IIE Transactions (2005) **37**, 1119–1128
Copyright © "IIE"
ISSN: 0740-817X print / 1545-8830 online
DOI: 10.1080/07408170500288166

Free shipping and purchasing decisions in B2B transactions: A game-theoretic analysis

MINGMING LENG and MAHMUT PARLAR*

Department of Computing and Decision Science, Lingnan University, Tuen Mun, Hong Kong
E-mail: lengm@mcmaster.ca or parlar@mcmaster.ca

Received December 2003 and accepted January 2005

Free shipping offers by eBusiness companies have become an effective means of attracting and keeping customers. Many business-to-consumer and business-to-business (B2B) companies now offer free shipping to buyers who spend more than a specific amount. In this paper we consider a B2B environment and assume that the buyer may be enticed to increase her purchase amount in order to qualify for free shipping. The seller's and the buyer's decisions (i.e., free shipping cutoff level and purchase amounts, respectively) affect each other's objective functions. Thus, we model the problem as a leader-follower game under complete information where the leader is the seller and the follower is the buyer. We assume that if the cutoff level announced by the seller is lower than the buyer's purchase amount, the seller absorbs the shipping cost. Otherwise, the buyer compares the values of two functions to determine whether she should increase her purchase amount to qualify for free shipping. We first determine the best response function for the buyer for any given value of the seller's cutoff level and present some structural results related to the response function. We then compute the Stackelberg solution for the leader-follower game and discuss the managerial implications of our findings. The results obtained are demonstrated with the help of two examples. We also present a complete sensitivity analysis for the Stackelberg solution and the objective function values for variations in the unit shipping cost.

1. Introduction

Although free shipping started out as a temporary marketing ploy to attract online shoppers to Internet sites during the 1999 holiday season, it has now become an integral part of doing business for many business-to-business (B2B) and business-to-consumer (B2C) companies. In recent surveys of Internet shoppers, the Boston Consulting Group found that along with guaranteed transaction security and price discounts, free shipping was one of the best means of enticing buyers to return to Internet sites (Bayles, 2001). In addition to the free shipping offers advertised by almost every B2C company, a large number of B2B companies also offer free shipping. For example, Natural Sense,[1] a Canadian aromatherapy products company, offers free shipping for online B2B orders over C$300. The printer and fax supplies company B2Bdirect.com[2] provides free shipping for orders over $200 and the trade show display company ShowstopperExhibits.com[3] is offering free ground shipping within the continental United States for any order amount. There is now even a new web site, www.freeshipping.com, that lists over 1000 online stores providing free shipping.

The recent prevalence of free shipping offers by the B2B companies may be partially attributed to the commercial availability of the Internet. Even though the traditional Electronic Data Interchange systems were expected to provide seamless interaction between sellers and buyers, this has not materialized and most companies have turned to the Internet to conduct their businesses online (Johnston and Mak, 2000). We expect the availability of free shipping offers to increase in the coming years in parallel with the continued growth of the Internet.

As different B2B sellers have begun offering free shipping for different purchase levels, a natural question to ask is the following: "what is the best cutoff level for purchase amounts at or above which the buyer receives free shipping?" Choosing a high cutoff level may result in some lost business for the seller since the buyers would have to spend more money than they initially intended in order to qualify for free shipping. On the other hand, setting a low cutoff point may entice a buyer to increase her purchase quantity and may generate higher gross revenues but this may also be costly for the seller since he has to absorb the shipping costs. Thus, the best cutoff level chosen by the seller must provide a tradeoff between the cost of lost business (for high cutoff levels) and cost of shipping (for low cutoff levels).

*Corresponding author
[1]http://www.naturalsense.com/naturalsenseb2b.htm
[2]http://www.b2bdirect-shop.com/Store/PolTerms.htm
[3]http://showstopperexhibits.com/

To our knowledge, there has been no previous attempt to model the interaction between the seller and the buyer in the free shipping problem. Thus, in order to model this problem in the B2B context we make some assumptions about the purchase behavior of the buyer. We assume that, in the absence of a free shipping offer, the buyer determines her optimal purchase quantity as a dollar amount y, by solving the problem of maximizing a "net revenue" function. But if the seller offers free shipping and announces a cutoff level x, the buyer determines her purchase amount in a different manner: (i) if the cutoff level x announced by the seller is lower than the buyer's purchase amount y, the buyer still purchases y and the seller absorbs the shipping cost; (ii) otherwise, the buyer compares the values of two functions to decide whether she should increase her purchase amount y to x in order to qualify for free shipping: more specifically, if the buyer's net revenue with y is larger than that obtained with x, then the buyer purchases y and pays for the shipping cost. Otherwise, the buyer increases her purchase quantity to x, and the seller absorbs the shipping cost. Furthermore, we assume that a third party (i.e., an external logistics company) is employed to ship the goods from the seller to the buyer. Hence, the shipping cost is assumed to be the same regardless of who pays for it.

Since the decisions made by the buyer and the seller affect their respective objectives, the free shipping problem can be modeled using a game-theoretic framework. In this paper we restrict our attention to a static game under complete information where each player's objective function is common knowledge between the players; see, Gibbons (1992, Chs. 1 and 2). In such a case each player consciously attempts to optimize his/her own objective recognizing that each objective function depends on both decision-makers' (players') decisions. In the B2B context, the seller would normally announce his decision first and the buyer would react to the announcement by choosing a purchase amount. Thus, it is reasonable to assume that in the game-theoretic analysis of the free shipping decision problem the seller is the leader and the buyer is the follower. For this scenario, we determine the Stackelberg strategy (Başar and Olsder, 1982) for each player in a static game of complete information which is played only once, i.e., the buyer makes a one-time purchase only.

We should note that the assumptions we have made regarding the B2B buyer's purchase behavior may not be directly applicable in the B2C context. For example, unlike the B2B buyers, individual buyers usually don't go to a B2C web site planning to spend a certain amount of money. Also, whereas a B2B buyer such as a university bookstore may not be averse to purchasing, say, a few more boxes of writing pads to qualify for free shipping, the B2C buyers normally do not purchase multiple copies of the same item. Finally, in the B2C context a large number of buyers make different purchase decisions independently of one another. Thus, the B2C seller can simply consider the collective decisions of the B2C buyers as a probability distribution rather than a strategic decision made by an individual customer. Thus, it would be incorrect to use our model to determine the free shipping policy for the B2C seller.

In Section 2 we introduce the objective functions of the seller and the buyer. In Section 3 we determine the best response function of the buyer: if the seller announces his cutoff level decision, then the buyer can determine her best response to this announcement by solving an optimization problem that maximizes her objective. In Section 3 we also provide some structural results for the buyer's best response. In particular, we show that if the seller announces his free shipping cutoff level, the buyer's best response is determined by two threshold values implying that for low, medium and high levels of announced cutoff level, the buyer behaves differently. In Section 4, we use the properties of the buyer's best response function and compute the Stackelberg strategy for each player. Section 5 describes the results of a sensitivity analysis where we examine the variations in the Stackelberg solution and the corresponding objective functions for changes in the unit shipping cost. The paper ends with a summary and discussion of future research avenues.

