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Lead-time reduction in a two-level supply chain: Non-cooperative equilibria vs. coordination with a profit-sharing contract

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ABSTRACT

This paper considers game-theoretic models of lead-time reduction in a two-level supply chain involving a manufacturer and a retailer. The retailer manages her inventory system using the order quantity, reorder point, continuous-review (q, r) policy. To satisfy the retailer's order, the manufacturer sets up his facility, implements a pre-determined production schedule and delivers finished products to the retailer. In our paper, the lead-time consists of three components: setup time, production time and shipping time, each being in a range between minimum and "normal" durations. The first two lead-time components are naturally determined by the manufacturer, whereas the shipping lead time may be chosen by the manufacturer or the retailer. We thus consider two problems according to who decides the shipping lead time, and for each problems in the non-cooperative setting, we obtain Pareto-optimal Nash and Stackelberg equilibria. We find that, for all games, the manufacturer should be responsible for the setup time and production time at their normal durations. Next, we develop a simple profit-sharing contract to achieve supply chain coordination. We show that, under our properly designed contract, the two supply chain members are better off, and thus, they would have no incentive to deviate from the global solution that maximizes the system-wide profit.

1. Introduction

The important role of lead-time reduction in supply chain and inventory management has been widely recognized by practitioners and academic researchers. Reducing lead times can result in more accurate forecasts, lower safety stocks and lower levels of out-of-stock items, smaller order sizes which lead to a reduction in finished goods inventory levels, a reduction in the bullwhip effect (defined as the increase in order variability in the higher echelons of the supply chain) and consequently lower costs. This has been demonstrated by, for example, Suri (1998) where we find that adopting the quick response manufacturing (QRM) strategy, many firms have cut lead times in all phases of their manufacturing and delivery systems, brought products to market more quickly and secured business prospects by helping compete in the rapidly changing economic arena. Granite Bay,[2] a US-based consulting firm, has reported that as a result of reducing their lead times by 50–80%, many world-class manufacturers have increased their market shares and improved profitability—Granite Bay regards lead-time reduction as the most important factor in achieving

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[2] http://www.granite-bay.com/reducing-lead-time.html (Last accessed October 2008.).

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world-class operations. Co-plenish Consulting Group[3] estimates that if a firm can reduce its lead time by 50%, it could realize a 48–57% reduction in total inventory, which means 75–108% faster turns and 48–57% less money tied up in inventory and exposed to obsolescence risks.

In our paper, we consider a two-level supply chain involving a manufacturer and a retailer and use game theory to answer the following question: How should the manufacturer's lead time and production quantity and the retailer's order policy parameters be determined in a way that is acceptable for both parties? In the supply chain we consider, the manufacturer produces a single item which is sold to the retailer. The manufacturer determines his optimal production quantity (Q), and the retailer implements a (q,r)-type inventory policy (as described in Hadley and Whitin, 1963; Nahmias, 1993) and chooses her optimal order quantity q and reorder point r. We assume in our paper that total lead-time (L) faced by the retailer consists of three independent components: setup time (L_1), production time (L_2) and shipping time (L_3); that is, $L = L_1 + L_2 + L_3$. We consider three lead-time components separately rather than as a single lead time because this is more realistic as we argue in the next section. For some publications that have considered separate lead-time components, see, for example, Ben-Daya and Raouf (1994) and Liao and Shyu (1991a). We should note that, in practice, the setup and production lead times are naturally determined by the manufacturer but the shipping lead-time L_3 may be chosen by the manufacturer or by the retailer. Thus, we consider the following two situations: (1) the manufacturer chooses the optimal shipping lead-time decision and absorbs a crashing cost if the resulting shipping lead time is less than some "normal" duration; (2) the retailer decides on the optimal shipping lead time and incurs the crashing cost.

After developing the expected (long-run average) profit functions for the retailer and the manufacturer, we consider two non-cooperative games with complete information (Gibbons, 1992, Chapter 1), where the two supply chain members have common knowledge of each other's objective functions and maximize their own expected profits. In some practical cases the two supply chain members may make their decisions simultaneously; while in other cases, a supply chain member may announce his decisions first, and the other responds by making her decisions accordingly. We consider both cases in our analysis. In this paper, we first investigate the problem where the manufacturer determines the shipping lead time. For this problem, we analyze two cases: in the first case the manufacturer and the retailer move simultaneously for which we determine the Pareto-optimal Nash equilibrium. In the second case, we consider the leader–follower game with the manufacturer as the leader and the retailer as the follower, and find the Stackelberg equilibrium. Then, for the problem where the retailer makes the shipping lead-time decision, we also analyze a Nash game and a Stackelberg game with the retailer as the leader and the manufacturer as the follower.

Next, we consider the cooperative game where the manufacturer and the retailer can negotiate to maximize supply chain-wide profits. This is achieved by formulating a system-wide objective as a function of the manufacturer's production quantity Q and the lead-time components $\mathbf{L} = (L_1, L_2, L_3)$, and the retailer's order quantity q and reorder point r. It follows that the global maximum system-wide profit is more than the sum of the profits of the manufacturer and the retailer if they do not cooperate. If cooperation improves the system-wide profits, how should the parties share the surplus and what kind of a mechanism will assure compliance? In order to eliminate the incentive for the players to deviate from the optimal solution, we present two criteria for supply chain coordination and develop a simple profit-sharing contract for the players to assure that the cooperation can be realized.

The paper is organized as follows. In Section 2 we provide a literature review on lead-time reduction and profit-sharing contracts, in order to point out the differences between our paper and previous publications. In Section 3, we consider two situations where the shipping lead time is determined by (i) the manufacturer and (ii) the retailer. For each situations, we formulate a "simultaneous-move" game and a leader–follower game in a non-cooperative setting; determine each supply chain member's best response given the other member's announced decision and then solve the two games (without side-payment) to find Pareto-optimal Nash and Stackelberg equilibria. In Section 4, we design a linear profit-sharing contract which allows both members to cooperate and maximizes the supply chain-wide profit. In Section 5, we summarize the paper and present some concluding remarks for possible extensions of the present research.

2. Literature review

In our paper we investigate the lead-time reduction in a two-level supply chain where the manufacturer and the retailer make their decisions with and without a profit-sharing contract. In order to draw a clear distinction between our paper and previously published literature, we now review the publications that are concerned with the lead-time reduction and/or profit-sharing contracts. Note that although our paper includes supply chain coordination with contracts, we do not review the publications related to this issue because others (e.g., Cachon, 2003; Leng and Parlar, 2005; Leng and Zhu, 2009; Tsay et al., 1998) have already presented such reviews.

2.1. Lead-time reduction

Given the importance of lead-time reduction, several authors have published papers dealing with different aspects of this issue. In two recent papers, Treville et al. (2004) have presented a qualitative discussion on the role of lead-time reduction while So and Zheng (2003) have

[3] http://www.coplenish.com/FreeStuffPages/LeadTime.pdf (Last accessed October 2008.).

provided a quantitative model to show the effect of lead time on supply chain performance.

Hill and Khosla (1992) proposed simple models to compare the costs and benefits of reducing lead time for a single manufacturer, and provide discussions on the calculation of optimal lead times. In these models the manufacturer determines the lead-time L which impacts the demand of the manufacturer's customers. The authors assumed the demand function as aL^{-b} and the manufacturer's cost of crashing the lead time as $\alpha L^{-\beta}$, where the parameters a, b, α, β are positive. Under these assumptions, the authors derived the optimal lead time, and performed a sensitivity analysis to examine the impacts of these four parameters on the manufacturer's decision. Ryu and Lee (2003) investigated a dual-sourcing problem in which two suppliers invest in their lead-time reduction. The authors computed the two suppliers' expected total cost per unit time for the case of no lead-time reduction and also that for the case of lead-time reduction. These two models were then compared to calculate the savings generated by crashing lead times. Wadhwa et al. (2005) used simulation to discuss the lead-time reduction for simple flexible manufacturing systems.

The publications discussed above only treated the lead time as a single variable; this largely simplifies the mathematical discussions but would result in unrealistic models and solutions, because in practice the lead time involves several components such as production lead time and shipping lead time. This approach of considering individual lead-time components separately is useful and important in the analysis of lead-time reduction for the following reasons: as Suri (1998) has shown, many firms reduce lead times in *all phases* of their manufacturing and delivery systems to implement their QRM strategy. For other literature regarding the QRM strategy, see, for example, Choi (2006) and Choi and Chow (2008). Moreover, Tersine and Hummingbird (1995) discussed the importance of lead-time reduction for a firm to gain competitive advantage, and showed that when firms shorten their lead times, they usually focus on the measures for reductions in setup times, throughput times and delivery times.

Thus, many researchers constructed their models by decomposing the total lead time into its individual components. In two earlier papers (Liao and Shyu, 1991a, b) have constructed models to determine the optimal lead time and reorder point for a buyer (e.g., a retailer) assuming that the buyer's order quantity is known. These authors assumed that demand faced by the buyer was normally and Poisson-distributed, respectively, and that the lead time could be decomposed into n components each assuming values between a minimum duration a_i and a "normal" time duration b_i. A crashing cost is incurred when the ith lead-time component is reduced from its normal duration. Ben-Daya and Raouf (1994) extended the model in Liao and Shyu (1991a) by assuming that the order quantity is also a decision variable (in addition to lead time) and computed the optimal order quantity. They showed that the optimal value of each lead-time component either equals the minimum duration a_i or normal duration b_i. Ouyang et al. (1996) in turn extended Ben-Daya and Raouf's (1994) model to incorporate costs of partial backorders and lost sales. They obtained the same analytical results for the optimal lead times as in Ben-Daya and Raouf (1994). More recently, Ben-Daya and Hariga (2003) investigated the problem of reducing lead time for a stochastic inventory system with learning consideration. The papers reviewed above assumed that lead time is a deterministic and controllable decision variable.

Since it is realistic and thus necessary to consider lead-time components separately, we shall also perform our analysis with three lead-time components (setup, production and shipping lead times) in our paper.

2.2. *Profit-sharing contracts*

We now survey the publications which developed profit-sharing contracts to coordinate supply chains. Under such contracts supply chain members share the system-wide profit that are made jointly by these members. Even though some publications (e.g., Cachon, 2003; Leng and Zhu, 2009; Tsay et al., 1998) reviewed side-payment contracts, they did not provide a survey on the applications of profit-sharing contracts in supply chain coordination. Since our paper is concerned with this issue, it is necessary to conduct the review in this section.

Jeuland and Shugan (1983) proposed an early paper to investigate supply chain coordination with a profit-sharing contract. The authors considered a two-level supply chain where a manufacturer receives fraction k_1 ($0<k_1<1$) of the chain-wide profit plus a fixed amount k_2 and a retailer receives fraction $1-k_1$ minus the fixed amount k_2. It was proved that the supply chain can achieve its maximum profit when the manufacturer and the retailer make their quantity and pricing decisions under the profit sharing contract. In a multi-period setting, Çanakoğlu and Bilgiç (2007) analyzed a two-echelon telecommunication supply chain, in which an operator determines the initial technology investment level and the capacity of the network for each period, and a vendor makes the final technology investment decision. The authors suggested a profit-sharing contract under which the operator and the vendor share the revenue and operating costs made throughout the periods along with initial technology investment. Moreover, a quantity discount scheme with buy-back was also developed in Çanakoğlu and Bilgiç (2007). As a result of proposing the above contracts, the operator chooses the same investment and capacity change decisions as the centralized solution; this means that supply chain coordination can be induced.

Shao and Ji (2009) investigated a decentralized assemble-to-order system in which multiple suppliers provide three components to an assembler who then assembles two substitutable products. Demand for the products is dependent of the selling prices determined by firms in the system. The authors showed that the decentralized system can be coordinated under a simple profit-sharing contract. Yu et al. (2009) analyze a

Stackelberg game for a two-level supply chain consisting of a manufacturer and multiple retailers. The manufacturer produces a single product and supplies it at the same wholesale price to multiple retailers who then sell the product in independent markets at retail prices. The authors developed a cooperative contract under which the extra profit generated by cooperation is shared among the manufacturer and his retailers, and showed that implementing this contract can result in supply chain coordination.

Anupindi et al. (2001) analyzed a two-stage decentralized distribution systems in which N retailers face stochastic demands. In the first (non-cooperative) stage, each retailer determines his order quantity to satisfy his own demand. For this stage, the authors found sufficient conditions for the existence of Nash equilibrium. In the second (cooperative) stage, the retailers transship products for the residual demands and allocate the corresponding additional profits. The authors applied the concept of core to the allocation of profit, and derived sufficient conditions for existence of the core. Granot and Sosic (2003) extended the results in Anupindi et al. (2001) to a three-stage model where the first and third stages are the same as the first and second stages in Anupindi et al. (2001), and in the second stage each retailer decides how much of his residual supply/demand he wants to share with the other retailers.

Kamrad and Siddique (2004) considered a decentralized supply chain in which a single producer orders components from a pool of potential suppliers. Using a real-options approach, the authors analyzed and valued supply contracts in a setting characterized by exchange rate uncertainty, order-quantity flexibility, supplier-switching options, profit sharing, and supplier reaction options. This paper endogenizes the extent and degree of profit sharing through the optimal policies that are obtained by solving the dual optimization problem for the suppliers and the producer, and also analyzed what induces the producer and the suppliers to accept flexibility in their contracts. Chauhan and Proth (2005) developed a provider–retailer partnership model that is based on profit sharing, assuming that customer demand depends on the retail price and tends to zero as the price of commodity approaches infinity. A scheme was proposed in this paper to maximize the system-wide profit, and divide the maximum profit among partners so that each has an allocation proportional to his risk.

Our review suggests that the profit-sharing contract is important and useful to supply chain coordination, and it is easy to implement this contract in practice because of its simple form.

2.3. Other lead-time-related literature

In addition to the papers reviewed above, there are other publications that are also related to lead time in supply chain operations. For example, in an early paper, Gerchak and Parlar (1991) assumed that lead time is random and analyzed the problem of investing in reducing lead-time randomness. For similar models on lead-time randomness reduction, see Gerchak (2000) and Ray et al. (2004). Moreover, Chen et al. (2006) proposed a recent paper to consider the coordination of a supply chain with a long lead time and demand information updating. Vlachos and Tagaras (2001) discussed two alternative ordering policies for a main and an emergency supply channel: an "early-ordering" policy that almost eliminates early stockouts in a replenishment cycle and a "late-ordering" policy that delays the emergency order decision until more demand information has been accumulated.

Based on our above review, we conclude that lead-time reduction is an important topic in supply chain management and the profit-sharing contract is a useful tool for supply chain coordination. However, we notice that the papers cited in Section 2.1 are only concerned with the optimization of the objective function of an *isolated* downstream member (e.g., a retailer) who determines what the lead time should be. Even though this could be a fact in reality, the lead time faced by the retailer may be determined by the manufacturer who incurs the crashing cost rather than by the retailer who may have no control over this variable. Moreover, the lead time has significant impacts on the operations and profits of both supply chain members. For instance, if the lead time becomes longer, the retailer would increase her reorder point to keep a desired service level (i.e., the probability that the customer demand during the lead time can be satisfied.) This results in a higher inventory-related cost for the retailer. On the other hand, to achieve a shorter lead time, the manufacturer may increase his investment on the lead-time reduction, thereby incurring a higher operating cost.

Since the lead-time decision made by a supply chain member impacts the performance of this member's supply chain partners, it is necessary to use game theory to analyze the lead-time reduction for a supply chain. Thus, in this paper we study a two-level supply chain involving a manufacturer and a retailer who make their quantity as well as lead-time decisions. From our review in Section 2.1, we find that it is more realistic to consider lead-time components separately rather than a single lead-time variable; so, we apply this modeling approach on lead time to our paper. In addition, since, in practice, the lead-time decision may be made by the manufacturer or by the retailer, we shall conduct our modeling and analysis for both cases. For each case, we discuss both Nash (simultaneous-move) game and Stackelberg (sequential-move) game.

3. Non-cooperative decisions without a profit-sharing contract: the Nash vs. Stackelberg equilibria

In this section we analyze the non-cooperative decisions of the manufacturer and the retailer. We consider two cases according to who determines the shipping lead-time L_3. For each case we consider, (i) a "simultaneous-move" game where the two supply chain members make their decisions without any communication; (ii) a leader–follower game in which the player who makes the shipping lead-time decision is the leader (who first announces his decision) and the other player is the follower (who responds to the leader's decision). For the former game, we find the Nash equilibrium; and for the latter, we find the Stackelberg equilibrium for the supply chain.

3.1. Analysis of the non-cooperative game where the manufacturer makes the shipping lead-time decision

We now consider the non-cooperative case where the manufacturer is responsible for shipping the items purchased by the retailer. For this case, the manufacturer determines all three lead-time components, i.e., the setup lead-time L_1, the production lead-time L_2 and the shipping lead-time L_3. The first two lead-time components are, of course, controlled by the manufacturer; but, we need to justify the assumption that the manufacturer manages the shipping lead-time L_3. In today's business world, many manufacturers sell their products online. In order to attract the retailers to place online orders, these manufacturers now commonly implement the "free-shipping" marketing strategy and they thus pay for the shipping costs; see, for example, www.freeshipping.com, which lists over 1,000 merchants who provide their free shipping services to all online orders. Thus, it is realistic to consider the games in which the manufacturer makes the shipping lead-time decision.

3.1.1. Best-response analyses for the retailer and the manufacturer

We now develop the expected profit functions for the manufacturer and the retailer in the two-level supply chain using the renewal reward theorem (RRT) and obtain the best-response functions for each player.

