568%)	2.3976 (4.642%)	3.1174 (5.739%)	1.0915
20%	5258.70 (4.770%)	1199.44 (3.532%)	0.7091 (9.384%)	2.3318 (4.577%)	3.0410 (5.660%)	1.0858
30%	4603.48 (4.818%)	1799.16 (3.532%)	0.6985 (9.195%)	2.2661 (4.509%)	2.9645 (5.576%)	1.0797
40%	3948.25 (4.882%)	2398.88 (3.532%)	0.6878 (9.000%)	2.2003 (4.437%)	2.8881 (5.489%)	1.0732
50%	3293.03 (4.972%)	2998.60 (3.532%)	0.6772 (8.800%)	2.1345 (4.360%)	2.8117 (5.396%)	1.0662
60%	2637.81 (5.107%)	3598.31 (3.532%)	0.6665 (8.595%)	2.0687 (4.279%)	2.7352 (5.299%)	1.0586
70%	1982.59 (5.332%)	4198.03 (3.532%)	0.6559 (8.383%)	2.0029 (4.193%)	2.6589 (5.196%)	1.0504
80%	1327.36 (5.781%)	4797.75 (3.532%)	0.6453 (8.166%)	1.9371 (4.101%)	2.5824 (5.088%)	1.0416
90%	672.14 (7.219%)	5397.47 (3.532%)	0.6346 (7.942%)	1.8713 (4.002%)	2.5059 (4.973%)	1.0320

	Impact of the Chinese Policy					
Market Share of Type-L Auto	$\hat{D}_H$	$\hat{D}_L$	$\hat{\Pi}_R$ $(\times 10^7)$	$\hat{\Pi}_M$ $(\times 10^7)$	$\hat{\Pi}$ $(\times 10^7)$	$\hat{\phi}$
10%	5912.77 (4.712%)	600.99 (3.752%)	0.7194 (9.513%)	2.3976 (4.639%)	3.1170 (5.725%)	1.0998
20%	5255.80 (4.712%)	1201.99 (3.752%)	0.7086 (9.310%)	2.3315 (4.563%)	3.0401 (5.632%)	1.0936
30%	4598.82 (4.712%)	1802.98 (3.752%)	0.6979 (9.100%)	2.2655 (4.481%)	2.9633 (5.534%)	1.0871
40%	3941.85 (4.712%)	2403.97 (3.752%)	0.6871 (8.886%)	2.1994 (4.395%)	2.8865 (5.430%)	1.0800
50%	3284.87 (4.712%)	3004.97 (3.752%)	0.6763 (8.666%)	2.1333 (4.304%)	2.8097 (5.322%)	1.0724
60%	2627.90 (4.712%)	3605.96 (3.752%)	0.6656 (8.439%)	2.0673 (4.208%)	2.7328 (5.207%)	1.0641
70%	1970.92 (4.712%)	4206.96 (3.752%)	0.6548 (8.206%)	2.0012 (4.105%)	2.6560 (5.087%)	1.0552
80%	1313.95 (4.712%)	4807.95 (3.752%)	0.6441 (7.966%)	1.9351 (3.995%)	2.5792 (4.959%)	1.0454
90%	656.97 (4.712%)	5408.94 (3.752%)	0.6333 (7.719%)	1.8690 (3.878%)	2.5024 (4.818%)	1.0347

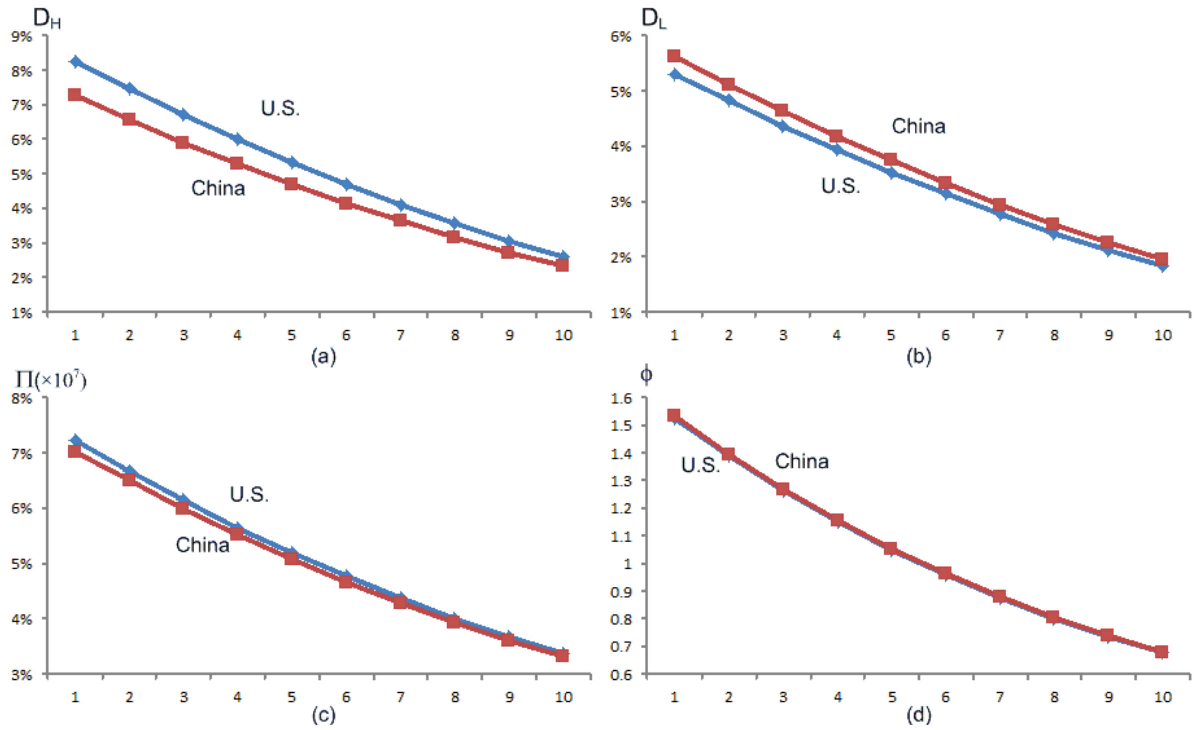


The impact of the market share of the Type-L automobile on: (a) the percentage change in Π, and (b) the profit-cost ratio ϕ.

Note that the effectiveness of the U.S. policy in improving the sales D_H depends on the market share of the Type-L automobile, which, however, does not affect its effectiveness in improving D_L. This occurs because we have fixed the income tax rate distribution of consumers in the Type-L automobile market segment so that the income tax rate distribution of consumers in the Type-H automobile market segment varies with the market share. Therefore, if the income tax rate distribution of consumers of an automobile type is independent of the market share, then the effectiveness of the U.S. policy in improving the sales of this automobile does not vary with the market share. Moreover, the effectiveness of the Chinese policy in improving the sales of each automobile type is independent of the market share of the Type-L automobile.

Table F: Normal Distribution for Consumers' Valuations: A sensitivity analysis on the means $E(\theta_H)$ and $E(\theta_L)$ of the consumers' normally-distributed valuations θ_H on the Type-H automobile and θ_L on the Type-L automobile. Each number inside the brackets is the percentage change in the value (which is given outside the brackets) after a tax reduction policy is implemented.

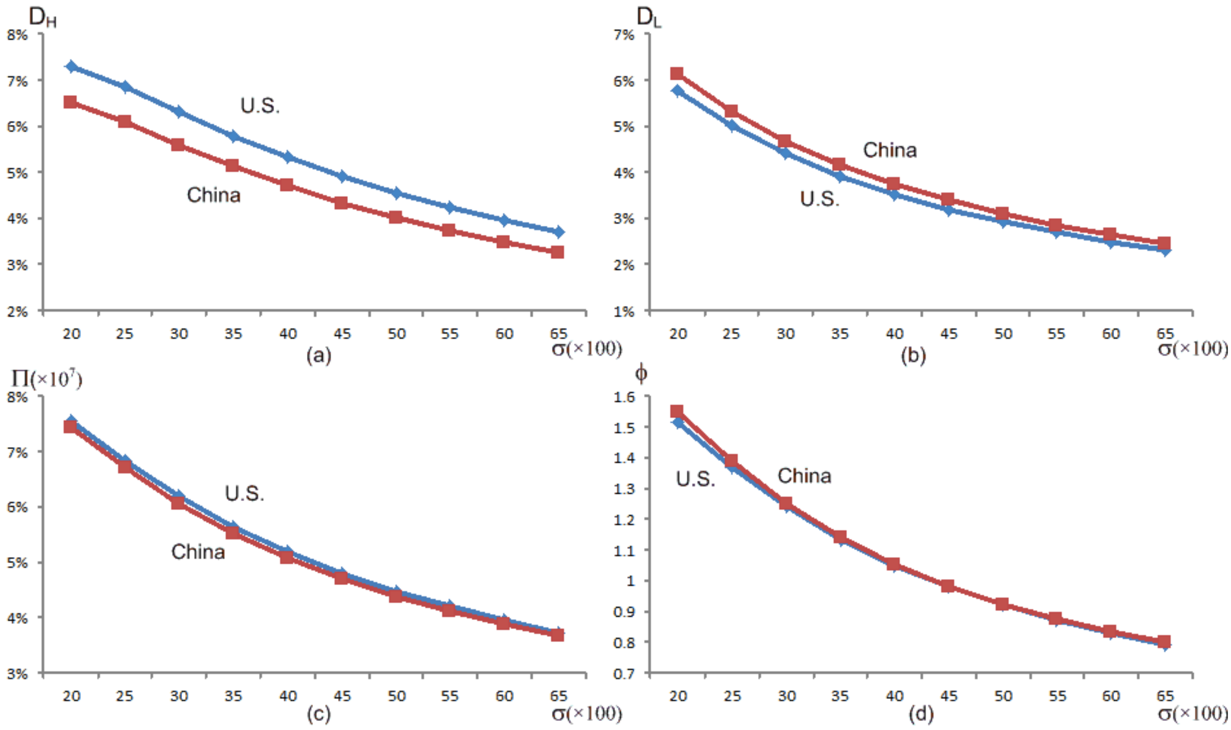
			Impact of the U.S. Policy					
Case	$E(\theta_H)$	$E(\theta_L)$	D_H	D_L	$\Pi_R(\times 10^7)$	$\Pi_M(\times 10^7)$	$\Pi(\times 10^7)$	ϕ
1	35,000	24,000	1398.39 (8.266%)	2816.41 (5.301%)	0.3560 (10.452%)	1.3693 (6.417%)	1.7253 (7.226%)	1.5262
2	35,500	24,500	1547.80 (7.473%)	3159.29 (4.828%)	0.4203 (9.898%)	1.5282 (5.817%)	1.9485 (6.672%)	1.3891
3	36,000	25,000	1696.47 (6.718%)	3507.45 (4.375%)	0.4917 (9.369%)	1.6884 (5.246%)	2.1801 (6.148%)	1.2648
4	36,500	25,500	1842.12 (6.004%)	3855.49 (3.943%)	0.5703 (8.864%)	1.8474 (4.704%)	2.4177 (5.656%)	1.1522
5	37,000	26,000	1982.59 (5.332%)	4198.03 (3.532%)	0.6559 (8.384%)	2.0029 (4.193%)	2.6588 (5.196%)	1.0504
6	37,500	26,500	2115.97 (4.702%)	4529.94 (3.144%)	0.7484 (7.928%)	2.1525 (3.713%)	2.9008 (4.769%)	0.9588
7	38,000	27,000	2240.66 (4.117%)	4846.57 (2.779%)	0.8474 (7.497%)	2.2942 (3.265%)	3.1416 (4.373%)	0.8764
8	38,500	27,500	2355.43 (3.576%)	5143.95 (2.439%)	0.9526 (7.090%)	2.4265 (2.850%)	3.3790 (4.011%)	0.8028
9	39,000	28,000	2459.43 (3.080%)	5418.93 (2.123%)	1.0634 (6.707%)	2.5480 (2.468%)	3.6114 (3.680%)	0.7373
10	39,500	28,500	2552.20 (2.629%)	5669.25 (1.833%)	1.1795 (6.348%)	2.6579 (2.118%)	3.8374 (3.382%)	0.6794
			Impact of the Chinese Policy					
Case	$E(\theta_H)$	$E(\theta_L)$	$\hat{D}_H$	$\hat{D}_L$	$\hat{\Pi}_R(\times 10^7)$	$\hat{\Pi}_M(\times 10^7)$	$\hat{\Pi}(\times 10^7)$	$\hat{\phi}$
1	35,000	24,000	1385.54 (7.272%)	2825.09 (5.626%)	0.3551 (10.191%)	1.3671 (6.245%)	1.7222 (7.036%)	1.5345
2	35,500	24,500	1534.97 (6.581%)	3168.24 (5.125%)	0.4194 (9.661%)	1.5261 (5.670%)	1.9454 (6.505%)	1.3962
3	36,000	25,000	1683.85 (5.924%)	3516.53 (4.645%)	0.4907 (9.153%)	1.6864 (5.120%)	2.1771 (6.003%)	1.2709
4	36,500	25,500	1829.89 (5.300%)	3864.56 (4.187%)	0.5692 (8.668%)	1.8456 (4.598%)	2.4148 (5.530%)	1.1575
5	37,000	26,000	1970.92 (4.712%)	4206.96 (3.752%)	0.6548 (8.206%)	2.0012 (4.105%)	2.6560 (5.087%)	1.0552
6	37,500	26,500	2105.02 (4.161%)	4538.58 (3.341%)	0.7473 (7.766%)	2.1510 (3.640%)	2.8982 (4.673%)	0.9630
7	38,000	27,000	2230.54 (3.647%)	4854.81 (2.954%)	0.8462 (7.350%)	2.2929 (3.206%)	3.1391 (4.291%)	0.8802
8	38,500	27,500	2346.22 (3.171%)	5151.68 (2.593%)	0.9514 (6.956%)	2.4253 (2.802%)	3.3767 (3.939%)	0.8062
9	39,000	28,000	2451.18 (2.734%)	5426.06 (2.258%)	1.0622 (6.585%)	2.5470 (2.429%)	3.6092 (3.618%)	0.7404
10	39,500	28,500	2544.94 (2.336%)	5675.74 (1.950%)	1.1783 (6.236%)	2.6571 (2.088%)	3.8353 (3.328%)	0.6822



The impact of the mean of consumers' valuations on: (a) the percentage change in D_H, (b) the percentage change in D_L, (c) the percentage change in Π, and (d) the profit-cost ratio ϕ.

Table G: Normal Distribution for Consumers' Valuations: A sensitivity analysis on the standard deviation σ_θ of the consumers' normally-distributed valuations θ_H on the Type-H automobile and θ_L on the Type-L automobile. Each number inside the brackets is the percentage change in the value (which is given outside the brackets) after a tax reduction policy is implemented.

	Impact of the U.S. Policy					
σ_θ	D_H	D_L	$\Pi_R(\times 10^7)$	$\Pi_M(\times 10^7)$	$\Pi(\times 10^7)$	ϕ
2000	2389.12 (7.306%)	4852.42 (5.764%)	0.4467 (14.459%)	2.3516 (6.346%)	2.7983 (7.563%)	1.5187
2500	2239.33 (6.869%)	4598.44 (5.020%)	0.4961 (12.210%)	2.2193 (5.712%)	2.7154 (6.843%)	1.3706
3000	2129.54 (6.326%)	4423.00 (4.416%)	0.5480 (10.576%)	2.1255 (5.125%)	2.6735 (6.198%)	1.2415
3500	2046.78 (5.803%)	4295.14 (3.928%)	0.6015 (9.343%)	2.0561 (4.620%)	2.6576 (5.653%)	1.1359
4000	1982.59 (5.332%)	4198.03 (3.532%)	0.6559 (8.384%)	2.0029 (4.193%)	2.6588 (5.196%)	1.0504
4500	1931.53 (4.918%)	4121.88 (3.205%)	0.7110 (7.617%)	1.9609 (3.831%)	2.6719 (4.812%)	0.9808
5000	1890.05 (4.555%)	4060.59 (2.932%)	0.7665 (6.990%)	1.9269 (3.523%)	2.6935 (4.487%)	0.9234
5500	1855.72 (4.238%)	4010.24 (2.701%)	0.8223 (6.463%)	1.8990 (3.259%)	2.7213 (4.207%)	0.8750
6000	1826.87 (3.959%)	3968.15 (2.503%)	0.8780 (5.998%)	1.8755 (3.030%)	2.7535 (3.958%)	0.8325
6500	1802.30 (3.713%)	3932.44 (2.332%)	0.9329 (5.555%)	1.8556 (2.830%)	2.7885 (3.726%)	0.7917
	Impact of the Chinese Policy					
σ_θ	$\hat{D}_H$	$\hat{D}_L$	$\hat{\Pi}_R(\times 10^7)$	$\hat{\Pi}_M(\times 10^7)$	$\hat{\Pi}(\times 10^7)$	$\hat{\phi}$
2000	2371.74 (6.525%)	4869.33 (6.133%)	0.4452 (14.096%)	2.3502 (6.281%)	2.7955 (7.453%)	1.5516
2500	2223.31 (6.104%)	4612.32 (5.338%)	0.4948 (11.916%)	2.2174 (5.625%)	2.7122 (6.719%)	1.3900
3000	2115.13 (5.606%)	4434.73 (4.693%)	0.5468 (10.332%)	2.1236 (5.032%)	2.6704 (6.076%)	1.2533
3500	2033.84 (5.134%)	4305.28 (4.174%)	0.6003 (9.137%)	2.0543 (4.528%)	2.6546 (5.536%)	1.1434
4000	1970.92 (4.712%)	4206.96 (3.752%)	0.6548 (8.206%)	2.0012 (4.105%)	2.6560 (5.087%)	1.0552
4500	1920.95 (4.343%)	4129.84 (3.405%)	0.7100 (7.461%)	1.9593 (3.748%)	2.6693 (4.710%)	0.9838
5000	1880.38 (4.021%)	4067.78 (3.114%)	0.7656 (6.853%)	1.9255 (3.444%)	2.6910 (4.392%)	0.9253
5500	1846.84 (3.739%)	4016.78 (2.869%)	0.8214 (6.347%)	1.8976 (3.185%)	2.7190 (4.120%)	0.8768
6000	1818.66 (3.492%)	3974.15 (2.658%)	0.8774 (5.921%)	1.8742 (2.960%)	2.7516 (3.886%)	0.8360
6500	1794.67 (3.274%)	3937.99 (2.476%)	0.9330 (5.559%)	1.8544 (2.764%)	2.7874 (3.683%)	0.8015



The impact of the standard deviation of consumers' valuations on: (a) the percentage change in D_H, (b) the percentage change in D_L, (c) the percentage change in Π, and (d) the profit-cost ratio ϕ.

Table H: Gamma Distribution for Consumers' Valuations: In Panel A, we calculate the expected sales of each type of automobile (D_H and D_L), the retailer's and the manufacturer's expected profits (Π_R and Π_M), the system-wide profit (Π), and the profit-cost ratio (ϕ) for the U.S. and the Chinese policies in the current U.S. setting. In Panel B, we provide the results for the sensitivity analysis on the cutoff level A (for the U.S. policy only). Note that $A > w_H$. In Panel C, we provide the results for the sensitivity analysis on the tax reduction percentage α (for the Chinese policy only).

Panel A

Policy	Sales of Type-H Automobile	Sales of Type-L Automobile	Retailer's Profit ($\times 10^7$)	Manufacturer's Profit ($\times 10^7$)	System-wide Profit ($\times 10^7$)	Profit-Cost Ratio
U.S.	1875.62	4284.32	0.6650	1.9887	2.6537	1.0843
Chinese	1864.37	4294.04	0.6641	1.9874	2.6515	1.0911

Panel B

	Impact of the U.S. Policy					
A	D_H	D_L	Π_R ($\times 10^7$)	Π_M ($\times 10^7$)	Π ($\times 10^7$)	ϕ
34500	1875.62	4284.32	0.6630	1.9887	2.6517	1.0806
37000	1875.62	4284.32	0.6645	1.9887	2.6532	1.0826
39500	1875.62	4284.32	0.6650	1.9887	2.6537	1.0840
42000	1875.62	4284.32	0.6650	1.9887	2.6537	1.0843
44500	1875.62	4284.32	0.6650	1.9887	2.6537	1.0843
47000	1875.62	4284.32	0.6650	1.9887	2.6537	1.0843
49500	1875.62	4284.32	0.6650	1.9887	2.6537	1.0843
52000	1875.62	4284.32	0.6650	1.9887	2.6537	1.0843
54500	1875.62	4284.32	0.6650	1.9887	2.6537	1.0843
57000	1875.62	4284.32	0.6650	1.9887	2.6537	1.0843

Panel C

	Impact of the Chinese Policy					
α	$\hat{D}_H$	$\hat{D}_L$	$\hat{\Pi}_R$ ($\times 10^7$)	$\hat{\Pi}_M$ ($\times 10^7$)	$\hat{\Pi}$ ($\times 10^7$)	$\hat{\phi}$
10%	1825.45	4217.28	0.6410	1.9497	2.5907	1.1166
18%	1861.50	4288.37	0.6624	1.9846	2.6470	1.0930
18.64%	1864.37	4294.04	0.6641	1.9874	2.6515	1.0911
25%	1892.83	4350.28	0.6816	2.0149	2.6965	1.0731
26%	1897.29	4359.10	0.6844	2.0192	2.7036	1.0704
34%	1932.79	4429.43	0.7069	2.0536	2.7605	1.0487
42%	1967.97	4499.31	0.7298	2.0878	2.8176	1.0278
50%	2002.80	4568.69	0.7532	2.1217	2.8749	1.0079
58%	2037.24	4637.53	0.7773	2.1552	2.9325	0.9887
66%	2071.26	4705.77	0.8018	2.1885	2.9903	0.9703
74%	2104.83	4773.38	0.8269	2.2213	3.0482	0.9527

Table I: Gamma Distribution for Consumers' Valuations: A sensitivity analysis on the sales tax rate t. Each number inside the brackets is the percentage change in the value (which is given outside the brackets) after a tax reduction policy is implemented.

	Impact of the U.S. Policy					
t	D_H	D_L	Π_R ($\times 10^7$)	Π_M ($\times 10^7$)	Π ($\times 10^7$)	ϕ
1%	2147.08 (0.816%)	4852.87 (0.606%)	0.8582 (1.443%)	2.2610 (0.680%)	3.1192 (0.889%)	0.9506
2%	2081.64 (1.753%)	4714.42 (1.284%)	0.8067 (2.978%)	2.1949 (1.450%)	3.0017 (1.856%)	0.9834
3%	2014.42 (2.818%)	4573.23 (2.038%)	0.7574 (4.607%)	2.1274 (2.314%)	2.8847 (2.906%)	1.0165
4%	1945.67 (4.020%)	4429.73 (2.871%)	0.7101 (6.334%)	2.0585 (3.276%)	2.7687 (4.043%)	1.0501
5%	1875.62 (5.367%)	4284.32 (3.787%)	0.6651 (8.166%)	1.9887 (4.340%)	2.6537 (5.273%)	1.0843
6%	1804.54 (6.866%)	4137.43 (4.788%)	0.6220 (10.108%)	1.9179 (5.512%)	2.5399 (6.601%)	1.1183
7%	1732.71 (8.526%)	3989.50 (5.878%)	0.5810 (12.165%)	1.8466 (6.795%)	2.4276 (8.033%)	1.1526
8%	1660.39 (10.358%)	3840.96 (7.061%)	0.5421 (14.344%)	1.7749 (8.195%)	2.3170 (9.574%)	1.1867
9%	1587.85 (12.371%)	3692.26 (8.341%)	0.5050 (16.650%)	1.7031 (9.717%)	2.2082 (11.229%)	1.2205
10%	1515.38 (14.575%)	3543.82 (9.722%)	0.4699 (19.091%)	1.6314 (11.365%)	2.1014 (13.004%)	1.2539

	Impact of the Chinese Policy					
t	$\hat{D}_H$	$\hat{D}_L$	$\hat{\Pi}_R$ ($\times 10^7$)	$\hat{\Pi}_M$ ($\times 10^7$)	$\hat{\Pi}$ ($\times 10^7$)	$\hat{\phi}$
1%	2145.01 (0.719%)	4854.68 (0.643%)	0.8580 (1.423%)	2.2608 (0.670%)	3.1188 (0.876%)	0.9533
2%	2077.40 (1.546%)	4718.13 (1.364%)	0.8063 (2.932%)	2.1945 (1.428%)	3.0008 (1.828%)	0.9866
3%	2007.92 (2.486%)	4578.91 (2.165%)	0.7568 (4.531%)	2.1266 (2.278%)	2.8835 (2.860%)	1.0208
4%	1936.82 (3.547%)	4437.42 (3.050%)	0.7094 (6.225%)	2.0575 (3.225%)	2.7670 (3.978%)	1.0557
5%	1864.37 (4.735%)	4294.04 (4.022%)	0.6641 (8.020%)	1.9874 (4.272%)	2.6515 (5.186%)	1.0911
6%	1790.86 (6.055%)	4149.18 (5.086%)	0.6209 (9.919%)	1.9163 (5.423%)	2.5373 (6.489%)	1.1270
7%	1716.58 (7.516%)	4003.25 (6.243%)	0.5798 (11.929%)	1.8447 (6.684%)	2.4245 (7.893%)	1.1632
8%	1641.82 (9.124%)	3856.65 (7.499%)	0.5407 (14.054%)	1.7727 (8.058%)	2.3134(9.402%)	1.1995
9%	1566.88 (10.886%)	3709.81 (8.856%)	0.5035 (16.300%)	1.7005 (9.549%)	2.2040 (11.022%)	1.2357
10%	1492.06 (12.811%)	3563.13 (10.320%)	0.4683 (18.674%)	1.6285 (11.163%)	2.0968 (12.757%)	1.2718

Table J: Gamma Distribution for Consumers' Valuations: A sensitivity analysis on the market share of the Type-L automobile. Each number inside the brackets is the percentage change in the value (which is given outside the brackets) after a tax reduction policy is implemented.