2. Objective functions of the players

In this section we describe the objective functions of the seller and the buyer. Since each player's objective function is influenced by the other player's decision, we develop procedures in Section 3 to compute the buyer's best response to an arbitrary decision of the seller. Information obtained on the best responses is later used in Section 4 to identify the Stackelberg equilibrium in a leader-follower scenario where the seller announces a Free Shipping (FS) cutoff level which is followed by the buyer's purchase amount decision.

We start by defining x as the seller's FS cutoff level (in \$) and y as the buyer's purchase amount (in \$). The total shipping cost of goods worth y dollars is given by a continuous function $C(y)$ for which we assume $C(0) = 0$, $0 < C'(y) < 1$, $C''(y) \le 0$ and $C(y) < y$. This implies that the shipping cost is increasing and concave in y but due to economies of scale the marginal cost is less than unity. Furthermore, we assume that the production cost $K(y)$ incurred by the seller has the property $K(0) = 0$, $0 < K'(y) < 1$, $K''(y) \le 0$ and $K(y) < y$, i.e., it is also an increasing and concave function in y. Since the seller's marginal revenue (i.e., one) should be more than his marginal cost we assume $1 > C'(y) + K'(y)$. Finally, since the seller's gross revenue should be larger than his total cost, we also assume that $y > C(y) + K(y)$.

When the buyer purchases goods worth y dollars, she sells them in a retail market and receives a gross revenue of $R(y)$ (in \$). There may be variable costs such as the inventory carrying costs, but we assume that they are deducted from the revenue. We assume $R(0) = 0$, $R'(y)|_{y=0} > 1$ (to eliminate trivial solutions) and $R''(y) < 0$, i.e., the gross revenue function is also increasing and strictly concave in its argument. We also assume that since the market demand is finite, there exists a $\bar{y}$ for which the gross revenue equals total purchase cost, i.e., $R(\bar{y}) = \bar{y}$. Note that a similar assumption about the form of the revenue function has been made by others; see, e.g., Erlenkotter and Trippi (1976) who developed a model that integrates capital investment decisions with output and pricing decisions. In most practical situations the seller's production cost $K(y)$ should be less than the buyer's gross revenue $R(y)$, thus the condition $K(y) < R(y)$ is assumed.

When two players arbitrarily select a FS cutoff level x and a purchase amount y, one of two things can happen:

1. If $x \le y$, then FS takes place and the seller absorbs the shipping cost and incurs the production cost. In this case the seller receives y dollars and spends $K(y)$ dollars for production and $C(y)$ dollars to ship the amount purchased, and the buyer pays y dollars and obtains a gross revenue of $R(y)$. Thus, the seller's net revenue is $y - C(y) - K(y)$ and the buyer's net revenue with FS is $R(y) - y$.
2. If $y \le x$, then the buyer has the option of increasing her purchase amount to the higher cutoff level in order to benefit from FS. If the buyer increases her purchase to the cutoff level x, then the buyer's and seller's net revenues are $R(x) - x$ and $x - C(x) - K(x)$, respectively. Otherwise, the buyer purchases y and the buyer's and seller's net revenues are $R(y) - y - C(y)$ and $y - K(y)$, respectively. Thus, when $y \le x$, the buyer determines her actual purchase amount by comparing $R(x) - x$ with $R(y) - y - C(y)$ as follows:
   (a) If
   $$R(x) - x \le R(y) - y - C(y),$$
   then the buyer stays with her original decision to purchase y dollars worth of goods and obtains a net revenue (without FS) of $R(y) - y - C(y)$. In this case the seller receives the net revenue of $y - K(y)$.
   (b) If
   $$R(x) - x \ge R(y) - y - C(y),$$
   then the buyer increases her purchase quantity to x and obtains a net revenue (with FS) of $R(x) - x$. In this case the seller receives a net revenue of $x - C(x) - K(x)$.

Remark 1. Let us define:

$$V(y) \equiv R(y) - y, \tag{1}$$

as the net revenue to the buyer when she purchases y dollars worth of goods and receives FS. Differentiating $V(y)$ we find $V'(y) = R'(y) - 1$. Since $V''(y) = R''(y) < 0$ and $R(\bar{y}) = \bar{y}$ for some $\bar{y} > 0$, the net revenue function is strictly concave with $V(0) = V(\bar{y}) = 0$. This implies that $V(y)$ has a unique maximizing value v satisfying $R'(v) = 1$. Thus, the net revenue function $V(y)$ starts at zero, increases until v, and then decreases and reaches zero at $\bar{y}$. Hence, $V(y)$ should be increasing at the point $y = 0$. The assumption of $R'(y)|_{y=0} > 1$ made above implies that $V'(y)|_{y=0} > 0$ which eliminates the trivial solution $y^* = 0$. Since the buyer would not be willing to purchase goods that would result in a negative revenue for $y > \bar{y}$, it follows that the feasible set of values for y is the interval $[0, \bar{y}]$. In light of this observation, the seller also limits his FS cutoff level to the interval $[0, \bar{y}]$. △

We now define $J_1(x, y)$ and $J_2(x, y)$ as the net revenue functions of the seller and buyer, respectively. Using the above arguments we have:

$$J_1(x, y) = \begin{cases} y - C(y) - K(y), & \text{if } x \le y \le \bar{y}, \\ x - C(x) - K(x), & \text{if } y \le x \le \bar{y} \text{ and } V(y) - C(y) \le V(x), \\ y - K(y), & \text{if } y \le x \le \bar{y} \text{ and } V(x) \le V(y) - C(y), \end{cases} \tag{2}$$

and

$$J_2(x, y) = \max\{V(y) - \mathbf{1}_{\{y<x\}}C(y),\ V(x)\}, \\ = \begin{cases} V(y), & \text{if } x \le y \le \bar{y}, \\ V(x), & \text{if } y \le x \le \bar{y} \text{ and } V(y) - C(y) \le V(x), \\ V(y) - C(y), & \text{if } y \le x \le \bar{y} \text{ and } V(x) \le V(y) - C(y). \end{cases} \tag{3}$$

where, as defined in Equation (1), the net revenue to the buyer is $V(y) = R(y) - y$.

Remark 2. As we assumed, the seller and the buyer in our model have a B2B relationship; for example, the seller may be supplying office products (such as writing pads) to a bookstore at wholesale prices. However, many office products companies (such as Office Depot and Staples) also have retail outlets where they sell products to the public at higher prices. Thus, the seller in our model is assumed to be such an entity which would be aware of the buyer's revenue function $R(y)$ since it would be similar to the seller's revenue function at its retail outlet.

What happens if there is more than one buyer in the market? In a competitive environment the revenue functions of all buyers should be similar to one another. Thus, we assume that all buyers have identical revenue functions and the seller can set his FS cutoff level assuming that $R(y)$ is common to all buyers. △

In this game-theoretic problem, the seller wants to maximize $J_1(x, y)$ and the buyer wants to maximize $J_2(x, y)$. The Stackelberg strategy for the seller (the leader) and the buyer (the follower) is found as follows: the buyer solves the optimization problem "max $J_2(x, y)$" for any value of x that may be chosen by the seller and determines her (buyer's) best response function $y(x)$ that maximizes her objective. The seller then solves the optimization problem "max $J_1(x, y(x))$" to determine the best FS cutoff level that will maximize his objective. The seller's objective function J_1 is fairly sensitive to his choice of the FS level x since J_1 is not only a function of x but also a function of the buyer's best response $y(x)$. In our presentation below, we will implement this general procedure to compute the Stackelberg strategy for the seller and the buyer in the FS game.