The retailer's expected profit function and best-response analysis: The two-level supply chain in this section serves a market with a single product. The retailer is the immediate downstream member of the manufacturer, and is responsible for direct sale of the product to the market. We assume that annual demand faced by the retailer is a normally distributed random variable with mean λ and variance σ^2. In order to avoid unrealistic settings, we assume that λ is sufficiently larger than σ. The retailer adopts the classical (q, r) continuous-review inventory policy with backorders where q and r denote the retailer's order quantity and reorder point, respectively. The retailer maximizes her expected profit subject to a service-level constraint.

We next compute the expected cycle profit and the expected cycle length and invoke RRT to find the retailer's expected long-run average profit $J_R(q, r; Q, \mathbf{L})$, where $\mathbf{L} = (L_1, L_2, L_3)$ is the vector of the manufacturer's lead-time decision variables. (In the sequel, where no confusion arises, we will shorten "expected long-run average profit" to "expected profit.") We also define h_R as the retailer's holding cost per item per unit time, π_R as the retailer's fixed penalty cost per backlogged item and X as the normally distributed lead-time demand with density function $f(x)$ having mean λL and standard deviation $\sigma\sqrt{L}$, where $L = L_1 + L_2 + L_3$. By using the RRT, we compute the retailer's expected profit function per time

$$J_R(q, r; Q, \mathbf{L}) = E(\text{cycle profit})E(\text{cycle time}) = \frac{\text{SR} - E(\text{HC}) - E(\text{BC}) - \text{PC}}{E(\text{cycle time})}, \tag{1}$$

aswhere SR and PC are two deterministic terms that denote the retailer's sale revenue and purchasing cost, respectively; $E(\text{HC})$ and $E(\text{BC})$, respectively, denote the retailer's per cycle expected holding cost and backorder cost. As in the original (q, r) model described in Hadley and Whitin (1963) (and the graph of the sample path process therein), the regenerative cycles in our model also commence when the inventory position drops to the reorder point r and an order is placed.

Expected cycle time: In each cycle, the retailer orders q units but the manufacturer produces Q units. If Q exceeds q, then the retailer receives all q units of the products; otherwise, the retailer receives only Q units and has this amount for sale during this cycle. Hence, in each cycle the retailer receives $\min(Q, q)$ units. Since the demand rate is λ, the expected cycle time is $E(\text{cycle time}) = \min(Q, q)/\lambda$.

Sales revenue (*SR*): To calculate the sales revenue, we observe that in each cycle $\min(Q, q)$ units are ordered by the retailer and all these units are eventually sold in the same cycle resulting in a revenue of $\text{SR} = v\min(Q, q)$.

Expected holding cost $E(\text{HC})$: To compute this cost, we make the standard simplifying approximation that backorders occur only a small fraction of time. (As Hadley and Whitin, 1963; Nahmias, 1993 indicate, this is a valid approximation when the backorder cost π is large relative to the holding cost h, which is usually the case in practical applications.) This has the implication that the on-hand inventory can be approximated by the inventory level, because we can ignore the case that the inventory level is negative, and thus compute the holding cost by using the inventory level. Since the average demand rate was assumed to be a constant λ, the expected inventory level varies linearly between $s = r - \lambda L$ (the safety stock) and $\min(Q, q) + s = \min(Q, q) + (r - \lambda L)$ and thus the average inventory during each cycle is approximately $\min(Q, q)/2 + (r - \lambda L)$. Since each cycle lasts an average of $\min(Q, q)/\lambda$ time units, we find the expected holding cost per cycle as $E(\text{HC}) = h_R[\min(Q, q)/2 + (r - \lambda L)]\min(Q, q)/\lambda$.

Expected backorder cost $E(\text{BC})$: This cost is computed as unit backorder cost times expected units backlogged per cycle; thus, we have $E(\text{BC}) = \pi_R \int_r^\infty (x - r)f(x)\,dx$.

Purchasing cost (*PC*): This cost includes two cost components: the fixed ordering cost and variable purchase cost. Letting A and w, respectively, denote per cycle fixed ordering cost incurred by the retailer and the unit wholesale price charged by the manufacturer, we compute the retailer's purchase cost per cycle as $\text{PC} = A + w\min(Q, q)$ where $w < v$.

Since the expected cycle length equals $\min(Q,q)/\lambda$, we find, from (1),

$$J_R(q,r;Q,\mathbf{L}) = (v-w)\lambda - h_R\left[\frac{\min(Q,q)}{2} + r - \lambda L\right] - \frac{\lambda}{\min(Q,q)}\left[\pi_R\int_r^{\infty}(x-r)f(x)\,\mathrm{d}x + A\right]. \quad (2)$$

In the two-level supply chain, the retailer determines her order quantity and reorder point to maximize the expected profit $J_R(q,r;Q,\mathbf{L})$ under a service-level constraint involving $\Pr(X\leqslant r)$, i.e., the probability that the lead-time demand will be satisfied. Thus, to find the optimal ordering decision, the retailer solves the constrained nonlinear programming problem: $\max_{q,r}J_R(q,r;Q,\mathbf{L})$, subject to $\Pr(X\leqslant r)=\rho$, where ρ is the retailer's desired service level; see, Hillier and Lieberman (2005), and Silver et al. (1998) for a discussion of the service level constraint.

Now, recalling that $X\sim N(\lambda L, \sigma\sqrt{L})$, we find that $\Pr(X\leqslant r) = \Phi[(r-\lambda L)/(\sigma\sqrt{L})]$ where Φ is the c.d.f. of the standardized normal r.v. so that the service-level constraint can be written as

$$r = \lambda L + \Phi^{-1}(\rho)\sigma\sqrt{L}, \quad (3)$$

where $\Phi^{-1}(\rho)$ denotes the safety *factor* (which depends on the value of ρ) and $\Phi^{-1}(\rho)\sigma\sqrt{L}$ is the safety *stock*; see, Hax and Candea (1984, p. 196) and Silver et al. (1998, p. 255). For simplicity, we denote the RHS of (3) by $\bar{r}$; that is, $\bar{r}\equiv\lambda L + \Phi^{-1}(\rho)\sigma\sqrt{L}$. The retailer's problem can thus be simply re-written as

$$\max_q J_R(q,\bar{r};Q,\mathbf{L}).$$

In order to find the Nash and Stackelberg strategies for the "simultaneous-move" and leader–follower games, respectively, we first derive the retailer's best response given that the manufacturer chooses a production quantity of Q and lead-time $L = L_1+L_2+L_3$.

The above results imply that, if the manufacturer increases his lead-time L, the retailer should accordingly increase her reorder point since $\bar{r} = \lambda L + \Phi^{-1}(\rho)\sigma\sqrt{L}$. Denoting the retailer's expected number of backorders per cycle by $B(r) = \int_r^{\infty}(x-r)f(x)\,\mathrm{d}x$, it can be shown that (Silver et al., 1998, p. 723)

$$B(r) = \sigma\sqrt{L}\Psi(\Phi^{-1}(\rho)), \quad (4)$$

where $\Psi(\Phi^{-1}(\rho)) \equiv \phi(\Phi^{-1}(\rho)) - \Phi^{-1}(\rho)(1-\rho)$ is the *unit normal linear loss* function (Porteus, 2002, Chapter 1) and ϕ represents the standard normal probability density function (p.d.f.).

We replace r in (2) with $\bar{r}$, and analyze the resulting function and find the retailer's best-response order quantity in the following proposition.

Proposition 1. *Given Q and $\mathbf{L}$, the retailer's best-response order quantity $\hat{q}(Q,\mathbf{L})$ is*

$$\hat{q}(Q,\mathbf{L}) = \min\left(Q, \sqrt{\frac{2\lambda[\pi_R\sigma\sqrt{L}\Psi(\Phi^{-1}(\rho))+A]}{h_R}}\right). \quad (5)$$

Proof. For a proof of this proposition and proofs of all subsequent propositions, corollaries and theorems, see Appendix A.

Proposition 1 indicates that the retailer's order quantity q should not exceed the manufacturer's production quantity Q, which is justified as follows: if $q>Q$, then the retailer's order cannot be satisfied and thus this player only receives Q units which are then available to satisfy end demand. This means that ordering q ($q>Q$) units of products is not useful to improving the retailer's performance.

Now, defining $k\equiv 2\lambda\pi_R\sigma\Psi(\Phi^{-1}(\rho))/h_R>0$ (a constant, independent of L), we can write $\bar{q} = \sqrt{k\sqrt{L}+2\lambda A/h_R}$, so that $\hat{q}(Q,\mathbf{L}) = \min(Q,\bar{q})$. Differentiating $\bar{q}$ w.r.t. L gives

$$\frac{\partial\bar{q}}{\partial L} = \frac{k}{4\sqrt{L}\sqrt{k\sqrt{L}+2\lambda A/h_R}}>0,$$

thus, $\bar{q}$ is increasing in the manufacturer's lead-time $L(=L_1+L_2+L_3)$, and the retailer's best-response order quantity $\hat{q}(Q,L)$ is non-decreasing in L announced by the manufacturer, as should be expected.

Substituting $q=\hat{q}(Q,\mathbf{L})$ and $r=\bar{r}$ into the retailer's objective function, (2) becomes

$$J_R(\hat{q}(Q,\mathbf{L}),\bar{r};Q,\mathbf{L}) = (v-w)\lambda - h_R\left[\frac{\hat{q}(Q,\mathbf{L})}{2} + \Phi^{-1}(\rho)\sigma\sqrt{L}\right] - \frac{\lambda}{\hat{q}(Q,\mathbf{L})}[\pi_R\sigma\sqrt{L}\Psi(\Phi^{-1}(\rho))+A].$$

Corollary 1. *The retailer's profit $J_R(\hat{q}(Q,\mathbf{L}),\bar{r};Q,\mathbf{L})$ is decreasing in L.*

The manufacturer's expected profit function and best-response analysis: We now develop the manufacturer's expected profit function in terms of his decision variables Q and $\mathbf{L}=(L_1,L_2,L_3)$, and find his best response to the retailer's decisions q and r. The manufacturer's profit $J_M(Q,\mathbf{L};q,r)$ is computed as sales revenue minus setup cost, production cost, shipping-related cost, holding cost and backorder cost. Denoting the six terms in $J_M(Q,\mathbf{L};q,r)$, respectively, as SR, SUC, PC, SC, HC and BC, and invoking the RRT, we have

$$J_M(Q,\mathbf{L};q,r) = \frac{E(\text{cycle profit})}{E(\text{cycle time})} = \frac{\text{SR}-\text{SUC}-\text{PC}-\text{SC}-\text{HC}-\text{BC}}{E(\text{cycle time})} \quad (6)$$

as the expected long-run average profit (objective) function of the manufacturer. Since the manufacturer delivers $\min(Q,q)$ units of products to the retailer, his expected cycle time is $\min(Q,q)/\lambda$.

Next, we compute the individual terms of the manufacturer's expected profit per cycle.

Sale revenue (SR): In each cycle, the manufacturer sells $\min(Q,q)$ units of products to the retailer at the wholesale price w. Hence, the manufacturer's cycle sale revenue is $\text{SR} = w\min(Q,q)$.

Setup cost (SUC): Recall that the setup time L_1 can assume values in the interval $[a_1,b_1]$ where a_1 and b_1 denote the shortest and "normal" durations, respectively. There is a fixed setup cost of K regardless of whether the setup time is normal or crashed. However, if the setup time is reduced to a level below b_1, the manufacturer incurs a crashing cost $R_1(L_1)>0$ with the property that $R_1'(L_1)<0$ and $R_1''(L_1)>0$ for $L_1 \in [a_1, b_1]$ and $R_1(b_1)=0$. Thus, the setup cost per cycle is $\mathrm{SUC} = K + R_1(L_1)$.

Production cost (*PC*): We assume that the manufacturer has sufficient capacity to produce any quantity ordered by the retailer in the normal lead-time b_2 at a cost of p per unit. However, it is also possible to crash the production time at some cost: if this time is crashed to $L_2 \in [a_2, b_2)$, the extra cost of producing one unit in L_2 is defined as $R_2(L_2)>0$ with the property that $R_2'(L_2)<0$ and $R_2''(L_2)>0$ for $L_2 \in [a_2, b_2]$ and $R_2(b_2)=0$. Thus, the cost of production in a cycle is $\mathrm{PC} = [p + R_2(L_2)]Q$. Note that we model the setup and production cost functions separately rather than a single, and joint, cost function because the setup cost is independent of the production quantity Q, whereas the production is increasing in Q.

Shipping-related cost (*SC*): We assume that the manufacturer is responsible for shipping and pays a third-party transportation firm two shipping-related costs: (i) the actual shipping cost and (ii) maintenance cost (which is a function of shipping quantity and shipping lead time) and may include, for example, the cost of insurance. The manufacturer ships $\min(Q, q)$ units in the normal lead-time b_3 at a shipping cost of s_1 per unit. However, the shipping lead time can be crashed at some cost. If the duration is crashed to $L_3 \in [a_3, b_3)$, an extra cost of shipping one unit in L_3 is defined as $R_3(L_3)>0$ with the property that $R_3'(L_3)<0$ and $R_3''(L_3)>0$ for $L_3 \in [a_3, b_3]$ and $R_3(b_3)=0$. The maintenance cost is computed as $s_2 \min(Q, q) L_3$, where s_2 denotes maintenance cost per item per unit time. Thus, we have

$$
\begin{aligned}
\mathrm{SC} &= \text{Shipping cost per cycle} + \text{Maintenance cost per cycle} \\
&= [s_1 + R_3(L_3)]\min(Q, q) + s_2 L_3 \min(Q, q) \\
&= [s_1 + R_3(L_3) + s_2 L_3]\min(Q, q)
\end{aligned}
$$

as the shipping cost in a cycle.

Holding cost (*HC*): If, in a cycle, the manufacturer's production quantity Q is greater than the retailer's order quantity q, then the manufacturer carries $Q-q$ units to the next cycle and absorbs the holding cost $h_M(Q-q)$, where h_M denotes the unit holding cost. On the other hand, if $Q \leqslant q$, then the holding cost is zero. Thus, the manufacturer's holding cost for each cycle can be computed as $\mathrm{HC} = h_M(Q-q)^+$.

Backorder cost (*BC*): For each cycle, if the retailer's order cannot be satisfied, then the unsatisfied part is backlogged and the manufacturer incurs the backorder cost $\mathrm{BC} = \pi_M(q-Q)^+$, where π_M denotes the unit backorder cost.

The manufacturer's expected profit function is thus

$$
\begin{aligned}
J_M(Q, \mathbf{L}; q, r) = w\lambda - \frac{K + R_1(L_1)}{\min(Q, q)}\lambda - \frac{[p + R_2(L_2)]Q}{\min(Q, q)}\lambda \\
- [s_1 + R_3(L_3) + s_2 L_3]\lambda \\
- \frac{h_M(Q-q)^+ + \pi_M(q-Q)^+}{\min(Q, q)}\lambda,
\end{aligned}
\tag{7}
$$

which does not involve the retailer's reorder point decision variable r. By comparing Q and q, we can re-write the manufacturer's profit function as

$$
\begin{aligned}
J_{M1} &\equiv J_M(Q, \mathbf{L}; q, r)|_{Q \leqslant q} \\
&= \left\{ w - \frac{K + R_1(L_1)}{Q} - [p + s_1 + R_2(L_2) + R_3(L_3) + s_2 L_3] \right. \\
&\quad \left. - \frac{\pi_M(q-Q)}{Q} \right\}\lambda,
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
J_{M2} &\equiv J_M(Q, \mathbf{L}; q, r)|_{Q \geqslant q} \\
&= \left\{ w - \frac{K + R_1(L_1)}{q} - \frac{[p + h_M + R_2(L_2)]Q}{q} \right. \\
&\quad \left. - [s_1 + R_3(L_3) + s_2 L_3] + h_M \right\}\lambda.
\end{aligned}
\tag{9}
$$

Next, we perform the best-response analysis for the manufacturer.

Proposition 2. *Given the values of q and r, the manufacturer's best-response production quantity is $\hat{Q}(q, r) = q$.*

Proposition 2 implies that the manufacturer's best-response production schedule is the lot-for-lot policy; that is, the manufacturer's optimal production quantity is equal to the retailer's order quantity. See Fig. 1 for a sample path of the manufacturer's and retailer's inventory level processes arising from their production and order processes, respectively.

Corollary 2. *The manufacturer's expected profit is increasing in the retailer's order quantity q.*

We next compute the best-response lead-time components for the manufacturer given that the retailer's decisions are q and r.

Proposition 3. *Given the values of q and r, we find that the best-response lead-time components are*

$$
\hat{L}_1 = b_1, \quad \hat{L}_2 = b_2, \quad \hat{L}_3 = \begin{cases} a_3 & \text{if } \tilde{L}_3 \leqslant a_3, \\ \tilde{L}_3 & \text{if } a_3 < \tilde{L}_3 < b_3, \\ b_3 & \text{if } b_3 \leqslant \tilde{L}_3, \end{cases}
\tag{10}
$$

where $\tilde{L}_3 = \{L_3 | s_2 = -R_3'(L_3)\}$.