	Impact of the U.S. Policy					
Market Share of Type-L Auto	D_H	D_L	Π_R ($\times 10^7$)	Π_M ($\times 10^7$)	Π ($\times 10^7$)	ϕ
10%	5594.46 (4.760%)	612.05 (3.787%)	0.7419 (8.829%)	2.2815 (4.681%)	3.0234 (5.669%)	1.0986
20%	4974.65 (4.798%)	1224.09 (3.787%)	0.7291 (8.728%)	2.2327 (4.630%)	2.9618 (5.610%)	1.0965
30%	4354.84 (4.847%)	1836.14 (3.787%)	0.7163 (8.623%)	2.1839 (4.577%)	2.9002 (5.548%)	1.0943
40%	3735.04 (4.912%)	2448.18 (3.787%)	0.7035 (8.515%)	2.1351 (4.522%)	2.8386 (5.484%)	1.0920
50%	3115.23 (5.003%)	3060.23 (3.787%)	0.6907 (8.403%)	2.0863 (4.464%)	2.7769 (5.417%)	1.0896
60%	2495.43 (5.139%)	3672.27 (3.787%)	0. 6779 (8.287%)	2.0375 (4.404%)	2.7153 (5.347%)	1.0869
70%	1875.62 (5.367%)	4284.32 (3.787%)	0. 6650 (8.166%)	1.9887 (4.340%)	2.6537 (5.273%)	1.0841
80%	1255.81 (5.822%)	4896.36 (3.787%)	0.6522 (8.041%)	1.9398 (4.274%)	2.5921 (5.197%)	1.0811
90%	636.01 (7.187%)	5508.41 (3.787%)	0.6394 (7.912%)	1.8910 (4.204%)	2.5304 (5.116%)	1.0779

	Impact of the Chinese Policy					
Market Share of Type-L Auto	$\hat{D}_H$	$\hat{D}_L$	$\hat{\Pi}_R$ ($\times 10^7$)	$\hat{\Pi}_M$ ($\times 10^7$)	$\hat{\Pi}$ ($\times 10^7$)	$\hat{\phi}$
10%	5593.11 (4.735%)	613.43 (4.022%)	0.7416 (8.790%)	2.2814 (4.677%)	3.0231 (5.657%)	1.1069
20%	4971.65 (4.735%)	1226.87 (4.022%)	0.7287 (8.672%)	2.2324 (4.617%)	2.9612 (5.586%)	1.1047
30%	4350.20 (4.735%)	1840.30 (4.022%)	0.7158 (8.551%)	2.1834 (4.554%)	2.8992 (5.513%)	1.1023
40%	3728.74 (4.735%)	2453.74 (4.022%)	0.7029 (8.425%)	2.1344 (4.488%)	2.8373 (5.436%)	1.0998
50%	3107.28 (4.735%)	3067.17 (4.022%)	0.6900 (8.294%)	2.0854 (4.419%)	2.7754 (5.356%)	1.0971
60%	2485.83 (4.735%)	3680.61 (4.022%)	0.6771 (8.159%)	2.0364 (4.347%)	2.7134 (5.273%)	1.0942
70%	1864.37 (4.735%)	4294.04 (4.022%)	0.6641 (8.020%)	1.9874 (4.272%)	2.6515 (5.186%)	1.0911
80%	1242.91 (4.735%)	4907.48 (4.022%)	0.6512 (7.874%)	1.9383 (4.193%)	2.5896 (5.095%)	1.0878
90%	621.46 (4.735%)	5520.91 (4.022%)	0.6383 (7.724%)	1.8893 (4.110%)	2.5276 (4.999%)	1.0842

Table K: Gamma Distribution for Consumers' Valuations: A sensitivity analysis on the shape k_1 of the consumers' valuations θ_H on the Type-H automobile and θ_L on the Type-L automobile. Each number inside the brackets is the percentage change in the value (which is given outside the brackets) after a tax reduction policy is implemented.

		Impact of the U.S. Policy					
k_H	k_L	D_H	D_L	$\Pi_R(\times 10^7)$	$\Pi_M(\times 10^7)$	$\Pi(\times 10^7)$	ϕ
65	45	1295.69 (8.027%)	2671.73 (5.997%)	0.3354 (10.404%)	1.2874 (6.754%)	1.6228 (7.489%)	1.6092
66	46	1442.65 (7.311%)	3074.23 (5.399%)	0.4051 (9.799%)	1.4633 (6.098%)	1.8684 (6.879%)	1.4571
67	47	1589.74 (6.629%)	3482.92 (4.830%)	0.4833 (9.221%)	1.6410 (5.476%)	2.1244 (6.306%)	1.3194
68	48	1734.75 (5.980%)	3889.08 (4.292%)	0.5701 (8.678%)	1.8173 (4.890%)	2.3873 (5.770%)	1.1955
69	49	1875.62 (5.367%)	4284.32 (3.787%)	0.6650 (8.166%)	1.9887 (4.340%)	2.6537 (5.273%)	1.0841
70	50	2010.48 (4.789%)	4661.08 (3.315%)	0.7679 (7.685%)	2.1523 (3.827%)	2.9202 (4.814%)	0.9843
71	51	2137.75 (4.248%)	5013.05 (2.878%)	0.8781 (7.234%)	2.3056 (3.350%)	3.1837 (4.393%)	0.8953
72	52	2256.16 (3.744%)	5335.42 (2.476%)	0.9952 (6.811%)	2.4467 (2.911%)	3.4418 (4.009%)	0.8162
73	53	2364.80 (3.278%)	5625.00 (2.109%)	1.1183 (6.416%)	2.5743 (2.509%)	3.6926 (3.662%)	0.7463
74	54	2463.11 (2.849%)	5880.22 (1.779%)	1.2467 (6.047%)	2.6877 (2.144%)	3.9345 (3.350%)	0.6849

		Impact of the Chinese Policy					
k_H	k_L	$\hat{D}_H$	$\hat{D}_L$	$\hat{\Pi}_R(\times 10^7)$	$\hat{\Pi}_M(\times 10^7)$	$\hat{\Pi}(\times 10^7)$	$\hat{\phi}$
65	45	1283.99 (7.051%)	2680.93 (6.362%)	0.3346 (10.148%)	1.2858 (6.619%)	1.6204 (7.329%)	1.6289
66	46	1430.80 (6.430%)	3083.86 (5.729%)	0.4043 (9.574%)	1.4617 (5.985%)	1.8660 (6.743%)	1.4724
67	47	1577.91 (5.836%)	3492.79 (5.127%)	0.4825 (9.028%)	1.6396 (5.382%)	2.1220 (6.189%)	1.3314
68	48	1723.13 (5.270%)	3898.98 (4.558%)	0.5692 (8.510%)	1.8159 (4.810%)	2.3850 (5.700%)	1.2047
69	49	1864.37 (4.735%)	4294.04 (4.022%)	0.6641 (8.020%)	1.9874 (4.272%)	2.6515 (5.186%)	1.0911
70	50	1999.75 (4.229%)	4670.43 (3.522%)	0.7670 (7.557%)	2.1510 (3.768%)	2.9180 (4.738%)	0.9897
71	51	2127.65 (3.755%)	5021.87 (3.059%)	0.8772 (7.122%)	2.3044 (3.299%)	3.1817 (4.325%)	0.8993
72	52	2246.80 (3.313%)	5343.57 (2.632%)	0.9943 (6.714%)	2.4456 (2.866%)	3.4399 (3.949%)	0.8193
73	53	2356.24 (2.904%)	5632.38 (2.243%)	1.1174 (6.334%)	2.5733 (2.469%)	3.6907 (3.609%)	0.7489
74	54	2455.39 (2.527%)	5886.78 (1.893%)	1.2459 (5.979%)	2.6868 (2.109%)	3.9328 (3.304%)	0.6872

Table L: Gamma Distribution for Consumers' Valuations: A sensitivity analysis on the scale k_2 of the consumers' valuations θ_H on the Type-H automobile and θ_L on the Type-L automobile. Each number inside the brackets is the percentage change in the value (which is given outside the brackets) after a tax reduction policy is implemented.

	Impact of the U.S. Policy					
k_2	D_H	D_L	$\Pi_R(\times 10^7)$	$\Pi_M(\times 10^7)$	$\Pi(\times 10^7)$	ϕ
503	1273.24 (8.414%)	3098.40 (5.541%)	0.3704 (10.172%)	1.4070 (6.499%)	1.7773 (7.244%)	1.5684
511	1429.20 (7.558%)	3405.84 (5.055%)	0.4354 (9.626%)	1.5577 (5.903%)	1.9931 (6.695%)	1.4244
519	1583.14 (6.767%)	3708.07 (4.601%)	0.5063 (9.108%)	1.7061 (5.345%)	2.2124 (6.183%)	1.2969
527	1732.63 (6.038%)	4001.83 (4.179%)	0.5830 (8.623%)	1.8503 (4.824%)	2.4333 (5.710%)	1.1841
535	1875.62 (5.367%)	4284.32 (3.787%)	0.6650 (8.166%)	1.9887 (4.340%)	2.6537 (5.273%)	1.0841
543	2010.44 (4.752%)	4553.27 (3.423%)	0.7522 (7.737%)	2.1199 (3.892%)	2.8721 (4.872%)	0.9955
551	2135.87 (4.191%)	4806.94 (3.086%)	0.8440 (7.335%)	2.2430 (3.478%)	3.0870 (4.505%)	0.9168
559	2251.08 (3.680%)	5044.08 (2.776%)	0.9401 (6.956%)	2.3574 (3.098%)	3.2975 (4.169%)	0.8469
567	2355.66 (3.218%)	5263.94 (2.490%)	1.0401 (6.599%)	2.4626 (2.750%)	3.5026 (3.864%)	0.7849
575	2449.53 (2.802%)	5466.19 (2.228%)	1.1434 (6.261%)	2.5584 (2.433%)	3.7019 (3.586%)	0.7296
	Impact of the Chinese Policy					
k_2	$\hat{D}_H$	$\hat{D}_L$	$\hat{\Pi}_R(\times 10^7)$	$\hat{\Pi}_M(\times 10^7)$	$\hat{\Pi}(\times 10^7)$	$\hat{\phi}$
503	1261.21 (7.390%)	3108.36 (5.880%)	0.3698 (10.001%)	1.4055 (6.383%)	1.7752 (7.117%)	1.5775
511	1417.08 (6.646%)	3415.92 (5.365%)	0.4347 (9.457%)	1.5562 (5.799%)	1.9909 (6.577%)	1.4327
519	1571.13 (5.957%)	3718.14 (4.885%)	0.5056 (8.947%)	1.7046 (5.253%)	2.2102 (6.076%)	1.3047
527	1720.92 (5.321%)	4011.77 (4.438%)	0.5822 (8.468%)	1.8489 (4.745%)	2.4310 (5.613%)	1.1914
535	1864.37 (4.735%)	4294.04 (4.022%)	0.6641 (8.020%)	1.9874 (4.272%)	2.6515 (5.186%)	1.0911
543	1999.78 (4.197%)	4562.68 (3.637%)	0.7512 (7.600%)	2.1187 (3.834%)	2.8699 (4.794%)	1.0023
551	2125.90 (3.705%)	4815.96 (3.280%)	0.8430 (7.208%)	2.2420 (3.430%)	3.0850 (4.436%)	0.9236
559	2241.88 (3.257%)	5052.65 (2.950%)	0.9391 (6.843%)	2.3565 (3.059%)	3.2956 (4.110%)	0.8539
567	2347.26 (2.850%)	5272.01 (2.647%)	1.0391 (6.503%)	2.4618 (2.720%)	3.5009 (3.814%)	0.7923
575	2441.95 (2.484%)	5473.73 (2.369%)	1.1426(6.186%)	2.5578 (2.410%)	3.7005 (3.547%)	0.7379

Appendix C Proofs

Proof of Theorem 1. To find the GNB retail price p_r^* for the consumer with the parameter θ and the tax rate γ_i, we should solve the constrained maximization problem in (1), where we need to compare the cutoff level A and the retail price p_r. Thus, we analyze the constrained problem for each of the following two cases, $p_r \le A$ and $p_r \ge A$; and then compare the maximum values obtained for the two cases to determine the optimal retail price maximizing Λ in (1). Note that when $p_r = A$, the function Λ is the same for the two cases.

1. If $p_r \le A$, then we can rewrite the maximization problem in (1) as

$$\max_{p_r} \Lambda \equiv \{\theta - [1+(1-\gamma_i)t]p_r\}^{1-\kappa}[(1-T)(p_r - w)]^{\kappa}, \text{ s.t. } \theta \ge [1+(1-\gamma_i)t]p_r \text{ and } p_r \ge w. \quad (38)$$

Differentiating Λ in (38) once and twice w.r.t. p_r, we have

$$\frac{d\Lambda}{dp_r} = (1-T)^{\kappa}\{\kappa\theta - [1+(1-\gamma_i)t][p_r - (1-\kappa)w]\}\{\theta - [1+(1-\gamma_i)t]p_r\}^{-\kappa}(p_r - w)^{\kappa-1}, \quad (39)$$

and

$$\begin{aligned}\frac{d^2\Lambda}{dp_r^2} &= -[1+(1-\gamma_i)t](1-T)^{\kappa}\{\theta - [1+(1-\gamma_i)t]p_r\}^{-\kappa}(p_r - w)^{\kappa-1} \\ &\quad + (1-T)^{\kappa}\{\kappa\theta - [1+(1-\gamma_i)t][p_r - (1-\kappa)w]\}\{[1+(1-\gamma_i)t]\kappa\{\theta \\ &\quad - [1+(1-\gamma_i)t]p_r\}^{-\kappa-1}(p_r - w)^{\kappa-1} + (\kappa-1)\{\theta - [1+(1-\gamma_i)t]p_r\}^{-\kappa}(p_r - w)^{\kappa-2}\}.\end{aligned}$$

We can find that at any point (the value of p_r) satisfying $d\Lambda/dp_r = 0$, $\kappa\theta - [1+(1-\gamma_i)t][p_r - (1-\kappa)w] = 0$, and thus the second-order derivative is $d^2\Lambda/dp_r^2 < 0$. It thus follows that when $p_r \le A$, Λ is a quasi-concave function of the retail price p_r. Temporarily ignore the constraints in (38). Equating $d\Lambda/dp_r$ in (39) to zero and solving the resulting equation for p_r, we find a unique maximizing value as $p_r = \kappa\theta/[1+(1-\gamma_i)t] + (1-\kappa)w$. Since $p_r \le A$, the optimal retail price for the problem (38) when $p_r \le A$, denoted by p_{r1}, can be written as

$$p_{r1} = \min\{A, \kappa\theta/[1+(1-\gamma_i)t] + (1-\kappa)w\}. \quad (40)$$

Next, we derive the condition under which p_{r1} in (40) satisfies the constraints in (38). It is easy to find that, in order to satisfy the constraints that $\theta \ge [1+(1-\gamma_i)t]p_r$ and $p_r \ge w$, the retail price must be determined such that $w \le p_r \le \theta/[1+(1-\gamma_i)t]$. However, this requires that $\theta/[1+(1-\gamma_i)t] \ge w$, or, $\theta \ge [1+(1-\gamma_i)t]w$. Otherwise, if $\theta < [1+(1-\gamma_i)t]w$, then no retail price can satisfy the condition that $w \le p_r \le \theta/[1+(1-\gamma_i)t]$, and the retailer and the consumer cannot complete any transaction.
We find from (40) that under the condition that $\theta \ge [1+(1-\gamma_i)t]w$, if $A \ge \kappa\theta/[1+(1-\gamma_i)t] + (1-\kappa)w$ or $\theta \le \hat{\theta}_i \equiv [A-(1-\kappa)w][1+(1-\gamma_i)t]/\kappa$, then $p_r = \kappa\theta/[1+(1-\gamma_i)t] + (1-\kappa)w$ must satisfy the constraints in (38). However, if $\hat{\theta}_i \le \theta \le \bar{\theta}$, then $p_{r1} = A$ and the consumer's and the retailer's functions can thus be written as

$$u(p_{r1};\theta,\gamma_i) = \theta - [1+(1-\gamma_i)t]A \quad \text{and} \quad \pi_R(p_{r1};\theta,\gamma_i) = (1-T)(A-w).$$

Since $A \ge w$, the retailer's profit $\pi_R(p_{r1};\theta,\gamma_i)$ must be greater than or equal to zero, i.e., $\pi_R(p_{r1};\theta,\gamma_i) \ge 0$. However, whether or not $u(p_{r1};\theta,\gamma_i)$ is non-negative depends on the sign of $\theta - [1+(1-\gamma_i)t]A$. Note that, for this case,

$$\theta \ge \hat{\theta}_i = [A-(1-\kappa)w][1+(1-\gamma_i)t]/\kappa \ge [1+(1-\gamma_i)t]A,$$

because $A \ge w$. This means that, if $\hat{\theta}_i \le \theta \le \bar{\theta}$, then the retailer and the consumer should be willing to complete their transaction with the retail price A.

Summarizing the above, we find that when $p_r \le A$, the optimal retail price is given as

$$p_{r1} = \begin{cases} \kappa\theta/[1+(1-\gamma_i)t] + (1-\kappa)w, & \text{if } [1+(1-\gamma_i)t]w \le \theta \le \hat{\theta}_i, \\ A, & \text{if } \hat{\theta}_i \le \theta \le \bar{\theta}. \end{cases} \tag{41}$$

2. If $p_r \ge A$, then we rewrite the maximization problem in (1) as

$$\max_{p_r} \Lambda \equiv [\theta-(1+t)p_r+\gamma_i tA]^{1-\kappa}[(1-T)(p_r-w)]^{\kappa}, \text{ s.t. } \theta \ge (1+t)p_r-\gamma_i tA \text{ and } p_r \ge w. \tag{42}$$

Similar to our analysis for the case that $p_r \le A$, we find that when $p_r \ge A$, Λ is also a quasi-concave function of the retail price p_r with a unique maximizing value p_{r2} as

$$p_{r2} = \max[A, \kappa(\theta+\gamma_i tA)/(1+t) + (1-\kappa)w]. \tag{43}$$

It is easy to find that, in order to satisfy the constraints in (42), the retail price must be determined such that $w \le p_r \le (\theta+\gamma_i tA)/(1+t)$. This requires that $\theta \ge (1+t)w - \gamma_i tA$. Otherwise, if $\theta < (1+t)w - \gamma_i tA$, then the retailer and the consumer cannot complete their transaction.
We find from (43) that under the condition that $\theta \ge (1+t)w-\gamma_i tA$, if $A \le \kappa(\theta+\gamma_i tA)/(1+t)+(1-\kappa)w$ or $\theta \ge \bar{\theta}_i \equiv [A-(1-\kappa)w](1+t)/\kappa-\gamma_i tA$ [which is not less than $(1+t)w-\gamma_i tA$, i.e., $\bar{\theta}_i \ge (1+t)w-\gamma_i tA$], then $p_{r2} = \kappa(\theta+\gamma_i tA)/(1+t)+(1-\kappa)w$, which must satisfy the constraints in (42). However, if $\theta \le \bar{\theta}_i$, then $p_{r2} = A$, and the consumer's and the retailer's functions can thus be written as

$$u(p_{r2};\theta,\gamma_i) = \theta - [1+(1-\gamma_i)t]A \quad \text{and} \quad \pi_R(p_{r2};\theta,\gamma_i) = (1-T)(A-w).$$

Since $A \ge w$, the retailer's profit $\pi_R(p_{r2};\theta,\gamma_i)$ must be greater than or equal to zero, i.e., $\pi_R(p_{r2};\theta,\gamma_i) \ge 0$. Noting that $\bar{\theta}_i \ge [1+(1-\gamma_i)t]A \ge (1+t)w-\gamma_i tA$, we find that, if $[1+(1-\gamma_i)t]A \le \theta \le \bar{\theta}_i$, then $u(p_{r2};\theta,\gamma_i)$ is non-negative and the consumer buys from the retailer with the retail price A. Otherwise, if $\theta < [1+(1-\gamma_i)t]A$, then the consumer abandons his or her purchase from the retailer.
In summary, we find that, when $p_r \ge A$, the optimal retail price is given as

$$p_{r2} = \begin{cases} A, & \text{if } [1+(1-\gamma_i)t]A \le \theta \le \bar{\theta}_i, \\ \kappa(\theta+\gamma_i tA)/(1+t) + (1-\kappa)w, & \text{if } \bar{\theta}_i \le \theta \le \bar{\theta}. \end{cases} \tag{44}$$

To find the optimal retail price p_r^* for the constrained maximization problem in (1), we need to compare the optimal retail price under the constraints that $p_r \le A$ and $p_r \ge A$. Note that $\hat{\theta}_i - \bar{\theta}_i = -\gamma_i t(1-\kappa)(A-w)/\kappa \le 0$. Thus, we find that $[1+(1-\gamma_i)t]w \le [1+(1-\gamma_i)t]A \le \hat{\theta}_i \le \bar{\theta}_i$. We consider the following five cases:

1. If $0 \le \theta < [1+(1-\gamma_i)t]w$, then we find from the above analysis that the two players cannot reach an agreement on the retail price.
2. If $[1+(1-\gamma_i)t]w \le \theta \le [1+(1-\gamma_i)t]A$, then, under the constraint that $p_r \le A$, the retailer and the consumer choose their retail price as $\kappa\theta/[1+(1-\gamma_i)t]+(1-\kappa)w$, as indicated by (41). However, under the constraint that $p_r \ge A$, the two players do not complete the transaction. Thus, for this case, the consumer should buy from the retailer with the retail price $\tilde{p}_r \equiv \kappa\theta/[1+(1-\gamma_i)t]+(1-\kappa)w$.
3. If $[1+(1-\gamma_i)t]A \le \theta \le \hat{\theta}_i$, then, under the constraint that $p_r \le A$, the retail price for the consumer is determined as $\tilde{p}_r$, as indicated by (41); and under the constraint that $p_r \ge A$, the retail price is A, as indicated by (44). Thus, the optimal retail price is $\tilde{p}_r$.
4. If $\hat{\theta}_i \le \theta \le \bar{\theta}_i$, then, as indicated by (41) and (44), the retail price is A under the constraint that $p_r \le A$ and A under the constraint that $p_r \ge A$. It thus follows that for $\theta \in [\hat{\theta}_i, \bar{\theta}_i]$, the optimal retail price is A.
5. If $\bar{\theta}_i \le \theta \le \bar{\theta}$, then we learn from (41) and (44) that the retail price is A under the constraint

that $p_r \leq A$, whereas the retail price is $\hat{p}_r \equiv \kappa(\theta+\gamma_i tA)/(1+t)+(1-\kappa)w$ under the constraint that $p_r \geq A$. Therefore, the optimal retail price for $\theta \in [\bar{\theta}_i, \bar{\theta}]$ is $\hat{p}_r$.

In conclusion, we obtain the optimal retail price as given in (2). This theorem is thus proved. ■

Proof of Theorem 2. This proof consists of two parts.

