Choosing an Online Retail Channel for a Manufacturer: Direct Sales or Consignment?[a]

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Abstract

We analyze a manufacturer's e-channel decision problem in which the manufacturer selects a direct-sales channel or a third-party consignment channel to complement his existing physical retail channel. We accordingly investigate two possible dual channels: a PD system involving a physical channel and a direct e-channel, and a PC system consisting of a physical channel and a consignment e-channel. For each system, we examine both a sequential-move game and a simultaneous-move game, as the manufacturer can strategically decide to announce his pricing decision before the physical retailer or to make his pricing decision with no communication with the physical retailer. Our analytical results indicate that, if the manufacturer's unit operating cost in the direct e-channel or the e-tailer's revenue allocation ratio in the consignment e-channel is sufficiently small, then the manufacturer has an incentive to adopt an e-channel. The manufacturer can always gain a higher profit by announcing his pricing decision before the physical retailer. If the manufacturer aims at increasing the demand, then he may choose the simultaneous-move game. Moreover, when the manufacturer selects an e-channel to increase his profit, he should adopt a direct e-channel if his unit e-channel operating cost is below a certain threshold that is dependent on the e-tailer's revenue allocation ratio, and adopt a consignment e-channel otherwise. A similar managerial insight is drawn when the manufacturer intends to increase the demand.

Key words: Direct sales; consignment; sequential-move game; simultaneous-move game.

1 Introduction

The rapid development of information technology has led many manufacturers to complement their existing physical retail channels with an online channel (or, e-channel), which provides them with an opportunity to serve more consumers who would otherwise have no intention to buy the manufacturers' products (Huang and Swaminathan 2009). A dual channel integrating an e-channel with a physical one has been a dominant sales structure for many companies in the past decade. However, the addition of an e-channel poses the important question of whether to set up a direct (proprietary) sales website or to contract with a third-party e-tailer. Different strategies have been adopted to tackle this issue. Many manufacturers—such as Hewlett & Packard, Lenovo, Compaq, Sony, Panasonic, Mattel, Pioneer Electronics, Cisco System, and Estee Lauder—sell products through their direct e-channels (Tsay and Agarwal 2004a, Kumar and Ruan 2006, and Chen, Kaya, and Ozer 2008). On the other hand, the largest English-language publisher Random House sells books online to readers through Amazon and Barnes & Noble (Dumrongsiri et al. 2008), and Enesco and Goebei utilize eBay to serve their consumers (Ow and Wood 2011). Dell Computer has been selling its products at computer stores, department stores, official supply stores, and its direct e-channel `DellAuction.com`, and has also started to use eBay as a marketing channel (Ow and Wood 2011). Moreover, Levi Strauss & Co., an American jeans and casual wear manufacturer, has terminated its direct-sales websites at `Levis.com` and `Dockers.com`, and handed online business over to some e-tail partners (Yoo and Lee 2010).

Motivated by the above practices, in this paper we focus on a dual channel problem in which a manufacturer selects an e-channel to complement an independent physical channel. That is, the manufacturer has been selling his products at a physical retailer's store, and considers to add an e-channel to serve his customers. As discussed above, there are two possible e-channels: (i) a direct e-channel, and (ii) a consignment e-channel in which the manufacturer sells through an e-tailer under a consignment contract. We denote the physical channel, the manufacturer's direct e-channel, and the consignment e-channel by P-channel, D-channel, and C-channel, respectively. The consignment contract in the C-channel involves a revenue sharing formula, which is consistent with the practice of many online marketplaces such as the online stores at `Amazon.com` and `ebay.com`; see Wang (2006), Chen, Cheng, and Chien (2011), Ryan, Sun, and Zhao (2012), and Abhishek, Jerath, and Zhang (2015). Under the contract, the manufacturer determines the retail price, and for each item sold, the e-tailer deducts a percentage of revenue and remits the balance to the manufacturer, as shown by the common practice by Amazon Marketplace, Kindle, iBook Store, and other e-tailers. Accordingly, we consider a consignment contract in the C-channel, in which the manufacturer makes his retail pricing decision given the e-tailer's revenue allocation ratio.

Since the C- and D-channels are two e-channel choices, the manufacturer has two possible dual channels: one consisting of a P-channel and a D-channel (simply called the "PD system"), and the other consisting of a P-channel and a C-channel (simply called the "PC system"). We aim to derive conditions for the manufacturer's e-channel selection and provide managerial insights which are expected to help practitioners make their dual channel decisions. In either the PD or the PC system, we consider the manufacturer's timing decision for the announcement of his retail price. There are two timing choices for the manufacturer. Under the first choice, the manufacturer determines and announces his retail price in the e-channel before the retailer discloses her retail price in the physical channel, which results in a sequential-move game under which the two firms make their pricing decisions “sequentially.” Under the second choice, the manufacturer and the retailer have no communication, and determine their retail prices “simultaneously,” which corresponds to a “simultaneous-move” game. For each of the PD and the PC systems, we derive the conditions under which the manufacturer willingly adopts an e-channel and thus operates a dual channel, and also compare the manufacturer’s profits and the demands in the two games to obtain the conditions under which the manufacturer prefers to play the sequential-move or the simultaneous-move game. Moreover, for each game, we compare the manufacturer’s profits and the demands in the PD and the PC systems to find which system is preferable to the manufacturer. In addition, we show that most of our major managerial insights still hold when the manufacturer makes his wholesale pricing decision in dual-channel supply chains.

The rest of the paper is organized as follows. We review the relevant literature in Section 2 to show the originality of our paper. In Section 3, we describe our problem and derive the dual-channel demand functions by analyzing consumers’ utilities drawn from their purchases in each channel. In Section 4, for each of the PD and the PC systems, we investigate the sequential-move and simultaneous-move games. Then, in Section 5, we compare the manufacturer’s profits and the demands in the two systems, and obtain the conditions under which the manufacturer prefers to adopt the D-channel (i.e., the PD system) or the C-channel (i.e., the PC system). In Section 6, we perform our game-theoretic analysis when the manufacturer makes his wholesale pricing decision in dual-channel supply chains. The paper ends with a summary of our major findings in Section 7. In addition, the proofs of all theorems, lemmas, propositions, and corollaries are relegated to Appendix A.

2 Literature Review

Our paper is related to the literature on dual-channel distribution. Observing the Internet’s great accessibility to consumers, a growing number of firms have launched e-channels along with their existing physical retail channels. The dual-channel hybrid of a physical channel and an e-channel has been drawing great attention from researchers. Chen et al.(2017) examined how a newly-added direct channel for a manufacturer influences a retailer Stackelberg supply chain and found that a dual-channel supply chain can enhance the profits of the manufacturer and the supply chain. Chiang, Chhajed, and Hess (2003) developed a consumer choice model and studied a pricing game involving a manufacturer and a retailer in a dual-channel supply chain. Assuming that consumers’ acceptance of an e-channel is homogeneous and the dual channel can only exist under certain conditions, the authors found that the manufacturer can employ a direct e-channel to mitigate double marginalization. For a dual-channel distribution problem, Xiao, Choi, and Cheng (2014) developed a retailer-Stackelberg pricing model to investigate a manufacturer’s product variety and channel structure strategies in a circular spatial market. They found that the motivation for the manufacturer to use dual channels decreases with the unit production cost, but increases with (i) the marginal cost of variety, (ii) the retailer’s marginal selling cost, and (iii) the customer’s fit cost. Assuming heterogeneous readers in their valuations of a book, Hua, Cheng, and Wang (2011) derived the conditions under which a publisher should sell only printed books, only e-books, or both of them.

Fruchter and Tapiero (2005) considered a manufacturer who sells a product through retail stores and an online store. Similar to Chiang, Chhajed, and Hess (2003), they assumed that consumers have a lower valuation for the product purchased online than for that bought in the physical channel.

Li et al. (2015) also made such an assumption because the consumers have a lower acceptance level for the online channel. Zhang et al. (2017) considered the setting in which consumers choose products based on quality and matchness, but they may assign different weights to quality and matchness according to the product type. A survey by Kacen, Hess, and Chiang (2013) provided further evidences that for many product categories consumers attach lower values to web-based purchases than the purchases in physical stores.

Based on the work of Chiang, Chhajed, and Hess (2003), Yan (2008) adopted a game-theoretic approach to investigate the strategic role of profit sharing between a manufacturer and a retailer, and found that both parties can benefit from a dual-channel profit-sharing strategy. Bernstein, Song, and Zheng (2008) performed an analysis to expose the value of the Internet as a selling channel for retailers and consumers. They studied the case in which a retailer manages both its retail stores and its proprietary online store, and also examined the case in which a traditional retailer aligns with a pure e-tailer to sell products online. The authors showed that the adoption of an online channel can reduce transaction costs and increase productivity. Moreover, as the additional market reached by the use of the Internet channel increases, the firms' total profits increase, although their profits from the original (physical) market decrease. Khouja, Park, and Cai (2010) analyzed the channel selection and pricing decisions for a manufacturer who has the options of selling through an online channel, his own retail channel, and/or an independent retail channel. They found that in a vertically integrated supply chain, the most critical factor is the variable cost of a product sold in the online channel compared with that sold in the retail channel; but, in the presence of an independent retailer, the size of the retail-captive consumer segment relative to the size of the hybrid consumer segment is a major factor.

Adopting a new channel is likely to cause channel conflicts (Cai 2010). Tsay and Agrawal (2004b) considered the channel conflicts that arise as a company undertakes direct sales and competes with its resellers. They concluded that the addition of a direct channel may not be detrimental to the reseller, especially when the direct channel has a significant disadvantage in the cost of sales effort and/or a significant advantage in the cost of fulfillment. Mukhopadhyay, Zhu, and Yue (2008) suggested that the value added by a retailer to the product may result in product differentiation to consumers and thus increase the manufacturer's and the retailer's profits. Xiao, Choi, and Cheng (2014) studied the case that an indirect channel sells a standard product whereas a direct channel offers a customized product to avoid the channel conflict. Li et al. (2016) developed an improved risk-sharing contract that can help coordinate a dual channel supply chain with a risk-averse retailer. Cattani et al. (2006) analyzed three prevalent price-matching strategies for a manufacturer to mitigate the channel conflict resulting from its introduction of an Internet channel and entry into the market as a competitor to its supply chain partner. The strategies include: (i) keeping wholesale prices as they were before, (ii) keeping retail prices as they were before, and (iii) selecting wholesale and retail prices that optimize the manufacturer's profit. The authors found that the first and the second strategies may be sub-optimal from the profit perspective, but they minimize disruptions to the traditional physical channel. Huang and Swaminathan (2009) studied the pricing strategies that are similar to those investigated by Cattani et al. (2006) for the case when a retailer adds an e-channel to an existing traditional channel. Ding, Dong, and Pan (2016)showed that, in a dual-channel supply chain involving a manufacturer and a retailer, a equal-pricing strategy and a price-matching strategy may not be optimal for the manufacturer.

Our paper is also relevant to the literature on the selection of Internet selling channels. As Yoo and Lee (2011) stated, an online store may be operated by a manufacturer (i.e., the manufacturer's direct online retailing), an existing independent physical retailer (i.e., bricks-and-clicks retailing), or a new third-party e-tailer (i.e., pure e-tailer selling). When a firm decides to directly access consumers through its online retailing system, it faces a key question of which e-channel (its direct e-channel or a third-party e-tailer) should be chosen. Bernstein, Song, and Zheng (2008) considered traditional retailers' decisions on expanding their existing operations through e-tailing. Abhishek, Jerath, and Zhang (2015) used a game-theoretic model to study an e-tailer's choice between agency selling (which provides manufacturers with direct access to their consumers but charges a fee for such an access) and reselling (which buys from manufacturers and sells to consumers online). Ow and Wood (2011) examined Dell Computer's choice problem of whether to sell excess inventory at its proprietary auction site or at eBay (which is a popular and well-established third-party auction site). Using the data collected on 557 auctions from `DellAuction.com` and 373 auctions from `ebay.com`, the authors performed an empirical study and found that the computers at `DellAuction.com` were sold for a price premium when compared to the computers sold by Dell at `ebay.com`.

Our paper differs from extant relevant publications in three aspects. First, we focus on the e-channel selection problem in which a manufacturer has an existing physical retail channel, and the manufacturer and a physical retailer make their pricing decisions in a sequential-move or simultaneous-move game setting when the manufacturer chooses an e-channel. Secondly, we consider consumers' purchasing channel decisions, and derive demand functions for the physical channel and the e-channel. Thirdly, we investigate the case that the manufacturer keeps the wholesale price in the physical channel unchanged when he adopts an e-channel, and also examine the case that the manufacturer chooses a wholesale price to maximize his profit in a dual-channel supply chain.

3 Problem Description and Consumer Choice Analysis

In this section, we first describe our research problem, and then analyze consumer choice models to derive demand functions for the manufacturer.

3.1 Problem Description

We consider an e-channel selection problem for a manufacturer who makes a product at the unit cost c_m and sells it to n potential heterogeneous consumers in a market. In addition to an extant P-channel, the manufacturer will adopt an e-channel to expand sales and improve his profit. As discussed in Section 1, in practice there are two common e-channel structures, i.e., the D- and C-channels. Therefore, in this problem the manufacturer needs to make a choice between the PD system (a combination of P- and D-channels) and the PC system (a combination of P- and C-channels).

Both systems involve a P-channel in which a physical retailer purchases the manufacturer's product at a wholesale price w and sells it to consumers at the retail price p_r. But, the e-channels in the two systems differ in their operations. In the D-channel of the PD system, the manufacturer directly sells his product to consumers at a retail price p_d. In the C-channel of the PC system, the manufacturer determines a retail price $\hat{p}_c$ and serves consumers through an e-tailer at the price $\hat{p}_c$. Hereafter, we use the symbol "ˆ" to indicate the PC system. As in the practice of Amazon Marketplace, the manufacturer and the e-tailer share a sales revenue according to an allocation ratio $\xi \in (0, 1)$ that is determined by the e-tailer. Specifically, when a consumer spends \$$\hat{p}_c$ to buy a unit of the manufacturer's product at the e-tailer's online store, the e-tailer retains the revenue $\xi\hat{p}_c$ and allocates the remaining amount $\hat{p}_c(1-\xi)$ to the manufacturer. Moreover, in reality, prior to trading with manufacturers, many e-tailers determine the value of ξ for each product category and announce the value at their websites. For instance, one can learn from Amazon's webpage titled "Selling on Amazon Fee Schedule" that the referral fee percentage is 15% for many product categories such as beauty, clothing and accessories, toys and games, outdoors, etc., and 8% for some of the other categories such as cell phone devices and consumer electronics. This evidence exposes a common practice that because the e-tailers such as Amazon and eBay serve a number of manufacturers, they cannot determine a revenue allocation ratio for every individual manufacturer but announce a single ratio applicable to all relevant manufacturers. Accordingly, it is reasonable to assume the exogeneity of the revenue allocation ratio in this paper.

In the above, $\xi\hat{p}_c$ can be *per se* viewed as the "percentage-based fee" paid by the manufacturer to the e-tailer for "using" a space at the e-tailer's online store. In addition, the e-tailer may charge the manufacturer a small fixed fee. For instance, besides a percentage-based fee, Amazon Marketplace charges a fixed fee of $39 per month from each seller who expects to sell regularly over a long time horizon at its online store. After paying the fee, the sellers can access certain infrastructure services such as easy interfaces for uploading and displaying product information. Because even small professional sellers have the monthly sales with a value much higher than many thousands of dollars, the fixed fee is negligibly small and thus, similar to Wang, Jiang, and Shen (2004), Ryan, Sun, and Zhao (2012), and Abhishek, Jerath, and Zhang (2015), in this paper we do not consider any fixed fee paid by the manufacturer to the e-tailer.

3.2 Consumer Choice Analysis and Demand Functions

When the manufacturer adopts an e-channel, each consumer can purchase a product in the P-channel or the e-channel (the D- or C-channel). Next, we develop consumer choice models, which are then analyzed to derive the demand function for each channel. We begin by computing a consumer's net utility drawn from his or her purchase in the P-channel. Let v denote the consumer's valuation of the product bought from the P-channel. Similar to Chiang, Chhajed, and Hess (2003), we consider heterogeneous consumers in their valuations of the product. In our paper, V is a uniformly distributed random variable on the support $[0, \bar{v}]$, where $\bar{v}$ represents the highest one among all consumers' valuations. The uniformly distributed valuation has been widely used for analytical tractability in many publications such as Balasubramanian (1998), Chiang, Chhajed, and Hess (2003), Kumar and Ruan (2006), Dumrongsiri et al. (2008), Khouja, Park, and Cai (2010), Yan and Ghose (2010), Yan, Wang, and Zhou (2010), and Hua, Cheng, and Wang (2011), Ofek, Katona, and Sarvary (2011), and Ryan, Sun, and Zhao (2012). The physical retailer should decide on her retail price p_r such that $p_r \leq \bar{v}$; this condition ensures some consumers' willingness to buy at the physical store. Thus, a consumer with the valuation $v \in [0, \bar{v}]$ can obtain the surplus $u_r = v - p_r$ from purchasing a product in the P-channel. The consumer may be willing to buy in the P-channel if and only if his or her surplus is non-negative (i.e., $v \geq p_r$).