As we noted above, when the shipping cost is a continuous function, the net revenue functions $J_1(x, y)$ and $J_2(x, y)$ of the players have to be expressed in terms of piecewise function consisting of three terms. If the shipping cost becomes a step-function then the net revenue functions would have to be expressed as piecewise functions consisting of a large number of terms depending on the definition of the specific step-function. Thus, it would be quite difficult to obtain any insights into the model and its solution. However, in that case, one can still solve the problem numerically and determine the Stackelberg strategy for both players using the procedure described in the previous paragraph.

3. Buyer's best response function

With the players' objective functions given by Equations (2) and (3), we now examine the optimal decision of the buyer in response to an arbitrary decision of the seller. In other words, if the seller announces his FS cutoff level decision as $x = \hat{x}$, we find the best response y_R that the buyer should choose to maximize her objective function. This result will be useful when we consider the leader-follower Stackelberg strategy in Section 4.

We now assume that the seller has announced his FS cutoff level $\hat{x}$ and in light of this announcement the buyer has to find her best response decision $y = y_R$ that maximizes the objective function $J_2(\hat{x}, y)$ given in Equation (3). First, note that if the buyer decides to choose a $y \in [\hat{x}, \bar{y}]$, then her objective function assumes the form:

$$J_{21}(\hat{x}, y) \equiv V(y), \quad \text{for } y \in [\hat{x}, \bar{y}].$$

On the other hand, if the buyer chooses a $y \in [0, \hat{x}]$, then her objective is:

$$J_{22}(\hat{x}, y) \equiv \begin{cases} V(\hat{x}), & \text{if } y \le \hat{x} \le \bar{y} \text{ and } V(y) - C(y) \le V(\hat{x}), \\ V(y) - C(y), & \text{if } y \le \hat{x} \le \bar{y} \text{ and } V(\hat{x}) \le V(y) - C(y). \end{cases}$$

Hence, the optimal solution y to maximize $J_{22}(\hat{x}, y)$ is given by:

$$\arg\max_{y\in[0,\hat{x}]} J_{22}(\hat{x}, y) = \begin{cases} \hat{x}, & \text{if } \max_{y\in[0,\hat{x}]} V(y) - C(y) \le V(\hat{x}), \\ \arg\max_{y\in[0,\hat{x}]} V(y) - C(y), & \text{if } V(\hat{x}) \le \max_{y\in[0,\hat{x}]} V(y) - C(y). \end{cases} \quad (4)$$

To identify the buyer's best response, we first recall from remark 1 that the function $V(y) = R(y) - y$ is concave and maximized at a point $v \in (0, \bar{y})$ that satisfies $R'(v) = 1$. Given the value of v, we consider two possible cases (Propositions 1 and 2 below). These correspond to the relative values of v and $\hat{x}$, and identify the best response for the buyer in each case.

We first consider a lemma that determines the value maximizing $V(y)$ for $\hat{x} \le y \le \bar{y}$.

Lemma 1. *For $\hat{x} \le y \le \bar{y}$, the value $y^{\#}$ that maximizes the buyer's net revenue function $V(y)$ (or, $J_{21}(\hat{x}, y)$) is given as*:

$$y^{\#} = \begin{cases} v, & \text{if } \hat{x} \le v, \\ \hat{x}, & \text{if } v \le \hat{x}, \end{cases}$$

with the corresponding maximum values:

$$V(y^{\#}) = \begin{cases} V(v), & \text{if } \hat{x} \le v, \\ V(\hat{x}), & \text{if } v \le \hat{x}. \end{cases}$$

Proof. The result follows by noting that $V(y)$ is a concave function which increases over $[0, v]$ and decreases over $[v, \bar{y}]$ with $V(0) = V(\bar{y}) = 0$. When $\hat{x} \le v$ and for $\hat{x} \le y \le \bar{y}$ (as in Fig. 1(a)), the $V(y)$ function first increases until v and then decreases, so it is optimal to choose $y^{\#} = v$ which

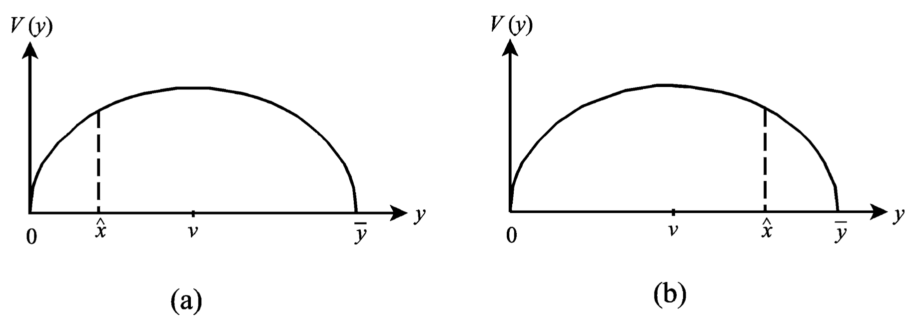

Fig. 1. (a) When $\hat{x} \le v$, the buyer's net revenue function $V(y)$ is maximized at $y^{\#} = v$; (b) when $\hat{x} \ge v$ the $V(y)$ function is maximized at $y^{\#} = \hat{x}$.

maximizes $V(y)$. Similarly, when $v \leq \hat{x}$ and for $\hat{x} \leq y \leq \bar{y}$ (as in Fig. 1(b)), the $V(y)$ function is decreasing, thus it is maximized at $y^{\#} = \hat{x}$. ■

The next proposition determines the optimal purchase quantity for the buyer when $\hat{x} \leq v$.

Proposition 1. *If $\hat{x} \leq v$, [i.e., if the seller decides on a FS cutoff level $\hat{x}$ that is less than or equal to the value v that maximizes $V(y)$], then the buyer's best response is to purchase goods worth v dollars, i.e., to choose $y_R = v$ and benefit from FS.*

Proof. First, referring to Fig. 1 (a and b) used in Lemma 1, note that for any $y \in [0, \hat{x}]$ we have $V(y) - C(y) < V(\hat{x}) - C(y) < V(\hat{x})$. Thus,

$$J_{22}(\hat{x}, y) = V(\hat{x}) < V(v) = J_{21}(\hat{x}, v).$$

Since ordering more than $\hat{x}$ and receiving FS results in a higher objective function value for the buyer than ordering less than $\hat{x}$, it follows that when $\hat{x} \leq v$ the buyer's best response is $y_R = v$. ■

The next proposition identifies the buyer's best response y_R when $v \leq \hat{x}$.