$\boxed{\textbf{Part I}}$ **Change in $\bar{\mu}$ after the policy is implemented.**

We can compare $\mu^0(\gamma_i)$ and $\mu(\gamma_i)$ [which is given as in (4)], and note that $\mu^0(\gamma_i)$ is equal to $\mu(\gamma_i)$ when $\gamma_i = 0$. We calculate the first-order derivative of $\mu(\gamma_i)$ w.r.t γ_i as,

$$
\begin{aligned}
\frac{\partial\mu(\gamma_i)}{\partial\gamma_i} &= \frac{\partial}{\partial\gamma_i}\left[\int_{\tilde{\theta}_i}^{\hat{\theta}_i}\tilde{p}_r f(\theta)d\theta + \left(F(\bar{\theta}_i)-F(\hat{\theta}_i)\right)A + \int_{\bar{\theta}_i}^{\bar{\theta}}\hat{p}_r f(\theta)d\theta\right]\Big/\left(1-F(\tilde{\theta}_i)\right)\\
&= \frac{t\kappa}{[1+(1-\gamma_i)t]\left(1-F(\tilde{\theta}_i)\right)}\left\{\left(1-\frac{\tilde{\theta}_i f(\tilde{\theta}_i)}{1-F(\tilde{\theta}_i)}\right)\int_{\tilde{\theta}_i}^{\hat{\theta}_i}\frac{\theta}{1+(1-\gamma_i)t}f(\theta)d\theta+\right.\\
&\quad+\int_{\bar{\theta}_i}^{\bar{\theta}}\left\{\frac{A\left[1+(1-\gamma_i)t\right]}{1+t}-\frac{\tilde{\theta}_i f(\tilde{\theta}_i)}{1-F(\tilde{\theta}_i)}\left(\frac{\theta+\gamma_i tA}{1+t}-w\right)\right\}f(\theta)d\theta\\
&\quad\left.-\frac{\tilde{\theta}_i f(\tilde{\theta}_i)}{1-F(\tilde{\theta}_i)}\left[\left(F(\bar{\theta}_i)-F(\hat{\theta}_i)\right)\frac{A-w}{\kappa}-\left(F(\hat{\theta}_i)-F(\tilde{\theta}_i)\right)w\right]\right\}\\
&= \frac{t\kappa}{[1+(1-\gamma_i)t]\left(1-F(\tilde{\theta}_i)\right)}\left\{\frac{1}{1+(1-\gamma_i)t}\int_{\tilde{\theta}_i}^{\hat{\theta}_i}\theta f(\theta)d\theta+\frac{A\left[1+(1-\gamma_i)t\right]}{1+t}\left(1-F(\bar{\theta}_i)\right)\right.\\
&\quad-\frac{\tilde{\theta}_i f(\tilde{\theta}_i)}{1-F(\tilde{\theta}_i)}\left[\frac{1}{1+(1-\gamma_i)t}\int_{\tilde{\theta}_i}^{\hat{\theta}_i}\left(\theta-\tilde{\theta}_i\right)f(\theta)d\theta\right.\\
&\quad\left.\left.+\frac{1}{1+t}\int_{\bar{\theta}_i}^{\bar{\theta}}\left(\theta-\bar{\theta}_i\right)f(\theta)d\theta+\left(1-F(\hat{\theta}_i)\right)\frac{A-w}{\kappa}\right]\right\}.
\end{aligned}
$$

It follows that $\partial\mu(\gamma_i)/\partial\gamma_i < 0$ when

$$
\frac{\tilde{\theta}_i f(\tilde{\theta}_i)}{1-F(\tilde{\theta}_i)} > \frac{\frac{1}{1+(1-\gamma_i)t}\int_{\tilde{\theta}_i}^{\hat{\theta}_i}\theta f(\theta)d\theta+\frac{A[1+(1-\gamma_i)t]}{1+t}\left(1-F(\bar{\theta}_i)\right)}{\frac{1}{1+(1-\gamma_i)t}\int_{\tilde{\theta}_i}^{\hat{\theta}_i}\left(\theta-\tilde{\theta}_i\right)f(\theta)d\theta+\frac{1}{1+t}\int_{\bar{\theta}_i}^{\bar{\theta}}\left(\theta-\bar{\theta}_i\right)f(\theta)d\theta+\left(1-F(\hat{\theta}_i)\right)\frac{A-w}{\kappa}}. \tag{45}
$$

As Lariviere [23] discussed, $\theta f(\theta)/(1-F(\theta))$ with $\theta = \tilde{\theta}_i$ on the left hand side of the above inequality is the "generalized failure rate," which is also the elasticity of demand, and is greater than 1 when the consumer demand is price elastic. Demand for automobiles is generally price elastic. For example, Goodwin et al. [18] showed that the price elasticity of Ford compact automobile is 2.8. When (45) holds for all i ($1 \leq i \leq n$), we can find that $\mu(\gamma_i) < \mu^0(\gamma_i)$, and $\bar{\mu} = \sum_{i=1}^{n+1}(\delta_i \times \mu(\gamma_i)) < \sum_{i=1}^{n+1}(\delta_i \times \mu^0(\gamma_i)) = \bar{\mu}^0$.

$\boxed{\textbf{Part II}}$ **Impact of the cutoff level A on D, Π_M, Π_R, and $\bar{\mu}$:**

When the cutoff level $A \geq w$, increasing A will not affect the value of $\tilde{\theta}_i$, i.e., the minimum valuation of the consumers who are willing to purchase an automobile. This means that increasing A when $A \geq w$ will not affect the expected sales D and thus the manufacturer's expected profit Π_M.

We now examine the impact of A on the expected retail price $\bar{\mu}$ and the retailer's expected profit Π_R in the automobile market segment. Taking the first-order derivatives of $\bar{\mu}$ in (3) and Π_R

in (5) w.r.t. the cutoff level A, we obtain that

$$\frac{\partial \bar{\mu}}{\partial A} = \sum\nolimits_{i=1}^{n} \left(\delta_i \times \frac{\partial \mu(\gamma_i)}{\partial A}\right),$$
$$\frac{\partial \Pi_R}{\partial A} = B(1-T)\sum\nolimits_{i=1}^{n} \left[\delta_i \times \left(1 - F(\tilde{\theta}_i)\right) \times \frac{\partial \mu(\gamma_i)}{\partial A}\right].$$

From Theorem 1, we find the following results. If $A \geq \kappa\bar{\theta}/[1+(1-\gamma_i)t]+(1-\kappa)w$, then $\hat{\theta}_i \geq \bar{\theta}$, and the negotiated retail price p_r^* for the consumer with the tax rate γ_i is $\tilde{p}_r$ for any $\theta \in [\tilde{\theta}_i, \bar{\theta}]$. It follows that

$$\mu(\gamma_i) = \left(\int_{\tilde{\theta}_i}^{\bar{\theta}} \tilde{p}_r f(\theta) d\theta\right) \Big/ \left(1 - F(\tilde{\theta}_i)\right) \quad \text{and} \quad \frac{\partial \mu(\gamma_i)}{\partial A} = 0.$$

If $\left[\kappa\bar{\theta} + (1-\kappa)(1+t)w\right]/(1+t-\kappa\gamma_i t) \leq A \leq \kappa\bar{\theta}/[1+(1-\gamma_i)t]+(1-\kappa)w$, then $\hat{\theta}_i \leq \bar{\theta} \leq \bar{\theta}_i$, and

$$\mu(\gamma_i) = \left[\int_{\tilde{\theta}_i}^{\hat{\theta}_i} \tilde{p}_r f(\theta) d\theta + \left(1 - F(\hat{\theta}_i)\right) A\right] \Big/ \left(1 - F(\tilde{\theta}_i)\right) \quad \text{and} \quad \frac{\partial \mu(\gamma_i)}{\partial A} = 1.$$

If $A \leq \left[\kappa\bar{\theta} + (1-\kappa)(1+t)w\right]/(1+t-\kappa\gamma_i t)$, then $\bar{\theta} \geq \bar{\theta}_i$, and

$$\frac{\partial \mu(\gamma_i)}{\partial A} = \frac{F(\bar{\theta}_i) - F(\hat{\theta}_i)}{1 - F(\tilde{\theta}_i)} + \frac{\kappa\gamma_i t \left(1 - F(\bar{\theta}_i)\right)}{(1+t)\left(1 - F(\tilde{\theta}_i)\right)} > 0.$$

Hence, if $A \geq \bar{A} \equiv \kappa\bar{\theta}/[1+(1-\gamma_n)t]+(1-\kappa)w$, then $\partial\bar{\mu}/\partial A = 0$ and $\partial \Pi_R/\partial A = 0$; if $A < \bar{A}$, then $\partial\bar{\mu}/\partial A > 0$ and $\partial \Pi_R/\partial A > 0$.

The above results imply that, for a type of automobile, when the cutoff level A is sufficiently large such that $A \geq \bar{A}$, increasing the value of A will not affect the expected retail price $\bar{\mu}$ and the retailer's expected after-tax profit Π_R; but, when $A < \bar{A}$, increasing the value of A will raise both $\bar{\mu}$ and Π_R.

This theorem is thus proved. ■

Proof of Theorem 3. We find from (14) that $\phi \geq 1$ if $S - (1-T)\sum_{j\in\{H,L\}}(B_j x_j) \geq 0$, which can be specified, using (12) and (13), as follows:

$$\eta \equiv \sum\nolimits_{j\in\{H,L\}} \left[B_j \sum\nolimits_{i=1}^{n+1} \left(\delta_i^j \times \eta_j(\gamma_i)\right)\right] \geq \eta_0, \tag{46}$$

where

$$\eta_j(\gamma_i) \equiv \left(1 - F_j(\tilde{\theta}_i^j)\right) \times [(1-\gamma_i t)\mu_j(\gamma_i) - c_j] + \gamma_i \times t \times \int_{\bar{\theta}_i^j}^{\bar{\theta}_j} (\hat{p}_r^j - A) f_j(\theta) d\theta, \tag{47}$$

and

$$\eta_0 \equiv \sum\nolimits_{j\in\{H,L\}} \left[B_j \left(1 - F_j(\theta_0^j)\right) \times (\mu_0^j - c_j)\right].$$

Note that η_0 is the value of η when $\gamma_i = 0$ (for $i = 1, \ldots, n+1$) for any value of A (or equivalently, when the U.S. policy is not executed). In the above formula for η_0, $\theta_0^j \equiv (1+t)w_j$. Next, we derive a sufficient condition for the cost-effectiveness of the U.S. policy following two steps.

$\boxed{\textbf{Step 1}}$ **Show that $A = w_H$ or any $A \geq \bar{A}_H$ are local maximizers of η.**

To examine the property of the function η, we calculate the first- and second-order derivatives of η w.r.t. A as

$$\frac{\partial \eta}{\partial A} = \sum_{j\in\{H,L\}} B_j \sum_{i=1}^{n} \left(\delta_i^j \Gamma_j(\gamma_i)\right),$$

$$\frac{\partial^2 \eta}{\partial A^2} = \sum_{j\in\{H,L\}} \frac{B_j}{\kappa_j} \sum_{i=1}^{n} \left\{\delta_i^j \left[f_j(\bar{\theta}_i^j)\left(1-\frac{\kappa_j\gamma_i t}{1+t}\right)(1+t-\kappa_j\gamma_i t) - f_j(\hat{\theta}_i^j)(1-\gamma_i t)[1+(1-\gamma_i)t]\right]\right\},$$

where

$$\Gamma_j(\gamma_i) \equiv (1-\gamma_i t)\left(1-F_j(\hat{\theta}_i^j)\right) - \left(1-\frac{\kappa_j\gamma_i t}{1+t}\right)\left(1-F_j(\bar{\theta}_i^j)\right).$$

For any $\gamma_i \in (0,1)$, when $A = w_j$, the threshold $\hat{\theta}_i = \bar{\theta}_i$, thus

$$\Gamma_j(\gamma_i) = -\gamma_i t\left(1-F_j(\hat{\theta}_i^j)\right)\left(1-\frac{\kappa_j}{1+t}\right) < 0.$$

It follows that when $A \leq \max\{A|\Gamma_j(\gamma_i) \leq 0 \text{ for } \gamma_i \in (0,1)\}$, $\partial\eta/\partial A < 0$. The value of η can be a local maximum at $A = w_H$ since we only consider $A \geq w_H$.

When $A \geq \bar{A}_H \equiv \kappa_H\bar{\theta}_H/[1+(1-\gamma_n)t] + (1-\kappa_H)w_H$, we find that $\hat{\theta}_i^j \geq \bar{\theta}_j$ (the maximum valuation for the Type-j automobile), for $i = 1, 2, \ldots, n$ and $j = H, L$; hence $\Gamma_j(\gamma_i) = 0$, and $\partial\eta/\partial A = 0$, meaning that any $A \geq \bar{A}_H$ results in the same value of η. Because $\hat{\theta}_i^j \leq \bar{\theta}_i^j \leq \bar{\theta}_j$, it is obvious that when $A \to \bar{A}_H$ and $A \leq \bar{A}_H$, $\Gamma_j(\gamma_i) \geq 0$ and $\partial\eta/\partial A \geq 0$, indicating that the value of η is at least a local maximum at any $A \geq \bar{A}_H$.

We calculate the value of η when $A = w_H$ and that when $A = \bar{A}_H$, and obtain that

$$\eta|_{A=w_H} - \eta|_{A=\bar{A}_H} = \sum_{j\in\{H,L\}} \left[B_j \sum_{i=1}^{n} \delta_i^j \times \left(\eta_j(\gamma_i)|_{A=w_H} - \eta_j(\gamma_i)|_{A=\bar{A}_H}\right)\right],$$

where

$$\begin{aligned}
\eta_H(\gamma_i)|_{A=w_H} - \eta_H(\gamma_i)|_{A=\bar{A}_H} &= \int_{\tilde{\theta}_i^H}^{\bar{\theta}_H} \left\{\left[\frac{\kappa_H(\theta+\gamma_i t w_H)}{1+t} + (1-\kappa_H-\gamma_i t)w_H\right]\right. \\
&\quad \left. -(1-\gamma_i t)\left[\frac{\kappa_H\theta}{1+(1-\gamma_i)t} + (1-\kappa_H)w_H\right]\right\} f_H(\theta)d\theta \\
&= \frac{\kappa_H\gamma_i t^2}{(1+t)\left[1+(1-\gamma_i)t\right]} \left\{\int_{\tilde{\theta}_i^H}^{\bar{\theta}_H} \theta f_H(\theta)d\theta - w_H\left[1+(1-\gamma_i)t\right]\left(1-F_H(\tilde{\theta}_i^H)\right)\right\} \\
&\geq 0;
\end{aligned}$$

and

$$\begin{aligned}
\eta_L(\gamma_i)|_{A=w_H} - \eta_L(\gamma_i)|_{A=\bar{A}_H} = (1-\gamma_i t)\left[\int_{\hat{\theta}_i^L}^{\bar{\theta}_i^L} (w_H - \tilde{p}_r^L) f_L(\theta)d\theta\right] + \frac{\kappa_L\gamma_i t}{1+t}\int_{\bar{\theta}_i^L}^{\bar{\theta}_L} \left[\frac{t\theta}{1+(1-\gamma_i)t}\right. \\
\left. +w_H - (1+t)\frac{w_H-(1-\kappa)w_L}{\kappa_L}\right] f_L(\theta)d\theta. \qquad (48)
\end{aligned}$$

In the above equation, $\int_{\hat{\theta}_i^L}^{\bar{\theta}_i^L} (w_H - \tilde{p}_r^L) f_L(\theta)d\theta \leq 0$; when $\theta = \bar{\theta}_i^L$,

$$\left[\frac{t\theta}{1+(1-\gamma_i)t} + w_H - (1+t)\frac{w_H-(1-\kappa_L)w_L}{\kappa_L}\right] = -\frac{(1+t)(1-\kappa_L)(1-\gamma_i t)}{\kappa_L[1+(1-\gamma_i)t]}(w_H - w_L) < 0,$$

and thus, the term

$$\int_{\tilde{\theta}_i^L}^{\bar{\theta}_L} \left[\frac{t\theta}{1+(1-\gamma_i)t} + w_H - (1+t)\frac{w_H-(1-\kappa_L)w_L}{\kappa_L}\right] f_L(\theta)d\theta$$

can be positive or negative. Therefore, the sign of $\eta_L(\gamma_i)|_{A=w_H} - \eta_L(\gamma_i)|_{A=\bar{A}_H}$ and that of $\eta|_{A=w_H} - \eta|_{A=\bar{A}_H}$ can be either positive or negative.

Let $\Delta\eta_L(A|\gamma_i) \equiv \eta_L(\gamma_i)|_A - \eta_L(\gamma_i)|_{A=\bar{A}_H}$, which can be specified as in (48) by replacing w_H with A. The first-order derivative of $\Delta\eta_L(A|\gamma_i)$ is calculated as,

$$\begin{aligned}\frac{\partial\Delta\eta_L(A|\gamma_i)}{\partial A} &= (1-\gamma_i t)\left[1+\frac{A-w_L}{\kappa_L[1+(1-\gamma_i)t]}([1+(1-\gamma_i)t]^2 - \gamma_i t\,(1+t-\kappa_L\gamma_i t)\right.\\ &\quad \left.\times(1-\kappa_L))\right]\left(F_L(\bar{\theta}_i^L) - F_L(\hat{\theta}_i^L)\right) + \gamma_i t\left[-\left(1-\frac{\kappa_L}{1+t}\right)\right.\\ &\quad \left.+\frac{A-w_L}{\kappa_L[1+(1-\gamma_i)t]}\,(1+t-\kappa_L\gamma_i t)\,(1-\kappa_L)\,(1-\gamma_i t)\right]\left(1-F_L(\bar{\theta}_i^L)\right).\end{aligned}$$

We find that at $A = w_L$, $\hat{\theta}_i^L = \bar{\theta}_i^L$, and

$$\partial\Delta\eta_L(A|\gamma_i)/\partial A|_{A=w_L} = -\gamma_i t\left(1-\frac{\kappa_L}{1+t}\right)(1-F_L(\bar{\theta}_i^L)) < 0;$$

and, when $A > w_L$ and $A - w_L$ is sufficiently large, $\partial\Delta\eta_L(A|\gamma_i)/\partial A > 0$. Therefore, it follows that, depending on the values of B_H and B_L (i.e., the sizes of the high-end and low-end automobile market segments) and how close the value of w_H is to w_L, the net benefit of the tax-reduction policy when $A = w_H$ may be higher or lower than that when $A = \bar{A}_H$. According to the proof of Theorem 2, we find that $\eta|_A$ for any $A > \bar{A}_H$ is equal to $\eta|_{A=\bar{A}_H}$ because increasing the value of A when $A > \bar{A}_H$ will not affect the retail price $\mu_j(\gamma_i)$. Therefore, the optimal A maximizing η could be equal to w_H or any $A \geq \bar{A}_H$ (in particular, $A = \infty$).

$\boxed{\textbf{Step 2}}$ **Derive the sufficient condition for the cost-effectiveness of the U.S. policy.**

When $A = \bar{A}_H$, we have

$$\eta|_{A=\bar{A}_H} = \sum\nolimits_{j\in\{H,L\}}\left\{B_j\sum\nolimits_{i=1}^{n+1}\delta_i^j\left[(1-\gamma_i t)\left(\int_{\tilde{\theta}_i^j}^{\bar{\theta}_j}\tilde{p}_r^j f_j(\theta)d\theta\right) - c_j\left(1-F_j(\tilde{\theta}_i^j)\right)\right]\right\}, \tag{49}$$

and

$$\begin{aligned}\eta|_{A=\bar{A}_H} - \eta_0 &= \sum\nolimits_{j\in\{H,L\}}\left\{B_j\sum\nolimits_{i=1}^{n}\delta_i^j\left\{\frac{\kappa_j}{1+(1-\gamma_i)t}\left[(1-\gamma_i t)\int_{\tilde{\theta}_i^j}^{\theta_0^j}(\theta-\tilde{\theta}_i^j)f_j(\theta)d\theta\right.\right.\right.\\ &\quad \left.\left.\left.-\gamma_i t\int_{\theta_0^j}^{\bar{\theta}_j}\left(\frac{t\theta}{1+t}-\tilde{\theta}_i^j\right)f_j(\theta)d\theta\right] - \gamma_i t w_j\left(1-F_j(\tilde{\theta}_i^j)\right) + (w_j-c_j)\left(F_j(\theta_0^j)-F_j(\tilde{\theta}_i^j)\right)\right\}\right\}.\end{aligned}$$

When $A = w_H$, we have

$$\begin{aligned}\eta|_{A=w_H} &= \sum\nolimits_{j\in\{H,L\}}\left\{B_j\sum\nolimits_{i=1}^{n+1}\delta_i^j\left\{\frac{\kappa_H}{1+t}\int_{\tilde{\theta}_i^H}^{\bar{\theta}_H}(\theta-\tilde{\theta}_i^H)f_H(\theta)d\theta + [w_H(1-\gamma_i t)-c_H]\left(1-F_H(\tilde{\theta}_i^H)\right)\right.\right.\\ &\quad +\frac{\kappa_L}{1+t}\left[\frac{(1-\gamma_i t)(1+t)}{1+(1-\gamma_i)t}\int_{\tilde{\theta}_i^L}^{\hat{\theta}_i^L}\left(\theta-\tilde{\theta}_i^L\right)f_L(\theta)d\theta + \int_{\tilde{\theta}_i^L}^{\bar{\theta}_L}\left[\theta-\tilde{\theta}_i^L+\gamma_i t(w_H-w_L)\right]f_L(\theta)d\theta\right]\\ &\quad \left.\left.+(w_H-w_L)\left[F_L(\bar{\theta}_i^L)-(1-\gamma_i t)F_L(\hat{\theta}_i^L)-\gamma_i t\right] + [w_L(1-\gamma_i t)-c_L]\left(1-F_L(\tilde{\theta}_i^L)\right)\right\}\right\},\end{aligned}$$

and

$$\eta|_{A=w_H} - \eta_0 = \sum\nolimits_{j\in\{H,L\}} \left\{ B_j \sum\nolimits_{i=1}^{n} \delta_i^j \left\{ \frac{\kappa_H}{1+t} \left[\int_{\tilde{\theta}_i^H}^{\bar{\theta}_H} (\theta - \tilde{\theta}_i^H) f_H(\theta) d\theta - \int_{\theta_0^H}^{\bar{\theta}_H} (\theta - \theta_0^H) f_H(\theta) d\theta \right] \right.\right.$$
$$- w_H \gamma_i t \left(1 - F_H(\tilde{\theta}_i^H)\right) + (w_H - c_H)\left(F_H(\theta_0^H) - F_H(\tilde{\theta}_i^H)\right) + \frac{\kappa_L}{1+t}\left[\frac{(1-\gamma_i t)(1+t)}{1+(1-\gamma_i)t}\right.$$
$$\left. \times \int_{\tilde{\theta}_i^L}^{\hat{\theta}_i^L} \left(\theta - \tilde{\theta}_i^L\right) f_L(\theta) d\theta + \int_{\hat{\theta}_i^L}^{\bar{\theta}_L} \left[\theta - \tilde{\theta}_i^L + \gamma_i t (w_H - w_L)\right] f_L(\theta) d\theta - \int_{\theta_0^L}^{\bar{\theta}_L} (\theta - \theta_0^L) f_L(\theta) d\theta \right]$$
$$+ (w_H - w_L)\left[F_L(\bar{\theta}_i^L) - (1-\gamma_i t) F_L(\hat{\theta}_i^L) - \gamma_i t\right] - w_L \gamma_i t \left(1 - F_L(\tilde{\theta}_i^L)\right)$$
$$\left.\left. + (w_L - c_L)\left(F_L(\theta_0^L) - F_L(\tilde{\theta}_i^L)\right)\right\}\right\}.$$

Therefore, $\eta|_{A=\bar{A}_H} - \eta_0 \geq 0$ if $\sum_{j\in\{H,L\}} \left(B_j \kappa_j \varphi_j^1(\gamma)\right) \geq \sum_{j\in\{H,L\}} \left(B_j \lambda_j^1(\gamma)\right)$, and $\eta|_{A=w_H} - \eta_0 \geq 0$ if $\sum_{j\in\{H,L\}} \left(B_j \kappa_j \varphi_j^2(\gamma)\right) \geq \sum_{j\in\{H,L\}} \left(B_j \lambda_j^2(\gamma)\right)$, where $\varphi_j^l(\gamma)$ and $\lambda_j^l(\gamma)$ are defined as in the theorem.