If a consumer buys in an e-channel, then he or she may obtain a surplus that differs from u_r, because, different from physical store shopping, the consumer cannot receive the product immediately after placing an order online and thus has no opportunity to make any physical inspection during the lead time. The disutility from online shopping is dependent on the characteristics of products and the nature of online channels, and it has been becoming smaller as a result of the improvement of network technology and online service level. Nevertheless, many consumers still prefer physical retail stores to Internet channels mainly because of their desires for service, shopping experience, perceived risk, offline habit, etc., as shown by a number of empirical studies (e.g., Liang and Huang 1998, Chu et al. 2010, Kollmann, Kuckertz, and Kayser 2012, and Kacen, Hess, and Chiang 2013). We can also learn from some publications (e.g., Chiang, Chhajed, and Hess 2003 and Song et al. 2014) that a consumer enjoys less from buying a product online than at a physical store. Accordingly, in this paper each consumer possesses a lower valuation for the product when buying it from the e-channel (i.e., the D- or C-channel) than from the P-channel.

A product with a valuation v in the P-channel has an online value θv, where $\theta \in (0, 1)$ is a measure of consumers' e-channel acceptance. Hence $v(1-\theta)$ reflects the disutility when a consumer makes an online shopping. The value of θ is dependent on the characteristics of the product, consumers' perceptions of the performance of online stores on different attributes such as quality, transaction costs, and uncertainty, as well as the importance of each attribute. In an empirical study by Kacen, Hess, and Chiang (2013), parameter θ is called the consumer's acceptance index of online stores, and is calculated as the ratio of the reservation price of buying a product online to that of buying the product at a physical store. When the value of θ is close to 0, consumers have very poor perceptions of the performance of online stores and thus much lower reservation price for the e-channel than for the P-channel. When the value of θ is close to 1, consumers' performance perceptions and reservation prices for the online and physical stores are almost equal. Kacen, Hess, and Chiang (2013) empirically investigated six product categories, and found that for the category of "books," the consumer's acceptance index of online stores is $\theta = 0.92$, which is the highest among the six categories. The acceptance index implies that the reservation price for online bookstores is 8% less than that for physical bookstores.

Using the above, we can find that, if a consumer with the valuation $v \in [0, \bar{v}]$ buys the product online, then he or she can draw a surplus as $u_i = \theta v - p_i$, where p_i is the retail price in the e-channel and is smaller than or equal to $\theta \bar{v}$, and i represents the e-channel adopted by the manufacturer, i.e., $i = d$ (the D-channel) or c (the C-channel). Therefore, each consumer with a valuation greater than or equal to p_i/θ can obtain a non-negative surplus from his or her online purchase and may be willing to buy through the e-channel. Comparing the consumer's valuations of the product from the P- and the e-channels, we obtain the following proposition, which is depicted by Figure 1 and can also be obtained by using the results given by Chiang, Chhajed, and Hess (2003).

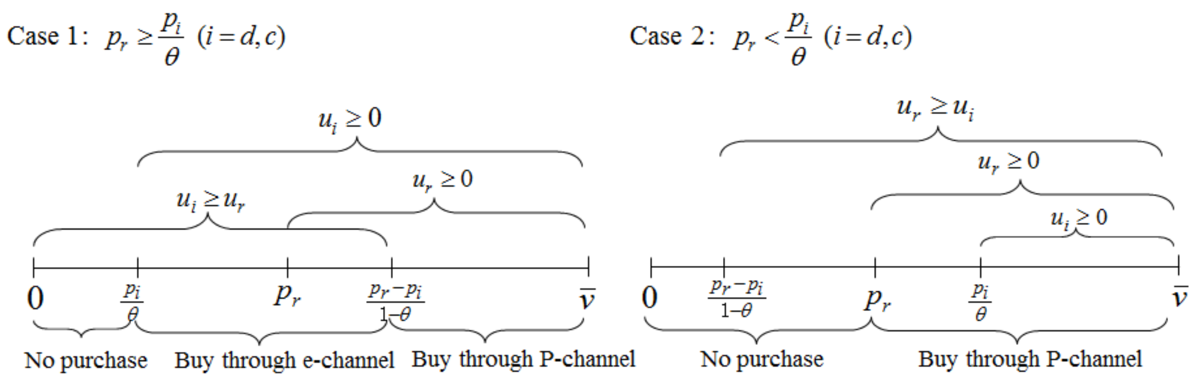

Figure 1: Consumers' channel preference.

Proposition 1 Given the retail prices p_r in the P-channel and p_i in the e-channel ($i = d, c$), a consumer with valuation v for the product in the P-channel makes a purchase decision as follows.

1. When $p_r \geq p_i/\theta$, we find that if $v < p_i/\theta$, then the consumer purchases nothing; if $p_i/\theta \leq v < (p_r - p_i)/(1-\theta)$, then the consumer buys in the e-channel; otherwise, if $(p_r - p_i)/(1-\theta) \leq v \leq \bar{v}$, then the consumer shops in the P-channel.
2. When $p_r < p_i/\theta$, we find that if $v < p_r$, then the consumer buys nothing; otherwise, if $p_r \leq v \leq \bar{v}$, then the consumer buys in the P-channel. ■

Using the above proposition, we can compute the expected demand faced by the manufacturer in each channel in the PD and the PC systems. Denoting the expected demands in the P-channel and e-channel of the PD (PC) system by D_p ($\hat{D}_p$) and D_d ($\hat{D}_c$), respectively, we have

$$[D_p \mid D_d] = \begin{cases} \left[\dfrac{n}{\bar{v}}\left(\bar{v} - \dfrac{p_r - p_d}{1-\theta}\right) \,\middle|\, \dfrac{n}{\bar{v}}\left(\dfrac{p_r - p_d}{1-\theta} - \dfrac{p_d}{\theta}\right)\right], & \text{if } p_r \geq \dfrac{p_d}{\theta}, \\ \left[\dfrac{n}{\bar{v}}(\bar{v} - p_r) \,\middle|\, 0\right], & \text{if } p_r < \dfrac{p_d}{\theta}; \end{cases} \tag{1}$$

and

$$\left[\hat{D}_p \,\middle|\, \hat{D}_c\right] = \begin{cases} \left[\dfrac{n}{\bar{v}}\left(\bar{v} - \dfrac{\hat{p}_r - \hat{p}_c}{1-\theta}\right) \,\middle|\, \dfrac{n}{\bar{v}}\left(\dfrac{\hat{p}_r - \hat{p}_c}{1-\theta} - \dfrac{\hat{p}_c}{\theta}\right)\right], & \text{if } \hat{p}_r \geq \dfrac{\hat{p}_c}{\theta}, \\ \left[\dfrac{n}{\bar{v}}(\bar{v} - \hat{p}_r) \,\middle|\, 0\right], & \text{if } \hat{p}_r < \dfrac{\hat{p}_c}{\theta}. \end{cases} \tag{2}$$

Observing the demand functions, we find that when $p_r < p_d/\theta$ and $\hat{p}_r < \hat{p}_c/\theta$, no consumer intends to buy online. Therefore, the manufacturer has an incentive to adopt an e-channel and implement a dual-channel distribution system, when the retail prices p_r, $\hat{p}_r$, p_d, and $\hat{p}_c$ are determined such that

$$p_d/\theta \leq p_r < \bar{v} \quad \text{and} \quad \hat{p}_c/\theta \leq \hat{p}_r < \bar{v}. \tag{3}$$

4 Game-Theoretic Analysis of Dual-Channel Supply Chains

As discussed in Section 3, the manufacturer has an existing P-channel, and decides on whether to launch an e-channel (i.e., the D-channel or C-channel) as an additional distribution channel. Thus, to find an optimal e-channel decision for the manufacturer, we should analyze two dual-channel systems (i.e., the PD and the PC systems), and then compare our results for these two systems. Similar to Yao and Liu (2005), we analyze the dual channel problem in the situation that the manufacturer does not change his wholesale price in the physical channel when he adopts an e-channel. Accordingly, we first calculate the manufacturer's optimal wholesale price in a single-channel supply chain which only involves the P-channel. It is easy to obtain the optimal wholesale price as $w = (\bar{v} - c_r + c_m)/2$, where c_r and c_m represent the retailer's unit selling cost and the manufacturer's unit product acquisition cost, respectively.

4.1 Analysis in the PD System

In this system, the manufacturer complements his existing P-channel with a D-channel. Each consumer can buy a product from either the P-channel or the D-channel, which implies that the two channels "compete" for consumers. When the manufacturer sells one unit of product in the D-channel, he incurs an operating cost c_d. As the empirical evidences in Garicano and Kaplan (2001) and Zhu (2004) indicate that the Internet can help reduce sellers' transaction costs, we assume that $c_d < c_r$, which actually reflects the differences between the two channels in stocking costs, employee labor costs, facility maintenance costs, etc.

When the manufacturer launches the D-channel, the manufacturer and the physical retailer determine their retail prices p_d and p_r to maximize their individual profits. The manufacturer's profit, denoted by π_m, includes his profits in the two channels, and thus can be calculated as $\pi_m = D_p(w - c_m) + D_d(p_d - c_m - c_d)$, where D_p and D_d are given in (1). Similarly, the physical retailer's profit π_r can be computed as $\pi_r = D_p(p_r - w - c_r)$. Substituting the results in (1) into the above profit functions, we can re-write the manufacturer's and the physical retailer's profits as

$$\pi_m = \frac{n}{\bar{v}}(w - c_m)\left(\bar{v} - \frac{p_r - p_d}{1-\theta}\right) + \frac{n}{\bar{v}}(p_d - c_m - c_d)\left(\frac{p_r - p_d}{1-\theta} - \frac{p_d}{\theta}\right), \tag{4}$$

$$\pi_r = \frac{n}{\bar{v}}(p_r - w - c_r)\left(\bar{v} - \frac{p_r - p_d}{1-\theta}\right). \tag{5}$$

In practice, the manufacturer and the physical retailer can make their retail pricing decisions either "sequentially" or "simultaneously." Specifically, in the sequential (leader-follower game) setting, the manufacturer first determines his online retail price p_d and announces it to the physical retailer, who then makes her pricing decision p_r. In the simultaneous ("simultaneous-move" game) setting, the two firms make their retail pricing decisions with no communication. Next, we investigate both games to find in which setting the manufacturer can obtain a higher profit and a higher demand. Hereafter, we use the superscripts "L" and "S" in notations to represent the sequential- and simultaneous-move games, respectively.

4.1.1 Analysis of the Sequential-Move Game in the PD System

When the manufacturer and the physical retailer play a sequential-move game, their profit maximization problems can be written as $\max_{p_d \le \theta p_r} \pi_m$ and $\max_{p_r \ge p_d/\theta} \pi_r$, respectively. Using backward induction, we find that, given the manufacturer's retail price p_d, the physical retailer's best-response is

$$p_r(p_d) = \max\left\{\frac{p_d}{\theta}, \frac{a + p_d}{2}\right\}, \quad \text{where } a \equiv (1-\theta)\bar{v} + w + c_r. \tag{6}$$

Using (6) to replace p_r in the manufacturer's profit function, we can derive the corresponding equilibrium retail prices (p_d^L, p_r^L) as given in the following proposition.

Proposition 2 When the manufacturer acts as the leader in the PD system, we can find the unique retail prices in Stackelberg equilibrium as

$$p_d^L = \min\left\{\frac{\theta a}{2-\theta}, \frac{\theta a + \theta b + (2-\theta)(c_d + c_m)}{2(2-\theta)}\right\} \text{ and } p_r^L = \frac{a + p_d^L}{2}, \tag{7}$$

where $b \equiv w - c_m$. ■

Using the above proposition, we can find the resulting demands and two firms' profits.

Proposition 3 For the sequential-move game in the PD system, we obtain the following results.

1. If the manufacturer's unit operating cost in the D-channel c_d is given such that $c_d \geq c_d^L$, where

$$c_d^L \equiv \frac{\theta a - \theta b - c_m(2-\theta)}{2-\theta}, \tag{8}$$

then the demands in the P- and D-channels are $D_p^L = n[\bar{v} - a/(2-\theta)]/\bar{v}$ and $D_d^L = 0$, respectively; and, the manufacturer's total profit in the two channels (which is also his profit in the P-channel) is $\pi_m^L = nb[\bar{v} - a/(2-\theta)]/\bar{v}$. In this case, both D_p^L and π_m^L are independent of c_d.

2. If $c_d < c_d^L$, then the demands in the two channels are computed as

$$D_p^L = n - \frac{n[a(4-3\theta) - \theta b - (2-\theta)(c_d + c_m)]}{4\bar{v}(2-\theta)(1-\theta)} \text{ and } D_d^L = \frac{n[\theta a - \theta b - (2-\theta)(c_d + c_m)]}{4\bar{v}\theta(1-\theta)}.$$

We then find the total demand and the manufacturer's total profit in the two channels as

$$\begin{aligned} D_{PD}^L &= D_p^L + D_d^L = n - \frac{n[\theta a + \theta b + (2-\theta)(c_d + c_m)]}{2\bar{v}\theta(2-\theta)}, \\ \pi_m^L &= nb - \frac{n}{8\bar{v}\theta(1-\theta)(2-\theta)}[2\theta ab(4-3\theta) - \theta^2 a^2 - \theta^2 b^2 \\ &\quad + 2\theta(2-\theta)(a-b)(c_m + c_d) - (2-\theta)^2(c_m + c_d)^2]. \end{aligned} \tag{9}$$

Moreover, when $c_d < c_d^L$, we find that as c_d increases, (i) both p_r^L and p_d^L increase, and the increase in p_d^L is greater than that in p_r^L; (ii) D_p^L increases, whereas both D_d^L and D_{PD}^L decrease; and (iii) the manufacturer's profit in the D-channel decreases, his profit in the P-channel increases, and π_m^L decreases. ■

The above proposition indicates an interesting finding that the manufacturer's profit in the P-channel is non-decreasing in his operating cost in the D-channel (i.e., c_d). This is justified as follows. Responding to a higher value of c_d, the manufacturer raises his retail price in the D-channel, and the physical retailer also increases the retail price in the P-channel. But, to compete with the D-channel, the physical retailer would raise the retail price at a smaller incremental rate than the manufacturer. As a result, although the demand in the D-channel reduces, the demand in the P-channel rises. Moreover, since the reduction in the manufacturer's profit in the D-channel cannot be offset by the increase in his profit in the P-channel, the manufacturer's total profit in the two channels is decreased.

4.1.2 Analysis of the Simultaneous-Move Game in the PD System

We consider a simultaneous-move game in which the manufacturer and the retailer determine their retail prices p_d and p_r with no communication. Differentiating π_r in (5) and π_m in (4) w.r.t. p_r and p_d, respectively, and setting the derivatives to zero, we find the two firms' best-response retail prices as

$$p_r(p_d) = \max\left\{\frac{p_d}{\theta}, \frac{a + p_d}{2}\right\} = \begin{cases} (a + p_d)/2, & \text{if } p_d < \theta a/(2-\theta), \\ p_d/\theta, & \text{if } p_d \geq \theta a/(2-\theta); \end{cases} \tag{10}$$

and

$$p_d(p_r) = \min\left\{\theta p_r, \frac{\theta b + \theta p_r + c_m + c_d}{2}\right\} = \begin{cases} \theta p_r, & \text{if } p_r < (\theta b + c_m + c_d)/\theta, \\ (\theta b + \theta p_r + c_m + c_d)/2, & \text{if } p_r \geq (\theta b + c_m + c_d)/\theta. \end{cases} \tag{11}$$

Proposition 4 When the manufacturer and the retailer play a simultaneous-move game in the PD system, we obtain the following results.

1. If $c_d \geq c_d^S$, where

$$c_d^S \equiv \frac{\theta a - (2-\theta)(\theta b + c_m)}{2-\theta}, \tag{12}$$

then the retail prices in Nash equilibrium are $p_d^S = \theta a/(2-\theta)$ and $p_r^S = a/(2-\theta)$, the demands in the P- and D-channels are $D_p^S = n[\bar{v} - a/(2-\theta)]/\bar{v}$ and $D_d^S = 0$, respectively; and the manufacturer's profit is $\pi_m^S = nb[\bar{v} - a/(2-\theta)]/\bar{v}$.

2. If $c_d < c_d^S$, then the retail prices in Nash equilibrium are

$$p_r^S = \frac{2a + \theta b + c_d + c_m}{4-\theta} \quad \text{and} \quad p_d^S = \frac{\theta a + 2(\theta b + c_d + c_m)}{4-\theta}.$$

As a result, the demands in the P-channel and the D-channel are

$$D_p^S = n - \frac{n}{\bar{v}}\frac{a\,(2-\theta) - (\theta b + c_d + c_m)}{(1-\theta)(4-\theta)} \text{ and } D_d^S = \frac{n}{\bar{v}}\frac{\theta a - (2-\theta)\,(\theta b + c_d + c_m)}{\theta\,(1-\theta)\,(4-\theta)},$$

and the total demand in the two channels is

$$D_{PD}^S = n - \frac{n}{\bar{v}}\frac{\theta a + 2(\theta b + c_d + c_m)}{\theta(4-\theta)}.$$

The manufacturer's profit is thus computed as

$$\begin{aligned} \pi_m^S &= nb - \frac{n}{\bar{v}\theta(1-\theta)(4-\theta)^2}\left\{2\theta ab(4-3\theta) - \theta^2 a^2 - \theta^3 b^2 + 2\theta a(2-\theta)(c_d + c_m)\right. \\ &\quad \left. -\theta b(4 - 3\theta + \theta^2)(c_d + c_m) - (2-\theta)^2(c_d + c_m)^2\right\}. \quad \blacksquare \end{aligned} \tag{13}$$

According to Propositions 3 and 4, we find that if the value of c_d is sufficiently high such that $c_d > c_d^L$ in the sequential-move game and $c_d > c_d^S$ in the simultaneous-move game, then the "real" online price p_d/θ is higher than the price p_r in the P-channel and thus, there is no demand in the D-channel. That is, the manufacturer can realize sales in the P- and D-channels when the manufacturer's unit operating cost c_d in the D-channel is below a certain threshold (i.e., c_d^L in the sequential-move game and c_d^S in the simultaneous-move game). This result implies the following remark.