The manufacturer's best-response total lead time is thus computed as

$$
\hat{L} = \sum_{i=1}^{3} \hat{L}_i = b_1 + b_2 + \hat{L}_3.
\tag{11}
$$

Remark 1. The result obtained in Proposition 3 indicates that it is not desirable for the manufacturer to crash the setup and production times L_1 and L_2, respectively. The intuitive reasoning behind this is as follows: for fixed values of lead-times L_2 and L_3, reducing the setup time L_1 results in a positive crash cost for the manufacturer since $R'(L_1)<0$ it also results in a decrease in the retailer's order quantity [from (37)] and hence an increase in the number of cycles per unit time with the corresponding increase in the manufacturer's setup cost. Thus, it is optimal for the manufacturer not to crash the setup lead time and keep it at the normal duration b_1. The same argument also applies to the production time indicating that it is optimal to keep it at its normal duration b_2.

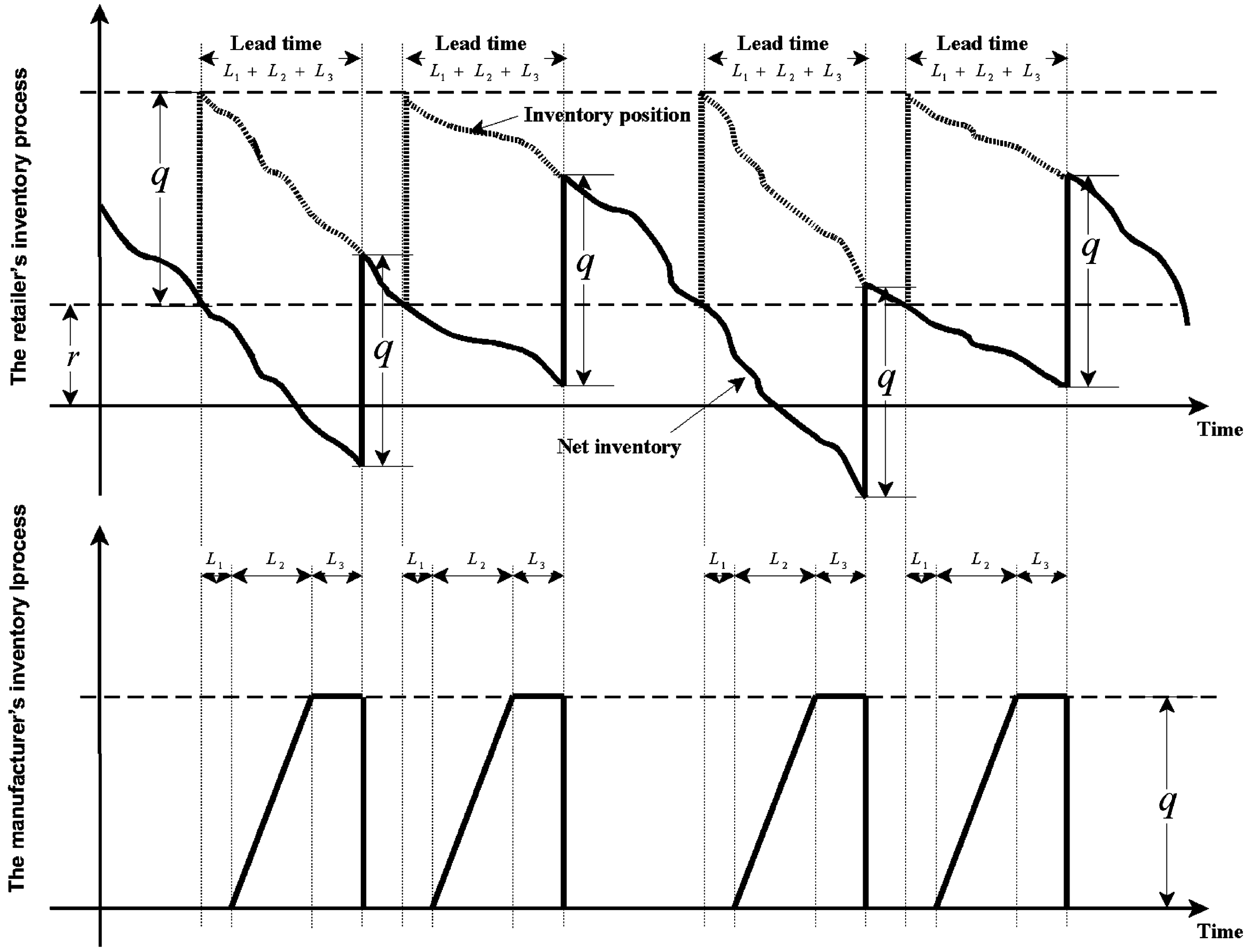

Fig. 1. The manufacturer's production process and the retailer's ordering process.

3.1.2. Nash and Stackelberg equilibria

By using the best-response results we obtained for the manufacturer and the retailer, we now compute (i) the Nash equilibrium $(Q^N, \mathbf{L}^N, q^N, r^N)$ for the "simultaneous-move" game and (ii) the Stackelberg equilibrium $(Q^S, \mathbf{L}^S, q^S, r^S)$ for the leader–follower game.

Theorem 1. *In the "simultaneous-move" game, there are multiple Nash equilibria among which only one is Pareto optimal. The unique Pareto-optimal Nash equilibrium solution for the manufacturer's lead time and production decisions is given as*

$$L_1^N = b_1, \quad L_2^N = b_2, \quad L_3^N = \begin{cases} a_3 & \text{if } \tilde{L}_3 \leqslant a_3, \\ \tilde{L}_3 & \text{if } a_3 < \tilde{L}_3 < b_3, \\ b_3 & \text{if } b_3 \leqslant \tilde{L}_3, \end{cases}$$

and

$$Q^N = \sqrt{\frac{2\lambda[\pi_R \sigma \sqrt{L^N} \Psi(\Phi^{-1}(\rho)) + A]}{h_R}},$$

where $\tilde{L}_3 = \{L_3 | s_2 = -R_3'(L_3)\}$, *and* $L^N = \sum_{i=1}^3 L_i^N = b_1 + b_2 + L_3^N$ *is the Pareto-optimal Nash lead time. For the retailer, the Pareto-optimal Nash equilibrium solution for her reorder point and order quantity is*

$$r^N = \lambda L^N + \Phi^{-1}(\rho)\sigma\sqrt{L^N}, \tag{12}$$

$$q^N = Q^N = \sqrt{\frac{2\lambda[\pi_R \sigma \sqrt{L^N} \Psi(\Phi^{-1}(\rho)) + A]}{h_R}}. \tag{13}$$

In Theorem 1, we applied the concept of Pareto-optimal Nash equilibrium to characterize the decisions of the manufacturer and the retailer. For an early discuss of this concept, see Nakayama (1980).

Next, we formulate a leader–follower game and find the Stackelberg equilibrium. We replace q and r in the manufacturer's objective $J_M(Q, \mathbf{L}; q, r)$ in (7) with the retailer's best responses $\hat{q}(Q, \mathbf{L})$ and $\bar{r}$, thus reducing it to a function of Q and $\mathbf{L} = (L_1, L_2, L_3)$. This gives

$$\begin{aligned} & J_M(Q, \mathbf{L}; \hat{q}(Q, \mathbf{L}), \bar{r}) \\ & \quad = \left\{ w - \frac{K + R_1(L_1) + [p + R_2(L_2)]Q + h_M[Q - \hat{q}(Q, \mathbf{L})]}{\hat{q}(Q, \mathbf{L})} \right. \\ & \qquad \left. - [s_1 + R_3(L_3) + s_2 L_3] \right\} \lambda, \end{aligned} \tag{14}$$

where $\hat{q}(Q,\mathbf{L}) = \min(Q,\bar{q})$ and

$$\bar{q} = \sqrt{\frac{2\lambda[\pi_R\sigma\sqrt{L}\Psi(\Phi^{-1}(\rho)) + A]}{h_R}},$$

with the usual constraints on the lead-times L_i, for $i = 1,2,3$. Note that the term involving the manufacturer's backorder cost π_M does not appear in (14) since $\hat{q}(Q,L) = \min(Q,\bar{q}) \leqslant Q$ implies $\pi_M[\hat{q}(Q,L) - Q]^+ = 0$.

Theorem 2. *The Stackelberg equilibrium solution for the manufacturer's lead time and production decisions is*

$$L_1^S = b_1, \quad L_2^S = b_2, \quad L_3^S = \begin{cases} a_3 & \text{if } \bar{L}_3 \leqslant a_3, \\ \bar{L}_3 & \text{if } a_3 < \bar{L}_3 < b_3, \\ b_3 & \text{if } b_3 \leqslant \bar{L}_3, \end{cases} \tag{15}$$

$$Q^S = \sqrt{\frac{2\lambda[\pi_R\sigma\sqrt{L^S}\Psi(\Phi^{-1}(\rho)) + A]}{h_R}}.$$

where $\bar{L}_3$ is the unique solution of the nonlinear equation $\partial J_M(\bar{q},\mathbf{L};\bar{q},\bar{r})/\partial L_3 = 0$ and $L^S = \sum_{i=1}^3 L_i^S = b_1 + b_2 + L_3^S$ is the Stackelberg lead time.

Corollary 3. *The manufacturer's expected profit under Stackelberg equilibrium is*

$$\begin{aligned} J_M(Q^S,\mathbf{L}^S;q^S,r^S) &= w\lambda - [K + R_1(b_1)]\sqrt{\frac{\lambda h_R}{2[\pi_R\sigma\sqrt{L^S}\Psi(\Phi^{-1}(\rho)) + A]}} \\ &\quad - [p + s_1 + R_2(b_2) + R_3(L_3^S) + s_2L_3^S]\lambda. \end{aligned} \tag{16}$$

Theorem 3. *The Stackelberg equilibrium solution for the retailer's order quantity and reorder point is*

$$q^S = \sqrt{\frac{2\lambda[\pi_R\sigma\sqrt{L^S}\Psi(\Phi^{-1}(\rho)) + A]}{h_R}}$$

and

$$r^S = \lambda L^S + \Phi^{-1}(\rho)\sigma\sqrt{L^S}$$

and with the corresponding expected profit

$$\begin{aligned} J_R(q^S,r^S;Q^S,\mathbf{L}^S) &= (v - w)\lambda - h_R\left[\frac{q^S}{2} + \Phi^{-1}(\rho)\sigma\sqrt{L^S}\right] \\ &\quad - \frac{\lambda[\pi_R\sigma\sqrt{L^S}\Psi(\Phi^{-1}(\rho)) + A]}{q^S}. \end{aligned} \tag{17}$$

From Theorems 2 and 3, we find that, when the manufacturer takes the role of "leader", the setup and production lead times should be reduced to their lower bounds, and the manufacturer's production quantity Q equals the retailer's order quantity q. Moreover, when we compare the results in Theorems 2 and 3 with those in Theorem 1, we can find that the difference between Nash and Stackelberg games is the shipping lead-time L_3. Note that the production (order) quantity and reorder point are functions of three lead-time components. Hence, Nash and Stackelberg games differ in the shipping lead time.

Next, we compare the equilibrium shipping lead times obtained for Nash and Stackelberg games.

Theorem 4. *When the manufacturer makes the shipping decision, Nash shipping lead time L_3^N must be less than or equal to Stackelberg shipping lead time L_3^S, i.e., $L_3^N \leqslant L_3^S$. More specifically, if $L_3^S = a_3$, then $L_3^N = L_3^S$; otherwise, if $a_3 < L_3^S \leqslant b_3$, then $L_3^N < L_3^S$.*

3.2. Analysis of the non-cooperative game where the retailer makes the shipping lead-time decision

In this section we assume that the shipping lead-time component L_3 is determined by the retailer rather than the manufacturer who thus only makes the decisions on the setup and production lead-times—i.e., L_2 and L_3. We make this assumption and perform the corresponding game analysis because of the following fact: In reality, the retailer may be more powerful than the manufacturer. As an example,[4] Wal-Mart, a leading world-wide retailer, has over 6,000 global suppliers, and 80% of these suppliers are from China. Many China-based suppliers are small producers scattered in the south China. For such a case, the retailer is able to control the shipping lead time and may invest in the reduction of this lead-time component so as to meet the retailer's needs in its marketing and inventory management.

In Section 3.1.1 we already developed the retailer's and the manufacturer's profit functions in which the shipping lead time is chosen by the manufacturer. Now, under the assumption that the retailer makes the shipping lead-time decision, we revise the profit functions (2) and (7) (given in Section 3.1.1) and analyze the resulting Nash and Stackelberg games. Since the shipping cost is, in this section, absorbed by the retailer, we move the shipping cost term from (7) to (2) and consequently have these two players' profit functions as follows:

$$\begin{aligned} J_R(q,r,L_3;Q,L_1,L_2) &= (v - w)\lambda - h_R\left[\frac{\min(Q,q)}{2} + r - \lambda L\right] \\ &\quad - \frac{\lambda}{\min(Q,q)}\left[\pi_R\int_r^\infty (x - r)f(x)\,dx + A\right] \\ &\quad - [s_1 + R_3(L_3) + s_2L_3]\lambda, \end{aligned} \tag{18}$$

$$\begin{aligned} J_M(Q,L_1,L_2;q,r,L_3) &= w\lambda - \frac{K + R_1(L_1)}{\min(Q,q)}\lambda - \frac{[p + R_2(L_2)]Q}{\min(Q,q)}\lambda \\ &\quad - \frac{h_M(Q - q)^+ + \pi_M(q - Q)^+}{\min(Q,q)}\lambda. \end{aligned} \tag{19}$$

Similarly in Section 3.1, we next find each player's best-response function which shall be then used to compute the Nash and Stackelberg equilibrium solutions.

3.2.1. Best-response analyses for the retailer and the manufacturer

We first compute the retailer's best response given that the manufacturer's decisions are Q, L_1 and L_2. Note that, as discussed in Section 3.1.1, the optimal reorder point r is $\bar{r} = \lambda L + \Phi^{-1}(\rho)\sigma\sqrt{L} = \lambda(L_1 + L_2 + L_3) + \Phi^{-1}(\rho)\sigma\sqrt{L_1 +}\ L_2 + L_3$, which is independent of the quantity decision q but dependent of the shipping lead-time decision L_3.

[4] http://www.pbs.org/wgbh/pages/frontline/shows/walmart/view/ (Last accessed October 2008.)

Next, we temporarily fix the value of L_3 and find the optimal solution $q(L_3)$ for the retailer. By comparing (2) and (18) we can find that the difference between the retailer's profit functions $J_R(q,r;Q,\mathbf{L})$ (in Section 3.1) and $J_R(q,r,L_3;Q,L_1,L_2)$ (in this section) is only as follows: $J_R(q,r,L_3;Q,L_1,L_2)$ involves the shipping cost term—i.e., $-[s_1+R_3(L_3)+s_2L_3]\lambda$—but $J_R(q,r;Q,\mathbf{L})$ does not. Note that this term is independent of the quantity decision variable q. So, we can conclude that, when the retailer is responsible for the shipping lead-time decision, this player still chooses the optimal solution that is given in Proposition 1. That is, given that the manufacturer's decisions are Q, L_1 and L_2, the retailer's best-response quantity decision $q(L_3)$ for the shipping lead-time L_3 is

$$q(L_3)=\min\left(Q,\sqrt{\frac{2\lambda[\pi_R\sigma\sqrt{L}\Psi(\Phi^{-1}(\rho))+A]}{h_R}}\right). \quad (20)$$

Substituting $q(L_3)$ and $\bar{r}$ into (18) gives

$$\begin{aligned}&J_R(q(L_3),\bar{r},L_3;Q,L_1,L_2)\\&=(v-w)\lambda-h_R\left(\frac{q(L_3)}{2}+\Phi^{-1}(\rho)\sigma\sqrt{L_1+L_2+L_3}\right)\\&\quad-\frac{\lambda}{q(L_3)}\left(\pi_R\int_{\bar{r}}^{\infty}(x-\bar{r})f(x)\,\mathrm{d}x+A\right)\\&\quad-[s_1+R_3(L_3)+s_2L_3]\lambda,\end{aligned}$$

which is too complicated for us to determine its concavity or unimodality. Since $L_3\in[a_3,b_3]$, we can conclude that the retailer's best-response (optimal) shipping lead-time $\hat{L}_3$ must exist. Using $\hat{L}_3$ we can then find the best-response order quantity $\hat{q}$ and reorder point $\hat{r}$ as follows:

$$\hat{q}=\min\left(Q,\sqrt{\frac{2\lambda[\pi_R\sigma\sqrt{L_1+L_2+\hat{L}_3}\Psi(\Phi^{-1}(\rho))+A]}{h_R}}\right), \quad (21)$$

$$\hat{r}=\lambda(L_1+L_2+\hat{L}_3)+\Phi^{-1}(\rho)\sigma\sqrt{L_1+L_2+\hat{L}_3}. \quad (22)$$

Next, we compute the manufacturer's best-response production quantity $\hat{Q}$, setup and production lead-times $\hat{L}_i$ ($i=1,2$), given that the retailer's decisions are q, r and L_3. Similar to our above analysis for the retailer, we find that the sole difference between the manufacturer's profit functions (7) and (19) is that the shipping cost—that is, $-[s_1+R_3(L_3)+s_2L_3]\lambda$—is involved in (7) but not considered in (19). Note that this shipping cost is independent of the manufacturer's decision variables Q, L_1 and L_2. Hence, the best-response solutions are the same as those given in Propositions 2 and 3, i.e.,

$$\hat{Q}=q,\quad \hat{L}_1=b_1,\quad \hat{L}_2=b_2. \quad (23)$$

3.2.2. Nash and Stackelberg equilibria

We now use the best-response analytic results in Section 3.2.1 to find the Nash and Stackelberg equilibrium solutions. More specifically, similarly in Section 3.1.2, we shall conduct the analyses for two games—the "simultaneous-move" game (a.k.a. Nash game) and the sequential-move game (a.k.a. Stackelberg game). In the former game two players make their decisions without communication; but in the latter game, the retailer acts as the Stackelberg leader and announces his decisions on q, r and L_3 to the manufacturer who then determines the values of Q, L_1 and L_2.