As shown in the main body of this paper, the highest sales tax rate t is 9.75% in Los Angeles. Thus for $\theta \leq \bar{\theta}_j$, $\frac{t}{1+t}\theta - \tilde{\theta}_i^j < 0$ because the highest valuation for the Type-j automobile (i.e., $\bar{\theta}_j$) is unlikely to be more than 10 times the minimum valuation of consumers who purchase the automobile (i.e., $\tilde{\theta}_i^j$). Moreover, we find from online Table A that the largest value of γ_i is 0.33, so $\gamma_i t < 0.033$. Therefore, $\varphi_j^1(\gamma)$ should be positive. It is obvious that $\varphi_j^2(\gamma) > 0$ because $\tilde{\theta}_i^j < \theta_0^j$. The sign of $\lambda_j^1(\gamma)$ depends on whether or not $\sum_{i=1}^n \left[\delta_i^j \gamma_i t w_j \left(1 - F_j(\tilde{\theta}_i^j)\right)\right]$ is greater than $\sum_{i=1}^n \left[\delta_i^j (w_j - c_j)\left(F_j(\theta_0^j) - F_j(\tilde{\theta}_i^j)\right)\right]$. If $\lambda_j^1(\gamma) < 0$ for $j = H, L$, then $\kappa_j \varphi_j^1(\gamma)$ must be greater than $\lambda_j^1(\gamma)$ when the U.S. tax reduction policy with a sufficiently large cutoff level (e.g., $A = \bar{A}_H$) or the policy without a cutoff level (i.e., $A = \infty$) is executed. If $\lambda_j^1(\gamma) < \varphi_j^1(\gamma)$, then a sufficient condition for the policy to be cost effective is $\sum_{j\in\{H,L\}} \left(B_j \kappa_j \varphi_j^1(\gamma)\right) \geq \sum_{j\in\{H,L\}} \left(B_j \lambda_j^1(\gamma)\right)$. If $\lambda_j^1(\gamma) > 1$ for $j = H, L$, then a cost-effective tax reduction policy may not exist. Similar discussion can be made for the sign of $\lambda_j^2(\gamma)$. This theorem is thus proved. ■

Proof of Theorem 4. According to Theorem 2, we find that the profit increase $\Pi_M^j + \Pi_R^j$ corresponding to the Type-j automobile is increasing in the cutoff level A when $A < \bar{A}_j \equiv \kappa_j \bar{\theta}_j / [1 + (1-\gamma_n)t] + (1-\kappa_j) w_j$ $(j = H, L)$; since $\bar{A}_L < \bar{A}_H$, the system-wide additional profit S is increasing in A when $A < \bar{A}_H$. Moreover, when $A \geq \bar{A}_H$, S and thus the profit-cost ratio ϕ remain constant. Theorem 3 indicates that if the two conditions in (15) in Theorem 3 are satisfied, then there must exist a cutoff level such that $\phi \geq 1$. We also note from the proof of Theorem 3 that one of the conditions in Theorem 3 is derived when $A = \bar{A}_H$; that is, when both conditions in Theorem 3 apply, the optimal cutoff level maximizing S must be greater than or equal to $\bar{A}_H$. It thus follows that the constraint that $A \geq w$ is redundant. ■

Proof of Theorem 5. To find the retail price negotiated by the consumer and the retailer, we construct the GNB model as

$$\begin{aligned} \max_{p_r} \quad & \Lambda = \{\theta - [1 + (1-\alpha) \times t] \times p_r\}^{1-\kappa} [(1-T)(p_r - w)]^{\kappa} \\ \text{s.t.} \quad & \theta - [1 + (1-\alpha) \times t] \times p_r \geq 0 \quad \text{and} \quad (1-T)(p_r - w) \geq 0. \end{aligned}$$

Differentiating Λ once and twice w.r.t. p_r gives

$$\frac{\partial \Lambda}{\partial p_r} = -(1-\kappa)[1+(1-\alpha)\times t]\{\theta - [1+(1-\alpha)\times t]\times p_r\}^{-\kappa}[(1-T)(p_r-w)]^{\kappa}$$
$$+\,\kappa(1-T)\{\theta - [1+(1-\alpha)\times t]\times p_r\}^{1-\kappa}[(1-T)(p_r-w)]^{\kappa-1},$$
$$\frac{\partial^2 \Lambda}{\partial p_r^2} = -\kappa(1-\kappa)[1+(1-\alpha)\times t]^2\{\theta - [1+(1-\alpha)\times t]\times p_r\}^{-\kappa-1}[(1-T)(p_r-w)]^{\kappa}$$
$$-\,\kappa(1-\kappa)(1-T)^2\{\theta - [1+(1-\alpha)\times t]\times p_r\}^{1-\kappa}[(1-T)(p_r-w)]^{\kappa-2}.$$

Note that $\partial^2\Lambda/\partial p_r^2 \le 0$; thus, Λ is a concave function with a unique optimal solution as $\hat{p}_r$ in (25). Using the argument in the proof of Theorem 1, we can then prove this theorem. ■

Proof of Theorem 6. We examine the impact of α on the expected retail price $\hat{\mu}$ and the retailer's and the manufacturer's expected profits $\hat{\Pi}_R$ and $\hat{\Pi}_M$. Taking the first-order derivative of $\hat{\mu}$ in (27) w.r.t. the reduction percentage α gives

$$\frac{\partial \hat{\mu}}{\partial \alpha} = \left\{ \frac{wt\kappa}{[1+(1-\alpha)t]\left(1-F(\hat{\theta})\right)} \right\} \xi, \text{ where } \xi \equiv f(\hat{\theta})\hat{\theta} + \left[\frac{1}{\hat{\theta}} - \frac{f(\hat{\theta})}{1-F(\hat{\theta})}\right]\int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta)d\theta.$$

Note that $\partial\hat{\mu}/\partial\alpha$ may be positive or negative, depending on the condition that $\xi \ge 0$. Since $\int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta)d\theta > \hat{\theta}\left(1-F(\hat{\theta})\right)$, we find that if $1-F(\hat{\theta}) > \hat{\theta}f(\hat{\theta})$, or, $\hat{\theta}f(\hat{\theta})\Big/\left(1-F(\hat{\theta})\right) < 1$, then $\xi > 1-F(\hat{\theta}) > 0$; otherwise, $\xi < 1-F(\hat{\theta})$. As Lariviere [23] discussed, $\hat{\theta}f(\hat{\theta})\Big/\left(1-F(\hat{\theta})\right)$ is a "generalized failure rate," which is smaller than 1 when the consumer demand is price inelastic.

The first-order derivative of ξ w.r.t. α is obtained as follows:

$$\begin{aligned}
\frac{\partial \xi}{\partial \alpha} &= \frac{tw}{1-F(\hat{\theta})}\left\{\left[\frac{1-F(\hat{\theta})}{(\hat{\theta})^2} + f'(\hat{\theta}) + \frac{\left(f(\hat{\theta})\right)^2}{1-F(\hat{\theta})}\right]\int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta)d\theta \right.\\
&\quad \left. -\,\hat{\theta}\left(1-F(\hat{\theta})\right)\left[\frac{\left(f(\hat{\theta})\right)^2}{1-F(\hat{\theta})} + f'(\hat{\theta})\right]\right\}\\
&\ge \frac{tw}{1-F(\hat{\theta})}\left\{\left[\frac{1-F(\hat{\theta})}{(\hat{\theta})^2} + f'(\hat{\theta}) + \frac{\left(f(\hat{\theta})\right)^2}{1-F(\hat{\theta})}\right]\hat{\theta}\left(1-F(\hat{\theta})\right) \right.\\
&\quad \left. -\,\hat{\theta}\left(1-F(\hat{\theta})\right)\left[\frac{\left(f(\hat{\theta})\right)^2}{1-F(\hat{\theta})} + f'(\hat{\theta})\right]\right\}\\
&= \frac{tw}{\hat{\theta}}\left(1-F(\hat{\theta})\right),
\end{aligned}$$

which is non-negative. Therefore, ξ is increasing in α.

It is easy to find that $\hat{D} = B\left(1-F(\hat{\theta})\right)$—where $\hat{\theta} = [1+(1-\alpha)t]w$—is increasing in α. We differentiate $\hat{\Pi}_R$ in (28) and $\hat{\Pi}_M$ in (29) w.r.t. α, and have

$$\frac{\partial \hat{\Pi}_R}{\partial \alpha} = \frac{B\kappa tw(1-T)}{1+(1-\alpha)t}\left[[1+(1-\alpha)t]f(\hat{\theta})(w-1) + \frac{\int_{\hat{\theta}}^{\bar{\theta}} \theta f(\theta)d\theta}{\hat{\theta}}\right] \ge 0,$$
$$\frac{\partial \hat{\Pi}_M}{\partial \alpha} = Btw(1-T)(w-c)f(\hat{\theta}) \ge 0,$$

which imply that, if the reduction percentage α is increased, then both the retailer's and the manufacturer's expected profits rise. This theorem is thus proved. ■

Proof of Theorem 7. Similar to the proof of Theorem 3, we find that $\hat{\phi} \geq 1$ if and only if $\hat{S} - \sum_{j\in\{H,L\}}(B_j\hat{x}_j)(1-T) \geq 0$ or the condition in Theorem 7 is satisfied.

Because the second term of τ_j^1 is equal to the first term of τ_j^1 at $\alpha = 0$, we calculate the first-order derivative of the first term w.r.t. α as,

$$\frac{\partial}{\partial\alpha}\left[\frac{1-\alpha t}{1+(1-\alpha)t}\int_{\hat{\theta}_j}^{\bar{\theta}_j}(\theta-\hat{\theta}_j)f_j(\theta)d\theta\right] = \frac{t}{[1+(1-\alpha)t]^2}\int_{\hat{\theta}_j}^{\bar{\theta}_j}\Big\{[1+(1-\alpha)t]\,\hat{\theta}_j - t\theta\Big\}f_j(\theta)d\theta.$$

Note that $\hat{\theta}_j > w$ and the original vehicle sales tax rate t is 10%. Since the maximum valuation of a consumer (i.e., $\bar{\theta}_j$) is unlikely to be as high as 10 times the wholesale price w, we find that $[1+(1-\alpha)t]\,\hat{\theta}_j - t\theta > 0$ for $\theta \in [\hat{\theta}_j, \bar{\theta}_j]$, and the above first-order derivative is positive. It follows that τ_j^1 is generally increasing in α and is positive.

Similarly, we calculate the first-order derivative of the first term in τ_j^2 w.r.t. α as

$$\frac{\partial}{\partial\alpha}\Big\{[(1-\alpha t)w_j - c_j]\Big(1-F_j(\hat{\theta}_j)\Big)\Big\} = -tw\Big(1-F_j(\hat{\theta}_j)\Big)\left\{1-\frac{1-\alpha t - c_j/w_j}{1+(1-\alpha)t}\frac{\hat{\theta}_j f_j(\hat{\theta}_j)}{1-F_j(\hat{\theta}_j)}\right\}.$$

Since the manufacturer's profit margin cannot be very large, c_j/w_j cannot be very small. Unless $\hat{\theta}_j f_j(\hat{\theta}_j)\Big/\Big(1-F_j(\hat{\theta}_j)\Big)$ is much greater than 1 (i.e., the demand is very price elastic), we find that $\frac{\partial}{\partial\alpha}\Big\{[(1-\alpha t)w_j - c_j]\Big(1-F_j(\hat{\theta}_j)\Big)\Big\} < 0$, indicating that τ_j^2 is generally decreasing in α and is negative.

This theorem is thus proved. ■

Proof of Theorem 8. This theorem consists of two parts, in which we derive the negotiated retail price p_H for the consumer who prefers a Type-H automobile and p_L for the consumer who prefers a Type-L automobile.

$\boxed{\text{Part I}}$ The Negotiated Retail Price p_H for the Consumer who Prefers a Type-H Automobile

The consumer with the valuation θ_H decides to buy a Type-H automobile if and only if his or her net gain $u(p_H;\theta_H,\gamma_i)$ is non-negative and no less than that drawn from purchasing a Type-L automobile, i.e., $u(p_H;\theta_H,\gamma_i) \geq \max\{u(p_L;\theta_L,\gamma_i),0\}$. That is, to find the retail price p_H for the consumer, we should solve the constrained maximization problem in (37), where we need to compare $u(p_L;\theta_L,\gamma_i)$ and 0 to decide on the disagreement payoff $\max\{u(p_L;\theta_L,\gamma_i),0\}$. Thus, we analyze two possible cases: (i) $u(p_L;\theta_L,\gamma_i) \geq 0$ and (ii) $u(p_L;\theta_L,\gamma_i) \leq 0$.

Case (i): The negotiated retail price p_H when $u(p_L;\theta_L,\gamma_i) \geq 0$

The consumer's disagreement payoff for his or her bargaining with the retailer over the retail price of a Type-H automobile is $u_H^0 = u(p_L;\theta_L,\gamma_i)$, because, if the consumer does not buy a Type-H automobile, then he or she should bargain with the retailer over the price p_L for the Type-L automobile. When $u(p_L;\theta_L,\gamma_i) \geq 0$, the consumer will buy a Type-L automobile at the price p_L. Thus the retailer's disagreement payoff for the bargaining of the price p_H is $v_H^0 = (1-T)(p_L - w_L)$.

Summarizing the above, we should use a backward approach to find the negotiated price p_H and derive the condition assuring that the consumer and the retailer trade the Type-H automobile. Specifically, we take the following two steps to solve our problem.

Step 1: Determine the disagreement payoff for Type-H automobile bargaining. Following our above argument, to find u_H^0 and v_H^0, we should analyze the Type-L automobile bargaining problem in which the disagreement payoffs for the consumer and the retailer are zero. Therefore, the generalized Nash bargaining (GNB) model for the negotiation of p_L is,

$$\begin{array}{rl}\max_{p_L} & \Lambda_L \equiv [\theta_L - (1+t)p_L + \gamma_i t\min(A,p_L)]^{1-\kappa}[(1-T)(p_L-w_L)]^{\kappa} \\ \text{s.t.} & \theta_L - (1+t)p_L + \gamma_i t\min(A,p_L) \geq 0 \quad \text{and} \quad (1-T)(p_L-w_L) \geq 0.\end{array}$$

Using the result in Theorem 1, we can find p_L as follows. If $0 \le \theta_L < \tilde{\theta}_i^L \equiv [1+(1-\gamma_i)t]w_L$, then the retailer and the consumer cannot reach an agreement on the retail price. If $\theta \ge \tilde{\theta}_i^L$, then the negotiated retail price p_L^* is determined as

$$p_L^* = \begin{cases} \tilde{p}_L \equiv \dfrac{\kappa\theta_L}{1+(1-\gamma_i)t} + (1-\kappa)w_L, & \text{if } \tilde{\theta}_i^L \le \theta_L \le \hat{\theta}_i^L, \\ A, & \text{if } \hat{\theta}_i^L \le \theta_L \le \bar{\theta}_i^L, \\ \hat{p}_L \equiv \dfrac{\kappa(\theta_L + \gamma_i t A)}{1+t} + (1-\kappa)w_L, & \text{if } \bar{\theta}_i^L \le \theta_L \le \bar{\theta}_L, \end{cases}$$

where $\hat{\theta}_i^L \equiv [A-(1-\kappa)w_L][1+(1-\gamma_i)t]/\kappa$ and $\bar{\theta}_i^L \equiv [A-(1-\kappa)w_L](1+t)/\kappa - \gamma_i t A$. Thus $u_H^0 = u(p_L^*;\theta_L,\gamma_i)$ and $v_H^0 = (1-T)(p_L^* - w_L)$.

Step 2: Find the negotiated retail price p_H. Using u_H^0 and v_H^0, we can obtain the negotiated retail price p_H for the consumer, by solving the GNB problem as follows:

$$\begin{array}{rl} \max_{p_H} & \Lambda_H \equiv [\theta_H - (1+t)p_H + \gamma_i t\min(A,p_H) - u_H^0]^{1-\kappa}[(1-T)(p_H-w_H) - v_H^0]^\kappa \\ \text{s.t.} & \theta_H - (1+t)p_H + \gamma_i t \min(A,p_H) \ge u_H^0 \quad \text{and} \quad (1-T)(p_H - w_H) \ge v_H^0. \end{array} \tag{50}$$

To solve the above problem, we need to compare the cutoff level A and the retail price p_H. Thus, we analyze the problem for each of the cases, $p_H \le A$ and $p_H \ge A$; and then compare the maximum values obtained for the two cases to determine the optimal retail price maximizing Λ_H in (50). Note that when $p_H = A$, the function Λ_H is the same for the two cases.

1. If $p_H \le A$, then we can rewrite the maximization problem in (50) as

$$\begin{array}{rl} \max_{p_H} & \Lambda_H \equiv \{\theta_H - [1+(1-\gamma_i)t]p_H - u_H^0\}^{1-\kappa}[(1-T)(p_H-w_H) - v_H^0]^\kappa \\ \text{s.t.} & \theta_H - [1+(1-\gamma_i)t]p_H \ge u_H^0 \quad \text{and} \quad (1-T)(p_H-w_H) \ge v_H^0. \end{array} \tag{51}$$

Let $U \equiv \theta_H - [1+(1-\gamma_i)t]p_H - u_H^0$ and $V \equiv (1-T)(p_H - w_H) - v_H^0$. Differentiating Λ_H in (50) once and twice w.r.t. p_H, we have

$$\frac{d\Lambda_H}{dp_H} = -[1+(1-\gamma_i)t](1-\kappa)U^{-\kappa}V^\kappa + \kappa(1-T)U^{1-\kappa}V^{\kappa-1},$$

and

$$\begin{aligned} \frac{d^2\Lambda_H}{dp_H^2} &= -\kappa(1-\kappa)[1+(1-\gamma_i)t]^2U^{-\kappa-1}V^\kappa - \kappa(1-\kappa)(1-T)[1+(1-\gamma_i)t]U^{-\kappa}V^{\kappa-1} \\ &\quad - \kappa(1-\kappa)[1+(1-\gamma_i)t](1-T)U^{-\kappa}V^{\kappa-1} - \kappa(1-\kappa)(1-T)^2U^{1-\kappa}V^{\kappa-2} \\ &= -\kappa(1-\kappa)U^{-\kappa-1}V^{\kappa-2}\left\{[1+(1-\gamma_i)t]V + (1-T)U\right\}^2 < 0. \end{aligned}$$

Thus, when $p_H \le A$, Λ_H is a concave function of p_H. Temporarily ignore the constraints in (51). Equating $d\Lambda_H/dp_H$ to zero and solving the resulting equation for p_H, we find a unique maximizing value as

$$p_H = \kappa\left(\theta_H - u_H^0\right)/[1+(1-\gamma_i)t] + (1-\kappa)\left[w_H + v_H^0/(1-T)\right].$$

Since $p_H \le A$, the optimal retail price for the problem (51), denoted by p_{H1}, can be written as

$$p_{H1} = \min\{A, \kappa\left(\theta_H - u_H^0\right)/[1+(1-\gamma_i)t] + (1-\kappa)\left[w_H + v_H^0/(1-T)\right]\}. \tag{52}$$

Next, we derive the condition under which p_{H1} in (52) satisfies the constraints in (51). The retail price satisfying the constraints must be such that

$$w_H + v_H^0/(1-T) \le p_H \le \left(\theta_H - u_H^0\right)/[1+(1-\gamma_i)t]. \tag{53}$$

For the case that $p_H \leq A$ to occur, it is required that $w_H + v_H^0/(1-T) \leq A$, i.e.,

$$\theta_L \leq \check{\theta}_i^L \equiv [1+(1-\gamma_i)t]\left[A-(w_H-\kappa w_L)\right]/\kappa < \hat{\theta}_i^L.$$

Condition (53) requires that $\left(\theta_H - u_H^0\right)/[1+(1-\gamma_i)t] \geq w_H + v_H^0/(1-T)$, or, $\theta_H \geq \tilde{\theta}_i^h$, where

$$\tilde{\theta}_i^h \equiv \theta_L + [1+(1-\gamma_i)t](w_H - w_L).$$

Otherwise, if $\theta_H < \tilde{\theta}_i^h$, then no retail price can satisfy condition (53), and the retailer and the consumer will not complete a transaction of the Type-H automobile.

We find from (52) that under the condition that $\theta_H \geq \tilde{\theta}_i^h$, if $A \geq \kappa\left(\theta_H - u_H^0\right)/[1+(1-\gamma_i)t] + (1-\kappa)\left[w_H + v_H^0/(1-T)\right]$ or $\theta_H \leq \hat{\theta}_i^h$, where

$$\hat{\theta}_i^h \equiv [1+(1-\gamma_i)t]\left\{A-(1-\kappa)\left[w_H + v_H^0/(1-T)\right]\right\}/\kappa + u_H^0,$$

then $p_H = \kappa\left(\theta_H - u_H^0\right)/[1+(1-\gamma_i)t] + (1-\kappa)\left[w_H + v_H^0/(1-T)\right]$ must satisfy the constraints in (51). However, if $\hat{\theta}_i^h \leq \theta_H \leq \bar{\theta}_H$, then $p_{H1} = A$, and the consumer's and the retailer's payoffs are

$$u(p_{H1};\theta_H,\gamma_i) = \theta_H - [1+(1-\gamma_i)t]A \quad \text{and} \quad \pi_R(p_{H1};\theta_H,\gamma_i) = (1-T)(A-w_H).$$

When $\theta_H \geq \hat{\theta}_i^h$, $p_{H1} = A$ satisfies condition (53), thus $A \geq w_H + v_H^0/(1-T)$. It follows that the retailer's profit $\pi_R(p_{H1};\theta_H,\gamma_i) \geq v_H^0$ and $u(p_{H1};\theta_H,\gamma_i) \geq u_H^0$. This means that, if $\hat{\theta}_i^h \leq \theta_H \leq \bar{\theta}_H$, the retailer and the consumer should be willing to complete their transaction with the retail price A.

Summarizing the above, we find that when $p_H \leq A$ and $\tilde{\theta}_i^L \leq \theta_L \leq \check{\theta}_i^L$, the optimal retail price is given as

$$p_{H1} = \begin{cases} \kappa\left(\theta_H - u_H^0\right)/[1+(1-\gamma_i)t] \\ \quad +(1-\kappa)\left[w_H + v_H^0/(1-T)\right], & \text{if } \tilde{\theta}_i^h \leq \theta_H \leq \hat{\theta}_i^h, \\ A, & \text{if } \hat{\theta}_i^h \leq \theta_H \leq \bar{\theta}_H. \end{cases} \tag{54}$$

2. If $p_H \geq A$, then we can rewrite the maximization problem in (50) as

$$\begin{array}{rl} \max_{p_H} & \Lambda_H \equiv [\theta_H - (1+t)p_H + \gamma_i tA - u_H^0]^{1-\kappa}[(1-T)(p_H - w_H) - v_H^0]^\kappa \\ \text{s.t.} & \theta_H - (1+t)p_H + \gamma_i tA \geq u_H^0 \quad \text{and} \quad (1-T)(p_H - w_H) \geq v_H^0. \end{array} \tag{55}$$

Similar to our analysis for the case that $p_H \leq A$, we find that when $p_H \geq A$, Λ_H is also a concave function of p_H with a unique maximizing value p_{H2} as

$$p_{H2} = \max\left\{A, \kappa(\theta_H + \gamma_i tA - u_H^0)/(1+t) + (1-\kappa)\left[w_H + v_H^0/(1-T)\right]\right\}. \tag{56}$$

In order to satisfy the constraints in (55), the retail price must be determined such that $w_H + v_H^0/(1-T) \leq p_H \leq (\theta_H - u_H^0 + \gamma_i tA)/(1+t)$. This requires $\theta_H \geq \left[w_H + v_H^0/(1-T)\right](1+t) - \gamma_i tA + u_H^0$. Otherwise, the retailer and the consumer cannot complete their transaction of the Type-H automobile.