Remark 1 If the manufacturer incurs a sufficiently high operating cost c_d in the D-channel, then he should not adopt the D-channel but only sells his products in the P-channel; otherwise, the manufacturer can achieve sales in the two channels. Specifically, it behooves the manufacturer to consider a dual-channel system if $c_d < c_d^L$ ($c_d < c_d^S$) in a sequential-move (simultaneous-move) game. ■

One may note that, in the above remark, the inequalities (i.e., $c_d < c_d^L$ and $c_d < c_d^S$)—which induce the manufacturer to operate a dual-channel system—are actually needed to ensure that the condition in (3) is satisfied for the positive demands in both the P-channel and the D-channel. In addition, we find that $c_d^S < c_d^L$ because $c_d^L - c_d^S = 2b/(2-\theta) > 0$.

Using Proposition 4, we can examine how the manufacturer's unit operating cost c_d affects the retail prices, demands, and the manufacturer's profits in both channels.

Corollary 1 If $c_d < c_d^S$, then as the value of c_d increases, (i) the retail prices p_r^S and p_d^S rise, and the increase in p_d^S is higher than that in p_r^S; (ii) the demand D_p^S increases, whereas the demands D_d^S and D_{PD}^S decrease; (iii) the manufacturer's profit in the D-channel decreases whereas that in the P-channel increases, and the manufacturer's total profit π_m^S decreases. ■

4.1.3 The Game Selection in the PD System

In Sections 4.1.1 and 4.1.2, we have investigated a sequential-move game and a simultaneous-move game, respectively. An important question arises as follows: when the manufacturer decides to adopt a D-channel in addition to an existing P-channel, should he choose a sequential-move game or a simultaneous-move game? To address the question, in the PD system we need to compare the total demand and the manufacturer's profit in the sequential-move game with those in the simultaneous-move game.

Theorem 1 In the PD system, we obtain the following results.

1. If $c_d \geq c_d^L$, then $D_d^S = D_d^L = 0$, and the demands and the manufacturer's total profits in the two channels are the same in both games, i.e., $D_{PD}^L = D_{PD}^S$, and $\pi_m^L = \pi_m^S$.
2. If $\bar{c}_d \leq c_d < c_d^L$, where $\bar{c}_d \equiv [\theta a - b(4-3\theta) - c_m(2-\theta)]/(2-\theta)$, then the manufacturer's profit and the total demand in the sequential-move game are higher than those in the simultaneous-move game, i.e., $\pi_m^L > \pi_m^S$, and $D_{PD}^L > D_{PD}^S$.
3. If $c_d < \bar{c}_d$, then the manufacturer's profit in the sequential-move game is higher than that in the simultaneous-move game, i.e., $\pi_m^L > \pi_m^S$, whereas the total demand in the sequential-move game is lower than that in the simultaneous-move game, i.e., $D_{PD}^L < D_{PD}^S$. ■

We learn from the above theorem that, when the manufacturer considers a dual-channel system and aims at the maximization of his profit, he should decide to announce his online retail price before the physical retailer determines her retail price, gaining the *first-mover advantage*. However, if the manufacturer aims to realize a higher demand, then the time of announcing his retail pricing decision will depend on his unit operating cost in the D-channel c_d. Specifically, if c_d is smaller than $\bar{c}_d$, then the manufacturer needs to announce his retail price "as the same time as" the physical retailer announces her price; there is no first-mover advantage. However, if $c_d \geq \bar{c}_d$, then the manufacturer should take the first-mover advantage by announcing his retail price prior to the physical retailer's price announcement.

4.2 Analysis in the PC System

In the PC system, the manufacturer serves consumers via both a third-party Internet marketplace and the existing independent physical store. In the C-channel, the manufacturer sells products directly to consumers on an e-tailer's marketplace under a *revenue-sharing consignment* contract.

The e-tailer offers the consignment contract by specifying a revenue allocation ratio $\xi \in (0,1)$. Under the contract, for each unit of the product sold, the e-tailer keeps a proportion ξ of the revenue for herself and remits the rest, $(1-\xi)$ proportion of the revenue, to the manufacturer.

We recall from Section 3.1 that as in practice, a unique revenue-allocation ratio usually applies to all the sellers whose products belong to a common product category, and all sellers have the knowledge of the revenue-allocation ratio prior to making an e-channel selection decision. Accordingly, the manufacturer makes his e-channel selection and pricing decisions, given the e-tailer's revenue allocation ratio ξ. Similar to Section 4.1, the manufacturer sets a retail price $\hat{p}_c$ in the C-channel to maximize his expected profit. In the PC system, the manufacturer and the physical retailer obtain their profits as $\hat{\pi}_m = \hat{D}_p(w - c_m) + \hat{D}_c[\hat{p}_c(1-\xi) - c_m]$ and $\hat{\pi}_r = \hat{D}_p(\hat{p}_r - w - c_r)$, respectively, which, using (2), can be specified as follows:

$$\hat{\pi}_m = \frac{nb}{\bar{v}}\left(\bar{v} - \frac{\hat{p}_r - \hat{p}_c}{1-\theta}\right) + \frac{n}{\bar{v}}[\hat{p}_c(1-\xi) - c_m]\left(\frac{\hat{p}_r - \hat{p}_c}{1-\theta} - \frac{\hat{p}_c}{\theta}\right), \tag{14}$$

$$\hat{\pi}_r = \frac{n}{\bar{v}}(\hat{p}_r - w - c_r)\left(\bar{v} - \frac{\hat{p}_r - \hat{p}_c}{1-\theta}\right). \tag{15}$$

Next, we investigate both a sequential-move game and a simultaneous-move game, and compare our analytic results to find which game makes the manufacturer better off.

4.2.1 Analysis of the Sequential-Move Game in the PC System

In this game, the manufacturer and the physical retailer sequentially make their retail pricing decisions in two steps. The manufacturer first announces his retail price $\hat{p}_c$ in the C-channel, and the retailer then determines her price $\hat{p}_r$ in the P-channel.

Proposition 5 When the manufacturer and the physical retailer play a sequential-move game in the PC system, we can find their retail prices in Stackelberg equilibrium, and also compute the resulting demands in the two channels and the manufacturer's profit.

1. If $\xi \geq \xi^L \equiv 1 - [\theta b + c_m(2-\theta)]/(\theta a)$, then the retail prices in Stackelberg equilibrium are $\hat{p}_c^L = \theta a/(2-\theta)$ and $\hat{p}_r^L = a/(2-\theta)$. As a result, the demands in the P-channel and the C-channel are $\hat{D}_p^L = n[\bar{v} - a/(2-\theta)]/\bar{v}$ and $\hat{D}_c^L = 0$, respectively, and the manufacturer's profit is $\hat{\pi}_m^L = nb[\bar{v} - a/(2-\theta)]/\bar{v}$.
2. If $\xi < \xi^L$, then the retail prices in Stackelberg equilibrium are

$$\hat{p}_r^L = \frac{1}{4(2-\theta)}\left[a(4-\theta) + \frac{\theta b + c_m(2-\theta)}{1-\xi}\right] \text{ and } \hat{p}_c^L = \frac{1}{2(2-\theta)}\left[\theta a + \frac{\theta b + c_m(2-\theta)}{1-\xi}\right].$$

The demands in the P- and C-channels are

$$\hat{D}_p^L = n - \frac{n}{4\bar{v}(1-\theta)(2-\theta)}\left[a(4-3\theta) - \frac{\theta b + c_m(2-\theta)}{1-\xi}\right],$$

$$\hat{D}_c^L = \frac{n}{4\bar{v}\theta(1-\theta)}\left[\theta a - \frac{\theta b + c_m(2-\theta)}{1-\xi}\right];$$

and the total demand is

$$\hat{D}_{PC}^L = n - \frac{n}{2\bar{v}\theta(2-\theta)}\left[\theta a + \frac{\theta b + c_m(2-\theta)}{1-\xi}\right].$$

The manufacturer's profit is

$$\begin{aligned}\hat{\pi}_m^L &= nb - \frac{n}{8\bar{v}\theta(1-\theta)(2-\theta)}\left\{2\theta ab(4-3\theta) - \theta^2 a^2(1-\xi)\right.\\&\left.+2\theta a c_m(2-\theta) - \frac{[\theta b + c_m(2-\theta)]^2}{1-\xi}\right\}. \quad \blacksquare\end{aligned}$$

4.2.2 Analysis of the Simultaneous-Move Game in the PC System

In the simultaneous-move game the manufacturer and the physical retailer determine their retail prices with no communication. Differentiating $\hat{\pi}_r$ in (15) w.r.t. $\hat{p}_r$ and $\hat{\pi}_m$ in (14) w.r.t. $\hat{p}_c$, setting them to zero, and solving the resulting equations, we find the two firms' best-response retail prices as

$$\hat{p}_r(\hat{p}_c) = \begin{cases} (a+\hat{p}_c)/2, & \text{if } \hat{p}_c < \theta a/(2-\theta), \\ \hat{p}_c/\theta, & \text{if } \hat{p}_c \geq \theta a/(2-\theta); \end{cases} \tag{16}$$

$$\hat{p}_c(\hat{p}_r,\xi) = \begin{cases} \theta\hat{p}_r, & \text{if } \hat{p}_r < (\theta b + c_m)/\left[\theta(1-\xi)\right], \\ [\theta b + \theta\hat{p}_r(1-\xi) + c_m]/2(1-\xi), & \text{if } \hat{p}_r \geq (\theta b + c_m)/\left[\theta(1-\xi)\right]. \end{cases} \tag{17}$$

Using the above we can calculate Nash equilibrium $(\hat{p}_c^S, \hat{p}_r^S)$ as well as the resulting demands and the manufacturer's profit.

Proposition 6 When the manufacturer and the retailer play the simultaneous-move game in the PC system, we obtain the following results.

1. If $\xi \geq \xi^S \equiv 1 - [(2-\theta)(\theta b + c_m)]/(\theta a)$, then the retail prices in Nash equilibrium are $\hat{p}_c^S = \theta a/(2-\theta)$ and $\hat{p}_r^S = a/(2-\theta)$. The demand in the C-channel is zero, and the manufacturer only gains the profit $\hat{\pi}_m^S = nb[\bar{v} - a/(2-\theta)]/\bar{v}$ in the P-channel.
2. If $\xi < \xi^S$, then the retail prices in Nash equilibrium are

$$\hat{p}_r^S = \frac{1}{4-\theta}\left(2a + \frac{\theta b + c_m}{1-\xi}\right) \quad \text{and} \quad \hat{p}_c^S = \frac{1}{4-\theta}\left[\theta a + \frac{2(\theta b + c_m)}{1-\xi}\right].$$

As a result, the demands in the P- and C-channels are

$$\begin{aligned}\hat{D}_p^S &= n - \frac{n}{\bar{v}(1-\theta)(4-\theta)}\left[a(2-\theta) - \frac{\theta b + c_m}{1-\xi}\right], \\ \hat{D}_c^S &= \frac{n}{\bar{v}\theta(1-\theta)(4-\theta)}\left[\theta a - \frac{(2-\theta)(\theta b + c_m)}{1-\xi}\right];\end{aligned}$$

and the total demand in the two channels is

$$\hat{D}_{PC}^S = \hat{D}_p^S + \hat{D}_c^S = n - \frac{n}{\bar{v}\theta(4-\theta)}\left[\theta a + \frac{2(\theta b + c_m)}{1-\xi}\right].$$

The manufacturer's profit is

$$\begin{aligned}\hat{\pi}_m^S &= nb - \frac{n}{\bar{v}\theta(1-\theta)(4-\theta)^2}\left\{2\theta ab(4-3\theta) - \theta^2 a^2(1-\xi)\right.\\&\left.+2\theta a c_m(2-\theta) - [\theta^2 b + c_m(2-\theta)^2]\frac{\theta b + c_m}{1-\xi}\right\}. \quad \blacksquare\end{aligned}$$

Observing Propositions 5 and 6, we find that for both the sequential-move and simultaneous-move games, when the e-tailer's revenue allocation ratio ξ is sufficiently high (viz., $\xi \geq \xi^L$ in the sequential-move game and $\xi \geq \xi^S$ in the simultaneous-move game), there is no demand in the C-channel, which takes place mainly because the "real" online price $\hat{p}_c/\theta$ is higher than the price in the P-channel $\hat{p}_r$.

Remark 2 If $\xi < \xi^L$ ($\xi < \xi^S$) in the sequential-move (simultaneous-move) game, then the manufacturer is willing to adopt a consignment e-channel in addition to his existing physical channel. This implies that the e-tailer should set her revenue allocation ratio ξ below a critical level in order to entice the manufacturer to use the e-channel. ■

4.2.3 The Game Selection in the PC System

In the PC system, the manufacturer can announce his online retail price before the physical retailer determines her retail price (i.e. the sequential-move game) or make a pricing decision with no communication with the retailer (i.e., the simultaneous-move game). Next, we examine which game makes the manufacturer better off.

Theorem 2 In the PC system, we derive the results regarding the manufacturer's game preference.

1. If $\xi \geq \xi^L$, then the demands and the manufacturer's total profits in the two channels are the same in both games, viz., the manufacturer has no preference between the two games.
2. If $\bar{\xi} \leq \xi < \xi^L$, where $\bar{\xi} \equiv 1 - [b(4-3\theta) + c_m(2-\theta)]/(\theta a) < \xi^S$, then both the manufacturer's total profit and the total demand in the sequential-move game are higher than those in the simultaneous-move game, which means that the manufacturer prefers the sequential-move game to the simultaneous-move game.
3. If $\xi < \bar{\xi}$, then the manufacturer's total profit in the sequential-move game is higher than that in the simultaneous-move game, whereas the total demand in the sequential-move game is lower than that in the simultaneous-move game. That is, from the profitability perspective, the manufacturer should prefer the sequential-move game; from the demand perspective, the manufacturer should prefer the simultaneous-move game. ■

As the above theorem implies, if the manufacturer adopts a consignment e-channel with an aim for a higher profit, then he should gain the first-mover advantage by announcing his online consignment retail price before the physical retailer determines her retail price. However, if the manufacturer aims for a larger demand, then he may or may not take the first-mover advantage, which depends on the revenue allocation ratio in the consignment contract. Specifically, if the allocation ratio is below the threshold $\bar{\xi}$, then the manufacturer should set the retail price with no communication with the retailer; otherwise, the manufacturer should announce his retail price before the retailer makes her decision and enjoy the first-mover advantage.

4.3 Summary of Analytic Results

We provide Table 1 to summarize our analytic results that are obtained for the sequential-move and simultaneous-move games in Sections 4.1 and 4.2.

	The PD System	
	The Sequential-Move Game	**The Simultaneous-Move Game**
Price		
P-channel	$p_r^L = \frac{a(4-\theta)+\theta b+(2-\theta)(c_d+c_m)}{4(2-\theta)}$	$p_r^S = \frac{2a+\theta b+c_d+c_m}{4-\theta}$
e-channel	$p_d^L = \frac{\theta a+\theta b+(2-\theta)(c_d+c_m)}{2(2-\theta)}$	$p_d^S = \frac{\theta a+2(\theta b+c_d+c_m)}{4-\theta}$
Demand		
P-channel	$D_p^L = n - \frac{n\left[a(4-3\theta)-\theta b-(2-\theta)(c_d+c_m)\right]}{4\bar{v}(2-\theta)(1-\theta)}$	$D_p^S = n - \frac{n\left[a(2-\theta)-(\theta b+c_d+c_m)\right]}{\bar{v}(1-\theta)(4-\theta)}$
e-channel	$D_d^L = \frac{n\left[\theta a-\theta b-(2-\theta)(c_d+c_m)\right]}{4\bar{v}\theta(1-\theta)}$	$D_d^S = \frac{n\left[\theta a-(2-\theta)(\theta b+c_d+c_m)\right]}{\bar{v}\theta(1-\theta)(4-\theta)}$
Total	$D_{PD}^L = n - \frac{n\left[\theta a+\theta b+(2-\theta)(c_d+c_m)\right]}{2\bar{v}\theta(2-\theta)}$	$D_{PD}^S = n - \frac{n\left[\theta a+2(\theta b+c_d+c_m)\right]}{\bar{v}\theta(4-\theta)}$
Profit		
Manufacturer	$\pi_m^L = nb - \frac{n}{8\bar{v}\theta(1-\theta)(2-\theta)}[2\theta ab(4-3\theta)-\theta^2a^2$ $+2\theta(2-\theta)(a-b)(c_m+c_d)-\theta^2b^2-(2-\theta)^2(c_m+c_d)^2]$	$\pi_m^S = nb - \frac{n}{\bar{v}\theta(1-\theta)(4-\theta)^2}[2\theta ab(4-3\theta)-\theta^2a^2+2\theta a(2-\theta)(c_d+c_m)$ $-(2-\theta)^2(c_d+c_m)^2-\theta^3b^2-\theta b(4-3\theta+\theta^2)(c_d+c_m)]$
	The PC System	
	The Sequential-Move Game	**The Simultaneous-Move Game**
Price		
P-channel	$\hat{p}_r^L = \frac{1}{4(2-\theta)}\left[(4-\theta)a+\frac{\theta b+c_m(2-\theta)}{1-\xi}\right]$	$\hat{p}_r^S = \frac{1}{4-\theta}\left(2a+\frac{\theta b+c_m}{1-\xi}\right)$
e-channel	$\hat{p}_c^L = \frac{1}{2(2-\theta)}\left[\theta a+\frac{\theta b+c_m(2-\theta)}{1-\xi}\right]$	$\hat{p}_c^S = \frac{1}{4-\theta}\left(\theta a+\frac{2(\theta b+c_m)}{1-\xi}\right)$
Demand		
P-channel	$\hat{D}_p^L = n - \frac{n\{a(4-3\theta)-[\theta b+c_m(2-\theta)]/(1-\xi)\}}{4\bar{v}(1-\theta)(2-\theta)}$	$\hat{D}_p^S = n - \frac{n}{\bar{v}(1-\theta)(4-\theta)}\left[a(2-\theta)-\frac{\theta b+c_m}{1-\xi}\right]$
e-channel	$\hat{D}_c^L = \frac{n}{4\bar{v}\theta(1-\theta)}\left[\theta a-\frac{\theta b+c_m(2-\theta)}{1-\xi}\right]$	$\hat{D}_c^S = \frac{n}{\bar{v}\theta(1-\theta)(4-\theta)}\left[\theta a-\frac{(2-\theta)(\theta b+c_m)}{1-\xi}\right]$
Total	$\hat{D}_{PC}^L = n - \frac{n}{2\bar{v}\theta(2-\theta)}\left[\theta a+\frac{\theta b+c_m(2-\theta)}{1-\xi}\right]$	$\hat{D}_{PC}^S = n - \frac{n}{\bar{v}\theta(4-\theta)}\left[\theta a+\frac{2(\theta b+c_m)}{1-\xi}\right]$
Profit		
Manufacturer	$\hat{\pi}_m^L = nb - \frac{n}{8\bar{v}\theta(1-\theta)(2-\theta)}$ $\times\left[2\theta ab(4-3\theta)-\theta^2a^2(1-\xi)+2\theta ac_m(2-\theta)-\frac{[\theta b+c_m(2-\theta)]^2}{1-\xi}\right]$	$\hat{\pi}_m^S = nb - \frac{n}{\bar{v}\theta(1-\theta)(4-\theta)^2}\{2\theta ab(4-3\theta)$ $-\theta^2a^2(1-\xi)+2\theta ac_m(2-\theta)-[\theta^2b+c_m(2-\theta)^2]\frac{\theta b+c_m}{1-\xi}\}$

Table 1: A summary of analytic results for the PD and PC systems in the sequential- and simultaneous-move games.