Theorem 5. *When the retailer makes the shipping lead-time decision, the Nash and Stackelberg equilibria are identical and they are computed as*

$$\begin{aligned}&Q^N=Q^S=q^N=q^S=\sqrt{\frac{2\lambda[\pi_R\sigma\sqrt{b_1+b_2+\check{L}_3}\Psi(\Phi^{-1}(\rho))+A]}{h_R}},\\&r^N=r^S=\lambda(b_1+b_2+\check{L}_3)+\Phi^{-1}(\rho)\sigma\sqrt{b_1+b_2+\check{L}_3},\\&L_i^N=L_i^S=b_i,\quad i=1,2;\quad L_3^N=L_3^S=\check{L}_3,\end{aligned}$$

where $\check{L}_3$ is the optimal solution of the following nonlinear problem:

$$\begin{aligned}\max_{L_3\in[a_3,b_3]}J_R(L_3)&=(v-w)\lambda-h_R(\hat{Q}+\Phi^{-1}(\rho)\sigma\sqrt{b_1+b_2+L_3})\\&\quad-[s_1+R_3(L_3)+s_2L_3]\lambda,\end{aligned}$$

in which

$$\hat{Q}=\sqrt{\frac{2\lambda[\pi_R\sigma\sqrt{b_1+b_2+L_3}\Psi(\Phi^{-1}(\rho))+A]}{h_R}}.$$

As Theorem 5 indicates, the Nash and Stackelberg equilibria are identical when the retailer determines the shipping lead-time L_3 and this player acts as the leader in the Stackelberg game. This result appears because of the following fact: in this section, the retailer is more powerful than the manufacturer, and controls the shipping lead-time L_3. As a result, the manufacturer's best-response decisions $\hat{L}_1$ and $\hat{L}_2$ are, respectively, b_1 and b_2 which are independent of the retailer's decisions, and the quantity decision $\hat{Q}$ equals the retailer's order quantity q.

Next, we compare the equilibrium shipping lead times determined in Section 3.1.2 when the manufacturer makes the shipping decision and those obtained in this section when the retailer is responsible for the shipping.

Theorem 6. *Nash and Stackelberg shipping lead times when the retailer makes the shipping decision—which are both equal to $\check{L}_3$, as shown in Theorem 5—must be less than or equal to those when the manufacturer makes the shipping decision. More specifically,*

1. *If Stackelberg equilibrium is equal to a_3 when the manufacturer makes the shipping decision, then Nash and Stackelberg solutions obtained in Sections* 3.1.2 *and* 3.2.2 *must be all equal to a_3.*
2. *If Stackelberg equilibrium is greater than a_3 but Nash equilibrium is equal to a_3 when the manufacturer makes the shipping decision, then Nash and Stackelberg solutions $\check{L}_3$ obtained in Section* 3.2.2 *must be also equal to a_3.*
3. *If both Nash and Stackelberg equilibrium are greater than a_3 when the manufacturer makes the shipping decision, then Nash and Stackelberg solutions $\check{L}_3$ obtained in Section* 3.2.2 *must be less than Nash and Stackelberg solutions obtained in Section* 3.1.2.

3.3. Numerical study and managerial implications

We consider several numerical examples and compute the manufacturer's and the retailer's equilibrium decisions and their profits. In addition, we present particular discussions on managerial implications generated by our numerical study.

We next begin by using a numerical example to illustrate our analysis in Section 3.1 in which the manufacturer determines all lead-time components including the shipping lead-time L_3.

Example 1. We use the following data in our computations:

λ	σ	K	A	h_R	h_M	π_R	π_M	p	w	v	s_1	s_2	ρ
600	80	700	150	12	10	140	80	15	50	70	10	6	0.9

The crashing cost functions are assumed to be represented in terms of an exponential function given as $R_i(L_i) = c_i(e^{b_i - L_i} - 1)$, for $L_i \in [a_i, b_i]$, with the parameters specified as $\mathbf{a} = (\frac{1}{365}, \frac{3}{365}, \frac{2}{365})$, $\mathbf{b} = (\frac{3}{365}, \frac{30}{365}, \frac{10}{365})$ and $\mathbf{c} = (0.4, 0.3, 0.1)$. The crashing cost functions are decreasing in L_i and at the normal time they are zero, i.e., $R_i(b_i) = 0$. Since the retailer's desired service level is set at $\rho = 0.9$, the safety factor is obtained as $\Phi^{-1}(\rho) = 1.28$ which gives $\Psi(\Phi^{-1}(\rho)) = 0.047$.

From our analysis in Section 3.1.2, we immediately find $L_1^N = L_1^S = b_1 = \frac{3}{365}$ and $L_2^N = L_2^S = b_2 = \frac{30}{365}$ since the optimal setup and production times involve no crashing. Next, we compute the Nash and Stackelberg solutions for the shipping time L_3. From Theorem 1, we solve equation $s_2 = -R_3'(L_3)$ to find $\tilde{L}_3 = -4.067 < a_3 = \frac{2}{365}$, so the Nash equilibrium is $L_3^N = a_3 = 2$ days. Similarly, from Theorem 2, we solve the equation

$$\frac{\partial J_M(\bar{q}, \mathbf{L}; \bar{q}, \bar{r})}{\partial L_3} = \left\{ \frac{K + R_1(L_1)}{\bar{q}^3} \frac{\lambda[\pi_R \sigma \Psi(\Phi^{-1}(\rho)) + 2A\sqrt{L_1 + L_2 + L_3}]}{2h_R\sqrt{L_1 + L_2 + L_3}} - R_3'(L_3) - s_2 \right\} \lambda = 0,$$

and find that $\bar{L}_3 = 0.014$ which is larger than $a_3 = \frac{2}{365}$ and smaller than $b_3 = \frac{10}{365}$; thus, we obtain the Stackelberg solution as $L_3^S = \bar{L}_3 \doteq 5.11$ days. We present our solutions in Table 1.

From Table 1, we find that, in Example 1, the reorder point and order quantity of the retailer in the Stackelberg game are higher than those in the "simultaneous-move" game; shipping time in Pareto-optimal Nash equilibrium is crashed to the minimum, whereas in Stackelberg equilibrium it is crashed to a time in the interval (a_3, b_3). Moreover, we notice that, in this example, both the manufacturer and the retailer are better off in the "simultaneous-move" game. This result may be counter-intuitive because the manufacturer takes the role of the "leader" and this player should thus gains more power to raise his benefits.

Table 1
Pareto-optimal Nash and Stackelberg equilibria and the resulting expected profits for the manufacturer and the retailer.

	Pareto-optimal Nash	Stackelberg
Manufacturer		
Production quantity (Q)	177.26	179.25
Setup lead time (L_1)	3 days	3 days
Production lead time (L_2)	30 days	30 days
Shipping lead time (L_3)	2 days	5.11 days
Expected profit (J_M)	\$12,609.49	\$12,605.83
Retailer		
Reorder point (r)	89.28	95.75
Order quantity (q)	177.26	179.25
Expected profit (J_R)	\$9,491.96	\$9,451.54

To justify the result in Example 1, we provide the following discussion: in the games developed in Section 3.1, the manufacturer absorbs the shipping-related cost which includes two components—shipping cost and the maintenance cost. As we discussed in Section 3.1.1, the shipping cost is computed based on the quantity of items transported from the manufacturer to the retailer, and the maintenance cost is calculated by using not only the shipping quantity but also the shipping lead-time L_3. In order to investigate the impact of shipping-related cost on the manufacturer's and the retailer's decisions, we should concentrate our attention on the maintenance cost. Note that, in our model, the notation "s_2" represents the maintenance cost per unit per unit time; in Example 1, the value of s_2 is \$6/unit/year, which means that the manufacturer pays \$6 if this players spends one year to transport one unit of the item to the retailer. When s_2 assumes a high value, the manufacturer incurs a considerable maintenance cost, and as a result, this player would have an incentive to crash the shipping lead-time L_3. However, if the manufacturer decides to reduce L_3, then he has to pay for the crashing cost and also, the retailer reduces her order quantity q (according to Theorems 1–3) and the sale quantity and profit of the manufacturer thus decline. Therefore, to make an decision on the shipping lead-time L_3, the manufacturer should compare the costs generated by crashing and keeping L_3. If the value of s_2 is so high that the cost savings made by keeping L_3 unchanged cannot offset the maintenance cost, then the manufacturer would invest in the reduction of L_3; otherwise, the manufacturer would choose a high lead-time value or even the normal shipping lead-time b_3.

In Example 1, when $s_2 = 6$, the manufacturer does not want to pay for high maintenance cost and instead crashes the shipping lead-time L_3. However, as we discussed above, the retailer would respond to the reduction of L_3 by decreasing her order quantity and hence, the manufacturer's sale quantity declines. The manufacturer thus needs to consider the impact of L_3 on the retailer's reaction while determining the value of L_3. In the "simultaneous-move" game, two players make their decisions without any communication. The retailer does not know which value the manufacturer shall set for the shipping lead-time L_3, even though the retailer may assume the reduction of L_3 due to the high value of s_2. For this case, the manufacturer decides to crash L_3 to the minimum $a_3 = 2$ days; but, the retailer does not largely reduce her order quantity. In this example, the manufacturer benefits from the lack of communication. However, in the sequential-move (Stackelberg) game, the manufacturer announces his decisions to the retailer before the latter determines her order quantity q. In order not to "trigger" a large reduction of q, the manufacturer reduces the value of L_3 to 5.11 days rather than the minimum 2 days. As a consequence, the manufacturer still absorbs a high maintenance cost, and this player's profit is thus smaller than that in the "simultaneous-move" game.

To illustrate our above discussion, we again consider Example 1 but change the value of s_2 to 2, i.e., $s_2 = \$2/\text{unit/year}$. Our results are given in Table 2.

From Table 2 we find that, when the value of s_2 is 2, the manufacturer is willing to absorb the maintenance cost rather than the cost of crashing L_3 in the sequential-move game, because the cost savings generated by setting $L_3 = b_3$ can compensate the manufacturer for his maintenance payment. But, in the "simultaneous-move" game, the manufacturer still crashes L_3 to the minimum a_3, since this player wants to take advantage of the fact that the retailer shall not largely reduce her order quantity q because of the lack of the manufacturer's lead-time decisions in advance.

Table 2
The computations for Example 1 when $s_2 = 2$.

	Pareto-optimal Nash	Stackelberg
Manufacturer		
Production quantity (Q)	177.26	182.21
Setup lead time (L_1)	3 days	3 days
Production lead time (L_2)	30 days	30 days
Shipping lead time (L_3)	2 days	10 days
Expected profit (J_M)	\$12,622.64	\$12,662.06
Retailer		
Reorder point (r)	89.28	105.87
Order quantity (q)	177.26	179.25
Expected profit (J_R)	\$9,491.96	\$9,391.23

To further demonstrate our above discussion, we next perform a sensitivity analysis to investigate the impact of s_2 on two players' profits. More specifically, we change the value of s_2 from \$1 to \$10 in steps of \$1, and compute the corresponding profits of the retailer and the manufacturer, as indicated in Fig. 2. When s_2 increases, the manufacturer's profit decreases. In addition, when s_2 is sufficiently small (e.g., $s_2 \leqslant 5$ in our example), the manufacturer gains more profit in the sequential-move (Stackelberg) game than in the "simultaneous-move" (Nash) game. However, when s_2 is large (e.g., $s_2 > 5$ in our example), the manufacturer's profit in the "simultaneous-move" game is no less than that in the sequential-move game. Note that, because the shipping lead-time L_3 has the upper bound b_3, the manufacturer's profits for both games are identical, as shown in Fig. 2. Moreover, because the manufacturer's decision on L_3 in the "simultaneous-move" game equals the lower bound a_3 for all values of s_2 in our study, the retailer's quantity decision is unchanged and thus this player's profit is constant independent of s_2. However, in the sequential-move game, when s_2 is sufficiently small, e.g., $s_2 \leqslant 5$, the retailer's profit is at a low level; but, when s_2 increases from 5 to 7, the retailer's profit jumps to a high level.

In addition, one may note from Tables 1 and 2 that the value of s_2 has a significant impact on the difference between two players' profits in Nash and Stackelberg games. More specifically, as Table 1 indicates, when $s_2 = \$6$, the manufacturer's profits in Nash and Stackelberg games are \$12,609.49 and \$12,605.83; this means that, when the manufacturer moves from Nash game to Stackelberg game, this player's profit decreases by $(\$12,609.49 - \$12,605.83)/\$12,609.49 \approx 0.03\%$. The retailer's profits decreases by 0.43% if moving from Nash game and Stackelberg game. Using Table 2, we find that, when $s_2 = \$2$, the manufacturer's profit increases by 0.31% and the retailer's profit decreases by 1.06%, if two players moves from Nash game to Stackelberg game. We compute the percentage changes of two players' profits for the values of s_2 (from 1 to 10), and present the results in Table 3. We find that, when $s_2 \geqslant 7$, the changes for both players are zero. This result appears because the shipping lead-time L_3 is equal to its upper bound b_3 when s_2 is sufficiently larger (e.g., $s_2 \geqslant 7$ in our numerical example).

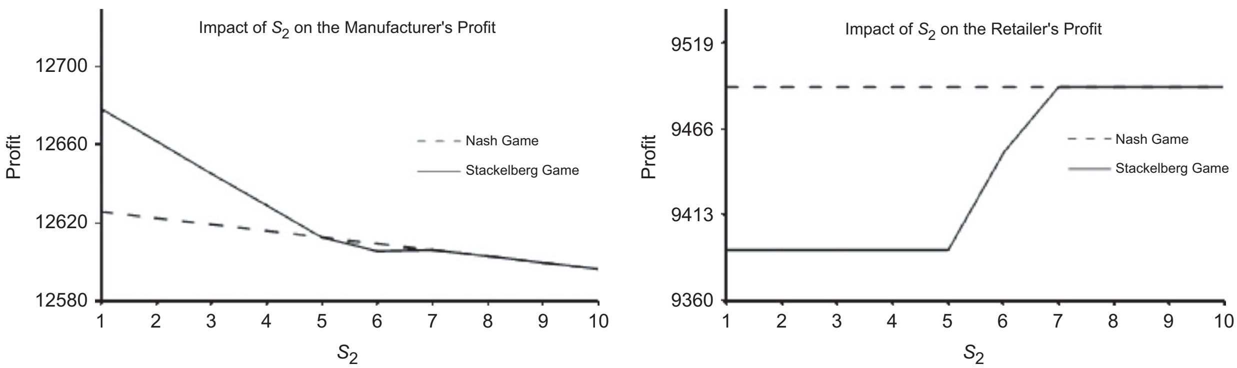

Fig. 2. The impacts of the parameter s_2 on the manufacturer's and the retailer's profits.

Table 3
The percentage changes in two players' profits for the values of s_2 from 1 to 10, when these two players change from Nash game to Stackelberg game.

s_2	Changes in manufacturer's profit (%)	Changes in retailer's profit (%)
1	0.42 (↑)	1.06 (↓)
2	0.31 (↑)	1.06 (↓)
3	0.21 (↑)	1.06 (↓)
4	0.1 (↑)	1.06 (↓)
5	0.0002 (↓)	1.06 (↓)
6	0.03 (↓)	0.43 (↓)
7	0	0
8	0	0
9	0	0
10	0	0

Note that the labels "↑" and "↓" mean the increase and the decrease in the percentages, respectively.

According to our above discussion, we draw managerial implications and present them in the following remark.

Remark 2. When the manufacturer is responsible for the shipping-related cost, the manufacturer may or may not takes the role of "leader" and announce his lead-time decisions to the retailer before the latter makes her ordering decision. More specifically, if the unit maintenance cost s_2 is sufficiently low, then the manufacturer should assume the "leader" role; otherwise, the manufacturer should not do so. In fact, because the retailer is always worse off in Stackelberg game, this player prefers Nash to Stackelberg games. Therefore, we can conclude that Nash equilibrium would be played when s_2 is sufficiently large; but, when s_2 is sufficiently low, Stackelberg would be played in the two-level supply chain.

Next, we consider the numerical problem in Example 1, and compute the Nash and Stackelberg solutions, and two players' profits for the games in which the retailer determines the shipping lead-time L_3. Note that, as Theorem 5 indicates, the Nash equilibrium is identical to the Stackelberg solution for this case.

Example 2. According to Theorem 5, we first maximize the function $J_R(L_3)$ subject to $L_3 \in [a_3, b_3]$ and find $\check{L}_3 = a_3 = 2$ days, which means that the retailer should crash the shipping lead time to the minimum bound a_3. Our solutions are presented in Table 4.

By comparing Examples 1 and 2, we find that these two players' decisions in Example 2 are the same as those for the Nash game in Example 1. However, the retailer's profit in Example 2 is lower than that in Example 1, whereas the manufacturer' profit in Example 2 is greater than that in Example 1. This happens because the retailer absorbs the shipping cost amounting to \$6,021.06. More specifically, in both cases, the setup and production lead times are not crashed but the shipping lead times are reduced to the minimum value a_3. Since the lead-time decisions are the same for both cases, the manufacturer's and the retailer's decisions are unchanged but the retailer pays the shipping cost \$6,021.06 because this player now controls the shipping lead time.