We find from (56) that under the condition that $\theta_H \geq \left[w_H + v_H^0/(1-T)\right](1+t) - \gamma_i tA + u_H^0$, if $A \leq \kappa(\theta_H + \gamma_i tA - u_H^0)/(1+t) + (1-\kappa)\left[w_H + v_H^0/(1-T)\right]$ or $\theta_H \geq \bar{\theta}_i^h$, where

$$\bar{\theta}_i^h \equiv \left\{A - (1-\kappa)\left[w_H + v_H^0/(1-T)\right]\right\}(1+t)/\kappa - \gamma_i tA + u_H^0,$$

then $p_{H2} = \kappa(\theta_H + \gamma_i tA - u_H^0)/(1+t) + (1-\kappa)\left[w_H + v_H^0/(1-T)\right]$, which must satisfy the constraints in (55). However, if $\theta_H \leq \bar{\theta}_i^h$, then $p_{H2} = A$, and the consumer's and the retailer's payoffs are

$$u(p_{H2};\theta_H,\gamma_i)=\theta_H-[1+(1-\gamma_i)t]A \quad \text{and} \quad \pi_R(p_{H2};\theta_H,\gamma_i)=(1-T)(A-w_H).$$

Similar to our analysis for the case that $p_H \le A$, when $\tilde{\theta}_i^L \le \theta_L \le \check{\theta}_i^L$, we find $p_{H2}=A \ge w_H+v_H^0/(1-T)$, and the retailer's profit $\pi_R(p_{H2};\theta_H,\gamma_i) \ge v_H^0$. When $[1+(1-\gamma_i)t]A+u_H^0 \le \theta_H \le \bar{\theta}_i^h$, $u(p_{H2};\theta_H,\gamma_i) \ge u_H^0$ and the consumer buys the Type-H automobile at the price A. Otherwise, if $\theta_H < [1+(1-\gamma_i)t]A+u_H^0$, then the consumer will not buy the Type-H automobile.

In summary, when $p_H \ge A$, the optimal retail price is given as

$$p_{H2}=\begin{cases} A, & \text{if } [1+(1-\gamma_i)t]A+u_H^0 \le \theta_H \le \bar{\theta}_i^h \\ & \text{and } \tilde{\theta}_i^L \le \theta_L \le \check{\theta}_i^L, \\ \kappa(\theta_H+\gamma_i tA-u_H^0)/(1+t) & \\ +(1-\kappa)\left[w_H+v_H^0/(1-T)\right], & \text{if } \bar{\theta}_i^h \le \theta_H \le \bar{\theta}_H. \end{cases} \tag{57}$$

To find the optimal retail price p_H^* for the constrained maximization problem in (50), we need to compare the optimal retail prices under the constraints that $p_H \le A$ and $p_H \ge A$. Note that $\hat{\theta}_i^h-\bar{\theta}_i^h=-\gamma_i t(1-\kappa)\left[A-w_H-v_H^0/(1-T)\right]/\kappa$. Thus, $\tilde{\theta}_i^h \le [1+(1-\gamma_i)t]A+u_H^0 \le \hat{\theta}_i^h \le \bar{\theta}_i^h$. In summary, when $u(p_L;\theta_L,\gamma_i)\ge 0$, i.e., $\theta_L \ge \tilde{\theta}_i^L=[1+(1-\gamma_i)t]w_L$, we consider the following five cases:

1. If $0 \le \theta_H < \tilde{\theta}_i^h$, then we find from the above analysis that the consumer will not buy a Type-H automobile.
2. If $\tilde{\theta}_i^h \le \theta_H \le [1+(1-\gamma_i)t]A+u_H^0$, then, under the constraint that $p_H \le A$, the retailer and the consumer choose the retail price as $\kappa\left(\theta_H-u_H^0\right)/[1+(1-\gamma_i)t]+(1-\kappa)\left[w_H+v_H^0/(1-T)\right]$, as indicated by (54). However, under the constraint that $p_H \ge A$, the two players do not complete the transaction. Thus, for this case, the consumer should buy from the retailer at the price $\tilde{p}_H \equiv \kappa\left(\theta_H-u_H^0\right)/[1+(1-\gamma_i)t]+(1-\kappa)\left[w_H+v_H^0/(1-T)\right]$.
3. If $[1+(1-\gamma_i)t]A+u_H^0 \le \theta_H \le \hat{\theta}_i^h$, then, under the constraint that $p_H \le A$, the retail price is $\tilde{p}_H$, as indicated by (54); and under the constraint that $p_H \ge A$, the retail price is A, as indicated by (57). Thus, the optimal retail price is $\tilde{p}_H$.
4. If $\hat{\theta}_i^h \le \theta_H \le \bar{\theta}_i^h$, then, as indicated by (54) and (57), the retail price is A under the constraint that $p_H \le A$ and A under the constraint that $p_H \ge A$. It thus follows that for $\theta \in [\hat{\theta}_i^h,\bar{\theta}_i^h]$, the optimal retail price is A.
5. If $\bar{\theta}_i^h \le \theta_H \le \bar{\theta}_H$, then we learn from (54) and (57) that the retail price is A under the constraint that $p_H \le A$, whereas the retail price is $\hat{p}_H \equiv \kappa(\theta_H+\gamma_i tA-u_H^0)/(1+t)+(1-\kappa)\left[w_H+v_H^0/(1-T)\right]$ under the constraint that $p_H \ge A$. Therefore, the optimal retail price for $\theta \in [\bar{\theta}_i^h,\bar{\theta}_H]$ is $\hat{p}_H$.

Case (ii): The negotiated retail price p_H when $u(p_L;\theta_L,\gamma_i)\le 0$

When $u(p_L;\theta_L,\gamma_i)\le 0$, the consumer will not buy a Type-L automobile because there is no negotiated retail price that can give the consumer a positive payoff. So the consumer's and the retailer's disagreement payoffs for the bargaining over the price p_H are $u_H^0=0$ and $v_H^0=0$, respectively. Then the negotiated retail price p_H can be obtained from the result in Case (i) by letting $u_H^0=v_H^0=0$. The sufficient condition for the case that $u(p_L;\theta_L,\gamma_i)\le 0$ is $\theta_L \le \tilde{\theta}_i^L=[1+(1-\gamma_i)t]w_L$.

$\boxed{\text{Part II}}$ The Negotiated Retail Price p_L for the Consumer who Prefers a Type-L Automobile

Similar to Part I analysis, when $u(p_H;\theta_H,\gamma_i)\ge 0$, i.e., $\theta_H \ge \tilde{\theta}_i^H \equiv [1+(1-\gamma_i)t]w_H$, the consumer's and the retailer's disagreement payoffs for the bargaining over p_L are $u_L^0=u(p_H^*;\theta_H,\gamma_i)$ and $v_L^0=(1-T)(p_H^*-w_H)$. We find four cases:

1. If $0 \le \theta_L < \tilde{\theta}_i^l \equiv \theta_H - [1 + (1-\gamma_i)t](w_H - w_L)$, then the consumer will not buy a Type-L automobile.
2. If $\tilde{\theta}_i^l \le \theta_L \le \hat{\theta}_i^l \equiv [1 + (1-\gamma_i)t]\left\{A - (1-\kappa)\left[w_L + v_L^0/(1-T)\right]\right\}/\kappa + u_L^0$, then the optimal retail price is $\tilde{p}_L \equiv \kappa\left(\theta_L - u_L^0\right)/[1 + (1-\gamma_i)t] + (1-\kappa)\left[w_L + v_L^0/(1-T)\right]$.
3. If $\hat{\theta}_i^l \le \theta_L \le \bar{\theta}_i^l \equiv \left\{A - (1-\kappa)\left[w_L + v_L^0/(1-T)\right]\right\}(1+t)/\kappa - \gamma_i t A + u_L^0$, then the optimal retail price is A.
4. If $\bar{\theta}_i^l \le \theta_L \le \bar{\theta}_L$, the optimal retail price is $\hat{p}_L \equiv \kappa(\theta_L + \gamma_i t A - u_L^0)/(1+t) + (1-\kappa)\left[w_L + v_L^0/(1-T)\right]$.

When $\theta_H \le \tilde{\theta}_i^H$, $u(p_H; \theta_H, \gamma_i) \le 0$, and $u_L^0 = v_L^0 = 0$. Then the negotiated retail price p_L can be obtained from the above result by letting $u_L^0 = v_L^0 = 0$. This theorem is thus proved. ■

Proof of Theorem 9. To find the retail price negotiated by the consumer and the retailer, we construct the GNB model as

$$\begin{array}{rl} \max_{\hat{p}_j} & \Lambda_j \equiv \left\{\theta_j - [1 + (1-\alpha)t]\hat{p}_j - \hat{u}_j^0\right\}^{1-\kappa}[(1-T)(\hat{p}_j - w_j) - \hat{v}_j^0]^\kappa \\ \text{s.t.} & \theta_j - [1 + (1-\alpha)t]\hat{p}_j \ge \hat{u}_j^0 \quad \text{and} \quad (1-T)(\hat{p}_j - w_j) \ge \hat{v}_j^0, \end{array}$$

Let $U \equiv \theta_j - [1 + (1-\alpha)t]\hat{p}_j - \hat{u}_j^0$ and $V \equiv (1-T)(\hat{p}_j - w_j) - \hat{v}_j^0$. Differentiating Λ_j once and twice w.r.t. $\hat{p}_j$, we have

$$\frac{d\Lambda_j}{d\hat{p}_j} = -[1 + (1-\alpha)t](1-\kappa)U^{-\kappa}V^\kappa + \kappa(1-T)U^{1-\kappa}V^{\kappa-1},$$

and

$$\frac{d^2\Lambda_j}{d\hat{p}_j^2} = -\kappa(1-\kappa)U^{-\kappa-1}V^{\kappa-2}\left\{[1 + (1-\alpha)t]V + (1-T)U\right\}^2 < 0.$$

Thus Λ_j is a concave function of $\hat{p}_j$. Using the argument in the proof of Theorem 8, we can then prove this theorem. ■

Appendix D Analytical Justifications for Remarks 1, 3, and 4

D.1 Justification for Remark 1

We provide our analytical justification for the results in Remark 1 as follows.

D.1.1 Changes in D, Π_M, and Π_R after the policy is implemented.

We examine the impact of the automobile tax reduction policy on the expected sales, the manufacturer's and the retailer's expected profits, and the average retail price in an automobile market segment. To do this, we need to compute the expected sales, the manufacturer's and the retailer's expected profits, and the average retail price before the policy is executed, which are denoted by D_0, Π_M^0, Π_R^0, and $\bar{\mu}^0$, respectively. Before the implementation of the policy, each consumer still bargains with the retailer over his or her retail price. Thus, we use (1) to develop the corresponding GNB model as

$$\begin{aligned} \max_{p_r} \quad & \Lambda_0 \equiv [\theta - (1+t)p_r]^{1-\kappa}[(1-T)(p_r - w)]^{\kappa} \\ \text{s.t.} \quad & \theta - (1+t)p_r \geq 0 \quad \text{and} \quad (1-T)(p_r - w) \geq 0. \end{aligned}$$

It is easy to find that the optimal retail price for the consumer with the valuation θ—which is denoted by p_r^0—is calculated as $p_r^0 = \kappa\theta/(1+t) + (1-\kappa)w$, and the consumer and the retailer successfully complete their transaction with the probability of $1 - F(\theta_0)$, where $\theta_0 \equiv (1+t)w$. As a result, we can calculate D_0, Π_M^0, Π_R^0, and $\bar{\mu}^0$ as

$$\begin{aligned} D_0 &= B\left(1 - F(\theta_0)\right), \\ \Pi_M^0 &= (1-T)(w-c)D^0 = B(1-T)(w-c)\left(1 - F(\theta_0)\right), \\ \Pi_R^0 &= B \times (1-T) \times \left[\int_{\theta_0}^{\bar{\theta}} (p_r^0 - w) f(\theta) d\theta\right], \\ \bar{\mu}^0 &= \mu^0(\gamma_i) = \left(\int_{\theta_0}^{\bar{\theta}} p_r^0 f(\theta) d\theta\right) \Big/ \left(1 - F(\theta_0)\right). \end{aligned}$$

Recall that the expected sales after the policy is executed are computed as $D = B \times \sum_{i=1}^{n+1}\left[\delta_i \times \left(1 - F(\tilde{\theta}_i)\right)\right]$. Since for $i \leq n$, $\tilde{\theta}_i = [1 + (1-\gamma_i)t]w < (1+t)w = \theta_0$, we find that,

$$D - D_0 = B\sum_{i=1}^{n+1}\left[\delta_i\left(1 - F(\tilde{\theta}_i)\right)\right] - B\left(1 - F(\theta_0)\right) = B\sum_{i=1}^{n}\left[\delta_i\left(F(\theta_0) - F(\tilde{\theta}_i)\right)\right] > 0.$$

Hence, $\Pi_M > \Pi_M^0 = (1-T)(w-c)D_0$. Next, we compare Π_R^0 and Π_R [which is given as in (5)]. Noting that for $i \leq n$ and any value of $\theta \in [0, \bar{\theta}]$, $\tilde{p}_r$ in (5) is greater than p_r^0, we find that when $\theta \in [\hat{\theta}_i, \bar{\theta}_i]$, $A > p_r^0$ because $\tilde{p}_r < A$ for $\theta \in [\hat{\theta}_i, \bar{\theta}_i]$; and when $\theta \in [\bar{\theta}_i, \bar{\theta}]$, $\hat{p}_r > p_r^0$ because $\tilde{p}_r < \hat{p}_r$ for $\theta \in [\bar{\theta}_i, \bar{\theta}]$. Therefore, we have

$$\begin{aligned} \Pi_R &> B(1-T)\sum_{i=1}^{n+1}\left\{\delta_i\left[\int_{\tilde{\theta}_i}^{\bar{\theta}} (p_r^0 - w) f(\theta) d\theta\right]\right\} \\ &> B(1-T)\sum_{i=1}^{n+1}\left\{\delta_i\left[\int_{\theta_0}^{\bar{\theta}} (p_r^0 - w) f(\theta) d\theta\right]\right\} \\ &= \Pi_R^0. \end{aligned}$$

D.1.2 Impact of the income tax rates γ_i and consumers' income tax rate distribution on D, Π_M, and Π_R.

It is obvious that D and Π_M are increasing in γ_i. We also find that

$$\frac{\partial \Pi_R}{\partial \gamma_i} = B(1-T)\delta_i \left\{ \int_{\tilde{\theta}_i}^{\hat{\theta}_i} \frac{\kappa\theta t}{[1+(1-\gamma_i)t]^2} f(\theta)d\theta + \frac{\kappa t A}{1+t}\left(1-F(\bar{\theta}_i)\right) \right\} > 0.$$

Thus, $D - D_0$, $\Pi_R - \Pi_R^0$, and $\Pi_M - \Pi_M^0$ are increasing in γ_i.

Note that δ_i is the probability that a consumer's income corresponds to the federal income tax rate γ_i, and the above proof has essentially shown that $D(\gamma_i) = B\left(1-F(\tilde{\theta}_i)\right)$, $\Pi_R(\gamma_i) = B(1-T)\left(1-F(\tilde{\theta}_i)\right)(\mu(\gamma_i)-w)$, and $\Pi_M(\gamma_i) = (1-T)(w-c)D(\gamma_i)$ are increasing in γ_i. Consider two income tax rate distributions $\{\delta_i\}$ and $\{\delta_i'\}$ such that their cumulative probabilities satisfy $\sum_{i=1}^m \delta_i \geq \sum_{i=1}^m \delta_i'$ or $1-\sum_{i=1}^m \delta_i \leq 1-\sum_{i=1}^m \delta_i'$ for $m = 1, 2, \ldots, n$ (i.e., the income with the tax rate distribution $\{\delta_i'\}$ is greater in the usual stochastic order, and $\{\delta_i\}$ is more skewed to the left). Compared with the distribution $\{\delta_i'\}$, $\{\delta_i\}$ has a larger probability for *small* value of income tax rate γ_i and a smaller probability for *large* value of γ_i. Because $D(\gamma_i)$, $\Pi_R(\gamma_i)$, and $\Pi_M(\gamma_i)$ are increasing in γ_i, we can obtain that

$$\begin{aligned}
D|_{\delta_i} &= \sum\nolimits_{i=1}^{n+1} (\delta_i D(\gamma_i)) < \sum\nolimits_{i=1}^{n+1} \left(\delta_i' D(\gamma_i)\right) = D|_{\delta_i'}, \\
\Pi_R|_{\delta_i} &= \sum\nolimits_{i=1}^{n+1} (\delta_i \Pi_R(\gamma_i)) < \sum\nolimits_{i=1}^{n+1} \left(\delta_i' \Pi_R(\gamma_i)\right) = \Pi_R|_{\delta_i'}, \\
\Pi_M|_{\delta_i} &= \sum\nolimits_{i=1}^{n+1} (\delta_i \Pi_M(\gamma_i)) < \sum\nolimits_{i=1}^{n+1} \left(\delta_i' \Pi_M(\gamma_i)\right) = \Pi_M|_{\delta_i'},
\end{aligned}$$

where $\{\delta_i\}$ and $\{\delta_i'\}$ can be treated as two different "weights." Our above conclusion must follow because for larger values of $D(\gamma_i)$, $\Pi_R(\gamma_i)$, and $\Pi_M(\gamma_i)$, the corresponding weight δ_i is smaller but the corresponding weight δ_i' is larger; and for sufficiently large values of $D(\gamma_i)$, $\Pi_R(\gamma_i)$, and $\Pi_M(\gamma_i)$, δ_i must be smaller than δ_i'. Therefore, the improvements $(D - D_0)$, $(\Pi_R - \Pi_R^0)$, and $(\Pi_M - \Pi_M^0)$ are larger when the income is greater in the usual stochastic order.

D.2 Justification for Remark 3

We learn from the proof of Theorem 3 that $\phi \geq 1$ if the following inequality is satisfied.

$$\eta = \sum\nolimits_{j\in\{H,L\}} \left[B_j \sum\nolimits_{i=1}^{n+1} \left(\delta_i^j \eta_j(\gamma_i)\right)\right] \geq \sum\nolimits_{j\in\{H,L\}} \left[B_j\left(1-F_j((1+t)w_j)\right)\left(\mu_0^j - c_j\right)\right], \qquad (58)$$

where $\eta_j(\gamma_i)$ is given in (47), and $\mu_0^j = \left\{\int_{(1+t)w_j}^{\bar{\theta}_j} [\kappa\theta/(1+t) + (1-\kappa_j)w_j] f_j(\theta)d\theta\right\} \Big/ \left(1-F_j((1+t)w_j)\right)$.

In our proof of Theorem 3, one of the conditions in (58) is derived by equating A to $\bar{A}_H$. From (49), we can obtain that

$$\begin{aligned}
\eta_L(\gamma_i)|_{A=\bar{A}_H} &= (1-\gamma_i t)\left[\int_{\tilde{\theta}_i^j}^{\bar{\theta}_j} \left(\frac{\kappa_j\theta}{1+(1-\gamma_i)t} + (1-\kappa_j)w_j\right) f_j(\theta)d\theta\right] - c_j\left(1-F_j(\tilde{\theta}_i^j)\right) \\
&> (1-\gamma_i t)\left[\int_{(1+t)w_j}^{\bar{\theta}_j} \left(\frac{\kappa_j\theta}{1+(1-\gamma_i)t} + (1-\kappa_j)w_j\right) f_j(\theta)d\theta\right] - c_j\left(1-F_j((1+t)w_j)\right) \\
&> (1-\gamma_i t)\left[\int_{(1+t)w_j}^{\bar{\theta}_j} \left(\frac{\kappa_j\theta}{1+t} + (1-\kappa_j)w_j\right) f_j(\theta)d\theta\right] - c_j\left(1-F_j((1+t)w_j)\right) \\
&= \left(1-F_j((1+t)w_j)\right)\left[(1-\gamma_i t)\mu_0^j - c_j\right],
\end{aligned}$$

and thus $\eta|_{A=\bar{A}_H} > \sum_{j\in\{H,L\}} \left\{B_j\left(1-F_j((1+t)w_j)\right)\left[\left(1-t\sum_{i=1}^{n}(\delta_i^j\gamma_i)\right)\mu_0^j - c_j\right]\right\}$.

Comparing the above inequality with (58), we find that, if $t\sum_{i=1}^{n}(\delta_i^j\times\gamma_i)$ is smaller, then $\eta|_{A=\bar{A}_H}$ is more likely to be greater than or equal to $\sum_{j\in\{H,L\}}\left[B_j\left(1-F((1+t)w_j)\right)(\mu_0^j - c_j)\right]$ and a cost-effective automobile tax reduction policy is more likely to exist. Moreover, since consumers with higher incomes are more likely to buy the high-end automobile, it generally holds that $\sum_{i=1}^{n}(\delta_i^L\times\gamma_i) < \sum_{i=1}^{n}(\delta_i^H\times\gamma_i)$. Given this fact, when B_L is much greater than B_H, a cost-effective policy is more likely to exist.

D.3 Justification for Remark 4

We first consider if the policy can increase the retail price for each consumer. Note from the proof of Remark 1 that $p_r^0 = \kappa\theta/(1+t) + (1-\kappa)w$, which must be smaller than $\hat{p}_r$. Then, we investigate the impact of the Chinese automobile tax reduction policy on the sales and the manufacturer's and the retailer's expected profits. From the proof of Remark 1, we find that when there is no tax reduction policy, the expected sales and the expected profits of the retailer and the manufacturer are computed as

$$D_0 = B\left(1-F(\theta_0)\right),\ \Pi_R^0 = B\kappa(1-T)\left(\frac{\displaystyle\int_{\theta_0}^{\bar{\theta}}\theta f(\theta)d\theta}{1+t} - w\right) \text{ and } \Pi_M^0 = B(1-T)(w-c)\left(1-F(\theta_0)\right),$$

where $\theta_0 = (1+t)w > [1+(1-\alpha)t]w = \hat{\theta}$. We then find that $1-F(\hat{\theta}) > 1-F(\theta_0)$ and thus, $\hat{D} > D_0$, $\hat{\Pi}_R > \Pi_R^0$ [because $\int_{\hat{\theta}}^{\bar{\theta}}\theta f(\theta)d\theta > \int_{\theta_0}^{\bar{\theta}}\theta f(\theta)d\theta$ and $1+(1-\alpha)t < 1+t$], and $\hat{\Pi}_M > \Pi_M^0$.

Appendix E Real Data-Based Numerical Experiments

Example 1 Suppose that the cutoff level for the U.S. policy is $A = \$49,500$ and the sales tax rate in the market $t = 5\%$. The retailer's bargaining power is $\kappa = 0.4$ (note that $\kappa \in [0,1]$). We assume consumers' valuations θ are normally distributed with the mean $E(\theta)$ and standard deviation σ_θ. Chen et al. [8] also considered a normal distribution for the consumers' valuations, and found that the standard deviations of consumers' valuations for three automobiles are between \$2,100 and \$6,600; thus we assume $\sigma_\theta = \$4,000$. As reported by `Edmunds.com` [13], the average retail price of a car or truck in the U.S. in 2009 was \$28,492, we can reasonably assume that the average valuation $E(\theta)$ of consumers is the average retail price \$28,492 plus the sales tax $5\% \times \$28,492 = \$1,424.6$. That is, $E(\theta) = \$29,916.6$. In addition, Albuquerque and Bronnenberg [1] calculated dealers' average gross margins as approximately \$1,600, based on individual car transaction data in the San Diego area (a location where most of the included car brands and types are in the medium- to high-end price segments). Accordingly, we assume that the retailer buys the automobile at the wholesale price $w = \$27,000$ (and gains an average margin of \$1,492). Let the maximum valuation $\bar{\theta} = \$55,000$. Since $\theta \in [0, \bar{\theta}]$, we truncate the normal distribution function at zero and at $\bar{\theta}$, and assume that the probability of negative values is added to that of zero and the probability of values greater than $\bar{\theta}$ is added to that of $\bar{\theta}$.

We denote the U.S. income tax rate distribution by P_i $(i = 1, \ldots, n+1)$, where $n = 5$ because, as online Table A indicates, there are five income brackets that correspond to five different federal tax rates for eligible consumers. Suppose $\delta_i = P_i$ $(i = 1, \ldots, n+1)$, i.e., the income tax rate distribution of consumers in the (generic) automobile market segment is the same as the U.S. income tax rate distribution. We find from (3) and (4) that, in order to calculate the expected retail price $\bar{\mu}$, we have to estimate the U.S. income tax rate distribution P_i $(i = 1, \ldots, n+1)$. Although we cannot get the data about the taxable income of each individual filer in the U.S., we can find from the tax statistics of Internal Revenue Service (IRS) [31] that the taxable incomes of single and joint filers in 2009 have mean values of \$25,597 and \$75,257, respectively, and their percentage distributions are given as in Panel B of online Table A. However, we still cannot obtain P_i $(i = 1, \ldots, 6)$, because the income brackets for the federal tax schedule (as given in Panel A of online Table A) are different from the income ranges for the statistics of taxable income distribution (as given in Panel B of Table A); moreover, they differ for the single and joint filers.