Proposition 2. *If $v \leq \hat{x}$, [i.e., the seller decides on a FS cutoff level $\hat{x}$ that exceeds the value v maximizing $V(y)$], then the buyer's best response is obtained as follows:*

1. *If the $C(y) + V(\hat{x})$ and $V(y)$ curves do not intersect (or, intersect only at one point), then the buyer's best response is to choose $y_R = \hat{x}$.*
2. *If the $C(y) + V(\hat{x})$ and $V(y)$ curves intersect at two points, say y_1 and y_2 with $y_1 < y_2$, then the buyer's best response y_R is found by maximizing $[V(y) - C(y)]$ with respect to y over the region $[y_1, y_2]$. In that case the value that maximizes the buyer's objective is given as:*

$$y^{\circ} = \arg \max_{y \in [y_1, y_2]} V(y) - C(y),$$

which is less than $\hat{x}$.

Proof. To show part 1 we refer to Fig. 2 and observe that if the two curves $C(y) + V(\hat{x})$ and $V(y)$ do not intersect (or, intersect at only one point), then for any $y \in [0, \bar{y}]$ we have

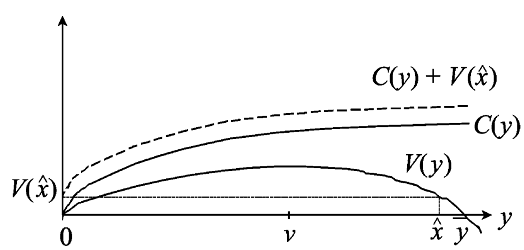

Fig. 2. The $C(y) + V(\hat{x})$ curve does not intersect $V(y)$.

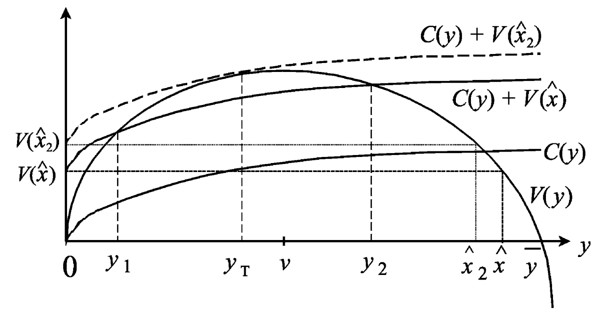

Fig. 3. The $C(y) + V(\hat{x})$ curve intersects $V(y)$ at two points y_1 and y_2.

$V(y) < C(y) + V(\hat{x})$, or, $V(y) - C(y) < V(\hat{x})$. This gives:

$$\max_{y \in [0, \hat{x}]} V(y) - C(y) \leq V(\hat{x}).$$

From Equation (4), we have that:

$$\arg \max_{y \in [0, \hat{x}]} J_{22}(\hat{x}, y) = \hat{x}, \text{ and } \max_{y \in [0, \hat{x}]} J_{22}(\hat{x}, y) = V(\hat{x}).$$

Furthermore, Lemma 1 shows that:

$$\arg \max_{y \in [\hat{x}, \bar{y}]} J_{21}(\hat{x}, y) = \hat{x}, \text{ and } \max_{y \in [\hat{x}, \bar{y}]} J_{21}(\hat{x}, y) = V(\hat{x}).$$

Since using $y = \hat{x}$ results in a better objective function value than any other y, it follows that the best response for the buyer is to order goods worth $\hat{x}$ and receive the FS.

To show part 2, we refer to Fig. 3 and consider the case when the two curves intersect at two points y_1 and y_2 with $y_1 < y_2$. Then for any $y \in [y_1, y_2]$, we have $V(y) > C(y) + V(\hat{x})$, that is, $V(y) - C(y) > V(\hat{x})$. Thus, from Equation (4) we have:

$$V(\hat{x}) < \max_{y \in [0, \hat{x}]} V(y) - C(y) = \max_{y \in [y_1, y_2]} V(y) - C(y).$$

Since

$$\max_{y \in [0, \hat{x}]} J_{22}(\hat{x}, y) = \max_{y \in [y_1, y_2]} V(y) - C(y),$$
$$\max_{y \in [\hat{x}, \bar{y}]} J_{21}(\hat{x}, y) = V(\hat{x}),$$

the optimal $y = y^{\circ}$ should be computed by solving the maximization problem $\max_{y \in [y_1, y_2]} V(y) - C(y)$. Obviously, we have $y^{\circ} < \hat{x}$. ■

To summarize, the buyer's best response y_R to the seller's decision $\hat{x}$ is:

$$y_R = \begin{cases} v, & \text{if } \hat{x} \leq v, \\ \hat{x}, & \text{if } v \leq \hat{x} \text{ and part 1 of Proposition 2 holds,} \\ y^{\circ}, & \text{if } v \leq \hat{x} \text{ and part 2 of Proposition 2 holds.} \end{cases} \quad (5)$$

The managerial implications of this result are as follows: when the cutoff point $\hat{x}$ is less than v (which maximizes the buyer's net revenue function $V(y)$) the buyer should purchase v units, maximize her net revenue function and take advantage of FS. But when the cutoff point $\hat{x}$ is raised to a level exceeding v, the buyer's best response changes: when $\hat{x}$ is only slightly higher than v, the $C(y) + V(\hat{x})$ curve (shown in Figs. 2 and 3) is likely to be above the $V(y)$ curve since $V(\hat{x})$ would be almost as high as the maximum value of $V(y)$. In this case part 1 of Proposition 2 would hold and the buyer would increase her purchase quantity to take advantage of FS so that $y_R = \hat{x}$. However, for much higher levels of the cutoff point $\hat{x}$ it may not be worthwhile for the buyer to immediately raise her purchase to $\hat{x}$. Since FS requires a large purchase, the buyer's best response would be to choose the level y° in the interval $[y_1, y_2]$. Thus, as $\hat{x}$ gradually increases from zero to $\bar{y}$, the buyer's reaction changes its structure at two threshold levels: (i) at $\hat{x}_1 = v_i$ and (ii) at some $\hat{x}_2$ for which $C(y) + V(\hat{x}_2)$ and $V(y)$ are tangential to one another at some point $y = y_T$.

Example 1. As an example, consider a case where the shipping cost function is linear, i.e., $C(y) = cy$ with $c \in (0, 1)$, and the buyer's gross revenue function is given as $R(y) = a\sqrt{y}$. We choose $c = 0.2$ and $a = 1$. For this "normalized" problem we obtain $V(y) = \sqrt{y} - y$ so that $\bar{y} = 1$, i.e., the players' decisions are constrained to take values in the unit interval and the value maximizing $V(y)$ is $v = 0.25$. Hence, we find $\hat{x}_1 = v = 0.25$ as the first threshold level where the buyer's response changes its structure. To determine the second threshold level $\hat{x}_2$, we first find the point $y = y_T$ where the two curves $C(y) + V(\hat{x}_2)$ and $V(y)$ are tangential to one another: equating the derivatives, we get $C'(y) = V'(y)$, or $y_T = 0.1736$ as the point where the two curves are tangential. The second threshold level $\hat{x}_2$ is then found by solving $C(y_T) + V(\hat{x}_2) = V(y_T)$, or $\hat{x}_2 = V^{-1}(V(y_T) - C(y_T)) \approx 0.4957$.