Table 4
The Pareto-optimal Nash and Stackelberg equilibria and the resulting expected profits for the manufacturer and the retailer when the retailer makes the shipping lead-time decision.

	Identical Nash and Stackelberg solutions
Manufacturer	
Production quantity (Q)	177.26
Setup lead time (L_1)	3 days
Production lead time (L_2)	30 days
Shipping lead time (L_3)	2 days
Expected profit (J_M)	\$18,630.54
Retailer	
Reorder point (r)	89.28
Order quantity (q)	177.26
Expected profit (J_R)	\$3,470.90

Table 5
Two players' expected profits when $s_1 = 0.02$ and $s_2 = 0.01$.

Who determines L_3?	Manufacturer		Retailer
Game	Nash game	Stackelberg game	Nash and Stackelberg games
Manufacturer's profit	\$18,682.77	\$18,682.77	\$18,630.54
Retailer's profit	\$9,391.23	\$9,391.23	\$9,478.59

When the unit shipping cost s_1 and the unit maintenance cost s_2 is very low, the retailer's profit in the games in which the player controls L_3 and absorbs the shipping-related cost would be greater than that in the games in which the manufacturer is responsible for the shipping-related cost. To illustrate this, we present the following example.

Example 3. We again consider the numerical problem in Example 1 but reduce the values of s_1 and s_2 to 0.02 and 0.01, respectively. Our solutions are given in Table 5.

From Table 5 we find that the retailer's profit is higher when the retailer determines the shipping lead-time L_3 than when the manufacturer is responsible for the shipment. Our managerial insights can thus be drawn in the following remark.

Remark 3. When the unit shipping cost s_1 and the unit maintenance cost s_2 are sufficiently low, the retailer has an incentive to control the shipping lead-time L_3 because this power results in more profit for the retailer. However, when s_1 and s_2 are high, the retailer's profit decreases if this player is responsible for shipping the purchased items. As a result, for such a case, the retailer should not take the power for controlling the shipment and instead transfers it to the manufacturer.

4. Supply chain coordination with a profit-sharing contract

In the non-cooperative scenarios presented above, we determined Pareto-optimal Nash and Stackelberg equilibria for the manufacturer and the retailer. It is, of course, possible to formulate a cooperative game where the two players can cooperate and increase the total (expected) profit incurred by the supply chain. Such an outcome gives rise to some interesting issues of surplus sharing between the players. One possibility is that even though the cooperative solution will raise the total system-wide profit, it may reduce the profit of one of the players; another possibility is where both players increase their profits. In both cases, it is important to establish a method whereby the surpluses are shared equitably between the players by a side-payment from one player to the other so that they will have an incentive to cooperate. In this section we propose a linear profit-sharing contract within the cooperative game to analyze the surplus sharing problem between the manufacturer and the retailer. For an early application of this contract in the cooperative inventory policies in a two-stage supply chain, see Jeuland and Shugan (1983).

4.1. Definition of the profit-sharing contract

We define $J^C(Q, \mathbf{L}, q, r)$ as the system-wide profit jointly achieved by the manufacturer and the retailer, i.e.,

$$J^C(Q, \mathbf{L}, q, r) = J_R[Q, \mathbf{L}, q, r] + J_M[Q, \mathbf{L}, q, r], \tag{24}$$

where $J_R[Q, L, q, r]$ and $J_M[Q, L, q, r]$, respectively, means the retailer's and the manufacturer's profits. Note that we now use the notations $J_i[\cdot]$ instead of $J_i(\cdot)$, for $i = R, M$, because, in Section 3, we considered two cases in which the shipping lead-time decision is made by different players. More specifically, in Section 3.1, the manufacturer determines the shipping lead-time L_3 and absorbs the shipping-related cost. Thus, the profit functions $J_R[\cdot]$ and $J_M[\cdot]$ are $J_R(q, r; Q, L)$ in (2) and $J_M(Q, L; q, r)$ in (7), respectively. In Section 3.2, the retailer is responsible for the shipment; so, $J_R[\cdot]$ and $J_M[\cdot]$ are given as $J_R(q, r, L_3; Q, L_1, L_2)$ in (18) and $J_M(Q, L_1, L_2; q, r, L_3)$ in (19). Even though the functions $J_R[\cdot]$ and $J_M[\cdot]$ take different forms for Sections 3.1 and 3.2, the system-wide profit function $J^C(Q, L, q, r)$ are the same for these two cases. In this section, we shall design a linear profit-sharing contract under which the two supply chain members can cooperate by choosing the globally optimal solution (Q^*, L^*, q^*, r^*) that maximizes the chain-wide profit $J^C(Q, L, q, r)$.

Since the manufacturer and the retailer now negotiate with each other to reach an agreement on the linear profit-sharing contract, it is no longer possible for them to act as a leader and a follower. Therefore, the game with profit-sharing terms is now considered as a "simultaneous-move" game and hence the optimal decisions announced by the two members should be the Nash equilibrium solution $(Q^{CN}, L^{CN}, q^{CN}, r^{CN})$. Since, in Section 3, (Q^N, L^N, q^N, r^N) denotes the Nash equilibrium for the non-cooperative "simultaneous-move" game, we now use the superscript "CN" to represent the Nash equilibrium for the cooperative game with profit-sharing terms.

4.1.1. Conditions required for a workable profit-sharing contract

We now focus on the design of a linear profit-sharing contract which would result in an expected profit of $J_R^C[Q, L, q, r]$ and $J_M^C[Q, L, q, r]$ for the retailer and the manufacturer, respectively. In order for the contract to be workable, two criteria must be satisfied:

Criterion 1. With the profit-sharing terms, the Nash equilibrium should be identical to the optimal solution that maximizes the supply chain-wide expected profit $J^C(Q, \mathbf{L}, q, r)$, i.e.,

$$(Q^{CN}, \mathbf{L}^{CN}, q^{CN}, r^{CN}) = (Q^*, \mathbf{L}^*, q^*, r^*) \equiv \arg\max_{Q,\mathbf{L},q,r} J^C(Q, \mathbf{L}, q, r).$$

Criterion 2. Each supply chain member's expected profit in the cooperative game with profit-sharing terms should be no less than his/her maximum expected profit in Nash or Stackelberg game without the profit-sharing contract, i.e.,

$$J_R^C[Q^{CN}, \mathbf{L}^{CN}, q^{CN}, r^{CN}] \geqslant J_R[Q^e, \mathbf{L}^e, q^e, r^e]$$

and

$$J_M^C[Q^{CN}, \mathbf{L}^{CN}, q^{CN}, r^{CN}] \geqslant J_M[Q^e, \mathbf{L}^e, q^e, r^e],$$

where $(Q^e, \mathbf{L}^e, q^e, r^e)$ represents an equilibrium in a non-cooperative game (Nash or Stackelberg game), i.e.,

$$(Q^e, \mathbf{L}^e, q^e, r^e) = \begin{cases} (Q^N, \mathbf{L}^N, q^N, r^N) & \text{for Nash game,} \\ (Q^S, \mathbf{L}^S, q^S, r^S) & \text{for Stackelberg game.} \end{cases}$$

Remark 4. Criterion 1 ensures that neither supply chain member will have an incentive to deviate from the optimal solution which maximizes the system-wide profit. Otherwise, one or both members could deviate from the system-wide optimal solution to increase his/her individual expected profit at the expense of a lower supply chain-wide profit. Criterion 2 ensures that both players are better off with the cooperative solution than with the Nash and Stackelberg equilibria.

4.1.2. Linear profit-sharing contract

By using the above arguments, we construct the players' objective functions with a linear profit-sharing contract as

$$J_R^C[Q, \mathbf{L}, q, r] = \beta J^C(Q, \mathbf{L}, q, r) - \gamma, \tag{25}$$

$$J_M^C[Q, \mathbf{L}, q, r] = (1 - \beta) J^C(Q, \mathbf{L}, q, r) + \gamma, \tag{26}$$

where $\beta \in (0, 1)$ is the percentage of the chain-wide profit allocated to the retailer; $\gamma \in (-\infty, +\infty)$ is a constant transfer payment (from the retailer to the manufacturer) independent of the decision variables. We can easily find that, in the game with the objective functions (25) and (26), two players shall make their equilibrium decisions which are identical to the globally optimal solutions that maximize the chain-wide profit $J^C(Q, L, q, r)$. This means that, when the above profit-sharing contract (β, γ) applies, Criterion 1 is satisfied for any value of β in the range (0,1). However, a natural question arises as follows: Which value of β should be suggested to coordinate this supply chain? In order to answer this question, we first use the concept of Nash arbitration scheme to find the maximum profits $J_i^C[Q, L, q, r]$ $(i = R, M)$ that two players can achieve after sharing the system-wide profit $J^C(Q, L, q, r)$.

Nash arbitration scheme which was developed by Nash (1950) has widely been used to allocate surplus on the negotiation set on which any point satisfies the following two conditions: (i) it is Pareto optimal; (ii) it is at or above the security level of each player—which is also called *status quo* point representing the guaranteed payoff incurred by the player in a non-cooperative play. Nash proved the uniqueness of Nash arbitration scheme and showed that, for a game with players 1 and 2, this scheme can be obtained by solving

$$\max_{f_1 \geqslant f_1^0, f_2 \geqslant f_2^0} (f_1 - f_1^0)(f_2 - f_2^0) \quad \text{s.t. } (f_1, f_2) \in \mathscr{P}, \tag{27}$$

where f_i and f_i^0, respectively, denote player i's allocated surplus and this player's security level, $i = 1$, 2; and $\mathscr{P}$ is the set of Pareto-optimal solutions.

In our problem, the manufacturer and the retailer bargain for the allocation of chain-wide profit $J^C(Q, L, q, r)$. To find Nash arbitration scheme, we need to solve (27) but use subscripts "R" and "M" to replace "1" and "2". Since two players share $J^C(Q, L, q, r)$, the Pareto optimal set $\mathscr{P}$ is written as $\mathscr{P} = \{(f_R, f_M) | f_R + f_M = J^C(Q, L, q, r)\}$. Moreover, we notice that, in the games without the profit-sharing contract, the retailer and the manufacturer can, respectively, gain the profits $J_R[Q^e, L^e, q^e, r^e]$ and $J_M[Q^e, L^e, q^e, r^e]$. Hence, the security levels of two players should, naturally, be $f_i^0 = J_i[Q^e, L^e, q^e, r^e]$, for $i = R, M$.

Theorem 7. *When we use the profit-sharing contract to coordinate the two-level supply chain, Nash arbitration scheme suggests that the retailer and the manufacturer shall share the maximum chain-wide profit* $J^C(Q^*, L^*, q^*, r^*)$ *as follows*:

$$J_R^C[Q^*, L^*, q^*, r^*] = \frac{J^C(Q^*, L^*, q^*, r^*) - J_M[Q^e, \mathbf{L}^e, q^e, r^e] + J_R[Q^e, \mathbf{L}^e, q^e, r^e]}{2},$$

$$J_M^C[Q^*, L^*, q^*, r^*] = \frac{J^C(Q^*, L^*, q^*, r^*) + J_M[Q^e, \mathbf{L}^e, q^e, r^e] - J_R[Q^e, \mathbf{L}^e, q^e, r^e]}{2}.$$

From Theorem 7, we find, for player i $(i = R, M)$,

$$\begin{aligned} J_i^C[Q^*, L^*, q^*, r^*] &= \frac{J^C(Q^*, L^*, q^*, r^*) + J_i[Q^e, \mathbf{L}^e, q^e, r^e] - J_j[Q^e, \mathbf{L}^e, q^e, r^e]}{2} \\ &= J_i[Q^e, \mathbf{L}^e, q^e, r^e] \\ &\quad + \frac{J^C(Q^*, L^*, q^*, r^*) - J_i[Q^e, \mathbf{L}^e, q^e, r^e] - J_j[Q^e, \mathbf{L}^e, q^e, r^e]}{2}, \end{aligned} \tag{28}$$

where $j = R, M$ and $j \neq i$. Note that the second term in the RHS of (28) represents a half of the chain-wide profit surplus jointly achieved by two players under the profit-sharing contract. Obviously, the chain-wide profit surplus (i.e., $J^C(Q^*, L^*, q^*, r^*) - J_R[Q^e, L^e, q^e, r^e] - J_M[Q^e, L^e, q^e, r^e]$) is non-negative; so, we have

$$J_i^C[Q^*, L^*, q^*, r^*] \geqslant J_i[Q^e, \mathbf{L}^e, q^e, r^e] \quad \text{for player } i \ (i = R, M),$$

which proves that Criterion 2 is satisfied if we use Theorem 7 to allocate the maximum chain-wide profit $J^C(Q^*, L^*, q^*, r^*)$ between the retailer and the manufacturer. Moreover, we notice from Theorem 7 that Nash arbitration scheme assumes that the two players have equal bargaining power to negotiate the allocation of the system-wide profit surplus. In our paper, we make the assumption that the manufacturer and the retailer have equal bargaining power on the profit-sharing contract.

Next, we choose the proper contract parameters β and γ so that two players can receive their profits that are suggested in Theorem 7.

Theorem 8. *In order to coordinate the two-level supply chain, the contract parameters* β *and* γ *must satisfy the following condition*:

$$\gamma = \frac{(2\beta - 1)J^C(Q^*, \mathbf{L}^*, q^*, r^*) + J_M[Q^e, \mathbf{L}^e, q^e, r^e] - J_R[Q^e, \mathbf{L}^e, q^e, r^e]}{2}. \tag{29}$$

From (29) we find that, for a given value of β, a unique value of γ can be determined. Since we assume that both players have equal bargaining power on the profit-sharing contract, we can accordingly set $\beta = \frac{1}{2}$. Thus, we can compute γ as

$$\gamma = \frac{J_M[Q^e, \mathbf{L}^e, q^e, r^e] - J_R[Q^e, \mathbf{L}^e, q^e, r^e]}{2}, \tag{30}$$

which represents the amount that the retailer should transfer to the manufacturer. Note that, if $\gamma \geqslant 0$, then the manufacturer should make the side payment $|\gamma|$ to the retailer.

In conclusion, the profit-sharing contract for supply chain coordination can be described as: the retailer and the manufacturer agree to equally share the chain-wide profit $J^C(Q^*, L^*, q^*, r^*)$, and the retailer also agree to transfer $(J_M[Q^e, \mathbf{L}^e, q^e, r^e] - J_R[Q^e, \mathbf{L}^e, q^e, r^e])/2$ to the manufacturer. Under this contract, two players' profit functions are

$$J_R^C[Q, \mathbf{L}, q, r] = \frac{J^C(Q, \mathbf{L}, q, r)}{2} - \frac{J_M[Q^e, \mathbf{L}^e, q^e, r^e] - J_R[Q^e, \mathbf{L}^e, q^e, r^e]}{2},$$

$$J_M^C[Q, \mathbf{L}, q, r] = \frac{J^C(Q, \mathbf{L}, q, r)}{2} + \frac{J_M[Q^e, \mathbf{L}^e, q^e, r^e] - J_R[Q^e, \mathbf{L}^e, q^e, r^e]}{2}$$

and this supply chain coordination can be induced.

4.2. Maximization of the system-wide profit

In this section we present some results pertaining to the optimal solution that maximizes the system-wide objective function $J^C(Q, \mathbf{L}, q, r)$. Since the system-wide expected profit $J^C(Q, \mathbf{L}, q, r) = J_R[Q, \mathbf{L}, q, r] + J_M[Q, \mathbf{L}, q, r]$ is maximized subject to the constraint $r = \bar{r} = \lambda L + \Phi^{-1}(\rho)\sigma\sqrt{L}$, we can write the profit function as

$$\begin{aligned} J^C(Q, \mathbf{L}, q) = {} & \nu\lambda - h_R\left[\frac{\min(Q,q)}{2} + \bar{r} - \lambda L\right] \\ & - \frac{\lambda}{\min(Q,q)}\left[\pi_R \int_{\bar{r}}^{\infty}(x-\bar{r})f(x)\,\mathrm{d}x + A\right] \\ & - \frac{K + R_1(L_1) + [p + R_2(L_2)]Q}{\min(Q,q)}\lambda \\ & - [s_1 + R_3(L_3) + s_2L_3]\lambda \\ & - \frac{h_M(Q-q)^+ + \pi_M(q-Q)^+}{\min(Q,q)}\lambda. \end{aligned} \quad (31)$$

Next, for any given $\mathbf{L}$, we maximize $J^C(Q, \mathbf{L}, q)$ in (31) to find the optimal production quantity $Q^*(\mathbf{L})$ for the manufacturer and the optimal order quantity $q^*(\mathbf{L})$ for the retailer. We note that, in (31), $\bar{r} = \lambda L + \Phi^{-1}(\rho)\sigma\sqrt{L}$, which is independent of Q and q.

Theorem 9. *For given lead-time values $\mathbf{L} = (L_1, L_2, L_3)$, the manufacturer's optimal production quantity $Q^*(\mathbf{L})$ and the retailer's optimal order quantity $q^*(\mathbf{L})$ are equal and they are found as*

$$Q^*(\mathbf{L}) = q^*(\mathbf{L}) = \sqrt{\frac{2\lambda[\pi_R\sigma\sqrt{L}\Psi(\Phi^{-1}(\rho)) + A + K + R_1(L_1)]}{h_R}}, \quad (32)$$

where $\bar{r} = \lambda L + \Phi^{-1}(\rho)\sigma\sqrt{L}$.