For the single filers, noting that each filer's taxable income I is a random variable, we should find the estimated probability density function $g_S(I)$ that best fits the single filers' percentage distribution $Perc_i^S$ $(i = 1, \ldots, 9)$ in Panel B of online Table A while assuring that the expected value of I equals the mean taxable income of single filers. The best-fitting distribution is a normal distribution with the mean $\mu_S = 25,597$ and standard deviation $\sigma_S = 28,440.11$. We then use the corresponding cumulative distribution function $G_S(I)$ to calculate the income tax rate distribution P_i^S $(i = 1, \ldots, 4)$ for the eligible single filers as follows: $P_1^S = N(8350) = 27.21\%$, $P_2^S = N(33950) - N(8350) = 34.34\%$, $P_3^S = N(82250) - N(33950) = 36.13\%$, and $P_4^S = N(135000) - N(82250) = 2.31\%$.

Similarly, for the joint filers, we find that the best-fitting distribution $g_J(I)$ for their taxable incomes is a normal distribution with the mean $\mu_J = 75,257$ and standard deviation $\sigma_J = 62,219.34$. We then use the corresponding cumulative distribution function $G_J(I)$ to calculate the income tax rate distribution P_i^J $(i = 1, \ldots, 5)$ for the eligible joint filers as follows: $P_1^J = N(16700) = 17.33\%$, $P_2^J = N(67900) - N(16700) = 27.96\%$, $P_3^J = N(137050) - N(67900) = 38.67\%$, $P_4^J = N(208850) - N(137050) = 14.44\%$, and $P_5^J = N(260000) - N(208850) = 1.44\%$.

From Panel B of online Table A, we learn that the proportion of single filers and that of joint filers are 57.57% and 42.43%, respectively. Thus the U.S. income tax rate distribution P_i $(i = 1, \ldots, 6)$ is calculated as: for the eligible filers, $P_1 = 23.02\%$, $P_2 = 31.63\%$, $P_3 = 37.21\%$, $P_4 = 7.46\%$, $P_5 = 0.61\%$, and the percentage of ineligible filers is $P_6 = 1 - \sum_{i=1}^{5} P_i = 0.07\%$. Using (3), we calculate the expected retail price as $\bar{\mu} = \$28,518.17$.

Similarly, if $\kappa = 0.6$ and other parameters stay the same, then the expected retail price is

$\bar{\mu} = \$29,277.26$. ■

Example 2 We use a numerical example to illustrate our analytical result to the extent that the sales, and the manufacturer's and the retailer's profits in an automobile market segment are increased after the policy is executed. We consider the same parameter values as in Example 1, and assume that the manufacturer's unit production cost is $c = \$23,000$, the market size is $B = 10,000$. Moreover, we assume the corporate tax rate is $T = 25\%$, because both China and the U.S. used the corporate tax rate of 25% in 2008 and 2009 (see Global Finance [17]). We use (6) to calculate the expected sales as $D = 6,752.45$, and use (5) and (7) to compute the retailer's and the manufacturer's expected profits as $\Pi_R = \$7.6886 \times 10^6$ and $\Pi_M = \$2.0257 \times 10^7$, respectively.

According to our analysis in the proof of Remark 1, we find that, before the tax reduction policy is implemented, the expected sales, and the retailer's and the manufacturer's expected profits are $6,523.42$, $\$7.1426 \times 10^6$, and $\$1.9570 \times 10^7$, respectively. After the policy is executed, the sales, the retailer's and the manufacturer's profits are increased by 3.51%, 7.64%, and 3.51%, respectively. ■

Example 3 To illustrate our statement in Remark 3 which regards the cost effectiveness of the U.S. policy, we use the same parameter values of A, t, and T as those in Example 2, and consider the following parameter values for both the Type-L and the Type-H automobiles: $B_L = 7,000$, $B_H = 3,000$, $w_L = \$24,000$, $w_H = \$34,000$, $c_L = \$20,000$, $c_H = \$29,000$, and $\kappa_L = \kappa_H = 0.4$. Moreover, the mean valuations of the Type-L and the Type-H automobiles are $E(\theta_L) = 26,000$ and $E(\theta_H) = 37,000$; $\sigma_\theta = 4,000$ for both automobiles; $\bar{\theta}_L = \$50,000$, and $\bar{\theta}_H = \$60,000$. Note that the manufacturer's profit margin from the Type-L automobile is $w_L - c_L = \$4,000$, which is lower than that from the Type-H automobile $w_H - c_H = \$5,000$. This is in line with the practical fact that a manufacturer generally gains a higher profit margin from a high-end automobile than that from a low-end automobile. For consumers in the Type-L automobile market segment, the income tax rate distribution is $\delta_1^L = P_1 + 2\% = 25.02\%$, $\delta_2^L = P_2 + 1.55\% = 33.18\%$, $\delta_3^L = P_3 - 1\% = 36.21\%$, $\delta_4^L = P_4 - 2\% = 5.46\%$, $\delta_5^L = P_5 - 0.5\% = 0.11\%$, and $\delta_6^L = 1 - \sum_{i=1}^{5} \delta_i^L = 0.02\%$. Because the income tax rate of all consumers in two market segments follows the distribution P_i $(i = 1, \ldots, 6)$ and the Type-L automobile consumers account for $B_L/(B_L + B_H) = 70\%$ of the total market, we can find the income tax rate distribution of consumers in the Type-H automobile market segment as $\delta_1^H = 18.35\%$, $\delta_2^H = 28.02\%$, $\delta_3^H = 39.54\%$, $\delta_4^H = 12.13\%$, $\delta_5^H = 1.78\%$, and $\delta_6^H = 0.18\%$. Using (14), we can find the profit-cost ratio of the U.S. policy as $\phi = 1.0506$, which means that, as the federal income tax is reduced by \$1, the after-tax profit of the automobile supply chain would be increased by \$1.0506.

In the above base case, we change the sizes of both market segments to $B_L = 6,000$ and $B_H = 4,000$, and the income tax rate distribution of Type-H automobile consumers is accordingly changed to $\delta_1^H = 10.5\%$, $\delta_2^H = 39.3\%$, $\delta_3^H = 31.4\%$, $\delta_4^H = 13.5\%$, and $\delta_5^H = 5.3\%$. The profit-cost ratio of the U.S. policy is calculated as $\phi = 1.0589$. This implies that the market share for each automobile type can affect the cost effectiveness of the U.S. tax reduction policy.

When we change the income tax rate distribution of the Type-L automobile consumers so that it is more skewed to the right: $\delta_1^L = 1.3 \times P_1 = 19.5\%$, $\delta_2^L = 1.2 \times P_2 = 55.5\%$, $\delta_3^L = 0.8 \times P_3 = 21.9\%$, $\delta_4^L = 0.3 \times P_4 = 2.5\%$, and $\delta_5^L = 1 - \sum_{i=1}^{4} \delta_i^L = 0.6\%$. Given $B_L = 7,000$ and $B_H = 3,000$, the income tax rate distribution of the Type-H automobile consumers is changed to $\delta_1^H = 4.5\%$, $\delta_2^H = 24.6\%$, $\delta_3^H = 40.1\%$, $\delta_4^H = 22.2\%$, and $\delta_5^H = 8.6\%$. We find that $\phi = 1.0519$.

We also change the standard deviation of consumers' valuations from the current value $4,000$ to $3,500$, we find that the profit-cost ratio is $\phi = 1.1361$. Therefore, the degree of consensus among consumers can be a key factor that affects the cost effectiveness of the U.S. policy. Later, we will perform a sensitivity analysis to investigate the impact of consumers' valuations on the cost effectiveness of the U.S. policy. ■

Example 4 We learn from the base case in Example 3 that the U.S. policy with the parameter values given in the base case is cost effective. We now determine a feasible target and a corresponding budget for the government. To set the feasible additional system-wide profit generated by the policy, we should solve the constrained nonlinear problem in (23) and find the maximum additional profit that can be achieved as $S^* = 1.3132 \times 10^6$. Assuming that the policy intends to increase the profit by $\tilde{S} = 1.300 \times 10^6$, which is smaller than S^*, we solve the equation that $S = \tilde{S}$ and find the cutoff level $\tilde{A} = \$35,380$. We then substitute $\tilde{A}$ into the government's expense function $X \equiv \sum_{j\in\{H,L\}}(B_j x_j) - TS/(1-T)$ and determine the budget $\tilde{X} = 1.2446 \times 10^6$ that can result in the targeted system-wide profit increase $\tilde{S} = 1.300 \times 10^6$ and the corresponding profit-cost ratio $\phi = 1.0445$. ■

Example 5 For the Chinese policy, we do not need to consider the consumer's income level and the income tax-related parameters as in Example 2. We assume that the original sales tax is $t = 10\%$ and the tax reduction percentage is $\alpha = 50\%$ or $\alpha = 25\%$. To compare the U.S. and the Chinese policies, we use the same automobile with its parameter values in Example 2 to investigate the effectiveness of the Chinese policy.

Using (27), we calculate the expected retail price under the Chinese policy in 2009 (i.e., $\alpha = 50\%$) as $\$28,459.89$. Moreover, we use (28) and (29) to compute the retailer's and the manufacturer's expected profits as $\hat{\Pi}_R = \$7.1426 \times 10^6$ and $\hat{\Pi}_M = \$1.9570 \times 10^7$, respectively. We can also find the expected sales as $B(1 - F(\hat{\theta})) = 6,523.42$. Moreover, we calculate the expected retail price, the expected sales, and the retailer's and the manufacturer's expected profits before the policy is executed as \$28189.65, 5215.92, $\$4.6538 \times 10^6$, and $\$1.5648 \times 10^7$, respectively. It thus follows that after the Chinese policy is implemented, the retail price, the sales, and the retailer's and the manufacturer's profits increase by 0.96%, 25.07%, 53.48%, and 25.07%, respectively.

Since the Chinese tax reduction percentage α was 25% in 2010, we calculate the resulting expected retail price, expected sales, and retailer's and manufacturer's expected profits as \$28316.48, 5881.93, $\$5.8076 \times 10^6$, and $\$1.7646 \times 10^7$, respectively. Comparing the above results with the results before the policy is executed, we find that the retail price, the sales, and the retailer's and the manufacturer's profits would be increased by 0.45%, 12.77%, 24.79%, and 12.77%, respectively.

Moreover, when we compare the results for the cases of $\alpha = 50\%$ and $\alpha = 25\%$, we find that as α is increased from 25% to 50%, the expected retail price, the sales, and the retailer's and the manufacturer's profits would be increased by 0.51%, 10.91%, 22.99%, and 10.91%, respectively. Note that the expected retail price $\hat{\mu}$ is increased as the value of α rises, even though the consumer demand is price elastic because $\hat{\theta}f(\hat{\theta})/(1 - F(\hat{\theta})) = 4.80 > 1$ at $\alpha = 25\%$ and $\hat{\theta}f(\hat{\theta})/(1 - F(\hat{\theta})) = 4.01 > 1$ at $\alpha = 50\%$.

Next, we provide a numerical example to demonstrate our result in Theorem 6 that $\hat{\mu}$ is increasing in α when the consumer demand is price inelastic. Assuming that the mean and standard deviation of consumer valuation θ are \$36,000 and \$4,000, respectively, we find that $\hat{\theta}f(\hat{\theta})/(1 - F(\hat{\theta})) = 0.47$ at $\alpha = 50\%$ and $\hat{\theta}f(\hat{\theta})/(1 - F(\hat{\theta})) = 0.66$ at $\alpha = 25\%$, which means that the demand is price inelastic. We then find that the expected retail price $\hat{\mu} = \$30,014.70$ when $\alpha = 50\%$ is greater than $\hat{\mu} = \$29,730.64$ when $\alpha = 25\%$, as indicated by Theorem 6. In another example, the mean and the standard deviation of consumer valuation θ are assumed to be $\$26,000$ and \$500, respectively. We find that $\hat{\theta}f(\hat{\theta})/(1 - F(\hat{\theta})) = 277.65$ at $\alpha = 50\%$ and $\hat{\theta}f(\hat{\theta})/(1 - F(\hat{\theta})) = 360.33$ at $\alpha = 25\%$, which implies that the demand is price elastic. The expected retail prices are then calculated as $\hat{\mu} = \$27,037.55$ when $\alpha = 50\%$ and $\hat{\mu} = \$27,404.10$ when $\alpha = 25\%$. The result reveals that, when the demand is significantly price elastic, the expected retail price will be decreasing in α. ■

Example 6 For the Chinese policy, we do not consider the consumer's income level and the income tax-related parameters in Example 3. We assume that the original sales tax is $t = 10\%$. We use the same two automobiles with the corresponding parameter values in Example 3 to investigate the cost effectiveness of the Chinese policy with $\alpha = 50\%$ (used in 2009) or $\alpha = 25\%$ (used in 2010). We first examine the policy with $\alpha = 50\%$. According to Theorem 7, we calculate $\tau_L^1 = 558.35$, $\tau_L^2 = -218.76$, $\tau_H^1 = 831.34$, and $\tau_H^2 = -230.41$. We find that, $-\tau_L^2/\tau_L^1 = 0.39$ (which is smaller than $\kappa_L = 0.4$) and $-\tau_H^2/\tau_H^1 = 0.28$ (which is smaller than $\kappa_H = 0.4$). Thus, the Chinese policy with $\alpha = 50\%$ should be cost effective. In fact, we can use (31) to calculate the system-wide expected profit increase as $\hat{S} = \$6.6056 \times 10^6$, and also find the government's expense as $\hat{X} = \$6.2672 \times 10^6$. Therefore, when $\alpha = 50\%$, the profit-cost ratio is $\hat{\phi} = \hat{S}/\hat{X} = 1.0540$. Similarly, when $\alpha = 25\%$, $\tau_L^1 = 263.70$, $\tau_L^2 = -72.90$, $\tau_H^1 = 384.42$, and $\tau_H^2 = -40.00$, which result in $-\tau_L^2/\tau_L^1 = 0.28 < \kappa_L$ and $-\tau_H^2/\tau_H^1 = 0.10 < \kappa_H$. Thus the Chinese policy with $\alpha = 25\%$ is also cost effective. Similar to the above, we can find that $\hat{S} = \$3.2280 \times 10^6$, $\hat{X} = \$2.6586 \times 10^6$, and $\hat{\phi} = \hat{S}/\hat{X} = 1.2142$.

If the retailer's bargaining power for the Type-j automobile is $\kappa_j = 0.20$ for $j = H, L$, then the Chinese policy is not cost effective when α is either 50% or 25%, because $\hat{\phi} = 0.8536$ when $\alpha = 50\%$ and $\hat{\phi} = 0.9889$ when $\alpha = 25\%$. However, when we set the tax reduction percentage α to be 20% (i.e., the sales tax rate is reduced by 20%), we find that $\tau_L^1 = 208.47$, $\tau_L^2 = -52.60$, $\tau_H^1 = 302.47$, and $\tau_H^2 = -20.32$. Since $\sum_{j\in\{H,L\}} B_j(\kappa_j \tau_j^1 + \tau_j^2) = 44,180 > 0$, according to Theorem 7, the Chinese policy with $\alpha = 20\%$ is cost effective with $\hat{\phi} = 1.0208$. ■

Example 7 Assuming that the tax reduction percentage α is unknown, we use other parameter values in Example 6 to determine a proper reduction percentage $\hat{\alpha}$ that results in the desired feasible targets and the corresponding budget. Using Corollary 1, we find the optimal tax reduction percentage α^* (maximizing both the additional sales ς^C and the profit increase S^C) as $\alpha^* = 0.603$, which means that, in order to assure the cost effectiveness of the Chinese policy, China should reduce its sales tax rate by *at most* 60.3% [that is, the *lowest* feasible sales tax rate should be $(1 - 60.3\%) \times 10\% = 3.97\%$]. Using $\alpha^* = 0.603$, we can calculate the corresponding maximum additional sales of the Type-H and the Type-L automobiles as 599.58 and 1001.63, and the maximum system-wide profit increase as $\$8.0270 \times 10^6$. Assuming that China attempts to target the feasible additional sales $\hat{\varsigma}_H^C = 400$ and $\hat{\varsigma}_L^C = 800$, we solve the equation that $\hat{\varsigma}_j = \hat{\varsigma}_j^C$ and find that $\hat{\alpha}_H^1 = 39.60\%$ and $\hat{\alpha}_L^1 = 47.96\%$, thus $\hat{\alpha}_1 = \max(\hat{\alpha}_H^1, \hat{\alpha}_L^1) = 47.96\%$ and the corresponding expense $X_1^C = \$5.9374 \times 10^6$. Similarly, we assume that the targeted profit increase is $\hat{S}^C = \$6.0 \times 10^6$, and find that $\hat{\alpha}_2 = 45.58\%$ and the corresponding expense $X_2^C = \$5.5605 \times 10^6$.

As discussed above, China should determine its budget as $\max(X_1^C, X_2^C) = \$5.9374 \times 10^6$, in order to achieve the targeted additional sales $\hat{\varsigma}_H^C = 400$ and $\hat{\varsigma}_L^C = 800$ as well as the targeted profit increase $\hat{S}^C = \$6.0 \times 10^6$. The corresponding tax reduction percentage is $\alpha = 47.96\%$, which results in the profit-cost ratio $\hat{\phi} = 1.0654$. ■

Example 8 We examine the impact of the U.S. tax reduction policy when the manufacturer is a strategic decision maker who may respond to the policy by changing the wholesale price.

We consider the same parameter values as in Example 2. Following the first two steps in Section 6.2, we find that the optimal wholesale price is $w = \$28300.24$. We then use (6) to calculate the expected sales as $D = 5462.70$, and use (5) and (7) to compute the retailer's and the manufacturer's expected profits as $\Pi_R = \$5.3021 \times 10^6$ and $\Pi_M = \$2.1715 \times 10^7$, respectively. Thus the supply chain profit is $\Pi = \$2.7017 \times 10^7$.

We can also find the optimal wholesale price before the policy is implemented by solving the equation in the second step in Section 6.2 with all $\gamma_i = 0$, which is $w_0 = \$28143.75$. Then, according to our analysis in Appendix D.1, we find that, before the policy is implemented, the expected sales are 5364.19, and the expected profits of the retailer, the manufacturer, and the supply chain are $\$5.1008 \times 10^6$, $\$2.0694 \times 10^7$, and $\$2.5795 \times 10^7$, respectively.

After the policy is executed, if the manufacturer fixes the wholesale price as w_0, we can calculate the sales as 5623.57, and the expected profits of the retailer, the manufacturer, and the supply chain as $\$5.5624 \times 10^6$, $\$2.1695 \times 10^7$, and $\$2.7257 \times 10^7$.

Comparing the above results, we find that a strategic manufacturer responds to the implementation of the U.S. policy by raising its wholesale price and gains a higher profit. As a result, although the U.S. policy can improve the sales and the profits of the retailer and the supply chain, the policy's impact is less than that when the manufacturer fixes its wholesale price. ■

Example 9 We examine the impact of the Chinese tax reduction policy when the manufacturer is a strategic decision maker who may respond to the policy by changing the wholesale price.

We consider the same parameter values as in Example 5 with the sales tax $t = 10\%$ and the tax reduction percentage $\alpha = 50\%$. Following the first two steps in Section 6.2, we find that the optimal wholesale price is $w = \$28143.75$. We then calculate the expected sales as $D = 5364.19$ and the retailer's and the manufacturer's expected profits as $\Pi_R = \$5.1008 \times 10^6$ and $\Pi_M = \$2.0694 \times 10^7$, respectively. Thus the supply chain profit is $\Pi = \$2.5795 \times 10^7$.

We can also find the optimal wholesale price before the policy is implemented by solving the equation in the second step in Section 6.2 with $\alpha = 0$, which is $w_0 = \$27380.02$. Then, according to our analysis in Appendix D.3, we find that, before the policy is implemented, the expected sales are 4799.19, and the expected profits of the retailer, the manufacturer, and the supply chain are $\$4.0829 \times 10^6$, $\$1.5765 \times 10^7$, and $\$1.9848 \times 10^7$, respectively.

After the policy is executed, if the manufacturer fixes the wholesale price as w_0, we can calculate the sales as 6148.16, and the expected profits of the retailer, the manufacturer, and the supply chain as $\$6.4202 \times 10^6$, $\$2.0197 \times 10^7$, and $\$2.6617 \times 10^7$.

Comparing the above results, we find that a strategic manufacturer responds to the implementation of the Chinese policy by raising the wholesale price and gains a higher profit. As a result, although the Chinese policy can improve the sales and the profits of the retailer and the supply chain, the policy's impact is less than that when the manufacturer fixes the wholesale price. ■

Appendix F Calculation of the Best-Fitting Distributions for the Taxable Incomes of U.S. Single and Joint Filers

To determine the joint federal income tax rate distribution P_i $(i = 1, \ldots, 6)$ for the single and joint filers in the U.S., we first find a probability distribution that best fits the percentage distribution of the U.S. single (or joint) filers' taxable incomes in Panel B of online Table A. Specifically, because each single (or joint) filer's taxable income I is a random variable, we can find the estimated probability density function $g_k(I)$ that best fits the percentage distribution $Perc_i^k$ ($k = S$ for single filers and $k = J$ for joint filers; $i = 1, \ldots, 9$) in Panel B of online Table A while assuring that the expected value of I equals the mean taxable income of the filers (i.e., μ_k). We do this by solving a nonlinear programming problem where the decision variables are the parameters of a tested density. We proceed as follows:

Step 1: We consider four commonly used distributions (gamma, lognormal, normal, Weibull). For each distribution, we solve the following minimization problem:

$$\min \Delta = \sum\nolimits_{i=1}^{9} \left[Perc_i^k - \int_{b_{i-1}}^{b_i} g_k(I)dI \right]^2, \quad \text{s.t.} \quad E(I) = \int_0^\infty I g_k(I)dI = \mu_k. \tag{59}$$

Here, the decision variables are the parameters of the fitting distribution; b_i $(i = 0, 1, \ldots, 9)$ denotes the taxable income ranges, $Perc_i^k$ ($k = S, J$, and $i = 1, \ldots, 9$) denotes the empirical percentage from Panel B of online Table A, and $\int_{b_{i-1}}^{b_i} g_k(I)dI$ means the estimated percentage of filers with taxable incomes in the range $[b_{i-1}, b_i]$ $(i = 1, \ldots, 9)$, where $k = S$ for single filers and $k = J$ for joint filers.

Step 2: We then compare the minimum sum of squared deviations obtained in Step 1 for all distributions and find the best-fitting distribution as the one with the smallest value of Δ.

Using the above two-step approach, we consider gamma, lognormal, normal, and Weibull distributions. For each distribution, we calculate the best-fitting parameter values and the minimum sum of squared deviations for single filers as shown in Table M. We find that the probability distribution that best fits the percentage distribution $Perc_i^S$ $(i = 1, \ldots, 9)$ provided by the U.S. Internal Revenue Service is the normal distribution with the mean $\mu_S = 25,597$ and the standard deviation $\sigma_S = 28,440.11$, which is rewritten as $\mathsf{Normal}(\mu_S, \sigma_S)$ or $\mathsf{Normal}(25,597, 28,440.11)$.

Table M: Single Filers: The best-fitting parameter values and the minimum sum of squared deviations for gamma, lognormal, normal, and Weibull distributions.

Distributions	**Best-Fitting Parameters**	**Minimum Sum of Squared Deviation**
Gamma(k_1, k_2)	Shape parameter: $k_1 = 25,730.40$ Scale parameter: $k_2 = 0.9948$	0.01909
Lognormal(μ, σ)	Mean of the variable I's logarithm: $\mu = 9.637$ Standard deviation of the variable I's logarithm: $\sigma = 1.013$	0.04291
Normal(μ, σ)	Mean of the variable I: $\mu = 25,597$ Standard deviation of the variable I: $\sigma = 28,440.11$	0.00242
Weibull(β, α)	Scale parameter: $\beta = 6,748.54$ Shape parameter: $\alpha = 25,599.19$	0.82377

Similarly, for each of the gamma, lognormal, normal, and Weibull distributions, we calculate the best-fitting parameter values and the minimum sum of squared deviations for the joint filers as shown in Table N. We find that the probability distribution that best fits the percentage distribution $Perc_i^J$ $(i = 1, \ldots, 9)$ provided by the U.S. Internal Revenue Service is the normal distribution with the mean $\mu_J = 75,257$ and the standard deviation $\sigma_J = 62,219.34$, which is rewritten as $\mathsf{Normal}(\mu_J,\sigma_J)$ or $\mathsf{Normal}(75,257,62,219.34)$.