5 Selection of the Online Channel

Using our analytic results in Section 4, we now examine the manufacturer's two critical questions as follows. First, we need to address the question of which e-channel the manufacturer should adopt in addition to an existing physical retail channel. That is, should the manufacturer adopt the PD or the PC system? The second question is concerned with what factors have a significant impact on the manufacturer's online channel selection and how those factors influence the decision.

We learn from Section 4 that the demand in an e-channel is zero in the sequential-move (simultaneous-move) game setting, when the manufacturer's unit operating cost c_d is sufficiently high such that $c_d \geq c_d^L$ ($c_d \geq c_d^S$) in the PD system, or the e-tailer's revenue allocation ratio ξ is sufficiently high such that $\xi \geq \xi^L$ ($\xi \geq \xi^S$) in the PC system. As a result, both the total demand and the manufacturer's total profit are the same in the PD and the PC systems, regardless of which game the manufacturer is involved.

Next, we begin by comparing the demand and the manufacturer's profit in the PD system with those in the PC system. Specifically, we perform our analysis under the following conditions: $c_d < c_d^L$ (in the PD system) and $\xi < \xi^L$ (in the PC system) in the sequential-move game, and $c_d < c_d^S$ (in the PD system) and $\xi < \xi^S$ (in the PC system) in the simultaneous-move game. In each game setting, we derive the conditions under which the manufacturer prefers to choose the PD or the PC system, and find the important factors that significantly influence the manufacturer's e-channel selection decision.

5.1 Comparison between the PD and the PC Systems in the Sequential-Move Game

We compare the demand and the manufacturer's profit in the PD system with those in the PC system, assuming that the manufacturer and the retailer play a sequential-move game.

Proposition 7 When the manufacturer adopts an e-channel in the sequential-move game (i.e., $c_d < c_d^L$ in the PD system and $\xi < \xi^L$ in the PC system), we find the following results regarding the comparison between the total demands and the manufacturer's profits in the two systems.

1. If $0 \leq c_d < \tilde{c}_{d1}^L \equiv \xi[\theta b + c_m(2-\theta)]/[(1-\xi)(2-\theta)]$, then $D_{PD}^L > \hat{D}_{PC}^L$; otherwise, if $\tilde{c}_{d1}^L \leq c_d < c_d^L$, then $D_{PD}^L \leq \hat{D}_{PC}^L$.
2. If $0 \leq c_d < \tilde{c}_{d2}^L \equiv (1-\sqrt{1-\xi})\{\theta a + [\theta b + c_m(2-\theta)]/\sqrt{1-\xi}\}/(2-\theta)$, then $\pi_m^L > \hat{\pi}_m^L$; otherwise, if $\tilde{c}_{d2}^L \leq c_d < c_d^S$, then $\pi_m^L \leq \hat{\pi}_m^L$.

In the above, $\tilde{c}_{d1}^L < \tilde{c}_{d2}^L < c_d^L$. ■

Proposition 7 indicates that when the manufacturer complements his physical channel with an e-channel, both his unit operating cost c_d in the D-channel and the e-tailer's revenue allocation ratio ξ have an impact on his profit and the total demand. Specifically, given a value of ξ, if c_d is sufficiently small such that $c_d < \tilde{c}_{d1}^L$, then the demand and the manufacturer's profit are higher in the PD system than those in the PC system; otherwise, the demand and the manufacturer's profit are higher in the PC system. This result may be attributed to the following fact. As shown in Proposition 3, in the PD system, the manufacturer's total profit is increasing in his operating cost in the D-channel c_d. We can also easily show that the manufacturer's total profit in the PC system is decreasing in the e-tailer's revenue allocation ratio ξ, and the manufacturer's total profits in the two systems are equal when $c_d = \xi = 0$.

Remark 3 When the manufacturer and the retailer make their retail pricing decisions sequentially, the manufacturer prefers to operate his own (proprietary) e-channel if c_d is sufficiently small and adopt a consignment e-channel otherwise. ■

5.2 Comparison between the PD and the PC Systems in the Simultaneous-Move Game

We now compare the demands and the manufacturer's profits in the two systems when the manufacturer and the retailer play a simultaneous-move game.

Proposition 8 In the simultaneous-move game setting, when the manufacturer adopts an e-channel (i.e., $c_d < c_d^S$ in the PD system and $\xi < \xi^S$ in the PC system), we compare the demands and the manufacturer's profits in the PD and the PC systems as follows.

1. If $0 \leq c_d < \tilde{c}_{d1}^S \equiv \xi(\theta b + c_m)/(1-\xi)$, then $D_{PD}^S > \hat{D}_{PC}^S$; otherwise, if $\tilde{c}_{d1}^S \leq c_d < c_d^S$, then $D_{PD}^S \leq \hat{D}_{PC}^S$.
2. If $0 \leq c_d < \tilde{c}_{d2}^S$, where $\tilde{c}_{d2}^S$ is a unique solution satisfying the equation $\pi_m^S - \hat{\pi}_m^S = 0$, then $\pi_m^S > \hat{\pi}_m^S$; otherwise, if $\tilde{c}_{d2}^S \leq c_d < c_d^S$, then $\pi_m^S \leq \hat{\pi}_m^S$.

In the above, $\tilde{c}_{d1}^S < \tilde{c}_{d2}^S < c_d^S$. ■

The above proposition delivers similar insights as Proposition 7 and Remark 3 (in the sequential-move game). That is, our insights about the manufacturer's preference on the PD or the PC system are consistent regardless of whether the manufacturer plays in the simultaneous-move or the sequential-move game.

5.3 The Manufacturer's Online Channel Selection

We learn from Propositions 7 and 8 that the manufacturer's e-channel selection decision depends on his unit operating cost in the D-channel (i.e., c_d) and the e-tailer's revenue allocation ratio in the C-channel (i.e., ξ). Given a revenue allocation ratio ξ, we draw Figure 2 to illustrate our analytic results concerning the comparison between the manufacturer's profits and the demands in the PD and the PC systems in the sequential-move and simultaneous-move games.

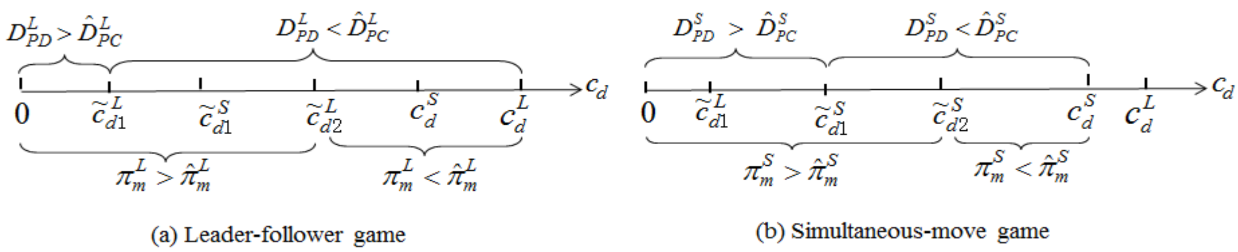

Figure 2: Comparison of the manufacturer's profits and the demands in the PD and PC systems.

Remark 4 When the manufacturer sells his products through a dual channel, he may adopt a direct e-channel or a consignment e-channel, which depends on the values of c_d and ξ.

1. When $0 \leq c_d \leq \tilde{c}_{d1}^L$, the manufacturer should adopt a direct e-channel, which can maximize his profit and the demand, regardless of which game the two firms play.

2. When $\tilde{c}_{d1}^L < c_d \leq \tilde{c}_{d1}^S$, the manufacturer prefers to adopt a direct e-channel if he aims at maximizing his profit, in both the sequential-move and simultaneous-move games. However, if the manufacturer aims at maximizing the demand, then he should adopt a consignment e-channel in the sequential-move game but adopt a direct e-channel in the simultaneous-move game.
3. When $\tilde{c}_{d1}^S < c_d \leq \min\left(\tilde{c}_{d2}^S, \tilde{c}_{d2}^L\right)$, the manufacturer should adopt a direct e-channel to maximize his profit but consider a consignment e-channel to maximize the demand, regardless of which game the two firms play.
4. When $\max\left(\tilde{c}_{d2}^S, \tilde{c}_{d2}^L\right) \leq c_d \leq c_d^S$, the manufacturer should adopt a consignment e-channel, which can maximize both his profit and the demand in the two games.
5. When $c_d^S < c_d \leq c_d^L$, the manufacturer should adopt a consignment e-channel and announce the retail price before the retailer. ■

The above remark indicates that the manufacturer should make his e-channel selection decision according to his objective, his e-channel unit operating cost, and the e-tailer's revenue allocation ratio. As an example, Levi Strauss & Co. discontinued its direct e-channel at `Levis.com` and `Dockers.com`, and handed online sales over to a few e-tailers due to the high operation costs and low demand in the e-channel in 1990s. The firm reopened its online store recently as the operation costs dropped and demand grew because of the rapid development of Internet technology and the increasing popularity of online shopping (Yoo and Lee 2010).

6 Game-Theoretic Analysis of Dual-Channel Supply Chains with the Wholesale Pricing Decision

In previous sections we have investigated the manufacturer's e-channel selection problem assuming that the wholesale price does not change when the manufacturer complements his physical channel with an e-channel. That is, in any dual-channel supply chain, the manufacturer always chooses the same wholesale price as that in the single-channel supply chain, which may differ from practical operations because the addition of an e-channel may entail a change in the wholesale price. In this section, we relax the assumption of unchanged wholesale price and analyze the dual-channel supply chain in which the manufacturer determines the retail price in the e-channel (i.e., p_i, $i = d, c$) and the wholesale price in the physical channel (i.e., w) to maximize his profit.

For the PD and PC systems, we analyze the sequential- and simultaneous-move games. In the sequential-move game, the manufacturer first determines p_i $(i = d, c)$ and w, and announces them to the physical retailer, who then makes her pricing decision p_r. In the simultaneous-move game, the two firms make their pricing decisions with no communication. For each game, we can obtain the two firms' equilibrium prices, the resulting demands, and the manufacturer's profit, as given in Table 2. Next, using these results, we examine whether our major insights obtained in Sections 4 and 5 still hold when the manufacturer makes his wholesale pricing decision in dual-channel supply chains. The major insights are concerned with (i) the game selection in the PD and the PC systems and (ii) the selection of the online channel.

	The PD System	
	The Sequential-Move Game	**The Simultaneous-Move Game**
Price		
P-channel	$w_{PD}^{L}=\frac{\bar{v}-c_r+c_m}{2}$	$w_{PD}^{S}=\frac{\bar{v}(8+\theta^2)-8c_r+c_m(8+\theta)-\theta c_d}{2(8+\theta)}$
	$p_r^{L}=\frac{\bar{v}(3-\theta)+c_r+2c_m+c_d}{4}$	$p_r^{S}=\frac{\bar{v}(12-2\theta-\theta^2)+4c_r+c_m(8+\theta)+c_d(4+\theta)}{2(8+\theta)}$
e-channel	$p_d^{L}=\frac{\theta\bar{v}+c_d+c_m}{2}$	$p_d^{S}=\frac{\theta\bar{v}(10-\theta)-2\theta c_r+c_m(8+\theta)+c_d(8+3\theta)}{2(8+\theta)}$
Demand		
P-channel	$D_p^{L}=n-\frac{n[\bar{v}(1-\theta)-c_r+c_d]}{4\bar{v}(1-\theta)}$	$D_p^{S}=\frac{n(2+\theta)[\bar{v}(1-\theta)-c_r+c_d]}{\bar{v}(1-\theta)(8+\theta)}$
e-channel	$D_d^{L}=\frac{n[\theta\bar{v}(1-\theta)+\theta c_r-2c_m(1-\theta)-c_d(2-\theta)]}{4\bar{v}\theta(1-\theta)}$	$D_d^{S}=\frac{n[\theta\bar{v}(1-\theta)(2+\theta)+6\theta c_r-c_m(8-7\theta-\theta^2)-c_d(8-\theta-\theta^2)]}{2\bar{v}\theta(1-\theta)(8+\theta)}$
Total	$D_{PD}^{L}=\frac{n(\theta\bar{v}-c_d-c_m)}{2\bar{v}\theta}$	$D_{PD}^{S}=\frac{n[3\theta\bar{v}(2+\theta)+2\theta c_r-c_m(8+\theta)-c_d(8+3\theta)]}{2\bar{v}\theta(8+\theta)}$
Profit		
Manufacturer	$\pi_m^{L}=\frac{n}{8\bar{v}\theta(1-\theta)}\{\theta\bar{v}^2(1-\theta^2)-2\theta\bar{v}c_r(1-\theta)+\theta c_r^2-4\theta\bar{v}c_m(1-\theta)$ $-2\theta c_d[\bar{v}(1-\theta)+c_r]+4c_mc_d(1-\theta)+2c_m^2(1-\theta)+c_d^2(2-\theta)\}$	$\pi_m^{S}=\frac{n}{4\bar{v}\theta(1-\theta)(8+\theta)}\{\theta\bar{v}^2(1-\theta)(2+\theta)^2-2\theta\bar{v}(1-\theta)[4c_r+c_m(8+\theta)+c_d(4+\theta)]$ $+4\theta c_r^2+c_m^2(8-7\theta+\theta^2)+c_d^2(8-3\theta+\theta^2)-8\theta c_rc_d+2c_mc_d(8-7\theta+\theta^2)\}$