For $\hat{x} \in [0.4957, 1]$, the two curves $C(y) + V(\hat{x}_2)$ and $V(y)$ intersect at two points. From Proposition 2, we have $y^\circ = \arg\max_{y\in[y_1,y_2]} V(y) - C(y) = 0.1736$. Note that as the shipping cost function is linear, we have $V''(y) - C''(y) = V''(y) \le 0$. Thus, the optimal solution y° can be obtained by solving $V'(y) - C'(y) = 0$, i.e., for this case, $y^\circ = y_T$.

Thus, the buyer's best response (as depicted in Fig. 4) is obtained as:

$$y_R = \begin{cases} 0.25, & \text{for } 0 \le \hat{x} \le 0.25, \\ \hat{x}, & \text{for } 0.25 \le \hat{x} \le 0.4957, \\ 0.1736, & \text{for } 0.4957 \le \hat{x} \le 1. \end{cases}$$

For low values of the cutoff level $\hat{x}$ less than $v = 0.25$, the buyer purchases her optimal amount $v = 0.25$ which maximizes $J_2(\hat{x}, y) = V(y)$. In this case the buyer does not pay for shipping.

Consider now the moderate levels of the cutoff level $\hat{x}$ between 0.25 and 0.4957. For this case we have, from Equation (3), $\max_{y\le\hat{x}\le 1}[V(y) - C(y)] \le V(\hat{x})$; i.e., purchasing $\hat{x}$ (where the seller absorbs the shipping cost) is better than purchasing any $y \le \hat{x}$ (where the buyer has to pay for shipping). Thus, the buyer is enticed to increase her purchase amount to above $v = 0.25$ in order to qualify for FS.

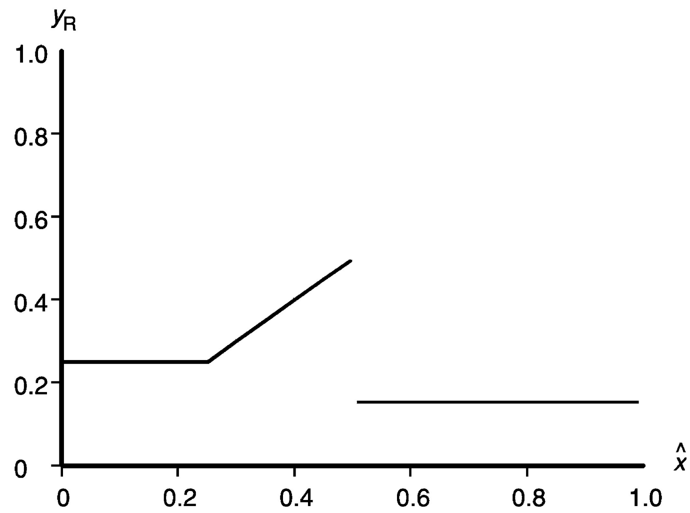

Fig. 4. The buyer's best response y_R following the seller's FS cutoff level announcement of $\hat{x}$.

If the cutoff level is greater than 0.4957, the buyer's purchase quantity becomes 0.1736 which is smaller than $v = 0.25$. At first sight, this case where the buyer reduces her purchase amount to a level below v for high values of the cutoff level may seem unintuitive. But some reflection reveals that if the cutoff level $\hat{x}$ is high and exceeds 0.4957, then we have, from Equation (3), $\max_{y\le\hat{x}\le 1}[V(y) - C(y)] \ge V(\hat{x})$; i.e., purchasing $\hat{x}$ (where the seller absorbs the shipping cost) is worse than purchasing the optimal y ($\le \hat{x}$) that maximizes $V(y) - C(y)$ (where the buyer has to pay for shipping). This results in an optimal purchase quantity that is lower than v.

4. Stackelberg solution

In the previous section we considered the optimal decisions of the buyer as a response to the seller's announced decision. A solution concept that uses the best responses and that seems to be reasonable in the present context is the Stackelberg strategy where one player assumes the role of the "leader" and the other is the "follower." Here, the leader announces his strategy first and the follower must make a decision to optimize her objective function after observing the leader's decision. But since the game is played under complete information, the leader can determine, *a priori*, the follower's response and optimize his objective accordingly. This solution concept was first introduced by the Austrian economist von Stackelberg (1934) and later used by economists (e.g., Intriligator, 1971; Gibbons, 1992) to analyze duopolistic competition. In recent years researchers in the marketing and operations research communities have also started using the Stackelberg strategy in different areas; see; e.g., Lal (1990) and Charnes *et al.* (1995) who analyze franchising coordination games and

Li *et al.* (2002) who present a game-theoretic model in a manufacturer-retailer supply chain. For a rigorous treatment of the Stackelberg strategy the text by Başar and Olsder (1982) can be consulted.

In our game-theoretic framework we assume that the leader is the B2B seller (e.g., Natural Sense) which announces his/its FS cutoff level $\hat{x}$. Given this information, we apply the method of backward induction to find the Stackelberg solutions. More specifically, at the first stage of the backward induction the buyer chooses an optimal purchase amount $y_R = f(\hat{x})$ as a function of $\hat{x}$ that maximizes her objective function $J_2(\hat{x}, y)$. Since the seller can determine the buyer's reaction y_R for each $\hat{x}$, in the second stage the seller must optimize his objective function $J_1(x, y)$ subject to the constraint $y = f(x)$, i.e., he must maximize $J_1(x, f(x))$ over $x \in [0, \bar{y}]$.

In order to analyze the Stackelberg solution for both players, we first note that as x varies, so do the endpoints of the interval $[y_1, y_2]$; see Fig. 3. Thus for the sake of generality we write the interval as $[y_1(x), y_2(x)]$ and the buyer's best response of, Equation (5) as:

$$y(x) = \begin{cases} v, & \text{if } x \le v, \\ x, & \text{if } v < x \text{ and part 1 of Proposition 2 holds,} \\ y^{\circ}, & \text{if } v < x \text{ and part 2 of Proposition 2 holds,} \end{cases} \tag{6}$$

where $\hat{x}$ is replaced by x, and

$$y^{\circ} = \arg \max_{y \in [y_1(x), y_2(x)]} V(y) - C(y). \tag{7}$$

The seller's problem in the first stage of the game is given as:

$$\max_{x \in [0, \bar{y}]} J_1(x, y(x)). \tag{8}$$

Once the seller determines his FS cutoff level (i.e., his Stackelberg decision) by solving Equation (8), he announces it as x_S. The buyer's order quantity Stackelberg decision is then computed from Equation (6).

Depending on the relative positions of the shipping cost function $C(y)$ and the buyer's net revenue function $V(y)$, the Stackelberg solution assumes different forms. The next two sections analyze these cases separately.

4.1. *Stackelberg solution when* C(y) *and* V(y) *intersect once*

In this section we consider the case where $C(y)$ and $V(y)$ intersect only once (i.e., at the origin) and $C(y) > V(y)$ for $y \in (0, \bar{y})$. (The case where the two curves intersect at the origin and are tangential to one another at one or more points is covered by the present discussion). Under this case, the Stackelberg solution assumes a particularly simple form obtained by the following theorem.