Table N: Joint Filers: The best-fitting parameter values and the minimum sum of squared deviations for gamma, lognormal, normal, and Weibull distributions.

Distributions	**Best-Fitting Parameters**	**Minimum Sum of Squared Deviation**
Gamma(k_1, k_2)	Shape parameter: $k_1 = 69,270.24$ Scale parameter: $k_2 = 1.086$	0.04149
Lognormal(μ, σ)	Mean of the variable I's logarithm: $\mu = 11.015$ Standard deviation of the variable I's logarithm: $\sigma = 0.654$	0.07095
Normal(μ, σ)	Mean of the variable I: $\mu = 75,257$ Standard deviation of the variable I: $\sigma = 62,219.34$	0.01268
Weibull(β, α)	Scale parameter: $\beta = 4,330.29$ Shape parameter: $\alpha = 75,267.03$	0.82983

Appendix G Discussions on τ_j^1 and τ_j^2 in Theorem 7

We find from (34) that, in the first term of τ_j^1 (i.e., $(1-\alpha t)\int_{\hat{\theta}_j}^{\bar{\theta}_j}(\theta-\hat{\theta}_j)f_j(\theta)d\theta/[1+(1-\alpha)t]$), $\int_{\hat{\theta}_j}^{\bar{\theta}_j}(\theta-\hat{\theta}_j)f_j(\theta)d\theta$ is considered as a Type-j automobile consumer's expected *extra* (sales tax-inclusive) valuation over the minimum valuation $\hat{\theta}_j$ for the purchased Type-j automobile when the tax reduction policy is implemented. Since $(1-\alpha)t$ is the sales tax under the Chinese policy, we can regard $\int_{\hat{\theta}_j}^{\bar{\theta}_j}(\theta-\hat{\theta}_j)f_j(\theta)d\theta/[1+(1-\alpha)t]$ as the consumer's extra before-tax valuation over $\hat{\theta}_j$. Therefore, the first term of τ_j^1 in (34) can be explained as the consumer's extra before-tax valuation excluding the tax reduction resulting from the Chinese policy. Similarly, in (34), the second term [i.e., $\int_{\theta_0^j}^{\bar{\theta}_j}(\theta-\theta_0^j)f_j(\theta)d\theta/(1+t)$] means the consumer's extra before-tax valuation when the Chinese policy is not implemented. It thus follows that τ_j^1 can be regarded as the expected increase in the extra before-tax valuation of a Type-j automobile consumer that is generated by the Chinese policy.

According to (35), we find that the first term of τ_j^2 can be rewritten as $[(w_j-c_j)-\alpha t w_j](1-F_j(\hat{\theta}_j))$, where (w_j-c_j) and $\alpha t w_j$ represent the manufacturer's unit profit from the Type-j automobile and a consumer's tax reduction based on the wholesale price. We can thus regard the first term as the manufacturer's expected profit from a Type-j automobile when a consumer's tax reduction on the wholesale price w_j is not included. The second term (i.e., $(w_j-c_j)(1-F_j(\theta_0^j)))$ refers to the manufacturer's expected profit from a Type-j automobile when the policy is not implemented. Hence, τ_j^2 given in (35) can be considered as the manufacturer's tax reduction-exclusive profit increase resulting from a Type-j automobile under the Chinese policy.

Appendix H Proof of the Result for the Chinese Policy

We learn from the result and proof of Theorem 7 that the profit-cost ratio $\hat{\phi}$ is increasing in $\sum_{j\in\{H,L\}}[B_j \times (\kappa_j\tau_j^1 + \tau_j^2)]$. When consumers' valuations for the Type-j automobile satisfy a normal distribution with the mean μ_j and standard deviation σ ($f_j(\theta)$ and $F_j(\theta)$ denote the p.d.f. and c.d.f.), we can show that

$$\begin{aligned}\kappa_j\tau_j^1 + \tau_j^2 = {}& \kappa_j\sigma^2\left[\frac{1-\alpha t}{1+(1-\alpha)t}f_j(\hat{\theta}_j) - \frac{1}{1+t}f_j(\theta_0^j)\right] \\ & + \left[\kappa_j\frac{1-\alpha t}{1+(1-\alpha)t}\left(\mu_j - \hat{\theta}_j\right) + (1-\alpha t)w_j - c_j\right]\left(1 - F_j(\hat{\theta}_j)\right) \\ & - \left[w_j - c_j + \frac{\kappa_j}{1+t}\left(\mu_j - \theta_0^j\right)\right]\left(1 - F_j(\theta_0^j)\right).\end{aligned}$$

It then follows that

$$\begin{aligned}&\frac{\partial(\kappa_j\tau_j^1 + \tau_j^2)}{\partial\sigma} \\ &= \kappa_j\sigma\left[\frac{1-\alpha t}{1+(1-\alpha)t}f_j(\hat{\theta}_j)\left[\left(\tfrac{\hat{\theta}_j-\mu_j}{\sigma}\right)^2 + 1\right] - \frac{1}{1+t}f_j(\theta_0^j)\left[\left(\tfrac{\theta_0^j-\mu_j}{\sigma}\right)^2 + 1\right]\right] \\ &+ \frac{1}{\sigma}\left[(1-\alpha t)w_j - c_j + \kappa_j\frac{1-\alpha t}{1+(1-\alpha)t}\left(\mu_j - \hat{\theta}_j\right)\right]\left(f_j(\hat{\theta}_j)\right)^2\left[1 - \left(\tfrac{\hat{\theta}_j-\mu_j}{\sigma}\right)^2\right] \\ &- \frac{1}{\sigma}\left[w_j - c_j + \frac{\kappa_j}{1+t}\left(\mu_j - \theta_0^j\right)\right]\left(f_j(\theta_0^j)\right)^2\left[1 - \left(\tfrac{\theta_0^j-\mu_j}{\sigma}\right)^2\right].\end{aligned}$$

In the above, $\hat{\theta}_j = [1+(1-\alpha)t]w_j$ and $\theta_0^j = (1+t)w_j$; thus, $\hat{\theta}_j < \theta_0^j$ and $\hat{\theta}_j$ is decreasing in α. Because μ_j is approximately equal to the average after-tax retail price, it should be greater than θ_0^j. For reasonable values of parameters such that $\hat{\theta}_j \geq \mu_j - \sigma$,

$$\begin{aligned}&\frac{\partial}{\partial\alpha}\left\{\frac{1-\alpha t}{1+(1-\alpha)t}f_j(\hat{\theta}_j)\left[\left(\frac{\hat{\theta}_j-\mu_j}{\sigma}\right)^2 + 1\right]\right\} \\ &= \frac{tf_j(\hat{\theta}_j)}{1+(1-\alpha)t}\left\{w_j\frac{\hat{\theta}_j-\mu_j}{\sigma^2}(1-\alpha t)\left[\left(\tfrac{\hat{\theta}_j-\mu_j}{\sigma}\right)^2 - 1\right]\right. \\ &\left. - \frac{t}{1+(1-\alpha)t}\left[\left(\tfrac{\hat{\theta}_j-\mu_j}{\sigma}\right)^2 + 1\right]\right\} \\ &< 0,\end{aligned}$$

and

$$\begin{aligned}&\frac{\partial}{\partial\alpha}\left[(1-\alpha t)w_j - c_j + \kappa_j\frac{1-\alpha t}{1+(1-\alpha)t}\left(\mu_j - \hat{\theta}_j\right)\right] \\ &= -tw_j\left(1 - \kappa_j\frac{1-\alpha t}{1+(1-\alpha)t}\right) - \frac{t^2}{[1+(1-\alpha)t]^2}\kappa_j\left(\mu_j - \hat{\theta}_j\right) \\ &< 0.\end{aligned}$$

It is easy to show that $(f_j(\theta))^2\{1-[(\theta-\mu_j)/\sigma]^2\}$ is increasing in θ when $\theta < \mu_j$ and $\theta \geq \mu_j - \sigma$. We can find that $\partial(\kappa_j\tau_j^1 + \tau_j^2)/\partial\sigma < 0$, which indicates that $\sum_{j\in\{H,L\}}[B_j \times (\kappa_j\tau_j^1 + \tau_j^2)]$ and thus $\hat{\phi}$ are decreasing in σ.

Appendix I Analytical Details Regarding the Consumers' Choices Between Two Automobiles

I.1 The U.S. Tax Reduction Policy

We consider the negotiated retail prices for the two automobiles when a consumer can choose between the Type-H and the Type-L automobiles. For the negotiation results, we define relevant notation in Table O.

$u_j^0 \equiv \max\{u(p_k;\theta_k,\gamma_i),0\}$ and $v_j^0 \equiv \max\{(1-T)(p_k-w_k),0\}$	$j,k=H,L; j\neq k$
$\tilde{\theta}_i^j \equiv [1+(1-\gamma_i)t]w_j$	
$\hat{\theta}_i^j \equiv [A-(1-\kappa)w_j][1+(1-\gamma_i)t]/\kappa$	$j=H$ for Type-H automobile
$\bar{\theta}_i^j \equiv [A-(1-\kappa)w_j](1+t)/\kappa-\gamma_i tA$	and $j=L$ for Type-L automobile
$\tilde{\theta}_i^h \equiv \theta_L+[1+(1-\gamma_i)t](w_H-w_L)$	
$\tilde{\theta}_i^l \equiv \theta_H-[1+(1-\gamma_i)t](w_H-w_L)$	
$\hat{\theta}_i^h \equiv [1+(1-\gamma_i)t]\left[A-(1-\kappa)w_H\right]/\kappa$	
$\hat{\theta}_i^l \equiv [1+(1-\gamma_i)t]\left[A-(1-\kappa)w_L\right]/\kappa$	
$\check{\theta}_i^H \equiv [1+(1-\gamma_i)t]\left[A-(w_L-\kappa w_H)\right]/\kappa$	
$\check{\theta}_i^L \equiv [1+(1-\gamma_i)t]\left[A-(w_H-\kappa w_L)\right]/\kappa$	
$\bar{\theta}_i^k \equiv \left\{A-(1-\kappa)\left[w_j+v_j^0/(1-T)\right]\right\}(1+t)/\kappa-\gamma_i tA+u_j^0$	$k=h$ and $j=H$ for Type-H automobile $k=l$ and $j=L$ for Type-L automobile

Table O: List of notations for the negotiated retail prices under the U.S. policy.

Theorem 8 Given the two automobiles' wholesale prices (w_H, w_L) and the U.S. tax reduction policy with the cutoff level A, the consumer with the valuation θ_j for the Type-j automobile ($j=H,L$) and a federal income tax rate $\gamma_i \in [0,1]$ negotiates with the retailer. The negotiated Type-H automobile retail price p_H^* and Type-L automobile retail price p_L^* are computed as follows.

1. If $(\theta_H,\theta_L) \in \mathbf{\Omega}_0 \equiv \{(\theta_H,\theta_L) : \theta_H < \tilde{\theta}_i^H, \theta_L < \tilde{\theta}_i^L\}$, then the consumer does not buy any automobile.
2. If $\theta_H \geq \tilde{\theta}_i^H$, then the consumer may decide to buy a Type-H automobile. The price p_H^* is shown in Table P.

Case	Conditions	Negotiated Retail Price p_H^*
1	$(\theta_H,\theta_L)\equiv\mathbf{\Omega}_{H1}\equiv\{(\theta_H,\theta_L):\tilde{\theta}_i^h\leq\theta_H\leq\hat{\theta}_i^h,\tilde{\theta}_i^L\leq\theta_L\leq\check{\theta}_i^L$ $\mid u_H^0=\theta_L-[1+(1-\gamma_i)t]\,\tilde{p}_L, v_H^0=(1-T)(\tilde{p}_L-w_L)\}$	$p_H^*=p_{H1}^*\equiv\kappa\left(\theta_H-u_H^0\right)/[1+(1-\gamma_i)t]$ $+(1-\kappa)\left[w_H+v_H^0/(1-T)\right]$
2	$(\theta_H,\theta_L)\equiv\mathbf{\Omega}_{H2}\equiv\{(\theta_H,\theta_L):\hat{\theta}_i^h\leq\theta_H\leq\bar{\theta}_i^h,\tilde{\theta}_i^L\leq\theta_L\leq\hat{\theta}_i^L$ $\mid u_H^0=\theta_L-[1+(1-\gamma_i)t]\,\tilde{p}_L, v_H^0=(1-T)(\tilde{p}_L-w_L)\}$	$p_H^*=p_{H2}^*\equiv A$
3	$(\theta_H,\theta_L)\equiv\mathbf{\Omega}_{H3}\equiv\{(\theta_H,\theta_L):\bar{\theta}_i^h\leq\theta_H\leq\bar{\theta}_H,\tilde{\theta}_i^L\leq\theta_L\leq\hat{\theta}_i^L$ $\mid u_H^0=\theta_L-[1+(1-\gamma_i)t]\,\tilde{p}_L, v_H^0=(1-T)(\tilde{p}_L-w_L)\}$	$p_H^*=p_{H3}^*\equiv\kappa(\theta_H+\gamma_i tA-u_H^0)/(1+t)$ $+(1-\kappa)\left[w_H+v_H^0/(1-T)\right]$
4	$(\theta_H,\theta_L)\equiv\mathbf{\Omega}_{H4}\equiv\{(\theta_H,\theta_L):\bar{\theta}_i^h\leq\theta_H\leq\bar{\theta}_H,\hat{\theta}_i^L\leq\theta_L\leq\bar{\theta}_i^L$ $\mid u_H^0=\theta_L-[1+(1-\gamma_i)t]\,A, v_H^0=(1-T)(A-w_L)\}$	$p_H^*=p_{H4}^*\equiv\kappa(\theta_H+\gamma_i tA-u_H^0)/(1+t)$ $+(1-\kappa)\left[w_H+v_H^0/(1-T)\right]$
5	$(\theta_H,\theta_L)\equiv\mathbf{\Omega}_{H5}\equiv\{(\theta_H,\theta_L):\bar{\theta}_i^h\leq\theta_H\leq\bar{\theta}_H,\bar{\theta}_i^L\leq\theta_L\leq\bar{\theta}_L$ $\mid u_H^0=\theta_L-(1+t)\hat{p}_L+\gamma_i tA, v_H^0=(1-T)(\hat{p}_L-w_L)\}$	$p_H^*=p_{H5}^*\equiv\kappa(\theta_H+\gamma_i tA-u_H^0)/(1+t)$ $+(1-\kappa)\left[w_H+v_H^0/(1-T)\right]$
6	$(\theta_H,\theta_L)\equiv\mathbf{\Omega}_{H6}\equiv\{(\theta_H,\theta_L):\tilde{\theta}_i^H\leq\theta_H\leq\hat{\theta}_i^H,\theta_L<\tilde{\theta}_i^L\}$	$p_H^*=p_{H6}^*\equiv\kappa\theta_H/[1+(1-\gamma_i)t]$ $+(1-\kappa)w_H$
7	$(\theta_H,\theta_L)\equiv\mathbf{\Omega}_{H7}\equiv\{(\theta_H,\theta_L):\hat{\theta}_i^H\leq\theta_H\leq\bar{\theta}_i^H,\theta_L<\tilde{\theta}_i^L\}$	$p_H^*=p_{H7}^*\equiv A$
8	$(\theta_H,\theta_L)\equiv\mathbf{\Omega}_{H8}\equiv\{(\theta_H,\theta_L):\bar{\theta}_i^H\leq\theta_H\leq\bar{\theta}_H,\theta_L<\tilde{\theta}_i^L\}$	$p_H^*=p_{H8}^*\equiv\kappa(\theta_H+\gamma_i tA)/(1+t)$ $+(1-\kappa)w_H$

Table P: The negotiated retail price p_H^* for the consumer who buys a Type-H automobile.

3. If $\theta_L \geq \tilde{\theta}_i^L$, then the consumer may decide to buy a Type-L automobile. The price p_L^* is shown in Table Q.

■

Using Theorem 8, we can derive the expected sales for the Type-j automobile ($j=H,L$) as,

$$D_j = B\left\{\sum_{i=1}^{n+1}\delta_i\left[\sum_{k=1}^{8}\left(\iint_{\mathbf{\Omega}_{jk}} f(\theta_H,\theta_L)d\theta_H d\theta_L\right)\right]\right\}. \tag{60}$$

Case	Conditions	Negotiated Retail Price p_L^*
1	$(\theta_H,\theta_L)\equiv\mathbf{\Omega}_{L1}\equiv\{(\theta_H,\theta_L):\tilde{\theta}_i^H\le\theta_H\le\check{\theta}_i^H,\tilde{\theta}_i^l\le\theta_L\le\hat{\theta}_i^l$ $\mid u_L^0=\theta_H-[1+(1-\gamma_i)t]\,\tilde{p}_H, v_L^0=(1-T)(\tilde{p}_H-w_H)\}$	$p_L^*=p_{L1}^*\equiv\kappa\left(\theta_L-u_L^0\right)/[1+(1-\gamma_i)t]$ $+(1-\kappa)\left[w_L+v_L^0/(1-T)\right]$
2	$(\theta_H,\theta_L)\equiv\mathbf{\Omega}_{L2}\equiv\{(\theta_H,\theta_L):\tilde{\theta}_i^H\le\theta_H\le\hat{\theta}_i^H,\hat{\theta}_i^l\le\theta_L\le\bar{\theta}_i^l$ $\mid u_L^0=\theta_H-[1+(1-\gamma_i)t]\,\tilde{p}_H, v_L^0=(1-T)(\tilde{p}_H-w_H)\}$	$p_L^*=p_{L2}^*\equiv A$
3	$(\theta_H,\theta_L)\equiv\mathbf{\Omega}_{L3}\equiv\{(\theta_H,\theta_L):\tilde{\theta}_i^H\le\theta_H\le\hat{\theta}_i^H,\bar{\theta}_i^l\le\theta_L\le\bar{\theta}_L$ $\mid u_L^0=\theta_H-[1+(1-\gamma_i)t]\,\tilde{p}_H, v_L^0=(1-T)(\tilde{p}_H-w_H)\}$	$p_L^*=p_{L3}^*\equiv\kappa(\theta_L+\gamma_itA-u_L^0)/(1+t)$ $+(1-\kappa)\left[w_L+v_L^0/(1-T)\right]$
4	$(\theta_H,\theta_L)\equiv\mathbf{\Omega}_{L4}\equiv\{(\theta_H,\theta_L):\hat{\theta}_i^H\le\theta_H\le\bar{\theta}_i^H,\bar{\theta}_i^l\le\theta_L\le\bar{\theta}_L$ $\mid u_L^0=\theta_H-[1+(1-\gamma_i)t]\,A, v_L^0=(1-T)(A-w_H)\}$	$p_L^*=p_{L4}^*\equiv\kappa(\theta_L+\gamma_itA-u_L^0)/(1+t)$ $+(1-\kappa)\left[w_L+v_L^0/(1-T)\right]$
5	$(\theta_H,\theta_L)\equiv\mathbf{\Omega}_{L5}\equiv\{(\theta_H,\theta_L):\bar{\theta}_i^H\le\theta_H\le\bar{\theta}_H,\bar{\theta}_i^l\le\theta_L\le\bar{\theta}_L$ $\mid u_L^0=\theta_H-(1+t)\hat{p}_H+\gamma_itA, v_L^0=(1-T)(\hat{p}_H-w_H)\}$	$p_L^*=p_{L5}^*\equiv\kappa(\theta_L+\gamma_itA-u_L^0)/(1+t)$ $+(1-\kappa)\left[w_L+v_L^0/(1-T)\right]$
6	$(\theta_H,\theta_L)\equiv\mathbf{\Omega}_{L6}\equiv\{(\theta_H,\theta_L):\theta_H<\tilde{\theta}_i^H,\tilde{\theta}_i^L\le\theta_L\le\hat{\theta}_i^L\}$	$p_L^*=p_{L6}^*\equiv\kappa\theta_L/[1+(1-\gamma_i)t]$ $+(1-\kappa)w_L$
7	$(\theta_H,\theta_L)\equiv\mathbf{\Omega}_{L7}\equiv\{(\theta_H,\theta_L):\theta_H<\tilde{\theta}_i^H,\hat{\theta}_i^L\le\theta_L\le\bar{\theta}_i^L\}$	$p_L^*=p_{L7}^*\equiv A$
8	$(\theta_H,\theta_L)\equiv\mathbf{\Omega}_{L8}\equiv\{(\theta_H,\theta_L):\theta_H<\tilde{\theta}_i^H,\bar{\theta}_i^L\le\theta_L\le\bar{\theta}_L\}$	$p_L^*=p_{L8}^*\equiv\kappa(\theta_L+\gamma_itA)/(1+t)$ $+(1-\kappa)w_L$

Table Q: The negotiated retail price p_L^* for the consumer who buys a Type-L automobile.

The total demand for the two automobiles is $D\equiv D_H+D_L$. Then, we can compute the manufacturer's and the retailer's expected profits. Specifically, the manufacturer's and the retailer's expected profits are obtained as,

$$\Pi_M=(1-T)(w-c)D, \tag{61}$$

and

$$\Pi_R=B(1-T)\sum\nolimits_{j=H,L}\left\{\sum\nolimits_{i=1}^{n+1}\delta_i\left[\sum\nolimits_{k=1}^{8}\left(\iint_{\mathbf{\Omega}_{jk}}(p_{jk}^*-w_j)f(\theta_H,\theta_L)d\theta_Hd\theta_L\right)\right]\right\}. \tag{62}$$

It follows that the system-wide expected profit is

$$\Pi=\Pi_M+\Pi_R,$$

and the system-wide profit increase is computed as

$$S=\Pi-\Pi_0,$$

where Π_0 is obtained from Π by letting $\gamma_i=0$ for $i=1,\ldots,n$.

Similar to Section 3.2, we can compute the government's net expense as $X\equiv B\sum_{j\in\{H,L\}}x_j-TS/(1-T)$, where

$$x_j\equiv t\sum\nolimits_{i=1}^{n+1}\delta_i\gamma_i\left[\sum\nolimits_{k=1}^{8}\left(\iint_{\mathbf{\Omega}_{jk}}p_{jk}^*f(\theta_H,\theta_L)d\theta_Hd\theta_L\right)\right].$$

Then, the profit-cost ratio can be calculated as

$$\phi\equiv S/X=(1-T)S\bigg/\left[B(1-T)\sum\nolimits_{j\in\{H,L\}}x_j-TS\right].$$

I.2 The Chinese Tax Reduction Policy

Similar to Section I.1, for the Chinese policy, we can derive the negotiated retail price $\hat{p}_j^*$ when the consumer chooses to buy a Type-j automobile as follows. For the negotiation results, we define relevant notation in Table R.

Theorem 9 Given the two automobiles' wholesale prices (w_H,w_L) and the Chinese tax reduction policy with the reduction percentage α, the consumer with the valuation θ_j for the Type-j auto-

$\hat{u}(\hat{p}_k;\theta_k) \equiv (1-\kappa)\{\theta_k - [1+(1-\alpha)t]w_k\}$	$k = H, L$
$\hat{u}_j^0 \equiv \max\{\hat{u}(\hat{p}_k;\theta_k), 0\}$ and $\hat{v}_j^0 \equiv \max\{(1-T)(\hat{p}_k - w_k), 0\}$	$j, k = H, L; j \neq k$
$\hat{\theta}_j \equiv [1+(1-\alpha)t]w_j$	$j = H, L$
$\hat{\theta}_h \equiv \theta_L + [1+(1-\alpha)t](w_H - w_L)$	
$\hat{\theta}_l \equiv \theta_H - [1+(1-\alpha)t]\,(w_H - w_L)$	

Table R: List of notations for the negotiated retail prices under the Chinese policy.

mobile ($j = H, L$) negotiates with the retailer. The negotiated Type-H automobile retail price $\hat{p}_H^*$ and Type-L automobile retail price $\hat{p}_L^*$ are computed as follows.

1. If $(\theta_H, \theta_L) \in \hat{\mathbf{\Omega}}_0 \equiv \{(\theta_H, \theta_L) : \theta_H < \hat{\theta}_H, \theta_L < \hat{\theta}_L\}$, then the consumer does not buy any automobile.
2. If $\theta_H \geq \hat{\theta}_H$, then the consumer may decide to buy a Type-H automobile. The price $\hat{p}_H^*$ is shown in Table S.