	The PC System	
	The Sequential-Move Game	**The Simultaneous-Move Game**
Price		
P-channel	$\hat{w}_{PC}^{L}=\frac{1}{8(1-\xi)-\theta(8-8\xi+\xi^2)}\{(1-\xi)[\bar{v}(1-\theta)(4-\theta\xi)$ $-c_r(4-4\theta+\theta\xi)]+2c_m(1-\theta)(2-\xi)\}$	$\hat{w}_{PC}^{S}=\frac{1}{2[8(1-\xi)-\theta(7-8\xi+\xi^2)-\theta^2]}\{(1-\xi)\{\bar{v}(1-\theta)(8-2\theta\xi+\theta^2)$ $-2c_r[4-\theta(4-\xi)]\}+c_m(1-\theta)(8+\theta-4\xi)\}$
	$\hat{p}_r^{L}=\frac{1}{8(1-\xi)-\theta(8-8\xi+\xi^2)}[2\bar{v}(1-\xi)(3-4\theta+\theta^2)$ $+c_r(2-2\theta-2\xi+\theta\xi)+c_m(1-\theta)(4-\xi)]$	$\hat{p}_r^{S}=\frac{1}{2(1-\xi)[8(1-\xi)-\theta(7-8\xi+\xi^2)-\theta^2]}\{\bar{v}(1-\theta)(1-\xi)[12(1-\xi)-\theta^2-\theta(2-4\xi)]$ $+2(1-\xi)[2-\theta(2-\xi)-2\xi]c_r+c_m[2\left(4-5\xi+\xi^2\right)-\theta\left(7-10\xi+2\xi^2\right)-\theta^2]\}$
e-channel	$\hat{p}_c^{L}=\frac{1}{8(1-\xi)-\theta(8-8\xi+\xi^2)}[\theta\bar{v}(1-\theta)(4-3\xi)$ $-\theta\xi c_r+4c_m(1-\theta)]$	$\hat{p}_c^{S}=\frac{1}{2(1-\xi)[8(1-\xi)-\theta(7+\theta-8\xi+\xi^2)]}\{c_m(1-\theta)(8+\theta-8\xi+\theta\xi)$ $+\theta\bar{v}(1-\theta)(1-\xi)[10-\theta(1-\xi)-6\xi]-2\theta c_r(1-\xi)(1-\theta+\xi)\}$
Demand		
P-channel	$\hat{D}_p^{L}=\frac{n[\bar{v}(2-2\theta+\theta\xi)(1-\xi)-2c_r(1-\xi)+\xi c_m]}{\bar{v}[8(1-\xi)-\theta(8-8\xi+\xi^2)]}$	$\hat{D}_p^{S}=\frac{n(2+\theta-2\xi)[\bar{v}(1-\xi)(2-2\theta+\theta\xi)-2c_r(1-\xi)+\xi c_m]}{2\bar{v}(1-\xi)[8(1-\xi)-\theta(7+\theta-8\xi+\xi^2)]}$
e-channel	$\hat{D}_c^{L}=\frac{n}{\theta\bar{v}[8(1-\xi)-\theta(8-8\xi+\xi^2)]}\{\theta\bar{v}[2(1-\theta)-\xi(3-2\theta)]$ $+\theta c_r(2-\xi)-c_m[4(1-\theta)+\theta\xi]\}$	$\hat{D}_c^{S}=\frac{n}{2\theta\bar{v}(1-\xi)[8(1-\xi)-(7+\theta-8\xi+\xi^2)\theta]}\{\theta\bar{v}(1-\xi)[2-\theta-\theta^2-6\xi+3\theta\xi]$ $+2\theta c_r(1-\xi)(3-\xi)+c_m[\theta^2-8(1-\xi)+\theta(7-11\xi+2\xi^2)]\}$
Total	$\hat{D}_{PC}^{L}=\frac{n\left\{\theta\bar{v}\left[(1-\theta)(4-5\xi)-\theta\xi^2\right]+\theta\xi c_r-4c_m(1-\theta)\right\}}{\theta\bar{v}[8(1-\xi)-\theta(8-8\xi+\xi^2)]}$	$\hat{D}_{PC}^{S}=\frac{n}{2\theta\bar{v}(1-\xi)[8(1-\xi)-\theta(7+\theta-8\xi+\xi^2)]}\{2\theta c_r(1-\xi)(1-\theta+\xi)+c_m(1-\theta)$ $\times[8(1-\xi)+\theta+\theta\xi]+\theta\bar{v}(1-\xi)\left[6-\theta^2(3-\xi)-10\xi-\theta\left(3-9\xi+2\xi^2\right)\right]\}$
Profit		
Manufacturer	$\hat{\pi}_m^{L}=\frac{n}{\theta\bar{v}[8(1-\xi)-\theta(8-8\xi+\xi^2)]}\{\theta\bar{v}^2(1-\theta)(1-\xi)(1+\theta-\theta\xi)$ $-\theta\bar{v}c_m[(1-\theta)(4-5\xi)-\theta\xi^2]-\theta\xi c_rc_m+2c_m^2(1-\theta)$ $+\theta c_r^2(1-\xi)-\theta\bar{v}c_r(1-\xi)(2-2\theta+\theta\xi)\}$	$\hat{\pi}_m^{S}=\frac{n}{4\bar{v}[8(1-\xi)-\theta(7+\theta-8\xi+\xi^2)]}\{(1-\xi)\{\bar{v}^2(1-\theta)[4+\theta^2+4\theta(1-\xi)]-4\bar{v}c_r(2-2\theta+\theta\xi)$ $+4c_r^2\}-2c_m\{\bar{v}[8-\theta^2-10\xi-\theta\left(7-10\xi+2\xi^2\right)]+2\xi c_r\}+c_m^2(1-\theta)(8+\theta-8\xi)/[\theta(1-\xi)]\}$

Table 2: A summary of analytic results for the PD and PC systems in the sequential- and simultaneous-move games when the manufacturer makes wholesale pricing decision.

6.1 The Game Selection in the PD and the PC Systems

We start with examining whether our insights in Sections 4.1.3 and 4.2.3 hold for our game analysis with the manufacturer's wholesale pricing decision in dual-channel supply chains. Using the results for the PD system in Table 2, we find that if $c_d < \bar{c}_d^L \equiv [(1-\theta)(\theta\bar{v} - 2c_m) + \theta c_r]/(2-\theta)$ in the sequential-move game, or $c_d < \bar{c}_d^S \equiv \Gamma/(8-\theta-\theta^2)$ (where $\Gamma \equiv \theta\bar{v}(1-\theta)(2+\theta)+6\theta c_r - c_m(8-7\theta-\theta^2)$) in the simultaneous-move game, then the demand in the D-channel is positive and the manufacturer should adopt an e-channel. Moreover, for the PC system, the demand in the C-channel is positive if, in the sequential-move game, $\xi < \bar{\xi}^L \equiv [2(1-\theta)(\theta\bar{v} - 2c_m) + \theta c_r]/\{\theta\bar{v}[(3-2\theta) + c_r + c_m]\}$, or in the simultaneous-move game, $\xi < \bar{\xi}^S \equiv \{\Lambda - \sqrt{\Lambda^2 - 4\theta\Gamma[3\bar{v}\,(2-\theta) + 2c_r + 2c_m]}\}/\{2\theta[3\theta\bar{v}(2-\theta) + 2c_r + 2c_m]\}$, where $\Lambda \equiv \theta\bar{v}(8-4\theta-\theta^2) + 8\theta c_r + c_m(11\theta - 8)$. The above results indicate that the results in Remarks 1 and 2 hold when the manufacturer makes his wholesale pricing decision in dual-channel supply chains. That is, the manufacturer should adopt an e-channel if he incurs a sufficiently low operating cost c_d in the D-channel or has a sufficiently small revenue allocation ratio ξ in the C-channel.

Next, we compare the manufacturer's profits in the sequential- and simultaneous-move games using our results in Table 2. In the PD system, the difference in the manufacturer's profits in the two games is computed as

$$\pi_m^L - \pi_m^S = \frac{n\theta}{8\bar{v}(1-\theta)(8+\theta)}\left[\bar{v}\,(1-\theta) + c_r - c_d\right]^2,$$

which is obviously positive. Similarly, in the PC system, we calculate the difference in the manufacturer's profits in the two games as

$$\hat{\pi}_m^L - \hat{\pi}_m^S = \frac{n\theta(1-\theta)\{\bar{v}(1-\xi)[2-\theta(2-\xi)] - 2c_r(1-\xi) + \xi c_m\}^2}{4\bar{v}(1-\xi)[8(1-\xi) - \theta(8-8\xi+\xi^2)][8(1-\xi) - \theta(7+\theta-8\xi+\xi^2)]}, \tag{18}$$

which is positive because $[8(1-\xi)-\theta(8-8\xi+\xi^2)][8(1-\xi)-\theta(7+\theta-8\xi+\xi^2)] > 0$, for $\xi \in [0, \bar{\xi}^S)$ and $\theta \in [0,1]$. This means that, in the PC system, the manufacturer prefers to play in the sequential-move game.

In addition, we compare the demands in the sequential- and simultaneous-move games. Using Table 2, we find that in the PD system, the difference in the demands in the two games is $D_{PD}^L - D_{PD}^S = n[\bar{v}(1-\theta) - c_r + c_d]/[\bar{v}(8+\theta)]$, which is positive when $\bar{v}(1-\theta) - c_r + c_d > 0$, or $c_d > \bar{c}_{d2} \equiv c_r - \bar{v}(1-\theta)$. That is, if the manufacturer's unit operating cost in the D-channel c_d is sufficiently small, then the demand in the sequential-move game is greater than that in the simultaneous-move game. Otherwise, the demand in the simultaneous-move game is higher. For the PC system, we find that if $\xi < \bar{\xi}_1 \equiv (-8+7\theta+\sqrt{17\theta^2 - 80\theta + 64})/(2\theta)$, then the demand in the sequential-move game is less than that in the simultaneous-move game; otherwise, the demand in the sequential-move game is greater.

Remark 5 According to our results above, we can draw managerial insights as follows.

1. In the PD system, the manufacturer always prefers the sequential-move game to the simultaneous-move game from the profitability perspective. From the demand perspective, if c_d is sufficiently large, then the manufacturer prefers to play in the sequential-move game; otherwise, the manufacturer is better off in the simultaneous-move game. The results are similar to those in Theorem 1.

2. We find that in the PC system, if $\xi \geq \bar{\xi}_1$, then the manufacturer prefers to play in the sequential-move game, from both the profitability and the demand perspectives. Otherwise, the manufacturer prefers to play in the sequential-move game from the profitability perspective, but prefers to play in the simultaneous-move game from the demand perspective. The results are similar to those in Theorem 2. ■

6.2 Numerical Analysis for the Selection of the Online Channel

We now examine whether the main results in Section 5 hold when the manufacturer makes his wholesale pricing decision in dual-channel supply chains. Since we cannot find any analytical results regarding the selection of the online channel, we have to perform numerical experiments to compare the results for the PD and the PC systems using Table 2. For our numerical experiments, $n = 1$, $\bar{v} = 1$, $\theta = 0.8$, $c_m = 0$, and $c_r = 0.025$. In addition, we assume the revenue allocation ratio as $\xi = 15\%$, which is a typical value for a large number of product categories sold through consignment contracts at `Amazon.com`.

When we change the value of c_d from 0 to 0.15, we use the formulas in Table 2 to calculate all the results, and find that the conclusions in Remark 4 hold with the cutoff levels $\tilde{c}_{d1}^L = 0.042$, $\tilde{c}_{d1}^S = 0.0472$, and $\tilde{c}_{d2}^L = 0.064$. We also conduct such calculations for other two common allocation ratio values (i.e., $\xi = 8\%$ and $\xi = 20\%$) used by `Amazon.com`. When $\xi = 8\%$, we obtain the conclusions in Remark 4 with the cutoff levels $\tilde{c}_{d1}^L = 0.018$, $\tilde{c}_{d1}^S = 0.0472$, $\tilde{c}_{d2}^L = 0.064$; and when $\xi = 20\%$, the conclusions in Remark 4 hold with $\tilde{c}_{d1}^L = 0.065$, $\tilde{c}_{d1}^S = 0.071$, and $\tilde{c}_{d2}^L = 0.088$. It thus follows that the conclusions in Remark 4 are very likely to hold when the manufacturer makes his wholesale pricing decision in dual-channel supply chains.

7 Summary and Concluding Remarks

In this paper, we analytically investigate a manufacturer's decision on selecting between a direct e-channel and a consignment e-channel to complement his existing traditional retail channel. Accordingly, we consider two types of dual channel: a PD system consisting of a physical channel and a direct e-channel, and a PC system consisting of a physical channel and a consignment e-channel. For each system, we analyze a sequential-move game and a simultaneous-move game, in which the manufacturer and the physical retailer determine their retail prices in the e-channel and the physical channel, respectively.

When the manufacturer's unit operating cost in the e-channel of the PD system or the e-tailer's revenue allocation ratio in the PC system is sufficiently small, the manufacturer is willing to adopt an e-channel in addition to the extant physical channel. For each system, by comparing the manufacturer's profit and the demand in the sequential-move game with those in the simultaneous-move game, we show that the manufacturer can always gain a higher profit by acting as a "leader" in setting his retail price. But, to increase the demand, the manufacturer should determine his retail price with no communication with the retailer, if his unit e-channel operating cost is low in the PD system or when the e-tailer's revenue allocation ratio is low in the PC system. Otherwise, the manufacturer should announce his retail price before the retailer.

We then investigate the manufacturer's e-channel selection decision by comparing his profits and the demands between the two systems in each game. Our analytical results reveal that, in each game, when the manufacturer aims at increasing his profit, he should adopt a direct e-channel if his unit e-channel operating cost is below a threshold given the e-tailer's revenue allocation ratio, and adopt a consignment e-channel otherwise. A similar conclusion can be obtained when the manufacturer intends to improve the demand. Both the manufacturer's e-channel operating cost and the e-tailer's revenue allocation ratio have a significant impact on the manufacturer's e-channel selection decision. We also find that most of our major managerial insights still hold when the manufacturer makes his wholesale pricing decision in dual-channel supply chains.

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References

Abhishek, V., Jerath, K. & Zhang, Z. J. (2015). Agency selling or reselling? Channel structures in electronic retailing, *Management Science* **62**(8): 2259–2280.

Balasubramanian, S. (1998). Mail versus mall: A strategic analysis of competition between direct marketers and conventional retailers, *Marketing Science* **17(3)**: 181–195.

Bernstein, F., Song, J. & Zheng, X. (2008). "Bricks-and-mortar" vs. "clicks-and-mortar": An equilibrium analysis, *European Journal of Operational Research* **187**(3): 671–690.

Cai, G. (2010). Channel selection and coordination in dual-channel supply chains, *Journal of Retailing* **86(1)**: 22–36.

Cattani, K., Gilland, W., Heese, H. S. & Swaninathan, J. (2006). Boiling frogs: Pricing strategies for a manufacturer adding a direct channel that competes with the traditional channel, *Production and Operations Management* **15(1)**: 40–56.

Chen, J., Cheng, H. & Chien, M. (2011). On channel coordination through revenue-sharing contracts with price and shelf-space dependent demand, *Applied Mathematical Modelling* **35(10)**: 4886–4901.

Chen, K. Y., Kaya, M. & Ozer, O. (2008). Dual sales channel management with service competition, *Manufacturing & Service Operations Management* **10**(4): 654–675.

Chen, X., Zhang, H., Zhang, M. & Chen, J. (2017). Optimal decisions in a retailer Stackelberg supply chain, *International Journal of Production Economics* **187**: 260–270.

Chiang, W. K., Chhajed, D. & Hess, J. D. (2003). Direct marketing, indirect profits: A strategic analysis of dual-channel supply-chain design, *Management Science* **49(1)**: 1–20.

Chu, J., Urriza, M. A., Javier, J., Calvo, C. & Chintagunta, P. K. (2010). An empirical analysis of shopping behavior across online and offline channels for grocery products: The moderating effects of household and product characteristics, *Journal of Interactive Marketing* **24**: 251–268.

Ding, Q., Dong, C. & Pan, Z. (2016). A hierarchical pricing decision process on a dual-channel problem with one manufacturer and one retailer, *International Journal of Production Economics* **175**: 197–212.

Dumrongsiri, A., Fan, M., Jain, A. & Moinzadeh, K. (2008). A supply chain model with direct and retail channels, *European Journal of Operational Research* **187**(3): 691–718.

Fruchter, G. E. & Tapireo, C. S. (2005). Dynamic online and offline channel pricing for heterogeneous customers in virtual acceptance, *International Game Theory Review* **7(2)**: 137–150.

Garicano, L. & Kaplan, S. (2001). The effects of business-to-business e-commerce on transaction costs, *The Journal of Industrial Economics* **49(4)**: 463–485.

Hua, G., Cheng, T. C. E. & Wang, S. (2011). Electronic books: To "e" or not to "e"? A strategic analysis of distribution channel choices of publishers, *International Journal of Production Economics* **129**(2): 338–346.

Huang, W. & Swaminathan, J. M. (2009). Introduction of a second channel: Implications for pricing and profits, *European Journal of Operational Research* **194**(1): 258–279.

Kacen, J. J., Hess, J. D. & Chiang, W. K. (2013). Bricks or clicks? Consumer attitudes toward traditional stores and online stores, *Global Economics and Management Review* **18(1)**: 12–21.

Khouja, M., Park, S. & Cai, G. (2010). Channel selection and pricing in the presence of retail-captive consumers, *International Journal of Production Economics* **125**(1): 84–95.

Kollmann, T., Kuckertz, A. & Kayser, I. (2012). Cannibalization or synergy? Consumers channel selection in online-offline multichannel systems, *Journal of Retailing and Consumer Services* **19**: 186–194.

Kumar, N. & Ruan, R. (2006). On manufacturers complementing the traditional retail channel with a direct online channel, *Quantitative Marketing & Economics* **4(3)**: 289–323.

Li, B., Hou, P., Chen, P. & Li, Q. (2016). Pricing strategy and coordination in a dual channel supply chain with a risk-averse retailer, *International Journal of Production Economics* **178**: 154–168.

Li, G., Huang, F., Cheng, T. C. E. & Ji, P. (2015). Competition between manufacturer's online customization channel and conventional retailer, *IEEE Transactions on Engineering Management* **62(2)**: 150–157.

Liang, T. & Huang, J. (1998). An empirical study on consumer acceptance of products in electronic markets: A transaction cost model, *Decision Support System* **24**: 29–43.

Mukhopadhyay, S. K., Zhu, X. & Yue, X. (2008). Optimal contract design for mixed channels under information asymmetry, *Production and Operations Management* **17(6)**: 641–650.

Ofek, E., Katona, Z. & Sarvary, M. (2011). "Bricks and clicks": The impact of product returns on the strategies of multichannel retailers, *Management Science* **31(1)**: 42–60.

Ow, T. T. & Wood, C. A. (2011). Which online channel is right? Online auction channel choice for personal computers in the presence of demand decay, *Electronic Commerce Research and Applications* **10**: 203–213.

Ryan, J. K., Sun, D. & Zhao, X. (2012). Competition and coordination in online marketplaces, *Production and Operations Management* **21(6)**: 997–1014.

Song, W., Wang, R., Fu, Y. & Peng, X. (2014). Consumer choice, firm performance and channel coordination in a dual-channel distribution system, *American Journal of Operations Research* **4**: 217–227.

Tsay, A. A. & Agrawal, N. (2004a). Channel conflict and coordination in the e-commerce age, *Production and Operations Management* **13**(1): 93–110.

Tsay, A. A. & Agrawal, N. (2004b). *Modeling conflict and coordination in multi-channel distribution systems: A review*, Kluwer Academic Publishers, Norwell, MA.

Wang, Y. (2006). Joint pricing-production decisions in supply chains of complementary products with uncertain demand, *Operations Research* **54(6)**: 1110–1127.

Wang, Y., Jiang, L. & Shen, Z. (2004). Channel performance under consignment contract with revenue sharing, *Management Science* **50(1)**: 34–47.