Theorem 1. *If $C(y)$ and $V(y)$ intersect only at $y = 0$, then the Stackelberg solution for both players is $(x_S, y_S) = (\bar{y}, \bar{y})$.*

Proof. In order to find a Stackelberg solution for the seller's decision, we first consider the buyer's reaction $y(x)$ to the seller's decision x and then maximize the seller's objective function $J_1(x, y(x))$.

When the seller's FS cutoff level x is less than v, i.e., when $x \le v$, we observe from Equation (6) that the buyer's best response is $y(x) = v$. In this case we find from Equation (2) that $J_1(x, y(x)) = v - C(v) - K(v)$ is the seller's objective function value, which is a constant.

However, when $v \le x \le \bar{y}$, the buyer's decision depends on the number of times the $C(y) + V(x)$ and $V(y)$ curves intersect. Since in this case the shipping cost curve $C(y)$ and the net revenue curve $V(y)$ intersect only at $y = 0$, for any $x \in (0, \bar{y})$ we have $C(y) + V(x) > V(y)$, i.e., the $C(y) + V(x)$ and $V(y)$ curves do not intersect. Referring to Equation (6), we see that the buyer's best response in this case is $y(x) = x$. On the other hand, from Equation (2), the seller's objective function value is given as $J_1(x, y(x)) = x - C(x) - K(x)$. Hence, the seller's objective function $J_1(x, y(x))$ can be written as:

$$J_1(x, y(x)) = \begin{cases} v - C(v) - K(v): & \text{if } x \le v, \\ \quad \text{(constant)}, & \\ x - C(x) - K(x), & \text{if } v \le x \le \bar{y}. \end{cases} \tag{9}$$

Note in Equation (9) that for $x \le v$, the seller's objective assumes a constant value whereas for $v \le x$, the objective is a monotonically increasing function of x for $x \in [v, \bar{y}]$ due to $C'(x) + K'(x) < 1$. Thus, the Stackelberg solution x_S for the seller is $x_S = \bar{y}$. Using Equation (6) we find that the Stackelberg solution y_S for the buyer is also $y_S = \bar{y}$. ■

Remark 3. In this case, the objective function values for the seller and the buyer are, respectively, $J_1(\bar{y}, \bar{y}) = \bar{y} - C(\bar{y}) - K(\bar{y}) > 0$ and $J_2(\bar{y}, \bar{y}) = V(\bar{y}) = 0$. For this unlikely case, due to a high shipping cost the supplier decides to provide FS only if the buyer is willing to purchase $\bar{y}$ units. In order to take advantage of FS the buyer then purchases $\bar{y}$ units resulting in a net revenue of $V(\bar{y}) = 0$. It is interesting to note that the buyer would also obtain a net revenue of zero when no trade takes place, i.e., when $x = y = 0$ we have $J_2(0, 0) = V(0) = 0$ in which case the seller's objective is reduced to $J_1(0, 0) = 0$.

Since either the Stackelberg solution or the "no trade" solution results in zero net revenue for the buyer, she may be able to enter into an agreement with the seller who obtains a positive revenue if the Stackelberg solution is used. Or, the seller could reduce the FS cutoff level by a small amount ε so that the buyer is willing to participate. That is, after the seller reduces his cutoff level $\bar{y}$ to $\bar{y} - \varepsilon$, the buyer's best response becomes $\bar{y} - \varepsilon$ resulting in $J_1(\bar{y} - \varepsilon, \bar{y} - \varepsilon) = (\bar{y} - \varepsilon) - C(\bar{y} - \varepsilon) - K(\bar{y} - \varepsilon) > 0$ and $J_2(\bar{y} - \varepsilon, \bar{y} - \varepsilon) = V(\bar{y} - \varepsilon) > 0$; a mutually beneficial outcome for both. △

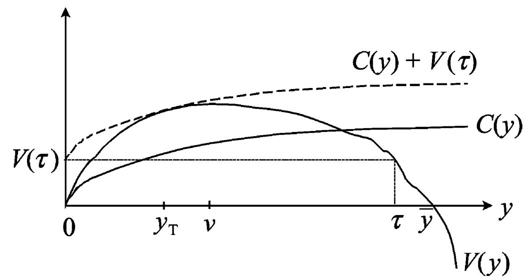

Fig. 5. The $C(y) + V(\tau)$ and $V(y)$ curves are tangential for some τ in the interval $[v, \bar{y}]$.

4.2. *Stackelberg solution when* C(y) *and* V(y) *intersect more than once*

We now consider the more complicated case in which the shipping cost curve $C(y)$ and the buyer's net revenue curve $V(y)$ intersect more than once. In this case the Stackelberg solution assumes a form that again depends on the relative positions of $C(y)$ and $V(y)$.

Suppose that the shipping cost curve $C(y)$ intersects the buyer's net revenue curve $V(y)$ at the origin and some other point(s) in the interval $(0, \bar{y})$. Then, referring to Fig. 5, we observe that for some $\tau \in [v, \bar{y}]$, the $C(y) + V(\tau)$ and $V(y)$ become tangential to one another at some point $y = y_T$. For $x \in (\tau, \bar{y}]$, we have that $C(y) + V(x) < C(y) + V(\tau)$, thus $C(y) + V(x)$ and $V(y)$ intersect more than once. Moreover, there exists some y such that $C(y) + V(x) < V(y)$, or, $V(x) < V(y) - C(y)$. On the other hand, for $x \in [v, \tau]$, we have $C(y) + V(x) > C(y) + V(\tau)$ implying that $C(y) + V(x)$ and $V(y)$ do not intersect.

The next theorem provides the Stackelberg solution for each player when $C(y)$ and $V(y)$ intersect more than once.

Theorem 2. *When $C(y)$ intersects $V(y)$ more than once, the Stackelberg solution (x_S, y_S) is obtained as*:

$$(x_S, y_S) = \begin{cases} (\tau, \tau), & \text{if } \tau - C(\tau) - K(\tau) \geq y^\circ - K(y^\circ), \\ (x^\circ, y^\circ), & \text{if } \tau - C(\tau) - K(\tau) \leq y^\circ - K(y^\circ), \end{cases} \tag{10}$$

where x° is an arbitrary value in the interval $[\tau, \bar{y}]$, and $y^\circ = \arg\max_{y \in [y_1(x), y_2(x)]} V(y) - C(y)$.

Proof. First, consider the case for $x \leq v$. In this case, as in Theorem 1 we find $J_1(x, y(x)) = v - C(v) - K(v)$, a constant.

The case $v \leq x \leq \bar{y}$ requires the analysis of two subcases: in the first case we have $x \in [v, \tau]$ and the $C(y) + V(x)$ and $V(y)$ curves do not intersect. In this region, the buyer's purchase quantity is always the same as the seller's FS cutoff decision as shown in part 1 of Proposition 2. Thus, we find $J_1(x, y(x)) = x - C(x) - K(x)$.

For the second case in which $x \in [\tau, \bar{y}]$ and τ is larger than v, we know from Equation (6) that the buyer's best response is $y^\circ = \arg\max_{y \in [y_1(x), y_2(x)]} V(y) - C(y)$. From the argument at the beginning of this section, we have $V(y^\circ) - C(y^\circ) > V(x)$ and from Proposition 2 we have that y° is less than x. Therefore, the seller's objective is $y^\circ - K(y^\circ)$ if he chooses $x \in [\tau, \bar{y}]$.