Case	Conditions	Negotiated Retail Price $\hat{p}_H^*$
1	$(\theta_H, \theta_L) \equiv \hat{\mathbf{\Omega}}_{H1} \equiv \{(\theta_H, \theta_L) : \hat{\theta}_h \leq \theta_H \leq \bar{\theta}_H, \hat{\theta}_L \leq \theta_L \leq \bar{\theta}_L\}$	$\hat{p}_H^* = \frac{\kappa\theta_H}{1+(1-\alpha)t} + (1-\kappa)w_H$
2	$(\theta_H, \theta_L) \equiv \hat{\mathbf{\Omega}}_{H2} \equiv \{(\theta_H, \theta_L) : \hat{\theta}_H \leq \theta_H \leq \bar{\theta}_H, \theta_L < \hat{\theta}_L\}$	$\hat{p}_H^* = \frac{\kappa\theta_H}{1+(1-\alpha)t} + (1-\kappa)w_H$

Table S: The negotiated retail price $\hat{p}_H^*$ for the consumer who buys a Type-H automobile.

3. If $\theta_L \geq \hat{\theta}_L$, then the consumer may decide to buy a Type-L automobile. The price $\hat{p}_L^*$ is shown in Table T.

Case	Conditions	Negotiated Retail Price $\hat{p}_L^*$
1	$(\theta_H, \theta_L) \equiv \hat{\mathbf{\Omega}}_{L1} \equiv \{(\theta_H, \theta_L) : \hat{\theta}_H \leq \theta_H \leq \bar{\theta}_H, \hat{\theta}_l \leq \theta_L \leq \bar{\theta}_L\}$	$\hat{p}_L^* = \frac{\kappa\theta_L}{1+(1-\alpha)t} + (1-\kappa)w_L$
2	$(\theta_H, \theta_L) \equiv \hat{\mathbf{\Omega}}_{L2} \equiv \{(\theta_H, \theta_L) : \theta_H < \hat{\theta}_H, \hat{\theta}_L \leq \theta_L \leq \bar{\theta}_L\}$	$\hat{p}_L^* = \frac{\kappa\theta_L}{1+(1-\alpha)t} + (1-\kappa)w_L$

Table T: The negotiated retail price $\hat{p}_L^*$ for the consumer who buys a Type-L automobile.

■

Using Theorem 9, we can derive the expected sales for the Type-j automobile ($j = H, L$) as,

$$\hat{D}_j = B\left[\sum_{k=1}^{2}\left(\iint_{\hat{\mathbf{\Omega}}_{jk}} f(\theta_H, \theta_L)d\theta_H d\theta_L\right)\right].$$

The total demand for the two automobiles is $\hat{D} \equiv \hat{D}_H + \hat{D}_L$. Then, we can compute the manufacturer's and the retailer's expected profits. Specifically, the manufacturer's and the retailer's expected profits are obtained as,

$$\hat{\Pi}_M = (1-T)(w-c)\hat{D},$$

and

$$\hat{\Pi}_R = B(1-T)\sum_{j=H,L}\left[\sum_{k=1}^{2}\left(\iint_{\hat{\mathbf{\Omega}}_j} (\hat{p}_{jk}^* - w_j)f(\theta_H, \theta_L)d\theta_H d\theta_L\right)\right].$$

It follows that the system-wide expected profit is

$$\hat{\Pi} = \hat{\Pi}_M + \hat{\Pi}_R,$$

and the system-wide profit increase is computed as

$$\hat{S} = \hat{\Pi} - \hat{\Pi}_0,$$

where $\hat{\Pi}_0$ is obtained from $\hat{\Pi}$ by letting $\alpha = 0$.

Similar to Section 4.2, we can compute the government's net expense as $\hat{X} = B \sum_{j \in \{H,L\}} \hat{x}_j - T\hat{S}/(1-T)$, where

$$\hat{x}_j \equiv \alpha t \sum_{k=1}^{2} \left(\iint_{\hat{\boldsymbol{\Omega}}_j} \hat{p}^*_{jk} f(\theta_H, \theta_L) d\theta_H d\theta_L \right).$$

Then, the profit-cost ratio can be calculated as

$$\hat{\phi} = \hat{S}/\hat{X} = (1-T)\hat{S} \Big/ \left[B(1-T) \sum_{j \in \{H,L\}} \hat{x}_j - T\hat{S} \right].$$

I.3 Sensitivity Analysis

Similar to our sensitivity analysis in Section 5.2 for the base case of two separate automobile market segments, we conduct a sensitivity analysis for the case of consumers' choices between the two automobiles. To compare the effectiveness of the U.S. and the Chinese tax reduction policies, we examine the impact of the decision variables of two policies (i.e., A and α), the sales tax rate t, the mean and standard deviation of consumers' valuations for each automobile ($E(\theta_j)$ and σ_j), and the correlation between θ_H and θ_L (i.e., ρ). Instead of considering two market segments, we now assume that the two automobiles are sold to a market with the size $B = 10,000$. Most of the parameter values are the same as those used in Section 5.2. We assume $E(\theta_H) = \$36,000$, $E(\theta_L) = \$26,000$, $\sigma_H = \sigma_L = 4,000$, $\rho = 0.5$, $w_H = \$35,000$, and $w_L = \$24,000$. To investigate the impact of ρ, we increase its value from 0 to 0.9 in increments of 0.1. Our sensitivity analysis results are presented below in Tables U–Y.

Table U: Consumers' Choices Between Two Automobiles: In Panel A, we calculate the expected sales of each type of automobile (D_H and D_L), the retailer's and the manufacturer's expected profits (Π_R and Π_M), the system-wide expected profit (Π), and the profit-cost ratio (ϕ) for the U.S. and the Chinese policies in the current U.S. setting. In Panel B, we provide the results for the sensitivity analysis on the cutoff level A (for the U.S. policy only). Note that $A > w_H$. In Panel C, we provide the results for the sensitivity analysis on the tax reduction percentage α (for the Chinese policy only).

Panel A

Policy	Sales of Type-H Automobile	Sales of Type-L Automobile	Retailer's Profit ($\times 10^7$)	Manufacturer's Profit ($\times 10^7$)	System-wide Profit ($\times 10^7$)	Profit-Cost Ratio
U.S.	2421.72	4621.92	0.77230	2.47635	3.24865	1.1300
Chinese	2421.57	4622.59	0.77210	2.47648	3.24858	1.1376

Panel B

	Impact of the U.S. Policy					
A	D_H	D_L	Π_R ($\times 10^7$)	Π_M ($\times 10^7$)	Π ($\times 10^7$)	ϕ
37000	1904.50	4621.92	0.66151	2.24360	2.90511	0.7292
39500	2410.54	4621.92	0.76815	2.47132	3.23947	1.1059
42000	2421.70	4621.92	0.77229	2.47634	3.24863	1.1299
44500	2421.72	4621.92	0.77230	2.47635	3.24865	1.1300
47000	2421.72	4621.92	0.77230	2.47635	3.24865	1.1300
49500	2421.72	4621.92	0.77230	2.47635	3.24865	1.1300
52000	2421.72	4621.92	0.77230	2.47635	3.24865	1.1300
54500	2421.72	4621.92	0.77230	2.47635	3.24865	1.1300
57000	2421.72	4621.92	0.77230	2.47635	3.24865	1.1300

Panel C

	Impact of the Chinese Policy					
α	$\hat{D}_H$	$\hat{D}_L$	$\hat{\Pi}_R$ ($\times 10^7$)	$\hat{\Pi}_M$ ($\times 10^7$)	$\hat{\Pi}$ ($\times 10^7$)	$\hat{\phi}$
10%	2344.02	4584.50	0.74486	2.43016	3.17502	1.1657
18%	2415.80	4619.85	0.77006	2.47307	3.24313	1.1396
18.64%	2421.57	4622.59	0.77210	2.47648	3.24858	1.1376
25%	2478.98	4649.19	0.79258	2.51030	3.30288	1.1177
26%	2488.03	4653.25	0.79584	2.51559	3.31143	1.1146
34%	2560.63	4684.71	0.82220	2.55770	3.37990	1.0907
42%	2633.56	4714.18	0.84915	2.59936	3.44851	1.0678
50%	2706.74	4741.66	0.87669	2.64053	3.51722	1.0459
58%	2780.13	4767.13	0.90482	2.68120	3.58602	1.0250
66%	2853.60	4790.58	0.93355	2.72132	3.65486	1.0049
74%	2927.25	4812.02	0.96287	2.76087	3.72373	0.9857

Table V: Consumers' Choices Between Two Automobiles: A sensitivity analysis on the sales tax rate t. Each number inside the brackets is the percentage change in the value (which is given outside the brackets) after a tax reduction policy is implemented.

	Impact of the U.S. Policy					
t	D_H	D_L	Π_R ($\times 10^7$)	Π_M ($\times 10^7$)	Π ($\times 10^7$)	ϕ
1%	3016.77 (1.150%)	4835.32 (0.177%)	0.9993 (1.430%)	2.8081 (0.645%)	3.8075 (0.850%)	0.9684
2%	2867.04 (2.449%)	4794.52 (0.447%)	0.9389 (2.952%)	2.7285 (1.384%)	3.6674 (1.781%)	1.0069
3%	2717.62 (3.912%)	4745.30 (0.814%)	0.8809 (4.574%)	2.6465 (2.222%)	3.5274 (2.800%)	1.0468
4%	2569.00 (5.554%)	4687.73 (1.282%)	0.8254 (6.301%)	2.5624 (3.166%)	3.3877 (3.913%)	1.0879
5%	2421.72 (7.392%)	4621.92 (1.857%)	0.7723 (8.138%)	2.4763 (4.221%)	3.2486 (5.126%)	1.1300
6%	2276.30 (9.445%)	4548.07 (2.543%)	0.7217 (10.092%)	2.3888 (5.393%)	3.1104 (6.447%)	1.1729
7%	2133.27 (11.731%)	4466.45 (3.346%)	0.6734 (12.170%)	2.2999 (6.688%)	2.9733 (7.882%)	1.2161
8%	1993.16 (14.274%)	4377.37 (4.270%)	0.6275 (14.377%)	2.2101 (8.111%)	2.8376 (9.437%)	1.2595
9%	1856.45 (17.098%)	4281.24 (5.320%)	0.5839 (16.721%)	2.1198 (9.667%)	2.7037 (11.118%)	1.3027
10%	1723.62 (20.229%)	4178.48 (6.504%)	0.5426 (19.209%)	2.0292 (11.363%)	2.5718 (12.931%)	1.3454

	Impact of the Chinese Policy					
t	$\hat{D}_H$	$\hat{D}_L$	$\hat{\Pi}_R$ ($\times 10^7$)	$\hat{\Pi}_M$ ($\times 10^7$)	$\hat{\Pi}$ ($\times 10^7$)	$\hat{\phi}$
1%	3016.77 (1.150%)	4835.35 (0.178%)	0.9993 (1.429%)	2.8081 (0.646%)	3.8075 (0.850%)	0.9695
2%	2867.04 (2.449%)	4794.64 (0.449%)	0.9388 (2.949%)	2.7286 (1.385%)	3.6674 (1.781%)	1.0093
3%	2717.59 (3.911%)	4745.56 (0.819%)	0.8808 (4.566%)	2.6466 (2.225%)	3.5274 (2.799%)	1.0507
4%	2568.93 (5.551%)	4688.17 (1.292%)	0.8252 (6.284%)	2.5625 (3.170%)	3.3877 (3.912%)	1.0936
5%	2421.57 (7.386%)	4622.59 (1.872%)	0.7721 (8.110%)	2.4765 (4.227%)	3.2486 (5.124%)	1.1376
6%	2276.02 (9.431%)	4549.01 (2.564%)	0.7214 (10.049%)	2.3889 (5.400%)	3.1103 (6.443%)	1.1826
7%	2132.81 (11.707%)	4467.67 (3.374%)	0.6730 (12.106%)	2.3001 (6.695%)	2.9731 (7.874%)	1.2283
8%	1992.44 (14.234%)	4378.90 (4.306%)	0.6270 (14.287%)	2.2103 (8.117%)	2.8373 (9.423%)	1.2744
9%	1855.42 (17.033%)	4283.05 (5.365%)	0.5833 (16.598%)	2.1199 (9.672%)	2.7031 (11.096%)	1.3206
10%	1722.20 (20.130%)	4180.55 (6.557%)	0.5418 (19.046%)	2.0292 (11.362%)	2.5710 (12.898%)	1.3666

Table W: Consumers' Choices Between Two Automobiles: A sensitivity analysis on the means $E(\theta_H)$ and $E(\theta_L)$ of the consumers' normally-distributed valuations θ_H on the Type-H automobile and θ_L on the Type-L automobile. Each number inside the brackets is the percentage change in the value (which is given outside the brackets) after a tax reduction policy is implemented.

			Impact of the U.S. Policy					
Case	$E(\theta_H)$	$E(\theta_L)$	D_H	D_L	$\Pi_R(\times 10^7)$	$\Pi_M(\times 10^7)$	$\Pi(\times 10^7)$	ϕ
1	34,000	24,000	1652.89 (10.317%)	3318.89 (3.963%)	0.4246 (10.243%)	1.7395 (6.588%)	2.1640 (7.286%)	1.7184
2	34,500	24,500	1849.93 (9.521%)	3661.61 (3.392%)	0.5001 (9.681%)	1.9310 (5.948%)	2.4311 (6.696%)	1.5457
3	35,000	25,000	2046.17 (8.768%)	3996.76 (2.850%)	0.5834 (9.142%)	2.1198 (5.339%)	2.7033 (6.138%)	1.3911
4	35,500	25,500	2237.92 (8.057%)	4318.57 (2.338%)	0.6742 (8.628%)	2.3027 (4.763%)	2.9769 (5.614%)	1.2530
5	36,000	26,000	2421.72 (7.392%)	4621.92 (1.857%)	0.7723 (8.138%)	2.4763 (4.221%)	3.2486 (5.126%)	1.1300
6	36,500	26,500	2594.59 (6.777%)	4902.62 (1.410%)	0.8771 (7.674%)	2.6384 (3.716%)	3.5155 (4.677%)	1.0210
7	37,000	27,000	2753.99 (6.208%)	5157.57 (0.998%)	0.9882 (7.236%)	2.7866 (3.250%)	3.7748 (4.265%)	0.9249
8	37,500	27,500	2898.20 (5.688%)	5384.85 (0.623%)	1.1050 (6.824%)	2.9196 (2.823%)	4.0247 (3.892%)	0.8407
9	38,000	28,000	3026.17 (5.219%)	5583.69 (0.284%)	1.2268 (6.439%)	3.0369 (2.437%)	4.2637 (3.559%)	0.7676
10	38,500	28,500	3137.52 (4.798%)	5754.38 (0.001%)	1.3530 (6.079%)	3.1382 (2.091%)	4.4912 (3.262%)	0.7046

			Impact of the Chinese Policy					
Case	$E(\theta_H)$	$E(\theta_L)$	$\hat{D}_H$	$\hat{D}_L$	$\hat{\Pi}_R(\times 10^7)$	$\hat{\Pi}_M(\times 10^7)$	$\hat{\Pi}(\times 10^7)$	$\hat{\phi}$
1	34,000	24,000	1652.42 (10.285%)	3319.24 (3.974%)	0.4244 (10.193%)	1.7394 (6.581%)	2.1637 (7.271%)	1.7349
2	34,500	24,500	1849.54 (9.498%)	3662.06 (3.405%)	0.4999 (9.637%)	1.9309 (5.946%)	2.4308 (6.685%)	1.5593
3	35,000	25,000	2045.86 (8.751%)	3997.30 (2.864%)	0.5832 (9.105%)	2.1198 (5.340%)	2.7030 (6.130%)	1.4023
4	35,500	25,500	2237.69 (8.046%)	4319.18 (2.352%)	0.6740 (8.595%)	2.3027 (4.766%)	2.9768 (5.610%)	1.2622
5	36,000	26,000	2421.57 (7.386%)	4622.59 (1.872%)	0.7721 (8.110%)	2.4765 (4.227%)	3.2486 (5.124%)	1.1376
6	36,500	26,500	2594.46 (6.771%)	4903.33 (1.425%)	0.8769 (7.650%)	2.6385 (3.723%)	3.5155 (4.676%)	1.0273
7	37,000	27,000	2753.93 (6.204%)	5158.29 (1.013%)	0.9881 (7.215%)	2.7868 (3.257%)	3.7748 (4.265%)	0.9301
8	37,500	27,500	2898.18 (5.686%)	5385.57 (0.636%)	1.1048 (6.806%)	2.9199 (2.831%)	4.0247 (3.892%)	0.8450
9	38,000	28,000	3026.16 (5.218%)	5584.37 (0.296%)	1.2266 (6.423%)	3.0371 (2.445%)	4.2637 (3.559%)	0.7710
10	38,500	28,500	3137.52 (4.798%)	5755.03 (0.005%)	1.3528 (6.066%)	3.1384 (2.099%)	4.4912 (3.262%)	0.7074

Table X: Consumers' Choices Between Two Automobiles: A sensitivity analysis on the standard deviation σ_θ of the consumers' normally-distributed valuations θ_H on the Type-H automobile and θ_L on the Type-L automobile. Each number inside the brackets is the percentage change in the value (which is given outside the brackets) after a tax reduction policy is implemented.

	Impact of the U.S. Policy					
σ_θ	D_H	D_L	$\Pi_R(\times 10^7)$	$\Pi_M(\times 10^7)$	$\Pi(\times 10^7)$	ϕ
2000	1614.22 (17.053%)	5924.43 (3.604%)	0.4523 (13.989%)	2.5037 (7.177%)	2.9560 (8.175%)	1.8903
2500	1919.50 (12.883%)	5411.25 (2.938%)	0.5285 (11.834%)	2.4872 (6.187%)	3.0157 (7.138%)	1.6105
3000	2136.84 (10.335%)	5061.92 (2.467%)	0.6081 (10.260%)	2.4802 (5.380%)	3.0883 (6.307%)	1.4020
3500	2298.00 (8.622%)	4810.63 (2.120%)	0.6897 (9.068%)	2.4773 (4.737%)	3.1669 (5.651%)	1.2473
4000	2421.72 (7.392%)	4621.92 (1.857%)	0.7723 (8.138%)	2.4763 (4.221%)	3.2486 (5.126%)	1.1300
4500	2519.46 (6.468%)	4475.32 (1.651%)	0.8558 (7.395%)	2.4764 (3.801%)	3.3322 (4.701%)	1.0390
5000	2598.52 (5.748%)	4358.29 (1.485%)	0.9399 (6.787%)	2.4769 (3.454%)	3.4167 (4.350%)	0.9668
5500	2663.73 (5.172%)	4262.80 (1.349%)	1.0243 (6.282%)	2.4775 (3.163%)	3.5018 (4.056%)	0.9085
6000	2718.40 (4.701%)	4183.45 (1.236%)	1.1090 (5.856%)	2.4783 (2.917%)	3.5874 (3.808%)	0.8605

	Impact of the Chinese Policy					
σ_θ	$\hat{D}_H$	$\hat{D}_L$	$\hat{\Pi}_R(\times 10^7)$	$\hat{\Pi}_M(\times 10^7)$	$\hat{\Pi}(\times 10^7)$	$\hat{\phi}$
2000	1613.49 (17.000%)	5927.35 (3.655%)	0.4517 (13.916%)	2.5043 (7.200%)	2.9560 (8.175%)	1.9197
2500	1919.05 (12.856%)	5413.09 (2.973%)	0.5282 (11.779%)	2.4875 (6.202%)	3.0157 (7.138%)	1.6299
3000	2136.54 (10.320%)	5063.16 (2.492%)	0.6079 (10.217%)	2.4804 (5.390%)	3.0883 (6.307%)	1.4156
3500	2297.79 (8.612%)	4811.53 (2.139%)	0.6894 (9.034%)	2.4775 (4.745%)	3.1668 (5.649%)	1.2572
4000	2421.57 (7.386%)	4622.59 (1.872%)	0.7721 (8.110%)	2.4765 (4.227%)	3.2486 (5.124%)	1.1376
4500	2519.35 (6.463%)	4475.84 (1.663%)	0.8557 (7.371%)	2.4765 (3.805%)	3.3321 (4.698%)	1.0450
5000	2598.43 (5.745%)	4358.71 (1.495%)	0.9397 (6.767%)	2.4769 (3.458%)	3.4166 (4.347%)	0.9717
5500	2663.66 (5.169%)	4263.15 (1.357%)	1.0241 (6.264%)	2.4776 (3.166%)	3.5017 (4.053%)	0.9124
6000	2718.34 (4.698%)	4183.73 (1.242%)	1.1089 (5.840%)	2.4784 (2.919%)	3.5872 (3.805%)	0.8638

Table Y: Consumers' Choices Between Two Automobiles: A sensitivity analysis on the correlation coefficient ρ of the consumers' normally-distributed valuations θ_H on the Type-H automobile and θ_L on the Type-L automobile. Each number inside the brackets is the percentage change in the value (which is given outside the brackets) after a tax reduction policy is implemented.

	Impact of the U.S. Policy					
ρ	D_H	D_L	$\Pi_R(\times 10^7)$	$\Pi_M(\times 10^7)$	$\Pi(\times 10^7)$	ϕ
-0.8	3885.91 (4.441%)	5372.09 (1.376%)	0.9910 (8.613%)	3.3603 (2.948%)	4.3513 (4.186%)	0.8658
-0.6	3643.06 (4.827%)	5181.46 (1.542%)	0.9627 (8.426%)	3.1938 (3.202%)	4.1565 (4.367%)	0.9166
-0.4	3434.24 (5.155%)	5033.59 (1.645%)	0.9336 (8.312%)	3.0555 (3.390%)	3.9891 (4.502%)	0.9549
-0.2	3235.55 (5.490%)	4909.04 (1.722%)	0.9030 (8.237%)	2.9287 (3.561%)	3.8317 (4.626%)	0.9901
0	3034.34 (5.875%)	4801.55 (1.783%)	0.8703 (8.188%)	2.8058 (3.731%)	3.6761 (4.755%)	1.0255
0.2	2817.91 (6.335%)	4710.64 (1.829%)	0.8344 (8.158%)	2.6812 (3.910%)	3.5156 (4.888%)	1.0633
0.4	2568.73 (6.961%)	4642.23 (1.855%)	0.7944 (8.143%)	2.5486 (4.109%)	3.3430 (5.040%)	1.1059
0.6	2248.92 (7.968%)	4618.08 (1.849%)	0.7482 (8.133%)	2.3974 (4.345%)	3.1456 (5.222%)	1.1569
0.8	1733.00 (10.280%)	4732.92 (1.785%)	0.6910 (8.087%)	2.1997 (4.643%)	2.8908 (5.446%)	1.2218

	Impact of the Chinese Policy					
ρ	$\hat{D}_H$	$\hat{D}_L$	$\hat{\Pi}_R(\times 10^7)$	$\hat{\Pi}_M(\times 10^7)$	$\hat{\Pi}(\times 10^7)$	$\hat{\phi}$
-0.8	3887.16 (4.475%)	5373.61 (1.405%)	0.9908 (8.590%)	3.3613 (2.979%)	4.3521 (4.205%)	0.8748
-0.6	3643.82 (4.849%)	5182.57 (1.563%)	0.9625 (8.402%)	3.1945 (3.224%)	4.1570 (4.378%)	0.9245
-0.4	3434.75 (5.171%)	5034.51 (1.663%)	0.9334 (8.287%)	3.0560 (3.407%)	3.9894 (4.509%)	0.9623
-0.2	3235.89 (5.501%)	4909.85 (1.739%)	0.9028 (8.212%)	2.9291 (3.575%)	3.8319 (4.631%)	0.9973
0	3034.34 (5.875%)	4802.29 (1.799%)	0.8699 (8.162%)	2.8061 (3.742%)	3.6761 (4.755%)	1.0327
0.2	2817.90 (6.334%)	4711.34 (1.844%)	0.8342 (8.131%)	2.6815 (3.919%)	3.5156 (4.888%)	1.0706
0.4	2568.65 (6.958%)	4642.90 (1.869%)	0.7942 (8.115%)	2.5488 (4.116%)	3.3430 (5.039%)	1.1133
0.6	2248.67 (7.956%)	4618.76 (1.864%)	0.7480 (8.104%)	2.3975 (4.349%)	3.1455 (5.218%)	1.1646
0.8	1732.48 (10.247%)	4733.71 (1.802%)	0.6908 (8.057%)	2.1997 (4.643%)	2.8906 (5.439%)	1.2296