Xiao, T., Choi, T. & Cheng, T. C. E. (2014). Product variety and channel structure strategy for a retailer-stackelberg supply chain, *European Journal of Operational Research* **233(1)**: 114–124.

Yan, R. (2008). Profit sharing and firm performance in the manufacturer-retailer dual-channel supply chain, *Electronic Commerce Research* **8(3)**: 155–172.

Yan, R. & Ghose, S. (2010). Forecast information and traditional retailer performance in a dual-channel competitive market, *Journal of Business Research* **63**: 77–83.

Yan, R., Wang, J. & Zhou, B. (2010). Channel integration and profit sharing in the dynamics of multi-channel firms, *Journal of Retailing and Consumer Services* **17**: 430–440.

Yao, D. & Liu, J. (2005). Competitive pricing of mixed retail and e-tail distribution channels, *Omega* **33**: 235–247.

Yoo, W. S. & Lee, E. (2011). Internet channel entry: A strategic analysis of mixed channel structures, *Marketing Science* **30(1)**: 29–41.

Zhang, T., Li, G., Cheng, T. C. E. & Lai, K. K. (2017). Welfare economics of review information: Implications for the online selling platform owner, *International Journal of Production Economics* **184**: 69–79.

Zhu, K. (2004). The complementarity of information technology infrastructure and e-commerce capability: A resource-based assessment of their business value, *Journal of Management Information Systems* **21(1)**: 167–202.

Appendix A Proofs

Proof of Proposition 1. In a dual channel, a consumer with the valuation v needs to compare the surpluses $u_r = v - p_r$ and $u_i = \theta v - p_i$ $(i = d, c)$ to make his or her purchase decision. If $v = (p_r - p_i)/(1 - \theta)$, then $v - p_r = \theta v - p_i$ and the consumer obtains an identical surplus from the two channels. In addition, if $p_i/\theta \le p_r$, then $p_r \le (p_r - p_i)/(1-\theta)$; otherwise, if $p_i/\theta > p_r$, then $p_r > (p_r - p_i)/(1-\theta)$. We accordingly consider two cases as indicated in Figure 1.

Case 1: $p_r \ge p_i/\theta$. For this case, $p_i/\theta \le p_r \le (p_r - p_i)/(1-\theta)$. As shown in Figure 1, if a consumer's valuation v is smaller than p_i/θ, then the consumer cannot enjoy any positive surplus from the purchase and thus, will not buy the product in any channel. If $p_i/\theta \le v \le (p_r - p_i)/(1-\theta)$, then the consumer will obtain a higher surplus from the online shopping and thus choose the e-channel (viz., the D- or the C-channel). Otherwise, if $(p_r - p_i)/(1-\theta) \le v \le \bar{v}$, then the consumer prefers to buy in the P-channel.

Case 2: $p_r < p_i/\theta$. We find that $(p_r - p_i)/(1-\theta) < p_r < p_i/\theta$. According to Figure 1, we find that, if $v < p_r$, then the consumer will not buy the product in any channel; otherwise, if $p_r \le v \le \bar{v}$, then the consumer will buy in the P-channel.

■

Proof of Proposition 2. We learn from (6) that the retailer's best-response retail price is

$$p_r(p_d) = \begin{cases} p_d/\theta, & \text{if } p_d \ge \theta a/(2-\theta), \\ (a+p_d)/2, & \text{if } p_d < \theta a/(2-\theta). \end{cases}$$

Since $p_r(p_d)$ depends on the value of p_d, we consider the following two cases:

1. If the manufacturer determines p_d such that $p_d \ge \theta a/(2-\theta)$, then the retailer's best response decision is $p_r(p_d) = p_d/\theta$. Substituting $p_r(p_d)$ into (4), we find that $\pi_m = nb(\bar{v} - p_d/\theta)/\bar{v}$. As $\partial \pi_m/\partial p_d = -nb/(\bar{v}\theta) < 0$, π_m is decreasing in p_d. Thus, the manufacturer's and the retailer's retail prices in equilibrium are calculated as

$$p_d^L = \frac{\theta a}{2-\theta} \text{ and } p_r^L = \frac{a}{2-\theta}.$$

   Using the above, we find that the demand in the D-channel is $D_d^L = 0$.
2. If the manufacturer's online retail price is set such that $p_d < \theta a/(2-\theta)$, then $p_r(p_d) = (a+p_d)/2$. Substituting $p_r(p_d)$ into (4) and solving the first-order condition $\partial \pi_m/\partial p_d = 0$, we find the manufacturer's retail price as

$$p_d^L = \frac{\theta a + \theta b + (2-\theta)(c_d + c_m)}{2(2-\theta)},$$

   which is the unique solution maximizing π_m, as $\partial^2 \pi_m/\partial p_d^2 = -b(2-\theta)/[\bar{v}\theta(1-\theta)] < 0$.

■

Proof of Proposition 3. We consider the following two cases.

1. If $c_d \ge c_d^L$, then $\theta a/(2-\theta) < [\theta a + \theta b + (2-\theta)(c_d + c_m)]/[2(2-\theta)]$, and we find from Proposition 2 that $p_d^L = \theta a/(2-\theta)$ and $p_r^L = a/(2-\theta)$. Using the retail prices, we compute the demand in the P-channel and that in the D-channel as $D_p^L = n - na/[\bar{v}\,(2-\theta)]$ and $D_d^L = 0$, respectively. The manufacturer's profit is $\pi_m^L = nb - nab/[\bar{v}(2-\theta)]$.
2. If $c_d < c_d^L$, then $p_d^L = [\theta a + \theta b + (2-\theta)(c_d + c_m)]/2(2-\theta)$. Using (7) we find the retail price in the P-channel as $p_r^L = [a(4-\theta) + \theta b + (2-\theta)(c_d + c_m)]/[4(2-\theta)]$. Moreover, we can compute the corresponding demands and the manufacturer's profit as given in this proposition.

   Next, we examine how the manufacturer's operating cost c_d affects the retail prices, demands, and the manufacturer's profits in both channels. We find

$$\frac{\partial p_r^L}{\partial c_d} = \frac{1}{4} > 0 \qquad \text{and} \qquad \frac{\partial p_d^L}{\partial c_d} = \frac{1}{2} > \frac{\partial p_r^L}{\partial c_d};$$

and

$$\frac{\partial D_p^L}{\partial c_d} = \frac{n}{4\bar{v}(1-\theta)} > 0, \ \frac{\partial D_d^L}{\partial c_d} = -\frac{n(2-\theta)}{4\bar{v}\theta(1-\theta)} < 0, \text{ and } \frac{\partial D_{PD}^L}{\partial c_d} = -\frac{n}{2\bar{v}\theta} < 0.$$

Moreover, the manufacturer's profit in the D-channel (i.e., $\pi_m^{L(D)}$) is calculated as

$$\pi_m^{L(D)} = \frac{n}{\bar{v}}(p_d^L - c_m - c_d)\left(\frac{p_r^L - p_d^L}{1-\theta} - \frac{p_d^L}{\theta}\right) = \frac{n}{\bar{v}}\frac{[\theta a - (2-\theta)(c_d + c_m)]^2 - \theta^2 b^2}{8\theta(1-\theta)(2-\theta)},$$

and his profit in the P-channel (i.e., $\pi_m^{L(P)}$) is calculated as

$$\pi_m^{L(P)} = \frac{nb}{\bar{v}}\left(\bar{v} - \frac{p_r - p_d}{1-\theta}\right) = \frac{nb}{4(2-\theta)\bar{v}}\left[4\bar{v}(2-\theta) - a(4-3\theta) + \theta b + (2-\theta)(c_d + c_m)\right].$$

We then compute the first-order derivatives

$$\begin{aligned}
\frac{\partial \pi_m^{L(D)}}{\partial c_d} &= -\frac{n}{4\bar{v}\theta(1-\theta)}\left[\theta a - (2-\theta)(c_d + c_m)\right], \ \frac{\partial \pi_m^{L(P)}}{\partial c_d} = \frac{nb}{4\bar{v}} > 0,\\
\frac{\partial \pi_m^L}{\partial c_d} &= -\frac{n}{4\bar{v}\theta(1-\theta)}[\theta a - \theta b - (2-\theta)(c_m + c_d)].
\end{aligned}$$

Since $c_d < c_d^L = [\theta a - \theta b - c_m(2-\theta)]/(2-\theta)$ and $\theta a - \theta b - (2-\theta)(c_m + c_d) > 0$. Thus $\partial \pi_m^{L(D)}/\partial c_d < 0$ and $\partial \pi_m^L/\partial c_d < 0$.

■

Proof of Proposition 4. We consider the following two cases:

1. If $c_d \geq c_d^S$, then $\theta a/(2-\theta) \leq \theta b + c_m + c_d$, and we have the following discussions.
   (a) If the manufacturer determines the retail price p_d subject to the constraint $p_d \geq \theta a/(2-\theta)$ and the retailer determines the retail price p_r subject to the constraint $p_r < (\theta b + c_m + c_d)/\theta$, we can use (10) and (11) to find the two firms' best-response retail price functions as $p_r(p_d) = p_d/\theta$ and $p_d(p_r) = \theta p_r$. It follows that the retail prices in Nash equilibrium satisfy the equation $p_r = p_d/\theta$, where $p_r \leq (\theta b + c_m + c_d)/\theta$ and $p_d \geq \theta a/(2-\theta)$. Substituting this equation into (1) gives the demand in the D-channel as $D_d = 0$, and the manufacturer's profit can be re-written as $\pi_m = nb\left(\bar{v} - p_d/\theta\right)/\bar{v}$. Differentiating π_m w.r.t. p_d yields $\partial \pi_m/\partial p_d = -nb/(\theta\bar{v}) < 0$, which means that π_m is decreasing in p_d. Thus, the two firms' retail prices in Nash equilibrium are $p_d^S = \theta a/(2-\theta)$ and $p_r^S = a/(2-\theta)$, and the manufacturer's profit is $\pi_m^S = nb[\bar{v} - a/(2-\theta)]/\bar{v}$.
   (b) If the manufacturer determines p_d under the constraint $p_d \geq \theta a/(2-\theta)$ and the retailer determines p_r subject to the constraint $p_r \geq (\theta b + c_m + c_d)/\theta$, then the two firms' best-response retail price functions are $p_r(p_d) = p_d/\theta$ and $p_d(p_r) = (\theta b + \theta p_r + c_m + c_d)/2$. Solving the equations, we obtain the unique retail prices in Nash equilibrium as

$$p_r^S = \frac{\theta b + c_m + c_d}{\theta} \quad \text{and} \quad p_d^S = \theta b + c_m + c_d.$$

   Using the above, we find that the demand in the D-channel is $D_d^S = 0$ and the manufacturer's profit is calculated as $\pi_m^S = nb[\bar{v} - (\theta b + c_m + c_d)/\theta]/\bar{v}$, which is smaller than

$nb[\bar{v}-a/(2-\theta)]/\bar{v}$ for the case that $c_d \geq c_d^S$.

(c) If the manufacturer determines p_d subject to $p_d < \theta a/(2-\theta)$ and the retailer determines p_r subject to $p_r < (\theta b+c_m+c_d)/\theta$, then the two firms' best-response retail price functions are $p_r(p_d) = (a+p_d)/2$ and $p_d(p_r) = \theta p_r$. Solving the equations, we obtain the retail prices in Nash equilibrium as

$$p_r^S = \frac{a}{2-\theta} \quad \text{and} \quad p_d^S = \frac{\theta a}{2-\theta};$$

and we then find that the demand in the D-channel is $D_d^S = 0$, and the manufacturer's maximum profit is $\pi_m^S = nb[\bar{v}-a/(2-\theta)]/\bar{v}$.

(d) If the manufacturer determines p_d subject to $p_d < \theta a/(2-\theta)$ and the retailer determines p_r subject to $p_r \geq (\theta b+c_m+c_d)/\theta$, then the two firms' best-response retail price functions are $p_r(p_d) = (a+p_d)/2$ and $p_d(p_r) = (\theta p_r+\theta b+c_m+c_d)/2$. When $p_r \geq (\theta b+c_m+c_d)/\theta$, if the value of p_d satisfies the best-response function $p_d(p_r) = (\theta p_r+\theta b+c_m+c_d)/2$, then it should also satisfy the inequality $p_d > \theta b+c_m+c_d$. Therefore, when $c_d \geq c_d^S$, i.e. $\theta b+c_m+c_d \geq \theta a/(2-\theta)$, there is no equilibrium solution of p_d under the constraint $p_d < \theta a/(2-\theta)$.

In summary, if $c_d \geq c_d^S$, then the demand in the D-channel is zero and the manufacturer achieves a profit only in the P-channel. The two firms' retail prices in Nash equilibrium are $p_d^S = \theta a/(2-\theta)$and $p_r^S = a/(2-\theta)$, and the manufacturer's maximum profit is $\pi_m^S = nb[\bar{v}-a/(2-\theta)]/\bar{v}$.

2. If $c_d < c_d^S$, then $\theta a/(2-\theta) > \theta b+c_m+c_d$, and we have the following discussions.

(a) If the manufacturer and the retailer determine their retail prices subject to the constraints $p_d \geq \theta a/(2-\theta)$ and $p_r < (\theta b+c_m+c_d)/\theta$, respectively, then the two firms' best-response retail price functions are $p_r(p_d) = p_d/\theta$ and $p_d(p_r) = \theta p_r$. Using the best responses, we can re-write the above two constraints as

$$\frac{\theta a}{2-\theta} \leq p_d < \theta b+c_m+c_d, \tag{19}$$

In this case $\theta a/(2-\theta) > \theta b+c_m+c_d$, the inequality in (19) is not satisfied, which means that there is no equilibrium solution if $p_d \geq \theta a/(2-\theta)$ and $p_r < (\theta b+c_m+c_d)/\theta$.

(b) If the manufacturer determines p_d under the constraint $p_d \geq \theta a/(2-\theta)$ and the retailer determines p_r subject to the constraint $p_r \geq (\theta b+c_m+c_d)/\theta$, then the two firms' best-response retail price functions are $p_r(p_d) = p_d/\theta$ and $p_d(p_r) = (\theta p_r+\theta b+c_m+c_d)/2$. Solving the equations gives the retail prices as

$$p_r^S = \frac{\theta b+c_m+c_d}{\theta} \quad \text{and} \quad p_d^S = \theta b+c_m+c_d.$$

When $c_d < c_d^S$, $p_d^S = \theta b+c_m+c_d < \theta a/(2-\theta)$, thus there is no equilibrium solution of p_d under the constraint $p_d \geq \theta a/(2-\theta)$.

(c) If the manufacturer determines p_d subject to $p_d < \theta a/(2-\theta)$ and the retailer determines p_r subject to $p_r < (\theta b+c_m+c_d)/\theta$, then the two firms' best-response retail price functions are $p_r(p_d) = (a+p_d)/2$ and $p_d(p_r) = \theta p_r$. Solving the equations, we obtain the retail prices as

$$p_r^S = \frac{a}{2-\theta} \quad \text{and} \quad p_d^S = \frac{\theta a}{2-\theta}. \tag{20}$$

When $c_d < c_d^S$, $p_r^S = a/(2-\theta) > (\theta b + c_m + c_d)/\theta$, thus there is no equilibrium solution of p_r under the constraint $p_r < (\theta b + c_m + c_d)/\theta$.

(d) If the manufacturer determines p_d subject to $p_d < \theta a/(2-\theta)$ and the retailer determines p_r subject to $p_r \geq (\theta b + c_m + c_d)/\theta$, then the two firms' best-response retail price functions are $p_r(p_d) = (a + p_d)/2$ and $p_d(p_r) = (\theta b + \theta p_r + c_m + c_d)/2$. Solving the equations yields the unique retail prices in Nash equilibrium as

$$p_r^S = \frac{2a + \theta b + c_d + c_m}{4-\theta} \quad \text{and} \quad p_d^S = \frac{\theta a + 2\left(\theta b + c_d + c_m\right)}{4-\theta}, \tag{21}$$

where p_d^S is a unique solution maximizing π_m^S because $\partial^2 \pi_m^S/\partial p_d^2 = -2n/[\bar{v}\theta(1-\theta)] < 0$, and p_r^S is a unique solution maximizing π_r^S because $\partial^2 \pi_r^S/\partial p_r^2 = -2n/[\bar{v}(1-\theta)] < 0$. Substituting p_r^S and p_d^S in (21) into (1), we can compute the corresponding demands and the manufacturer's profit as given in this proposition.