To summarize, the seller's objective function is:

$$J_1(x, y(x)) = \begin{cases} v - C(v) - K(v): & \text{if } x \leq v, \\ \quad \text{(constant)}, & \\ x - C(x) - K(x), & \text{if } v \leq x \leq \tau, \\ y^\circ - K(y^\circ): & \text{if } \tau \leq x \leq \bar{y}. \\ \quad \text{(constant)}, & \end{cases}$$

In order to find the seller's best decision, we compare the maximum value of each expression. Using arguments similar to those in Theorem 1, we know that in $[0, \tau]$ the seller's best decision is τ with the locally maximized objective value given as $J_1(\tau, \tau) = \tau - C(\tau) - K(\tau)$. In the interval $[\tau, \bar{y}]$, the seller's objective is $y^\circ - K(y^\circ)$ which is constant, then any value of x in $[\tau, \bar{y}]$ results in the same objective value for the seller. Hence, the seller can select an arbitrary value x° in $[\tau, \bar{y}]$, i.e., $x^\circ \in [\tau, \bar{y}]$. By comparing $J_1(\tau, \tau)$ with $y^\circ - K(y^\circ)$ we find the Stackelberg solution in the case where $C(y)$ and $V(y)$ intersect at more than one point. ■

Remark 4. In this case, the curve $C(y)$ intersects $V(y)$ more than once, which implies that for some purchase quantity the buyer can obtain a revenue higher than the shipping cost. Hence, even if the buyer pays the shipping cost, the buyer's net revenue (i.e., $V(y) - C(y)$) could be positive. Furthermore, if the cutoff level is lower than τ, the buyer can increase her net revenue by increasing the purchase quantity to the cutoff level. However, for a high cutoff level larger than τ, the buyer's net revenue is decreased if she increases her purchase amount to qualify for FS. As a result, the buyer has to choose an optimal solution by maximizing her net revenue of $V(y) - C(y)$. △

Example 2. We continue with the problem presented in example 1 but now consider the production cost function for the seller to be $K(y) = ky$ where $k \in (0, 1)$. For this example we choose $k = 0.4$. For the given parameter values, we find that the $C(y)$ and $V(y)$ curves intersect more than once, i.e., at zero and at 0.694. Thus, Theorem 2 applies and the Stackelberg solution (x_S, y_S) is found by using Equation (10). We know from example 1 that $\tau = \hat{x}_2 = 0.4957$ is the point for which the curves $C(y) + V(\tau)$ and $V(y)$ become tangential to one another. Since y° was computed as 0.1736, the Stackelberg solution can be found by simply comparing the values of $J_1(\tau, \tau)$ and $y^\circ - K(y^\circ)$. In particular, we find that at $\tau = 0.4957$ the seller's objective assumes the value $J_1(\tau, \tau) = 0.1983$, which is larger than $y^\circ - ky^\circ = 0.1042$. Thus, for this case the Stackelberg strategy for the two players is obtained as $(x_S, y_S) = (\tau, \tau) = (0.4957, 0.4957)$ with $J_1(\tau, \tau) = 0.1983$ and $J_2(\tau, \tau) = 0.2083$ as the players' objective function values.

5. Sensitivity analysis

In the previous examples with $(c, k, a) = (0.2, 0.4, 1)$ the Stackelberg solution was found to be (τ, τ) since $J_1(\tau, \tau) > y^\circ - ky^\circ$. For different parameter values the solution may move away to (τ, τ). In this section we present a sensitivity analysis and show that the results may be different to (τ, τ). As in example 2, we fix $a = 1$ and $k = 0.4$ and vary c to observe the variations in the players' Stackelberg decisions (x_S, y_S) and their respective objective functions (J_1, J_2). The analysis of variations in the unit shipping cost c reveals important insights about the nature of the Stackelberg solutions when the buyer's decisions are impacted by the FS policy.

Since we assumed that $C'(y) + K'(y) < 1$, we have $c + k < 1$. Thus, the feasible range for c is $(0, 1 - k)$, or $(0, 0.6)$ when $k = 0.4$. In this section, we let c vary in this range and obtain the Stackelberg solutions for both players as shown in Fig. 6.

This graph depicts two distinct regions defined by the point $c_1 = 0.491006$ where the Stackelberg solutions follow different patterns. In region 1, we have that $c \in (0, c_1]$ and $(x_S, y_S) = (\tau, \tau)$. Here, the buyer matches the seller's cutoff point and as c increases so does the cutoff point.

In region 2, the unit shipping cost c varies in the interval $(c_1, 0.6)$ and we find that the Stackelberg solutions deviate from (τ, τ). From Theorem 2, we have that $(x_S, y_S) = (x^\circ, y^\circ)$ where x° is an arbitrary value in the interval $[\tau, \bar{y}]$. In the sensitivity analysis, we assume $x^\circ = \bar{y} = 1$. In this interval, the seller has an incentive to set a high cutoff level to avoid incurring the shipping cost, since the unit shipping cost c is very high. In return, the buyer reacts by lowering her purchase quantity to very low levels (even lower than any quantity in region 1).

The effect of varying the unit shipping cost on the players' objectives is observed in Fig. 7. In region 1, as explained above, the buyer increases her purchase quantity

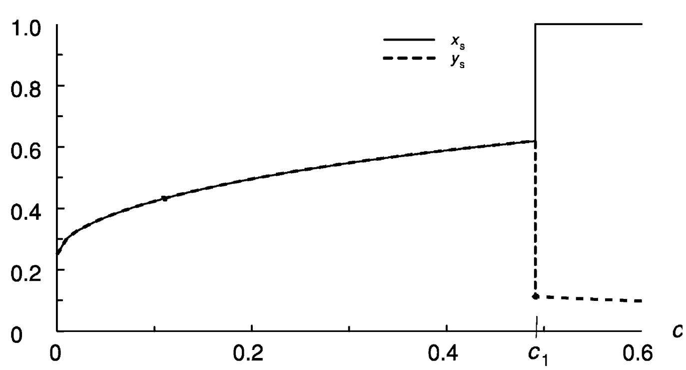

Fig. 6. Stackelberg strategies x_S and y_S when c is varied over $(0, 0.6)$. Regions 1 and 2 are defined, respectively, as the intervals $(0, c_1)$ and $(c_1, 0.6)$ where $c_1 = 0.491\ 06$.