As indicated in Theorem 9, the manufacturer and the retailer should make an identical quantity decision, which depends on three lead-time components. When we compare (32) with the equilibrium quantities in Section 3, we find that the globally optimal solution (32) includes the term $K + R_1(L_1)$ but the quantities in Section 3 do not. This occurs because of the fact: the fixed setup cost $K + R_1(L_1)$ is needed for the calculation of global quantity. However, in the non-cooperative games, this cost is absorbed by the manufacturer, but it does not impact the manufacturer's quantity decision which is always equal to the retailer's order quantity.

From Theorem 9 we have obtained $q^*(\mathbf{L})$ and $Q^*(\mathbf{L})$, which are given in terms of lead-time vector $\mathbf{L}$. Substituting $q^*(\mathbf{L})$ and $Q^*(\mathbf{L})$ into the system-wide profit function (31) and simplifying it, we have

$$\begin{aligned} J^C(\mathbf{L}) = {} & \nu\lambda - h_RQ^*(\mathbf{L}) - h_R\Phi^{-1}(\rho)\sigma\sqrt{L} \\ & - [p + R_2(L_2)]\lambda - [s_1 + R_3(L_3) + s_2L_3]\lambda, \end{aligned} \quad (33)$$

which only depends on the decision variables in $\mathbf{L}$.

Taking the first-order partial derivative of $J^C(L)$ in (33) w.r.t. L_1 gives

$$\frac{\partial J^C(\mathbf{L})}{\partial L_1} = -h_R\frac{\partial Q^*(\mathbf{L})}{\partial L_1} - \frac{h_R\Phi^{-1}(\rho)\sigma}{2\sqrt{L}} < 0,$$

which means that the chain-wide profit $J^C(L)$ is decreasing in L_1. Because $L_1 \in [a_1, b_1]$, the optimal setup lead time $L_1^* = a_1$. However, it is complicated to solve (33) for L_2^* and L_3^*. Since the function $J^C(L)$ is continuous in the bounded region $R = \{\mathbf{L}|L_i \in [a_i, b_i], i = 1,2,3\}$, the optimal lead-time components L_2^* and L_3^* that maximize $J^C(L)$ must exist. We denote the optimal lead-time vector by $\mathbf{L}^* = (L_1^*, L_2^*, L_3^*)$, and denote total lead time by $L^* = L_1^* + L_2^* + L_3^*$. Using $\mathbf{L}^*$, the optimal solutions r^*, Q^* and q^* are then obtained as

$$\begin{aligned} r^* &= \lambda L^* + \Phi^{-1}(\rho)\sigma\sqrt{L^*} \\ Q^* = q^* &= \sqrt{\frac{2\lambda[\pi_R\sigma\sqrt{L^*}\Psi(\Phi^{-1}(\rho)) + A + K + R_1(L_1^*)]}{h_R}}. \end{aligned} \quad (34)$$

In Section 3 we have shown that both equilibrium setup and production lead-time components should take their greatest values b_i, $i = 1,2$; this means that the globally optimal lead times L_i^* $(i = 1,2)$ must be less than or equal to the equilibrium solutions L_i^N and L_i^S $(i = 1,2)$. Next, we compare the globally optimal shipping lead time L_3^* with the equilibrium shipping decisions L_3^N and L_3^S.

Theorem 10. *When we compare the optimal shipping lead time L_3^* (that maximizes the chain-wide profit $J^C(\mathbf{L})$) with the equilibria L_3^N and L_3^S (that are obtained in Section 3), we have the following conclusions.*

1. *When the manufacturer makes the shipping decision, L_3^* must be less than or equal to L_3^N and L_3^S that are obtained in Section 3.1.2. More specifically,*
   (a) *If Stackelberg shipping lead time L_3^S is equal to a_3 when the manufacturer makes the shipping decision, then the globally optimal solution L_3^* and Nash and Stackelberg equilibria (i.e., L_3^N and L_3^S) obtained in Section 3.1.2 must be all equal to a_3.*
   (b) *If Stackelberg shipping lead time L_3^S is greater than a_3 but Nash solution L_3^N is equal to a_3 when the manufacturer makes the shipping decision, then the globally optimal solution L_3^* must be also equal to a_3; that is, for this case, L_3^* must be the same as Nash equilibrium L_3^N but be less than Stackelberg solution L_3^S obtained in Section 3.1.2.*
   (c) *If both Nash and Stackelberg shipping lead time (i.e., L_3^N and L_3^S) are greater than a_3 when the manufacturer makes the shipping decision, then the globally optimal solution L_3^* must be less than the equilibria L_3^N and L_3^S obtained in Section 3.1.2.*
2. *When the retailer makes the shipping decision, we can arrive to the following results:*
   (a) *If Nash equilibrium obtained in Section 3.1.2 (when the manufacturer makes the shipping decision) is equal to a_3, the optimal solution L_3^* must be equal to both Nash and Stackelberg solutions obtained in Section 3.2.2 (when the retailer makes the shipping decision).*
   (b) *Otherwise, if Nash equilibrium obtained in Section 3.1.2 is greater than a_3, then L_3^* may be less than, may be equal to or may be greater than L_3^N and L_3^S that are obtained in Section 3.2.2.*

Although Theorem 10 indicates that the optimal solution L_3^* is no more than the Nash and Stackelberg equilibrium solutions when the manufacturer makes the shipping decision, we cannot determine if L_3^* is less than the equilibria obtained in Section 3.2.2 when the retailer is responsible for the shipping. To justify our conclusion in Theorem 10, 2(b) we present three numerical examples as follows:

1. When we consider the numerical problem in Example 1 but only reduce the value of s_2 to 0.101, we find that the optimal solution L_3^* and the equilibria (obtained when the retailer makes the shipping decision) are all equal to a_3, even though, when the manufacturer makes the shipping decision, Nash equilibrium is greater than a_3.
2. When we still consider the numerical problem in Example 1 but reduce the values of s_2 and π_R to 0.101 and 0.01, respectively, we find that the optimal solution L_3^* is equal to $a_3 = 2$ days, but the equilibria (when the retailer makes the shipping decision) are both equal to 0.015 years or 5.475 days, which is greater than a_3. For this case, Nash equilibrium (when the manufacturer makes the shipping decision) equals 0.0174 years or 6.351 days.
3. We notice from the proof of Theorem 10 that, when $K + R_1(L_1)$ is sufficiently large so that $Q^*(L) \gg \hat{Q}$, the term $\partial J^C(L)/\partial L_3$ could be positive at the point $L_3 \in \{L_3 | \partial J_R(L_3)/\partial L_3 = 0\}$ and the globally optimal shipping lead-time L_3^* would thus be greater than the equilibrium shipping decision obtained when the retailer is responsible for the shipping. To show this, we consider the numerical problem in Example 1, but, respectively, reduce the values of s_2 and π_R to 0.101 and 0.1 and increase the value of K to a huge number (e.g., 7×10^{35}). We then find that the optimal solution L_3^* is equal to 0.0137 years or 5 days, which is greater than $a_3 = 2$ days, but the equilibria (when the retailer makes the shipping decision) are both equal to $a_3 = 2$ days. For this case, Nash equilibrium (when the manufacturer makes the shipping decision) still equals 0.0174 years or 6.351 days.

4.3. Numerical example

We illustrate our analysis of the cooperative game with a linear profit-sharing contract by using the data in Examples 1 and 2 given in Section 3.3.

As discussed in Section 4.1, the parameter $\beta = \frac{1}{2}$ and γ can be found by using (30). For different games, we present the values of γ in Table 6.

Next, we maximize the system-wide objective function $J^C(Q, \mathbf{L}, q, r)$ in (31) and obtain the solution as

$$(Q^*, r^*, q^*, L_1^*, L_2^*, L_3^*) = (302.98, 23.01, 302.98, \tfrac{1}{365}, \tfrac{3}{365}, \tfrac{2}{365}) \quad (35)$$

Table 6
Computation of γ.

Who determines L_3?	Manufacturer		Retailer
Game	Nash game	Stackelberg game	Nash and Stackelberg games
γ	\$1,558.77	\$1,577.15	\$7,579.82

with the maximum system-wide profit $J^C(Q^*, r^*, q^*, \mathbf{L}^*) = \$23,171.46$.

Next, we compare two players' optimal decisions for the games with and without the profit-sharing contract (β, γ). In particular, we compute the percentage changes in two players' order quantities Q and q, reorder point r and the shipping lead-time L_3. Note that we ignore the comparison on the setup and production lead times because they are the same for all cases. The comparison is given in Table 7 in which each percentage is calculated by using the formula

$$\% = \frac{\begin{pmatrix} \text{Optimal solution} & & \text{Equilibrium solution} \\ \text{for the game} & - & \text{for the game} \\ \text{with the} & & \text{without the} \\ \text{contract}(\beta,\gamma) & & \text{contract } (\beta,\gamma) \end{pmatrix}}{\text{Equilibrium solution for the game without the contract } (\beta,\gamma)}.$$

If the percentage is positive, then the decision is increased from the non-cooperative to cooperative cases; otherwise, the decision is decreased.

From Table 7, we find that, under the profit-sharing contract, the retailer largely increases her order quantity q but largely decreases her reorder point r (because all lead-time components are reduced to their lower bounds). As a result, the retailer reduces the number of orders per year, and also save the inventory-related cost during shorter lead time. The manufacturer accordingly increases his production quantity Q. As a result, the manufacturer achieves more sale profits.

Table 7
The percentage changes in two players' decisions when these players cooperate.

Who determines L_3?	Manufacturer		Retailer
Game	Nash game (%)	Stackelberg game (%)	Nash and Stackelberg games
Q	↑ 70.9	↑ 69	↑ 70.9
q	↑ 70.9	↑ 69	↑ 70.9
r	↓ 74	↓ 76	↓ 74
L_3	0	↓ 61	0

Note that the labels "↑" and "↓" mean the increase and the decrease in their decisions, respectively.

Table 8
The manufacturer's and the retailer's profits under the profit-sharing contract.

Who determines L_3?	Manufacturer		Retailer
Game	Nash game	Stackelberg game	Nash and Stackelberg games
Manufacturer's profit under the contract	\$13,144.5 [↑ 4.2%]	\$13,162.88 [↑ 4.5%]	\$19,165.55 [↑ 2.9%]
Retailer's profit under the contract	\$10,026.96 [↑ 5.6%]	\$10,008.58 [↑ 5.9%]	\$4,005.91 [↑ 15.6%]

The numbers in the brackets [·] indicate the percentage increase in each player's profit when the supply chain is coordinated under the profit-sharing contract.

Using Theorem 7, we can find the manufacturer's and the retailer's profits under the profit-sharing contract, are indicated in Table 8.

From Table 8, we find that both players are better off by cooperating for supply chain coordination. Furthermore, the percentage increase in the retailer's profit is greater than that in the manufacturer's profit. If the retailer is responsible for the shipment, this player should have more incentives to agree with the manufacturer for the profit-sharing contract.

5. Summary and concluding remarks

Lead-time reduction is one of the most important problems encountered in the efficient management of a supply chain. In this paper we considered game-theoretic analyses of lead-time reduction in a two-level supply chain, where a manufacturer determines his production quantity and the setup and production lead-time components and a retailer chooses her order quantity and reorder point. We considered two cases where the shipping lead time is determined by the manufacturer or by the retailer. For each case in the non-cooperative setting, we compute the Pareto-optimal Nash and Stackelberg equilibria for the "simultaneous-move" and sequential-moves games, respectively. Next, we considered a cooperative version of the same problem and designed a simple profit-sharing contract. This contract assures that a system-wide objective function is optimized while increasing the profits of both the manufacturer and retailer compared to the case in the non-cooperative setting. We showed that the supply chain can be coordinated with a properly designed profit-sharing contract.

The main contributions of our paper are summarized as follows:

1. We modeled setup, production and shipping cost functions in terms of the three lead-time components. We hope that these models would be useful for other scholars to analyze some relevant problems.
2. We used game theory to analyze the problem of lead-time reduction in supply chain management. Our review in Section 2 has shown that the publications reviewed are only concerned with a single firm's lead-time decision. As we discussed in Section 1, the lead-time decision of a supply chain member has significant impacts on the performance of the member's upstream and downstream parties. Thus, our game-theoretic analysis on lead-time reduction is necessary in supply chain management.
3. We developed a profit-sharing contract to induce the coordination of the two-echelon supply chain. This is necessary because improving the chain-wide performance is too important to be ignored in supply chain operations. Many practitioners and academics have already paid attention to designing various contracts for supply chain coordination. For the supply chain under study in our paper, we suggested a profit-sharing contract that is easy to implement, and showed that the contract can coordinate the supply chain.

In our paper, we assume that the retailer decides on the reorder point for a given service level, because treating the service level as the retailer's decision variable would make our model too complicated to be tractable. However, as a potential research topic in future, we would calculate optimal service level and thus reorder point for the supply chain discussed in this paper. Finding optimal service level is an interesting topic in supply chain management. For example, in a recent publication, Xiao and Yang (2008) developed a price–service competition model of two supply chains to investigate the optimal decisions of players under demand uncertainty. Each supply chain consists of one risk-neutral supplier and one risk-averse retailer who makes decisions on both price and its service level.

We showed in our paper that a properly designed profit-sharing contract could be an effective means of achieving supply chain coordination. Naturally, such a contract between the manufacturer and the retailer as described in this paper can also be applied to other problems with two competing decision makers. For example, in one of the early applications of game theory in supply chain management, Parlar (1988) considered two competing newsvendors facing random demands and showed the uniqueness of the Nash equilibrium. Although the author showed that cooperation between the two players can increase the system-wide profit, Parlar (1988) did not present an operational procedure to coordinate the supply chain with a properly designed contract. We hope to consider this problem in a future publication.

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Appendix A. Proofs

Proof of Proposition 1. For given values of the manufacturer's production quantity Q and lead-time $\mathbf{L}$, we maximize $J_R(q,\bar{r};Q,\mathbf{L})$ to find the retailer's best-response $\hat{q}(Q,\mathbf{L})$. Since the retailer actually receives $\min(Q,q)$ units of products, we compare Q and q, and analyze the profit function $J_R(q,\bar{r};Q,\mathbf{L})$ for each of the following two cases.

1. $Q\leqslant q$. For this case, the retailer's conditional profit function (denoted by J_{R1}) is
$$J_{R1}\equiv J_R(q,\bar{r};Q,\mathbf{L})|_{Q\leqslant q} = (v-w)\lambda - h_R\left(\frac{Q}{2}+\bar{r}-\lambda L\right) - \frac{\lambda}{Q}\left[\pi_R\int_{\bar{r}}^{\infty}(x-\bar{r})f(x)\,\mathrm{d}x + A\right],$$
which is a constant independent of q.
2. $Q\geqslant q$. For this case, we write the retailer's conditional profit function (denoted by J_{R2}) as
$$J_{R2}\equiv J_R(q,\bar{r};Q,\mathbf{L})|_{Q\geqslant q} = (v-w)\lambda - h_R\left(\frac{q}{2}+\bar{r}-\lambda L\right) - \frac{\lambda}{q}\left[\pi_R\int_{\bar{r}}^{\infty}(x-\bar{r})f(x)\,\mathrm{d}x + A\right].$$

(Note that $J_R(q,\bar{r};Q,L)$ is a continuous function of q, since at $Q=q$, we have $J_{R1}=J_{R2}$.) The first- and second-order derivatives of J_{R2} w.r.t. q are,

$$\frac{\partial J_{R2}}{\partial q}=-\frac{h_R}{2}+\frac{\lambda}{q^2}\left[\pi_R\int_{\bar{r}}^{\infty}(x-\bar{r})f(x)\,dx+A\right],$$
$$\frac{\partial^2 J_{R2}}{\partial q^2}=-\frac{2\lambda}{q^3}\left[\pi_R\int_{\bar{r}}^{\infty}(x-\bar{r})f(x)\,dx+A\right]<0, \quad (36)$$

which implies that J_{R2} is a concave function in q. We first ignore the constraint $Q\geqslant q$, and find the corresponding optimal solution $\bar{q}$. Setting $\partial J_{R2}/\partial q$ in (36) to zero and solving the resulting equation, we have

$$\bar{q}=\sqrt{\frac{2\lambda\left[\pi_R\int_{\bar{r}}^{\infty}(x-\bar{r})f(x)\,dx+A\right]}{h_R}}=\sqrt{\frac{2\lambda[\pi_R\sigma\sqrt{L}\Psi(\Phi^{-1}(\rho))+A]}{h_R}},$$

where we use (4) to find the second equality.When we consider the constraint $Q\geqslant q$, we have the retailer's best-response order quantity as

$$\hat{q}(Q,\mathbf{L})=\min(Q,\bar{q})=\min\left(Q,\sqrt{\frac{2\lambda[\pi_R\sigma\sqrt{L}\Psi(\Phi^{-1}(\rho))+A]}{h_R}}\right). \quad (37)$$

This proves the proposition. □

Proof of Corollary 1. We consider the following two cases:

1. When $\bar{q}\geqslant Q$, we find that $\hat{q}(Q,\mathbf{L})=Q$ and
$$J_R(\hat{q}(Q,\mathbf{L}),\bar{r};Q,\mathbf{L})=(v-w)\lambda-h_R\left[\frac{Q}{2}+\Phi^{-1}(\rho)\sigma\sqrt{L}\right]-\frac{\lambda}{Q}[\pi_R\sigma\sqrt{L}\Psi(\Phi^{-1}(\rho))+A],$$
which is decreasing in L.
2. When $\bar{q}<Q$, we find that $\hat{q}(Q,\mathbf{L})=\bar{q}$ and
$$J_R(\hat{q}(Q,\mathbf{L}),\bar{r};Q,\mathbf{L})=(v-w)\lambda-h_R\left[\sqrt{\frac{\lambda[\pi_R\sigma\sqrt{L}\Psi(\Phi^{-1}(\rho))+A]}{2h_R}}+\Phi^{-1}(\rho)\sigma\sqrt{L}\right]-\sqrt{\frac{\lambda h_R[\pi_R\sigma\sqrt{L}\Psi(\Phi^{-1}(\rho))+A]}{2}},$$
which is also decreasing in L. □

Proof of Proposition 2. In order to obtain the manufacturer's best-response production quantity, we analyze J_{M1} and J_{M2} given in (8) and (9).