■

Proof of Corollary 1. Using Proposition 4 we find that when $c_d < c_d^S$,

$$\frac{\partial p_r^S}{\partial c_d} = \frac{1}{4-\theta} > 0 \quad \text{and} \quad \frac{\partial p_d^S}{\partial c_d} = \frac{2}{4-\theta} > \frac{\partial p_r^S}{\partial c_d};$$

and

$$\frac{\partial D_p^S}{\partial c_d} = \frac{n}{\bar{v}(1-\theta)\left(4-\theta\right)} > 0, \; \frac{\partial D_d^S}{\partial c_d} = -\frac{n(2-\theta)}{\bar{v}\theta(1-\theta)\left(4-\theta\right)} < 0, \text{ and } \frac{\partial D_{PD}^S}{\partial c_d} = -\frac{2n}{\bar{v}\theta\left(4-\theta\right)} < 0.$$

Moreover, the manufacturer's profit in the D-channel $\pi_m^{S(D)}$ is calculated as

$$\pi_m^{S(D)} = (p_d^S - c_m - c_d) D_d^S = \frac{n\{[\theta a - (2-\theta)(c_d + c_m)]^2 + \theta^2 b\left[\theta a - (2-\theta)(c_d + c_m)\right] - 2\theta^2 b^2 (2-\theta)^2\}}{\bar{v}\theta(1-\theta)(4-\theta)^2},$$

and that in the P-channel $\pi_m^{S(P)}$ is calculated as

$$\pi_m^{S(P)} = b D_p^S = nb - \frac{nb\left[a(2-\theta) - \theta b - (c_d + c_m)\right]}{\bar{v}(1-\theta)(4-\theta)}.$$

Differentiating $\pi_m^{S(D)}$, $\pi_m^{S(P)}$, and π_m^S (which is the sum of $\pi_m^{S(D)}$ and $\pi_m^{S(P)}$) w.r.t. c_d, we find that

$$\frac{\partial \pi_m^{S(D)}}{\partial c_d} = -\frac{n(2-\theta)}{\bar{v}(1-\theta)(4-\theta)^2}\left\{2\left[\theta a - (c_d + c_m)\right] + \theta^2 b\right\},$$

which is negative because $c_d < c_d^S = \theta a - (2-\theta)(\theta b + c_m)/(2-\theta)$;

$$\frac{\partial \pi_m^{S(P)}}{\partial c_d} = \frac{nb}{\bar{v}(1-\theta)(4-\theta)} > 0;$$

and,

$$\frac{\partial \pi_m^S}{\partial c_d} = -\frac{n\{2(2-\theta)[\theta a - (2-\theta)(\theta b + c_m + c_d)] + \theta b(1-\theta)(4-\theta)\}}{\bar{v}(1-\theta)(4-\theta)^2} < 0,$$

which also follows the fact that $c_d < c_d^S = \theta a - (2-\theta)(\theta b + c_m)/(2-\theta)$. ■

Proof of Theorem 1. Using Propositions 3 and 4, we can draw the results as follows.

1. If $c_d \geq c_d^L$, then we find that $D_d^S = D_d^L = 0$, $D_p^L = D_p^S = n[\bar{v} - a/(2-\theta)]/\bar{v}$, and $\pi_m^L = \pi_m^S =$

$nb[\bar{v} - a/(2-\theta)]/\bar{v}$.

2. If $c_d^S \le c_d < c_d^L$, then we learn from Proposition 4 that $D_d^S = 0$, $D_{PD}^S = D_p^S = n[\bar{v} - a/(2-\theta)]/\bar{v}$, and $\pi_m^S = nb[\bar{v} - a/(2-\theta)]/\bar{v}$.
   To compare the total demands in the sequential-move and simultaneous-move games, we compute

$$\begin{aligned} D_{PD}^S - D_{PD}^L &= \frac{n}{\bar{v}}\left(\bar{v} - \frac{a}{2-\theta}\right) - \frac{n}{\bar{v}}\frac{\theta a + 2\theta b + 2(c_d + c_m)]}{\bar{v}\theta(4-\theta)} \\ &= -\frac{n}{\bar{v}}\frac{\theta a - \theta b - (2-\theta)(c_d + c_m)}{2\theta(2-\theta)}, \end{aligned}$$

   which is negative as $c_d < c_d^L$. Moreover,

$$\begin{aligned} \pi_m^S - \pi_m^L &= \frac{n}{8\bar{v}\theta(1-\theta)(2-\theta)}[2\theta ab(4-3\theta) - \theta^2 a^2 - \theta^2 b^2 + 2\theta(2-\theta)(a-b)(c_m + c_d) \\ &\quad -(2-\theta)^2(c_m + c_d)^2] - \frac{n}{\bar{v}}\frac{ab}{2-\theta} \\ &= -\frac{n}{\bar{v}}\frac{1}{8\theta(1-\theta)(2-\theta)}\left[\theta a - \theta b - (2-\theta)(c_m + c_d)\right]^2, \end{aligned}$$

   which is also negative. Thus, when $c_d^S \le c_d < c_d^L$, we find that $\pi_m^L > \pi_m^S$ and $D_{PD}^L \ge D_{PD}^S = D_p^S$.

3. If $c_d < c_d^S$, then, using Propositions 3 and 4, we have

$$\begin{aligned} \pi_m^L - \pi_m^S &= \frac{n}{\bar{v}\theta(1-\theta)(4-\theta)^2}[2\theta ab(4-3\theta) - \theta^2 a^2 - \theta^3 b - \theta b(4 - 3\theta + \theta^2)(c_d + c_m) \\ &\quad +2\theta a(2-\theta)(c_d + c_m) - (2-\theta)^2(c_d + c_m)^2] - \frac{n}{8\bar{v}\theta(1-\theta)(2-\theta)}[2\theta ab(4-3\theta) \\ &\quad -\theta^2 a^2 - \theta^2 b^2 + 2\theta(2-\theta)(a-b)(c_m + c_d) - (2-\theta)^2(c_m + c_d)^2], \end{aligned}$$

   or,

$$\pi_m^L - \pi_m^S = \frac{n\theta}{8\bar{v}(1-\theta)(2-\theta)(4-\theta)^2}\left[\theta a - (4-3\theta)b - (2-\theta)(c_m + c_d)\right]^2,$$

   which is positive. Thus, $\pi_m^L > \pi_m^S$. In addition, we find that

$$\begin{aligned} D_{PD}^L - D_{PD}^S &= \frac{n[\theta a + 2\theta b + 2(c_d + c_m)]}{\bar{v}\theta(4-\theta)} - \frac{n[\theta a + \theta b + (2-\theta)(c_d + c_m)]}{2\bar{v}\theta(2-\theta)} \\ &= -\frac{n}{2\bar{v}(2-\theta)(4-\theta)}[\theta a - b(4-3\theta) - (2-\theta)(c_d + c_m)], \end{aligned} \tag{22}$$

   which is negative, because if $c_d < \bar{c}_d$ (where $\bar{c}_d$ is defined as in the theorem), then $\theta a - b(4 - 3\theta) - (2-\theta)(c_d + c_m) > 0$.
   Next, to compare the thresholds c_d^S and $\bar{c}_d$, we compute

$$c_d^S - \bar{c}_d = \frac{\theta a - \theta b(2-\theta) - c_m(2-\theta)}{2-\theta} - \frac{\theta a - b(4-3\theta) - c_m(2-\theta)}{2-\theta} = \frac{b(4-\theta)(1-\theta)}{2-\theta},$$

   which is positive, because $\theta < 1$ and $b > 0$. That is, $c_d^S > \bar{c}_d$. Similarly, we can show that $D_{PD}^L \ge D_{PC}^S$ when $\bar{c}_d \le c_d < c_d^S$.
   Summarizing the above, we prove the theorem.

■

Proof of Proposition 5. As a response to the online retail price $\hat{p}_c$, the physical retailer determines her retail price $\hat{p}_r$ to maximize her profit in (15). That is, the retailer's best-response price is $\hat{p}_r(\hat{p}_c) = \arg\max_{\hat{p}_r \geq \hat{p}_c/\theta} \hat{\pi}_r(\hat{p}_c)$. We take the first- and second-order derivatives of $\hat{\pi}_r$ in (15) with respect to $\hat{p}_r$, and find that $\partial^2 \hat{\pi}/(\partial \hat{p}_r)^2 = -2b/[\bar{v}(1-\theta)] < 0$. Solving $\partial \hat{\pi}_r/\partial \hat{p}_r = 0$ yields $\hat{p}_r(\hat{p}_c) = (a + \hat{p}_c)/2$. Thus, the physical retailer's best response is

$$\hat{p}_r(\hat{p}_c) = \max\left\{\frac{\hat{p}_c}{\theta}, \frac{a + \hat{p}_c}{2}\right\}.$$

Next, we calculate the retail prices in Stackelberg equilibrium.

1. If the manufacturer determines his online retail price subject to $\hat{p}_c \geq \theta a/(2-\theta)$, then the retailer's best response is $\hat{p}_r(\hat{p}_c) = \hat{p}_c/\theta$. Substituting $\hat{p}_r(\hat{p}_c)$ into (14), we find that $\hat{\pi}_m = nb(\bar{v} - \hat{p}_c/\theta)/\bar{v}$. Differentiating $\hat{\pi}_m$ w.r.t. $\hat{p}_c$ gives $\partial \hat{\pi}_m/\partial \hat{p}_c = -nb/(\bar{v}\theta) < 0$, which means that $\hat{\pi}_m$ is decreasing in $\hat{p}_c$. Thus, for this case, the retail prices in Stackelberg equilibrium are

$$\hat{p}_c^L = \frac{\theta a}{2-\theta} \text{ and } \hat{p}_r^L = \frac{a}{2-\theta}. \tag{23}$$

2. If $\hat{p}_c < \theta a/(2-\theta)$, then $\hat{p}_r(\hat{p}_c) = (a + \hat{p}_c)/2$. Substituting $\hat{p}_r(\hat{p}_c)$ into (14) and solving the first-order condition $\partial \hat{\pi}_m/\partial \hat{p}_c = 0$, we find $\hat{p}_c^L = \{\theta a + [\theta b + c_m(2-\theta)]/(1-\xi)\}/[2(2-\theta)]$. Thus, for this case, the prices in Stackelberg equilibrium are

$$\begin{aligned} \hat{p}_c^L &= \frac{1}{2(2-\theta)}\left[\theta a + \frac{\theta b + c_m(2-\theta)}{1-\xi}\right], \\ \hat{p}_r^L &= \frac{1}{4(2-\theta)}\left[(4-\theta)a + \frac{\theta b + c_m(2-\theta)}{1-\xi}\right], \end{aligned} \tag{24}$$

which are unique, because $\partial^2 \hat{\pi}_m/(\partial \hat{p}_c)^2 = -b(2-\theta)(1-\xi)/[\bar{v}\theta(1-\theta)] < 0$.

According to the above, we find the equilibrium prices as

$$\begin{aligned} \hat{p}_c^L &= \hat{p}_c' \equiv \min\left\{\frac{\theta a}{2-\theta}, \frac{1}{2(2-\theta)}\left[\theta a + \frac{\theta b + c_m(2-\theta)}{1-\xi}\right]\right\}, \\ \hat{p}_r^L &= (a + \hat{p}_c')/2. \end{aligned} \tag{25}$$

Thus we have following discussions.

1. If $\xi < \xi^L$, i.e., $\theta a > [\theta b + c_m(2-\theta)]/(1-\xi)$, then $\{\theta a + [\theta b + c_m(2-\theta)]/(1-\xi)\}/[2(2-\theta)] < \theta a/(2-\theta)$, the two firms' retail prices in Stackelberg equilibrium are obtained as given in (24), and we can calculate the corresponding demands and the manufacturer's profit as shown in Item 1 of this proposition.
2. If $\xi \geq \xi^L$, then the two firms' retail prices in Stackelberg equilibrium are obtained as given in (23). We then find the demand in the P-channel and that in the C-channel as $\hat{D}_p^L = n[\bar{v} - a/(2-\theta)]/\bar{v}$ and $\hat{D}_c^L = 0$, respectively. The manufacturer's profit can be re-written as $\hat{\pi}_m^L = nb[\bar{v} - a/(2-\theta)]/\bar{v}$.

■

Proof of Proposition 6. We consider the following two cases.

1. If $\xi \geq \xi^S$, where ξ^S is defined as in this proposition, then $(\theta b + c_m)/(1-\xi) \geq \theta a/(2-\theta)$, and we have the following discussions.
   (a) If the manufacturer determines his retail price $\hat{p}_c$ subject to the constraint $\hat{p}_c \geq \theta a/(2-\theta)$ and the retailer determines her retail price $\hat{p}_r$ subject to the constraint $\hat{p}_r < (\theta b + c_m)/(1-$

ξ), then we can find from (16) and (17) that the two firms' best-response functions are $\hat{p}_r(\hat{p}_c) = \hat{p}_c/\theta$ and $\hat{p}_c(\hat{p}_r) = \theta\hat{p}_r$. As a result, the retail prices in Nash equilibrium are those satisfying the equation $\hat{p}_r = \hat{p}_c/\theta$, where $\hat{p}_r < (\theta b + c_m)/(1-\xi)$ and $\hat{p}_c \geq \theta a/(2-\theta)$. Substituting this equation into (2) yields that the demand in the C-channel is $\hat{D}_c^S = 0$, and the manufacturer's profit can be re-written as $\hat{\pi}_m = nb(\bar{v} - \hat{p}_c/\theta)/\bar{v}$. Differentiating $\hat{\pi}_m$ w.r.t. $\hat{p}_c$ gives that $\partial\hat{\pi}_m/\partial\hat{p}_c = -nb/\theta\bar{v} < 0$, which means that $\hat{\pi}_m$ is decreasing in $\hat{p}_c$. Thus, the two firms' retail prices in Nash equilibrium are $\hat{p}_c = \theta a/(2-\theta)$ and $\hat{p}_r^S = a/(2-\theta)$, and the manufacturer's maximum profit is $\hat{\pi}_m^S = nb[\bar{v} - a/(2-\theta)]/\bar{v}$.

(b) If the manufacturer determines $\hat{p}_c$ subject to the constraint $\hat{p}_c \geq \theta a/(2-\theta)$ and the retailer determines $\hat{p}_r$ subject to the constraint $\hat{p}_r \geq (\theta b + c_m)/[\theta(1-\xi)]$, then using (16) and (17) we compute the best-response retail price functions as $\hat{p}_r(\hat{p}_c) = \hat{p}_c/\theta$ and $\hat{p}_c(\hat{p}_r) = [\theta\hat{p}_r(1-\xi) + \theta b + c_m]/[2(1-\xi)]$. Solving the equations, we obtain the unique retail prices in Nash equilibrium as

$$\hat{p}_r^S = \frac{\theta b + c_m}{\theta(1-\xi)} \text{ and } \hat{p}_c^S = \frac{\theta b + c_m}{1-\xi}.$$

Using the above, we find that the demand in the C-channel is $\hat{D}_c^S = 0$ and the manufacturer's profit is calculated as $\hat{\pi}_m = nb\{\bar{v} - (\theta b + c_m)/[\theta(1-\xi)]\}/\bar{v}$, which is smaller than $nb[\bar{v} - a/(2-\theta)]/\bar{v}$ (i.e., the manufacturer's maximum profit when $\xi \geq \xi^S$).

(c) If the manufacturer determines $\hat{p}_c$ subject to $\hat{p}_c < \theta a/(2-\theta)$ and the retailer determines $\hat{p}_r$ subject to $\hat{p}_r < (\theta b + c_m)/[\theta(1-\xi)]$, then the best-response functions are $\hat{p}_r(\hat{p}_c) = (a + \hat{p}_c)/2$ and $\hat{p}_c(\hat{p}_r) = \theta\hat{p}_r$. Solving the two equations, we obtain the retail prices in Nash equilibrium as

$$\hat{p}_r^S = \frac{a}{2-\theta} \quad \text{and} \quad \hat{p}_c^S = \frac{\theta a}{2-\theta};$$

and we find that the demand in the C-channel is $\hat{D}_c^S = 0$, and the manufacturer's profit is $\hat{\pi}_m^S = nb[\bar{v} - a/(2-\theta)]/\bar{v}$.

(d) If the manufacturer determines $\hat{p}_c$ subject to $\hat{p}_c < \theta a/(2-\theta)$ and the retailer determines $\hat{p}_r$ subject to $\hat{p}_r \geq (\theta b + c_m)/[\theta(1-\xi)]$, then the two firms' best-response retail price functions are $\hat{p}_r(\hat{p}_c) = (a + \hat{p}_c)/2$ and $\hat{p}_c(\hat{p}_r) = [\theta\hat{p}_r(1-\xi) + \theta b + c_m]/[2(1-\xi)]$. When $\hat{p}_r \geq (\theta b + c_m)/[\theta(1-\xi)]$, $\hat{p}_c(\hat{p}_r) \geq (\theta b + c_m)/(1-\xi)$. Noting that when $\xi \geq \xi^S$, $(\theta b + c_m)/(1-\xi) \geq \theta a/(2-\theta)$, we conclude that there is no equilibrium solution under the constraint $\hat{p}_c < \theta a/(2-\theta)$.

Summarizing the above, we have the results for the case of $\xi \geq \xi^S$ as shown in this proposition.

2. If $\xi < \xi^S$, then $(\theta b + c_m)/(1-\xi) < \theta a/(2-\theta)$, and we have the following discussions.

(a) If the manufacturer and the retailer determine their retail prices subject to the constraints $\hat{p}_c \geq \theta a/(2-\theta)$ and $\hat{p}_r < (\theta b + c_m)/(1-\xi)$, respectively, then the two firms' best-response retail price functions are $\hat{p}_r(\hat{p}_c) = \hat{p}_c/\theta$ and $\hat{p}_c(\hat{p}_r) = \theta\hat{p}_r$. Using the best responses, we can re-write the above two constraints as

$$\frac{\theta a}{2-\theta} \leq \hat{p}_c < \frac{\theta b + c_m}{1-\xi}, \tag{26}$$

which is contrary to the condition $(\theta b + c_m)/(1-\xi) < \theta a/(2-\theta)$. Thus, there is no equilibrium solution if $\hat{p}_c \geq \theta a/(2-\theta)$ and $\hat{p}_r < (\theta b + c_m)/(1-\xi)$.

(b) If the manufacturer determines $\hat{p}_c$ subject to $\hat{p}_c \geq \theta a/(2-\theta)$ and the retailer determines $\hat{p}_r$ subject to the constraint $\hat{p}_r \geq (\theta b + c_m)/(1-\xi)$, then the best-response retail price functions are $\hat{p}_r(\hat{p}_c) = \hat{p}_c/\theta$ and $\hat{p}_c(\hat{p}_r) = [\theta\hat{p}_r(1-\xi) + \theta b + c_m]/[2(1-\xi)]$. Solving the equations, we obtain the retail prices as

$$\hat{p}_r^S = \frac{\theta b + c_m}{\theta(1-\xi)} \quad \text{and} \quad \hat{p}_c^S = \frac{\theta b + c_m}{1-\xi},$$

which implies that when $\xi < \xi^S$, $\hat{p}_c^S < \theta a/(2-\theta)$; thus, there is no equilibrium solution under the constraint $\hat{p}_c \geq \theta a/(2-\theta)$.