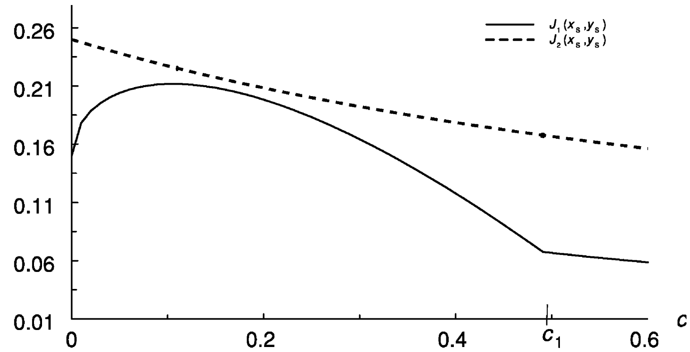

Fig. 7. Objective functions of the two players for different values of the unit shipping cost c. Regions 1 and 2 are defined, respectively, as the intervals $(0, c_1)$ and $(c_1, 0.6)$ where $c_1 = 0.491\ 06$.

to qualify for FS, i.e., $y_S = x_S = \tau$ where $\tau \in [v, \bar{y}]$ and the buyer's objective assumes the value $J_2(\tau, \tau) = V(\tau)$. From remark 1, we know that $V(y)$ decreases in the interval $[v, \bar{y}]$. As indicated in Fig. 6, the Stackelberg solution τ is increasing in $c \in [0, c_1]$. Hence, when c increases in region 1, the buyer's net revenue $V(\tau)$ decreases, as shown in Fig. 7. In region 1 where the buyer matches the seller's FS cutoff level, the seller's objective initially improves. However, as c grows larger (beyond about 0.10) the seller's objective decreases due to increases in the shipping costs he must pay.

In region 2 where $c \in (c_1, 0.6)$, the deterioration of both players' objectives continues and both players experience worsening values for J_1 and J_2. More specifically, in region 2, the buyer pays the shipping cost in addition to the purchase cost. Hence, the value of her objective becomes worse when c moves from region 1 to region 2. Furthermore, since increasing values of c result in higher total shipping costs, the buyer's objective (net revenue) decreases, as indicated in Fig. 7. In region 2, the seller's net revenue is the buyer's purchase quantity $y^\circ - ky^\circ$, where y° is obtained by solving the equation $V'(y) - C'(y) = 0$, or, $V'(y) = c$. Since $V''(y) < 0$ (from remark 1), we know that y° is decreasing in $c \in [c_1, \bar{y}]$. As a result, the value of the seller's objective function decreases over region 2. Moreover, since the buyer doesn't increase her purchase quantity to the cutoff level when the unit shipping cost $c \in [c_1, \bar{y}]$, the seller's objective function value in region 2 is lower than any other value in region 1.

6. Summary and concluding remarks

In this paper we presented a game-theoretic analysis of a FS problem between a seller and a buyer in the B2B context. If the seller's FS cutoff level is lower than the buyer's purchase amount, then the former absorbs the shipping costs. Otherwise, the buyer compares the values of two functions to determine whether she should increase her purchase amount to qualify for FS. The first step in our analysis involves the computation of a best response function for the buyer. We showed that if the seller announces his FS cutoff level first, then the buyer's best response depends on the shape of her net revenue function and the shape of the shipping cost function. In particular, the buyer's best response is determined by using a policy with two critical levels. Assuming that the seller is the leader and the buyer is the follower, the Stackelberg solution for this leader-follower game was computed using the properties of the buyer's best response functions. We presented two numerical examples and a sensitivity analysis along with the managerial implications for all the significant results obtained.

One of the crucial features of our model was the assumption that each player's objective function is common knowledge for both players; i.e., we modeled a game with complete information. As we showed in Equations (2) and (3), the objective functions of the decision-makers involve $C(y)$, $K(y)$ and $V(y) = R(y) - y$. Now, the seller may be able to estimate the buyer's net revenue function $V(y) = R(y) - y$ because the former may have a retail outlet and may be aware of the revenue function $R(y)$. Similarly, the buyer would be aware of the shipping cost function $C(y)$ because it would normally be posted on the seller's website. However, the buyer may not be aware of the exact form of the seller's production cost function $K(y)$. In this case we would no longer have a game of complete information and the resulting problem with asymmetric information would have to be solved using the techniques of Bayesian games with incomplete information; see, Gibbons (1992, ch. 3).

Throughout the paper we assumed that the buyer is a rational player who tries to maximize her own objective function; a reasonable assumption in the context of a B2B game problem with a single seller and a single buyer. Now consider the case of B2C shopping where there may be a large number of potential buyers each making his/her purchase decision independently. In this case the buyers' purchase order can be represented by a random variable Y with a probability distribution function $g(y)$. It would be interesting to formulate the problem under the B2C assumption and determine the optimal FS cutoff level for the seller.

Another interesting and related game-theoretic problem arises when two or more B2C sellers (e.g., Amazon.com and Barnesandnoble.com) compete to attract market share by setting their FS cutoff levels. Assuming a B2C environment, this problem could be analyzed to determine the Nash strategies for the players. We hope to examine these problems in the future.

Acknowledgement

The authors wish to thank Dr. Candi Yano and two anonymous referees for their insightful comments that helped improve the model and the exposition.

References

Başar, T. and Olsder, G.J. (1982) *Dynamic Noncooperative Game Theory,* Academic Press, London, UK.

Bayles, D.L. (2001) *E-Commerce Logistics and Fulfillment: Delivering the Goods*, Prentice Hall, Upper Saddle River, NJ.

Charnes, A., Huang, Z.M. and Mahajan, V. (1995) Franchising coordination with brand name considerations. *Research in Marketing*, **12**, 1–47.

Erlenkotter, D. and Trippi, R.R. (1976) Optimal investment scheduling with price-sensitive dynamic demand. *Management Science*, **23**(1), 1–11.

Gibbons. R. (1992) *Game Theory for Applied Economists*, Princeton University Press, Princeton, NJ.

Intriligator, M.D. (1971) *Mathematical Optimization and Economic Theory*, Prentice-Hall, Englewood Cliffs, NJ.

Johnston, R.B. and Mak, H.C. (2000) An emerging vision of internet-enabled supply-chain electronic commerce. *International Journal of Electronic Commerce*, **4**(4), 43–59.

Lal, R. (1990) Improving channel coordination through franchising. *Marketing Science*, **9**, 299–318.

Li, S.X., Huang, Z. and Ashley, A. (2002) Manufacturing-retailer supply chain cooperation through franchising: a chance-constrained game approach. *INFOR*, **40**(2) 131–148.

Von Stackelberg, H. (1934) *Marktform und Gleichgewicht*. Springer, Vienna, (an English translation of this book published by Oxford University Press entitled *The Theory of Market Economy,* appeared in 1952).

Biographies

Mingming Leng is an Assistant Professor in the Department of Computing and Decision Sciences at Lingnan University, Hong Kong. He holds a B. Eng. and an M. Eng. both in Management Science from the People's Republic of China, and a Ph.D. in Management Science/System from McMaster University's DeGroote School of Business, Canada. He is currently interested in the applications of game theory to supply chain-related problems. One of his papers which examines strategic issues in an ancient Chinese horse race problem was recently published in *Computers & Operations Research*. He has also published in *INFOR* a review paper on the game-theoretic applications in supply chain analysis.

Mahmut Parlar is a Professor of Management Science at McMaster University's Michael G. De-Groote School of Business. He holds a B.Sc. in Mathematics and an M.Sc. in Operations Research/Statistics both from the Middle East Technical University, Turkey, and a Ph.D. in Management Sciences from the University of Waterloo, Canada. His current research program is focused on the applications of stochastic modeling and game-theoretic analysis in supply chain management. His papers have appeared in the form of nearly 80 articles published in journals such as *Operations Research*, *Management Science*, *Naval Research Logistics*, *IIE Transactions* and others.