1. When $Q\leqslant q$, the manufacturer's expected profit function is J_{M1}. Taking the first partial derivative of J_{M1} with respect to Q, we have
$$\frac{\partial J_{M1}}{\partial Q}=\left[\frac{K+R_1(L_1)}{Q^2}+\frac{\pi_M q}{Q^2}\right]\lambda>0.$$
Thus, q is the manufacturer's optimal solution that maximizes $J_M(Q,\mathbf{L};q,r)$ subject to $Q\leqslant q$, i.e., $q=\operatorname{argmax}_{Q\in[0,q]}J_M(Q,\mathbf{L};q,r)$.
2. When $Q\geqslant q$, the manufacturer's expected profit function is J_{M2}. The first-order partial derivative of J_{M2} w.r.t. Q is
$$\frac{\partial J_{M2}}{\partial Q}=-\frac{[p+h_M+R_2(L_2)]\lambda}{q}<0,$$
which implies that, in order to maximize $J_M(Q,\mathbf{L};q,r)$ subject to $Q\geqslant q$, the manufacturer should choose q as his optimal solution, i.e., $q=\operatorname{argmax}_{Q\in[q,+\infty]}J_M(Q,\mathbf{L};q,r)$.

In conclusion, when the retailer announces her decisions as q and r, the manufacturer's best-response production quantity is q. □

Proof of Corollary 2. Replacing Q in (7) with q simplifies (7) to

$$J_M(q,\mathbf{L};q,r)=\left\{w-\frac{K+R_1(L_1)}{q}-[p+s_1+R_2(L_2)+R_3(L_3)+s_2L_3]\right\}\lambda, \quad (38)$$

where $L_i\in[a_i,b_i]$, $R_i(b_i)=0$ and $R_i(L_i)>0$ for $L_i\in[a_i,b_i)$, $i=1,2,3$. From (38) we find that the manufacturer's expected profit $J_M(q,\mathbf{L};q,r)$ is increasing in q. This means that, if the retailer increases her order quantity q, the manufacturer's expected profit increases. □

Proof of Proposition 3. The first-order partial derivative of $J_M(q,\mathbf{L};q,r)$ in (38) w.r.t. L_1 is

$$\frac{\partial J_M(q,\mathbf{L};q,r)}{\partial L_1}=-\frac{R_1'(L_1)}{q}\lambda>0,$$

which implies that $J_M(q,\mathbf{L};q,r)$ is an increasing function in L_1; thus, $\hat{L}_1=b_1$. Likewise, the best-response production lead time is $\hat{L}_2=b_2$.

The first- and second-order partial derivatives of $J_M(q,\mathbf{L};q,r)$ in (38) w.r.t. L_3 are

$$\frac{\partial J_M(q,\mathbf{L};q,r)}{\partial L_3}=-[R_3'(L_3)+s_2]\lambda,$$
$$\frac{\partial^2 J_M(q,\mathbf{L};q,r)}{\partial L_3^2}=-R_3''(L_3)\lambda<0, \quad (39)$$

which implies that $J_M(q,L;q,r)$ is concave in L_3. Thus, momentarily ignoring the constraint $a_3\leqslant L_3\leqslant b_3$ and solving $dJ_M(q,L;q,r)/dL_3=0$ gives $\tilde{L}_3$ as the unique positive solution of $-R_3'(L_3)=s_2$ (since $R_3'(L_3)<0$). However, since L_3 is defined over $[a_3,b_3]$, the best-response solution $\hat{L}_3$ can be at either end point a_3 or b_3, or at some interior point. We formalize this result as shown in (10). □

Proof of Theorem 1. From Proposition 3, we find that the manufacturer's best-response lead-time components are independent of the retailer's decisions q and r. Thus, the manufacturer's Nash equilibrium lead times are the same as the best responses given by (10) in Proposition 3.

Substituting L_i^N $(i = 1, 2, 3)$ into the retailer's best reorder point $\bar{r} = \lambda L + \Phi^{-1}(\rho)\sigma\sqrt{L}$, we find that the Nash equilibrium reorder point for the retailer as (12). Next, we utilize the best-response production quantity of the manufacturer in Proposition 1 and the best-response order quantity of the retailer in Proposition 2 to obtain the Nash equilibrium (Q^N, q^N) as the solution of the following two equations:

$$q^N = \min\left[Q^N, \sqrt{\frac{2\lambda[\pi_R\sigma\sqrt{L^N}\Psi(\Phi^{-1}(\rho)) + A]}{h_R}}\right],$$

$$Q^N = q^N.$$

This gives

$$Q^N = q^N \leqslant \sqrt{\frac{2\lambda[\pi_R\sigma\sqrt{L^N}\Psi(\Phi^{-1}(\rho)) + A]}{h_R}},$$

which indicates that, for the "simultaneous-move" game, there are multiple equilibrium quantities. However, as we showed in Corollary 2, the manufacturer prefers the retailer to increase her order quantity; and from the proof of Proposition 1, we find that the retailer's profit reaches its maximum when $q^N = \bar{q} = \sqrt{2\lambda[\pi_R\sigma\sqrt{L^N}\Psi(\Phi^{-1}(\rho)) + A]/h_R}$. Thus, both supply chain members prefer to choose their quantities as $\bar{q}$; this is Pareto optimal. We thus compute Pareto-optimal Nash equilibrium Q^N and q^N as (13). □

Proof of Theorem 2. We first analyze the profit function $J_M(Q, \mathbf{L}; \hat{q}(Q, \mathbf{L}), \bar{r})$ in (14) to find the manufacturer's Stackelberg production quantity Q^S by considering the following two cases:

1. When $Q \leqslant \bar{q}$, the manufacturer's profit function is

$$J_M(Q, \mathbf{L}; \hat{q}(Q, \mathbf{L}), \bar{r}) = \left\{w - \frac{K + R_1(L_1)}{Q} - [p + s_1 + R_2(L_2) + R_3(L_3) + s_2L_3]\right\}\lambda,$$

which is increasing in Q; thus, we increase the value of Q to $\bar{q}$.

2. When $Q \geqslant \bar{q}$, the manufacturer's profit function is

$$J_M(Q, \mathbf{L}; \hat{q}(Q, \mathbf{L}), \bar{r}) = \left\{w - \frac{K + R_1(L_1) + [p + R_2(L_2)]Q + h_M(Q - \bar{q})}{\bar{q}} - [s_1 + R_3(L_3) + s_2L_3]\right\}\lambda,$$

which is decreasing in Q so that the value of Q is $\bar{q}$.

In summary, we obtain the manufacturer's optimal production quantity as $Q^S = \bar{q}$. Since the retailer's best-response order quantity is $q^S = \min(Q^S, \bar{q})$, we have $q^S = \bar{q}$. Thus, we find that $Q^S = q^S = \bar{q}$.

Next, we substitute Q^S into (14), analyze the resulting function $J_M(\bar{q}, \mathbf{L}; \bar{q}, \bar{r})$ and find the Stackelberg lead-times L_i^S, $i = 1, 2, 3$. The first-order partial derivative of $J_M(\bar{q}, \mathbf{L}; \bar{q}, \bar{r})$ w.r.t. L_1 is

$$\frac{\partial J_M(\bar{q}, \mathbf{L}; \bar{q}, \bar{r})}{\partial L_1} = \left\{-\frac{R_1'(L_1)}{\bar{q}} + \frac{K + R_1(L_1)}{\bar{q}^3}\frac{\lambda[\pi_R\sigma\Psi(\Phi^{-1}(\rho)) + 2A\sqrt{L_1 + L_2 + L_3}]}{2h_R\sqrt{L_1 + L_2 + L_3}}\right\}\lambda > 0,$$

which implies that the Stackelberg equilibrium setup lead time is $L_1^S = b_1$.

The first-order partial derivative of $J_M(\bar{q}, \mathbf{L}; \bar{q}, \bar{r})$ w.r.t. L_2 is

$$\frac{\partial J_M(\bar{q}, \mathbf{L}; \bar{q}, \bar{r})}{\partial L_2} = \left[\frac{K + R_1(L_1)}{\bar{q}^3}\frac{\lambda[\pi_R\sigma\Psi(\Phi^{-1}(\rho)) + 2A\sqrt{L_1 + L_2 + L_3}]}{2h_R\sqrt{L_1 + L_2 + L_3}} - R_2'(L_2)\right]\lambda > 0,$$

which implies that, in the Stackelberg equilibrium, the optimal production lead time is $L_2^S = b_2$.

The first- and second-order partial derivatives of $J_M(\bar{q}, \mathbf{L}; \bar{q}, \bar{r})$ w.r.t. L_3 are

$$\frac{\partial J_M(\bar{q}, \mathbf{L}; \bar{q}, \bar{r})}{\partial L_3} = \left[\frac{K + R_1(L_1)}{\bar{q}^3}\frac{\lambda[\pi_R\sigma\Psi(\Phi^{-1}(\rho)) + 2A\sqrt{L_1 + L_2 + L_3}]}{2h_R\sqrt{L_1 + L_2 + L_3}} - R_3'(L_3) - s_2\right]\lambda \tag{40}$$

and

$$\frac{\partial^2 J_M(\bar{q}, \mathbf{L}; \bar{q}, \bar{r})}{\partial L_3^2} = -3\frac{K + R_1(L_1)}{\bar{q}^5} \times \left[\frac{\pi_R\sigma\Psi(\Phi^{-1}(\rho)) + 2A\sqrt{L_1 + L_2 + L_3}}{2h_R\sqrt{L_1 + L_2 + L_3}}\right]^2\lambda^3 - \frac{K + R_1(L_1)}{4\bar{q}^3}\frac{\lambda^2\pi_R\sigma\Psi(\Phi^{-1}(\rho))}{h_R(L_1 + L_2 + L_3)^{-3/2}} - R_3''(L_3)\lambda,$$

which is negative and the expected profit $J_M(\bar{q}, \mathbf{L}; \bar{q}, \bar{r})$ is thus a concave function in L_3. Setting (40) to zero and solving the resulting equation, we can find a unique optimal solution $\bar{L}_3$. Since L_3 is constrained to take values in the interval $[a_3, b_3]$, we arrive at the Stackelberg solution L_3^S, as shown in (15).

Substituting L_i^S $(i = 1, 2, 3)$ into $\bar{q}$ gives

$$Q^S = \sqrt{\frac{2\lambda[\pi_R\sqrt{L^S}\sigma\Psi(\Phi^{-1}(\rho)) + A]}{h_R}}. \qquad \square$$

Proof of Corollary 3. Follows from Theorem 2. □

Proof of Theorem 3. From Theorem 2, first we find that the Stackelberg order quantity of the retailer is

$$q^S = \sqrt{\frac{2\lambda[\pi_R\sqrt{L^S}\sigma\Psi(\Phi^{-1}(\rho)) + A]}{h_R}}.$$

Next, replacing L with L^S, we find the Stackelberg reorder point $r^S = \lambda L^S + \Phi^{-1}(\rho)\sigma\sqrt{L^S}$. We substitute Stackelberg equilibrium $(Q^S, \mathbf{L}^S, q^S, r^S)$ into (2) and obtain (17). □

Proof of Theorem 4. According to Theorems 1 and 2, we need to determine which of $\tilde{L}_3$ and $\bar{L}_3$ is larger, in order to compare L_3^N and L_3^S. We notice from Theorem 1 that $\tilde{L}_3$ is obtained by solving the equation $s_2 = -R_3'(L_3)$, i.e., $s_2 = -R_3'(\tilde{L}_3)$; and we find from Theorem 2 that $\bar{L}_3$ is the unique solution of the nonlinear equation $\partial J_M(\bar{q}, \mathbf{L}; \bar{q}, \bar{r})/\partial L_3 = 0$. Recalling from the proof of Theorem 2, we have

$$\begin{aligned}\frac{\partial J_M(\bar{q}, \mathbf{L}; \bar{q}, \bar{r})}{\partial L_3} &= \frac{K + R_1(L_1)}{\bar{q}^3} \frac{\lambda[\pi_R \sigma \Psi(\Phi^{-1}(\rho)) + 2A\sqrt{L_1 + L_2 + L_3}]}{2h_R\sqrt{L_1 + L_2 + L_3}}\lambda \\ &\quad - [R_3'(L_3) + s_2]\lambda,\end{aligned}$$

where

$$\bar{q} = \sqrt{\frac{2\lambda[\pi_R \sigma \sqrt{L_1 + L_2 + L_3}\Psi(\Phi^{-1}(\rho)) + A]}{h_R}}.$$

Substituting $\tilde{L}_3$ into the first-order derivative $\partial J_M(\bar{q}, \mathbf{L}; \bar{q}, \bar{r})/\partial L_3$ gives

$$\begin{aligned}\left.\frac{\partial J_M(\bar{q}, \mathbf{L}; \bar{q}, \bar{r})}{\partial L_3}\right|_{L_3 = \tilde{L}_3} &= \frac{K + R_1(L_1)}{\bar{q}^3} \frac{\lambda[\pi_R \sigma \Psi(\Phi^{-1}(\rho)) + 2A\sqrt{L_1 + L_2 + \tilde{L}_3}]}{2h_R\sqrt{L_1 + L_2 + \tilde{L}_3}}\lambda \\ &\quad - [R_3'(\tilde{L}_3) + s_2]\lambda \\ &= \frac{K + R_1(L_1)}{\bar{q}^3} \frac{\lambda[\pi_R \sigma \Psi(\Phi^{-1}(\rho)) + 2A\sqrt{L_1 + L_2 + \tilde{L}_3}]}{2h_R\sqrt{L_1 + L_2 + \tilde{L}_3}}\lambda \\ &> 0,\end{aligned}$$

which implies that the profit function $J_M(\bar{q}, \mathbf{L}; \bar{q}, \bar{r})$ is increasing in L_3 at the point $\tilde{L}_3$. Therefore, when we increase the value of L_3 from $\tilde{L}_3$ to a little higher value, the profit $J_M(\bar{q}, \mathbf{L}; \bar{q}, \bar{r})$ rises. Moreover, as shown in the proof of Theorem 2, the function $J_M(\bar{q}, \mathbf{L}; \bar{q}, \bar{r})$ is strictly concave in L_3, i.e., $\partial^2 J_M(\bar{q}, \mathbf{L}; \bar{q}, \bar{r})/\partial L_3^2 < 0$. It thus follows that the optimal solution $\bar{L}_3$ that maximizes $J_M(\bar{q}, \mathbf{L}; \bar{q}, \bar{r})$must be greater than $\tilde{L}_3$; that is, $\tilde{L}_3 < \bar{L}_3$. Because

$$L_3^N = \begin{cases} a_3 & \text{if } \tilde{L}_3 \leqslant a_3 \\ \tilde{L}_3 & \text{if } a_3 < \tilde{L}_3 < b_3 \\ b_3 & \text{if } b_3 \leqslant \tilde{L}_3 \end{cases} \quad \text{and} \quad L_3^S = \begin{cases} a_3 & \text{if } \bar{L}_3 \leqslant a_3, \\ \bar{L}_3 & \text{if } a_3 < \bar{L}_3 < b_3, \\ b_3 & \text{if } b_3 \leqslant \bar{L}_3, \end{cases}$$

we can arrive to the result as shown in this theorem. □

Proof of Theorem 5. We find from (23) that the manufacturer's best-response decisions $\hat{L}_1$ and $\hat{L}_2$ are the constant independent of the retailer's decisions, and the quantity decision $\hat{Q}$ equals the retailer's order quantity q which is determined by (21). This implies that the Nash and Stackelberg solutions of the setup and production lead times are identical; that is, $L_1^N = L_1^S = b_1$ and $L_2^N = L_2^S = b_2$.