(c) If the manufacturer determines $\hat{p}_c$ subject to $\hat{p}_c < \theta a/(2-\theta)$ and the retailer determines $\hat{p}_r$ subject to $\hat{p}_r < (\theta b + c_m)/\theta(1-\xi)$, then the best-response retail price functions are $\hat{p}_r(\hat{p}_c) = (a + \hat{p}_c)/2$ and $\hat{p}_c(\hat{p}_r) = \theta\hat{p}_r$. Solving the equations gives the retail prices as

$$\hat{p}_r^S = \frac{a}{2-\theta} \quad \text{and} \quad \hat{p}_c^S = \frac{\theta a}{2-\theta},$$

which means that when $\xi < \xi^S$, $\hat{p}_r^S > (\theta b + c_m)/\theta(1-\xi)$; thus, there is no equilibrium solution under the constraint $\hat{p}_r < (\theta b + c_m)/\theta(1-\xi)$.

(d) If the manufacturer determines $\hat{p}_c$ subject to $\hat{p}_c < \theta a/(2-\theta)$ and the retailer determines $\hat{p}_r$ subject to $\hat{p}_r \geq (\theta b + c_m)/\theta(1-\xi)$, then the best-response functions are $\hat{p}_r(\hat{p}_c) = (a+\hat{p}_c)/2$ and $\hat{p}_c(\hat{p}_r) = [\theta\hat{p}_r(1-\xi) + \theta b + c_m]/[2(1-\xi)]$. Solving the two functions yields the unique retail prices in Nash equilibrium as

$$\hat{p}_r^S = \frac{1}{4-\theta}\left(2a + \frac{\theta b + c_m}{1-\xi}\right) \text{ and } \hat{p}_c^S = \frac{1}{4-\theta}\left(\theta a + \frac{2(\theta b + c_m)}{1-\xi}\right).$$

Substituting $\hat{p}_r^S$ and $\hat{p}_c^S$ into (2), we can compute the corresponding demands and the manufacturer's profit as given in this proposition.

■

Proof of Theorem 2. According to Propositions 5 and 6, we obtain the following results.

1. If $\xi \geq \xi^L$, then $\hat{D}_c^S = \hat{D}_c^L = 0$, $\hat{D}_p^L = \hat{D}_p^S = n[\bar{v} - a/(2-\theta)]/\bar{v}$, and $\hat{\pi}_m^L = \hat{\pi}_m^S = nb[\bar{v} - a/(2-\theta)]/\bar{v}$.
2. If $\xi^S \leq \xi < \xi^L$, then $\hat{D}_c^S = 0$, $\hat{D}_{PC}^S = \hat{D}_p^S = n[\bar{v} - a/(2-\theta)]/\bar{v}$ and $\hat{\pi}_m^S = nb[\bar{v} - a/(2-\theta)]/\bar{v}$. To compare the total demands in the two games, we compute

$$\begin{aligned}\hat{D}_{PC}^S - \hat{D}_{PC}^L &= \frac{n}{2\bar{v}\theta(2-\theta)}\left[\theta a + \frac{\theta b + c_m(2-\theta)}{1-\xi}\right] - \frac{n}{\bar{v}}\frac{a}{2-\theta} \\ &= -\frac{n}{\bar{v}}\left[\theta a - \frac{\theta b + c_m(2-\theta)}{1-\xi}\right],\end{aligned}$$

which is negative since $\xi < \xi^L$. Moreover,

$$\begin{aligned}\hat{\pi}_m^S - \hat{\pi}_m^L &= \frac{n}{8\bar{v}\theta(1-\theta)(2-\theta)}\Big[2\theta ab(4-3\theta) - \theta^2 a^2(1-\xi) + 2\theta a c_m(2-\theta) \\ &\quad - \frac{[\theta b + c_m(2-\theta)]^2}{1-\xi}\Big] - \frac{n}{\bar{v}}\frac{a}{2-\theta} \\ &= -\frac{n(1-\xi)}{8\bar{v}\theta(1-\theta)(2-\theta)}\left[\theta a - \frac{\theta b + c_m(2-\theta)}{1-\xi}\right]^2,\end{aligned}$$

which is also negative. In addition, since

$$\xi^S - \bar{\xi} = \frac{(4-3\theta)b + c_m(2-\theta)}{\theta a} - \frac{(2-\theta)(\theta b + c_m)}{\theta a} = \frac{b(4-\theta-\theta^2)}{\theta a} > 0$$

as $\theta < 1$ and $b > 0$, we find that $\xi^S > \bar{\xi}$.

Therefore, when $\bar{\xi} < \xi^S \leq \xi < \xi^L$, we have the result as listed in Item 2 in this theorem.

3. If $\xi < \xi^S$, we learn from Propositions 5 and 6 that

$$\begin{aligned}
\hat{\pi}_m^L - \hat{\pi}_m^S &= \frac{n}{\bar{v}\theta(1-\theta)(4-\theta)^2}\{2\theta ab(4-3\theta) - \theta^2 a^2(1-\xi) + 2\theta a c_m(2-\theta) \\
&\quad -[(c_m 2-\theta)^2 + \theta^2 b]\frac{\theta b + c_m}{1-\xi}\Big\} - \frac{n}{8\bar{v}\theta(1-\theta)(2-\theta)}\{2\theta ab(4-3\theta) \\
&\quad -\theta^2 a^2(1-\xi) + 2\theta a c_m(2-\theta) - \frac{[\theta b + c_m(2-\theta)]^2}{1-\xi}\Big\} \\
&= \frac{n}{8\bar{v}\theta(1-\theta)(2-\theta)(4-\theta)^2}\{\theta^4 a^2(1-\xi) - 2\theta^3 ab(4-3\theta) - 2\theta^3 a c_m(2-\theta) \\
&\quad -8(2-\theta)[c_m(2-\theta)^2 + \theta^2 b]\frac{\theta b + c_m}{1-\xi} + (4-\theta)^2\frac{[\theta b + c_m(2-\theta)]^2}{1-\xi}\Big\} \\
&= \frac{n\theta(1-\xi)}{8\bar{v}(1-\theta)(2-\theta)(4-\theta)^2}\left[\theta a - \frac{b(4-3\theta) + c_m(2-\theta)}{1-\xi}\right]^2,
\end{aligned}$$

which is positive. Thus, $\hat{\pi}_m^L > \hat{\pi}_m^S$. In addition,

$$\begin{aligned}
\hat{D}_{PC}^L - \hat{D}_{PC}^S &= \frac{n}{\bar{v}\theta(4-\theta)}\left[\theta a + 2\frac{\theta b + c_m}{1-\xi}\right] - \frac{n}{2\bar{v}\theta(2-\theta)}\left[\theta a + \frac{\theta b + c_m(2-\theta)}{1-\xi}\right] \\
&= -\frac{n}{2\bar{v}(2-\theta)(4-\theta)}\left[\theta a - \frac{b(4-3\theta) + c_m(2-\theta)}{1-\xi}\right],
\end{aligned}$$

which may or may not be positive. If $\xi < \bar{\xi}$, where $\bar{\xi}$ is defined as in this theorem, then $\hat{D}_{PC}^L < \hat{D}_{PC}^S$; otherwise, $\hat{D}_{PC}^L \geq \hat{D}_{PC}^S$.

■

Proof of Proposition 7. Using Table 1, we can calculate the difference between the total demand in the PD system and that in the PC system as

$$D_{PD}^L - \hat{D}_{PC}^L = \frac{n}{2\bar{v}\theta(2-\theta)}\left\{\frac{\xi}{1-\xi}[\theta b + c_m(2-\theta)] - c_d(2-\theta)\right\},$$

which is positive, or, $D_{PD}^L > \hat{D}_{PC}^L$, if $\xi[\theta b + c_m(2-\theta)]/(1-\xi) - c_d(2-\theta) > 0$, viz., $c_d < \tilde{c}_{d1}^L$. Noting that

$$\begin{aligned}
c_d^L - \tilde{c}_{d1}^L &= \frac{\theta a - \theta b - c_m(2-\theta)}{2-\theta} - \frac{\xi}{1-\xi}\frac{\theta b + c_m(2-\theta)}{2-\theta} \\
&= \frac{1}{2-\theta}\left[\theta a - \frac{\theta b + c_m(2-\theta)}{1-\xi}\right],
\end{aligned}$$

which is positive when $\xi < \xi^L$, and thus, $c_d^L > \tilde{c}_{d1}^L$. Therefore, if $\tilde{c}_{d1}^L \leq c_d < c_d^L$, then $D_{PD}^L \leq \hat{D}_{PC}^L$.

Next, we compare the manufacturer's profits in the two systems as

$$\begin{aligned}
\pi_m^L - \hat{\pi}_m^L &= \frac{n}{8\bar{v}\theta(1-\theta)(2-\theta)} \left\{ -\theta^2 a^2 (1-\xi) + 2\theta a c_m (2-\theta) - \frac{[\theta b + c_m(2-\theta)]^2}{1-\xi} \right. \\
&\quad \left. + \theta^2 a^2 + \theta^2 b^2 - 2\theta(2-\theta)(a-b)(c_m+c_d) + (2-\theta)^2 (c_m+c_d)^2 \right\} \\
&= \frac{n}{8\bar{v}\theta(1-\theta)(2-\theta)} \left\{ [\theta a - \theta b - (2-\theta)(c_m+c_d)]^2 - (1-\xi)\left[\theta a - \frac{\theta b + c_m(2-\theta)]}{1-\xi}\right]^2 \right\} \\
&= \frac{n}{8\bar{v}\theta(1-\theta)(2-\theta)} \left\{ [\theta a - \theta b - (2-\theta)(c_m+c_d)] + \sqrt{1-\xi}\left[\theta a - \frac{\theta b + c_m(2-\theta)}{1-\xi}\right] \right\} \\
&\quad \times \left\{ \theta a \left(1-\sqrt{1-\xi}\right) - c_d(2-\theta) + \frac{\left(1-\sqrt{1-\xi}\right)[\theta b + c_m(2-\theta)]}{\sqrt{1-\xi}} \right\}.
\end{aligned}$$

When $\xi < \xi^L$ and $c_d < c_d^L$, $[\theta a - \theta b - (2-\theta)(c_m+c_d)] + \sqrt{1-\xi}\{\theta a - [\theta b + c_m(2-\theta)]/(1-\xi)\}$ is positive. Thus, if $\theta a(1-\sqrt{1-\xi}) - c_d(2-\theta) + (1-\sqrt{1-\xi})\,[\theta b + c_m(2-\theta)]\,/\sqrt{1-\xi} > 0$, or, $c_d < \tilde{c}_{d2}^L$ where $\tilde{c}_{d2}^L$ is defined as in this proposition, then $\pi_m^L > \hat{\pi}_m^L$.

In addition,

$$\begin{aligned}
c_d^L - \tilde{c}_{d2}^L &= \frac{1}{2-\theta}[\theta a - \theta b - c_m(2-\theta)] - \frac{1-\sqrt{1-\xi}}{2-\theta}\left[\theta a + \frac{\theta b + c_m(2-\theta)}{\sqrt{1-\xi}}\right] \\
&= \frac{\sqrt{1-\xi}}{2-\theta}\left[\theta a - \frac{\theta b + c_m(2-\theta)}{1-\xi}\right],
\end{aligned}$$

and

$$\begin{aligned}
\tilde{c}_{d1}^L - \tilde{c}_{d2}^L &= \frac{\xi}{(2-\theta)(1-\xi)}[\theta b + c_m(2-\theta)] - \frac{1-\sqrt{1-\xi}}{2-\theta}\left[\theta a + \frac{\theta b + c_m(2-\theta)}{\sqrt{1-\xi}}\right] \\
&= -\frac{1-\sqrt{1-\xi}}{2-\theta}\left[\theta a - \frac{\theta b + c_m(2-\theta)}{1-\xi}\right].
\end{aligned}$$

Because $\xi < \xi^L$, $\theta a > [\theta b + c_m(2-\theta)]/(1-\xi)$ and $\tilde{c}_{d1}^L < \tilde{c}_{d2}^L < c_d^L$. ∎

Proof of Proposition 8. Using Table 1, we compute

$$D_{PD}^S - \hat{D}_{PC}^S = \frac{2n}{\bar{v}\theta(4-\theta)}\left[\frac{\xi}{1-\xi}(\theta b + c_m) - c_d\right],$$

which is positive, or, $D_{PD}^S > \hat{D}_{PC}^S$, if $c_d < \tilde{c}_{d1}^S$. We also calculate

$$\begin{aligned}
c_d^S - \tilde{c}_{d1}^S &= \frac{1}{2-\theta}[\theta a - (2-\theta)(\theta b + c_m)] - \frac{\xi}{1-\xi}(\theta b + c_m) \\
&= \frac{1}{2-\theta}\left[\theta a - \frac{(2-\theta)(\theta b - c_m)}{1-\xi}\right].
\end{aligned}$$

According to Proposition 6, we find that, in the simultaneous-move game, a dual channel exists when $\xi < \xi^S$. It thus follows that $c_d^S - \tilde{c}_{d1}^S > 0$. Hence, if $\tilde{c}_{d1}^S < c_d \le c_d^S$, then $D_{PD}^S < \hat{D}_{PC}^S$.

Next, we compare the manufacturer's profits in the two systems as follows.

$$\begin{aligned}
\pi_m^S - \hat{\pi}_m^S &= \frac{n}{\bar{v}\theta(1-\theta)(4-\theta)^2}\Bigg\{-\theta^2 a^2(1-\xi) + 2\theta a c_m(2-\theta) - \frac{\left[c_m(2-\theta)^2+\theta^2 b\right](\theta b + c_m)}{1-\xi} \\
&\quad +\theta^2 a^2 + \theta^3 b^2 + \theta b(4-3\theta+\theta^2)(c_d+c_m) - 2\theta a(2-\theta)(c_d+c_m) + (2-\theta)^2(c_d+c_m)^2\Bigg\} \\
&= \frac{n}{\bar{v}\theta(1-\theta)(4-\theta)^2}\Bigg\{\xi\theta^2 a^2 - \frac{\left[c_m(2-\theta)^2+\theta^2 b\right](\theta b + c_m)}{1-\xi} \\
&\quad + \left[(2-\theta)^2(c_d+c_m)+\theta^2 b\right](\theta b + c_d + c_m) - 2\theta a c_d(2-\theta)\Bigg\}.
\end{aligned}$$

We note that

$$\frac{\partial \pi_m^S}{\partial c_d} = -\frac{n}{\bar{v}\theta(1-\theta)(4-\theta)^2}\left\{2(2-\theta)\left[\theta a - (2-\theta)(\theta b + c_d + c_m)\right] + \theta b\left(4-3\theta+\theta^2\right)\right\},$$

which is positive if $c_d < c_d^S$, i.e., $\theta a > (2-\theta)(\theta b + c_d + c_m)$. Thus, π_m^S is decreasing in c_d. In addition, when $c_d = 0$,

$$\begin{aligned}
(\pi_m^S - \hat{\pi}_m^S)|_{c_d=0} &= \frac{n}{\bar{v}\theta(1-\theta)(4-\theta)^2}\left\{\xi\theta^2 a^2 - \left[c_m(2-\theta)^2+\theta^2 b\right]\frac{\xi(\theta b + c_m)}{1-\xi}\right\} \\
&= \frac{n\xi}{\bar{v}\theta(1-\theta)(4-\theta)^2}\left[\theta^2 a^2 - \frac{\left[c_m(2-\theta)^2+\theta^2 b\right](\theta b + c_m)}{1-\xi}\right] \\
&> 0;
\end{aligned}$$

when $c_d = c_d^S$,

$$\begin{aligned}
(\pi_m^S - \hat{\pi}_m^S)|_{c_d=c_d^S} &= -(1-\xi)\left\{\theta a - \frac{1}{1-\xi}\left[c_m(2-\theta) + \frac{\theta^2 b}{2-\theta}\right]\right\}\left[\theta a - \frac{(2-\theta)(\theta b + c_m)}{1-\xi}\right] \\
&< 0;
\end{aligned}$$

and when $c_d = \tilde{c}_{d1}^S$,

$$\begin{aligned}
(\pi_m^S - \hat{\pi}_m^S)|_{c_d=\tilde{c}_{d1}^S} &= -\theta^2 a^2(1-\xi) - \left[c_m(2-\theta)^2+\theta^2 b\right]\frac{\theta b + c_m}{1-\xi} + \theta^2 a^2 + \theta^3 b^2 \\
&\quad +\theta b(4-3\theta+\theta^2)(c_d+c_m) - 2\theta a c_d(2-\theta) + (2-\theta)^2(c_d+c_m)^2 \\
&= \xi\left[\theta a - \frac{(2-\theta)(\theta b + c_m)}{(1-\xi)}\right]^2 > 0.
\end{aligned}$$

Therefore, there must exist a unique value of c_d in the interval $\left(\tilde{c}_{d1}^S, c_d^S\right)$, denoted by $\tilde{c}_{d2}^S$, which satisfies $\pi_m^S - \hat{\pi}_m^S = 0$. Accordingly, if $c_d < \tilde{c}_{d2}^S$, then $\pi_m^S - \hat{\pi}_m^S > 0$, i.e., $\pi_m^S > \hat{\pi}_m^S$; otherwise, if $\tilde{c}_{d2}^S < c_d < c_d^L$, then $\pi_m^S < \hat{\pi}_m^S$. ■