To find Nash equilibrium, we substitute the manufacturer's best-response quantity decision $\hat{Q} = q$ into the retailer' best-response order quantity (20). As a result, for the given shipping lead-time L_3, we find that

$$\hat{Q} = \hat{q} = \sqrt{\frac{2\lambda[\pi_R \sigma \sqrt{b_1 + b_2 + L_3}\Psi(\Phi^{-1}(\rho)) + A]}{h_R}}. \tag{41}$$

Moreover, we have $\hat{r} = \lambda(b_1 + b_2 + L_3) + \Phi^{-1}(\rho)\sigma\sqrt{b_1 + b_2 + L_3}$. Thus, the retailer's profit function in terms of L_3 is

$$\begin{aligned}J_R(L_3) &= (v - w)\lambda - h_R\left(\frac{\hat{Q}}{2} + \Phi^{-1}(\rho)\sigma\sqrt{L_1 + L_2 + L_3}\right) \\ &\quad - \frac{\lambda}{\hat{Q}}\left(\pi_R \int_{\hat{r}}^{\infty} (x - \hat{r}) f(x)\,dx + A\right) \\ &\quad - [s_1 + R_3(L_3) + s_2 L_3]\lambda.\end{aligned}$$

As we argued above, the optimal shipping lead-time L_3 that maximizes $J_R(L_3)$ must exist because the profit function is continuous in $L_3 \in [a_3, b_3]$. We denote this optimal solution by $\check{L}_3$. Therefore, $L_3^N = \check{L}_3$, and

$$Q^N = q^N = \sqrt{\frac{2\lambda[\pi_R \sigma \sqrt{b_1 + b_2 + \check{L}_3}\Psi(\Phi^{-1}(\rho)) + A]}{h_R}},$$

$$r^N = \lambda(b_1 + b_2 + \check{L}_3) + \Phi^{-1}(\rho)\sigma\sqrt{b_1 + b_2 + \check{L}_3}.$$

We can simplify the function $J_R(L_3)$ to

$$\begin{aligned}J_R(L_3) &= (v - w)\lambda - h_R(\hat{Q} + \Phi^{-1}(\rho)\sigma\sqrt{b_1 + b_2 + L_3}) \\ &\quad - [s_1 + R_3(L_3) + s_2 L_3]\lambda.\end{aligned}$$

In the Stackelberg game the retailer takes the "leader" role and announces the decision on q, r and L_3 to the manufacturer who then makes the corresponding decisions. To find the Stackelberg solution, we should substitute the manufacturer's best-response decisions into the retailer's profit function. Because the setup and production lead times are independent of the retailer's decisions, we only consider the impact of Q on the retailer's profit. Since $\hat{Q} = q$ in which q is determined by (20), we have

$$\hat{Q} = \hat{q} = \sqrt{\frac{2\lambda[\pi_R \sigma \sqrt{b_1 + b_2 + L_3}\Psi(\Phi^{-1}(\rho)) + A]}{h_R}},$$

which is identical to (41) for the Nash equilibrium. The following discussion is the same as that for the Nash game; so, this theorem proves. □

Proof of Theorem 6. From Theorem 5 we find that, when the retailer makes the shipping decision, the shipping lead times for the Nash and Stackelberg games are equal, i.e., $L_3^N = L_3^S = \check{L}_3$. Note that $\check{L}_3$ is the optimal solution of the following nonlinear problem:

$$\begin{aligned}\max_{L_3 \in [a_3, b_3]} J_R(L_3) &= (v - w)\lambda - h_R(\hat{Q} + \Phi^{-1}(\rho)\sigma\sqrt{b_1 + b_2 + L_3}) \\ &\quad - [s_1 + R_3(L_3) + s_2 L_3]\lambda,\end{aligned}$$

where

$$\hat{Q} = \sqrt{\frac{2\lambda[\pi_R \sigma \sqrt{b_1 + b_2 + L_3}\Psi(\Phi^{-1}(\rho)) + A]}{h_R}}.$$

Temporarily ignoring the constraint $L_3 \in [a_3, b_3]$, we calculate the first-order partial derivative of $J_R(L_3)$ w.r.t. L_3 as

$$\begin{aligned}\frac{\partial J_R(L_3)}{\partial L_3} &= -h_R\left(\frac{\partial \hat{Q}}{\partial L_3} + \frac{\Phi^{-1}(\rho)\sigma}{2\sqrt{b_1+b_2+L_3}}\right) - [R_3'(L_3)+s_2]\lambda \\ &= -\left(\frac{\lambda\pi_R\sigma\Psi(\Phi^{-1}(\rho))}{2\hat{Q}\sqrt{b_1+b_2+L_3}} + \frac{h_R\Phi^{-1}(\rho)\sigma}{2\sqrt{b_1+b_2+L_3}}\right) \\ &\quad - [R_3'(L_3)+s_2]\lambda.\end{aligned}$$

We recall from Section 3.1.2 that $\tilde{L}_3$ is obtained by solving the equation $R_3'(L_3)+s_2=0$; that is, $R_3'(\tilde{L}_3)+s_2=0$. Since $R_3''(L_3)>0$, we find that, when $L_3>\tilde{L}_3$, $R_3'(L_3)+s_2>0$. Thus, for any value of L_3 equal to or greater than $\tilde{L}_3$, we have

$$\begin{aligned}\left.\frac{\partial J_R(L_3)}{\partial L_3}\right|_{L_3 \geqslant \tilde{L}_3} &= -\left(\frac{\lambda\pi_R\sigma\Psi(\Phi^{-1}(\rho))}{2\hat{Q}\sqrt{b_1+b_2+L_3}} + \frac{h_R\Phi^{-1}(\rho)\sigma}{2\sqrt{b_1+b_2+L_3}}\right) \\ &\quad - [R_3'(L_3)+s_2]\lambda < 0,\end{aligned}$$

which implies that the optimal solution maximizing $J_R(L_3)$ must be less than $\tilde{L}_3$. Since $L_3 \in [a_3, b_3]$, we reach the result that $\check{L}_3$ is less than or equal to Nash equilibrium that is obtained in Section 3.1.2 when the manufacturer makes the shipping decision. As Theorem 4 indicates, in Section 3.1.2, Nash equilibrium is less than or equal to Stackelberg equilibrium. Since the shipping lead time $L_3 \in [a_3, b_3]$, we can prove this theorem. □

Proof of Theorem 7. To find Nash arbitration scheme, we need to solve the following constrained nonlinear problem:

$$\begin{aligned}\max_{J_R^C, J_M^C} \quad & (J_R^C[Q^*, L^*, q^*, r^*] - J_R[Q^e, \mathbf{L}^e, q^e, r^e])(J_M^C[Q^*, L^*, q^*, r^*] \\ & - J_M[Q^e, \mathbf{L}^e, q^e, r^e]) \\ \text{s.t.} \quad & J_R^C[Q^*, L^*, q^*, r^*] + J_M^C[Q^*, L^*, q^*, r^*] \\ & = J^C(Q^*, L^*, q^*, r^*).\end{aligned}$$

Using the constraint we have

$$J_R^C[Q^*, L^*, q^*, r^*] = J^C(Q^*, L^*, q^*, r^*) - J_M^C[Q^*, L^*, q^*, r^*]. \tag{42}$$

Replacing $J_R^C[Q^*, L^*, q^*, r^*]$ in the objective function with the RHS of the above equation, we can re-write our maximization problem as

$$\begin{aligned}&\max_{J_M^C}(J^C(Q^*, L^*, q^*, r^*) - J_M^C[Q^*, L^*, q^*, r^*] - J_R[Q^e, \mathbf{L}^e, q^e, r^e]) \\ &\quad \times (J_M^C[Q^*, L^*, q^*, r^*] - J_M[Q^e, \mathbf{L}^e, q^e, r^e]),\end{aligned}$$

and then find the optimal solution as

$$\begin{aligned}&J_M^C[Q^*, L^*, q^*, r^*] \\ &\quad = \frac{J^C(Q^*, L^*, q^*, r^*) + J_M[Q^e, \mathbf{L}^e, q^e, r^e] - J_R[Q^e, \mathbf{L}^e, q^e, r^e]}{2}.\end{aligned}$$

Using (42) we can easily compute $J_R^C[Q^*, L^*, q^*, r^*]$, as shown in this theorem. □

Proof of Theorem 8. Substituting the global solution $(Q^*, \mathbf{L}^*, q^*, r^*)$ into (25) and (26), we have

$$\begin{aligned}J_R^C[Q^*, \mathbf{L}^*, q^*, r^*] &= \beta J^C(Q^*, \mathbf{L}^*, q^*, r^*) - \gamma, \\ J_M^C[Q^*, \mathbf{L}^*, q^*, r^*] &= (1-\beta) J^C(Q^*, \mathbf{L}^*, q^*, r^*) + \gamma,\end{aligned}$$

and we compare the results with two players' profits in Theorem 7 to find

$$\begin{aligned}&\beta J^C(Q^*, \mathbf{L}^*, q^*, r^*) - \gamma \\ &\quad = \frac{J^C(Q^*, L^*, q^*, r^*) - J_M[Q^e, \mathbf{L}^e, q^e, r^e] + J_R[Q^e, \mathbf{L}^e, q^e, r^e]}{2}, \\ &(1-\beta) J^C(Q^*, \mathbf{L}^*, q^*, r^*) + \gamma \\ &\quad = \frac{J^C(Q^*, L^*, q^*, r^*) + J_M[Q^e, \mathbf{L}^e, q^e, r^e] - J_R[Q^e, \mathbf{L}^e, q^e, r^e]}{2}.\end{aligned}$$

Solving the above equations for β and γ gives (29). □

Proof of Theorem 9. To compute the optimal Q^* and q^* for given lead-time components L_i $(i = 1, 2, 3)$, we find from (31) that the system-wide profit depends on the comparison between Q and q:

1. $Q \leqslant q$. In this case, the system-wide profit function (31) is simplified to

$$\begin{aligned}J^C(Q, \mathbf{L}, q) = \nu\lambda - h_R\frac{Q}{2} - \frac{\lambda}{Q}\left[\pi_R\int_{\bar{r}}^{\infty}(x-\bar{r})f(x)\,dx + A\right] \\ - \frac{K + R_1(L_1) + \pi_M q}{Q}\lambda \\ - h_R(\bar{r} - \lambda L) - [p + R_2(L_2)]\lambda \\ - [s_1 + R_3(L_3) + s_2 L_3]\lambda + \pi_M\lambda.\end{aligned} \tag{43}$$

   The first-order partial derivative of $J^C(Q, \mathbf{L}, q)$ in (43) w.r.t. q is

$$\frac{\partial J^C(Q, \mathbf{L}, q)}{\partial q} = -\frac{\pi_M}{Q}\lambda < 0,$$

   which implies that $J^C(Q, \mathbf{L}, q)$ is decreasing in q. Thus, the optimal value of q equals Q.

2. $Q \geqslant q$. In this case, the system-wide profit function (31) is reduced to

$$\begin{aligned}J^C(Q, \mathbf{L}, q) = \nu\lambda - h_R\left(\frac{q}{2} + \bar{r} - \lambda L\right) \\ - \frac{\lambda}{q}\left[\pi_R\int_{\bar{r}}^{\infty}(x-\bar{r})f(x)\,dx + A\right] \\ - \frac{K + R_1(L_1)}{q}\lambda - \frac{[p + R_2(L_2)]Q}{q}\lambda \\ - [s_1 + R_3(L_3) + s_2 L_3]\lambda - \frac{h_M(Q-q)}{q}\lambda.\end{aligned} \tag{44}$$

   The first-order partial derivative of $J^C(Q, \mathbf{L}, q)$ (44) w.r.t. Q is

$$\frac{\partial J^C(Q, \mathbf{L}, q)}{\partial Q} = -\frac{[p + R_2(L_2)]\lambda}{q} - \frac{h_M\lambda}{q} < 0,$$

   which implies that, when $Q \geqslant q$, $J^C(Q, \mathbf{L}, q, r)$ is decreasing in Q; thus, the optimal value of Q equals q.

From the above analysis, we find that the optimal values of Q and q must be equal. Replacing q with Q in (31), we re-write the system-wide expected profit function as

$$J^C(Q,\mathbf{L}) = v\lambda - h_R\frac{Q}{2} - \frac{\lambda}{Q}\left[\pi_R\int_{\bar r}^{\infty}(x-\bar r)f(x)\,\mathrm{d}x + A\right] - \frac{K+R_1(L_1)}{Q}\lambda - h_R(\bar r - \lambda L) - [p+R_2(L_2)]\lambda - [s_1+R_3(L_3)+s_2L_3]\lambda. \quad (45)$$

Taking the first- and second-order derivatives of $J^C(Q,\mathbf{L})$ in (45) w.r.t. Q, we have

$$\frac{\partial J^C(Q,\mathbf{L})}{\partial Q} = -\frac{h_R}{2} + \frac{\lambda}{Q^2}\left[\pi_R\int_{\bar r}^{\infty}(x-\bar r)f(x)\,\mathrm{d}x + A\right] + \frac{K+R_1(L_1)}{Q^2}\lambda,$$

$$\frac{\partial^2 J^C(Q,\mathbf{L})}{\partial Q^2} = -\frac{2\lambda}{Q^3}\left[\pi_R\int_{\bar r}^{\infty}(x-\bar r)f(x)\,\mathrm{d}x + A\right] - \frac{2\lambda[K+R_1(L_1)]}{Q^3} < 0. \quad (46)$$

Thus, the function $J^C(Q,\mathbf{L})$ in (45) is strictly concave in Q. Equating (46) to zero and solving it, we obtain the optimal solution $Q^*(\mathbf{L})$ maximizing (45) as

$$Q^*(\mathbf{L}) = \sqrt{\frac{2\lambda[\pi_R\int_{\bar r}^{\infty}(x-\bar r)f(x)\,\mathrm{d}x + A + K + R_1(L_1)]}{h_R}},$$

where $\bar r = \lambda L + \Phi^{-1}(\rho)\sigma\sqrt{L}$. Since $q^*(\mathbf{L}) = Q^*(\mathbf{L})$ and $\int_{\bar r}^{\infty}(x-\bar r)f(x)\,\mathrm{d}x = \sigma\sqrt{L}\Psi(\Phi^{-1}(\rho))$, we arrive to the result (32). □

Proof of Theorem 10. We partially differentiate $J^C(\mathbf{L})$ w.r.t. L_3 and find

$$\frac{\partial J^C(\mathbf{L})}{\partial L_3} = -h_R\frac{\partial Q^*(\mathbf{L})}{\partial L_3} - \frac{h_R\Phi^{-1}(\rho)\sigma}{2\sqrt{L}} - [R_3'(L_3)+s_2]\lambda = -\left(\frac{\lambda\pi_R\sigma\Psi(\Phi^{-1}(\rho))}{2Q^*(\mathbf{L})} + \frac{h_R\Phi^{-1}(\rho)\sigma}{2}\right)\frac{1}{\sqrt{L}} - [R_3'(L_3)+s_2]\lambda,$$

where

$$Q^*(\mathbf{L}) = \sqrt{\frac{2\lambda\left[\pi_R\sigma\sqrt{L}\Psi(\Phi^{-1}(\rho)) + A + K + R_1(L_1)\right]}{h_R}}.$$

As argued in the proof of Theorem 6, $R_3'(L_3)+s_2>0$, for $L_3>\tilde L_3$. Thus, when $L_3\geqslant\tilde L_3$, we have

$$\left.\frac{\partial J^C(\mathbf{L})}{\partial L_3}\right|_{L_3\geqslant\tilde L_3} = -\left(\frac{\lambda\pi_R\sigma\Psi(\Phi^{-1}(\rho))}{2Q^*(\mathbf{L})} + \frac{h_R\Phi^{-1}(\rho)\sigma}{2}\right)\frac{1}{\sqrt{L}} - [R_3'(L_3)+s_2]\lambda < 0.$$

This means that the optimal solution L_3^* that maximizes $J^C(\mathbf{L})$ must be less than $\tilde L_3$. Since $L_3\in[a_3,b_3]$, we arrive to the result that L_3^* is less than or equal to the Nash equilibrium that is obtained in Section 3.1.2 when the manufacturer makes the shipping decision. As Theorem 4 indicates, when the manufacturer is responsible for the shipping, Nash equilibrium is less than or equal to Stackelberg equilibrium. Since the shipping lead time $L_3\in[a_3,b_3]$, we can arrive to the first item in this theorem.

Recall from Section 3.2.2 that, if ignoring the constraint $L_3\in[a_3,b_3]$, the equilibrium shipping decision, when the retailer is responsible for the shipping, satisfies the first-order condition

$$\frac{\partial J_R(L_3)}{\partial L_3} = -\left(\frac{\lambda\pi_R\sigma\Psi(\Phi^{-1}(\rho))}{2\hat Q} + \frac{h_R\Phi^{-1}(\rho)\sigma}{2}\right)\frac{1}{\sqrt{b_1+b_2+L_3}} - [R_3'(L_3)+s_2]\lambda,$$

where

$$\hat Q = \sqrt{\frac{2\lambda[\pi_R\sigma\sqrt{b_1+b_2+L_3}\Psi(\Phi^{-1}(\rho)) + A]}{h_R}}.$$

We notice that it is hard to compare the partial derivatives $\partial J^C(\mathbf{L})/\partial L_3$ with $\partial J_R(L_3)/\partial L_3$, because of the following facts: (i) the term $K+R_1(L_1)$ involves in $\partial J^C(\mathbf{L})/\partial L_3$ but does not appear in $\partial J_R(L_3)/\partial L_3$ and (ii) the globally optimal setup and production lead-time components L_1^* and L_2^* are smaller than the equilibrium solutions $L_1^N = b_1$ and $L_2^N = b_2$, respectively. Hence, we find that the solution that satisfies the equation $\partial J^C(\mathbf{L})/\partial L_3 = 0$ may be greater than or may be smaller than that satisfying the equation $\partial J_R(L_3)/\partial L_3 = 0$. However, we find from Theorem 6 that the equilibrium solutions obtained in Section 3.2.2 are less than those obtained in Section 3.1.2; and find from Theorem 4 that, when the manufacturer makes the shipping decision, Nash equilibrium is less than or equal to Stackelberg solution. Since the shipping lead time $L_3\in[a_3,b_3]$, we can reach part (a) in the second item of this theorem. □

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