Decision Support

Side-payment contracts in two-person nonzero-sum supply chain games: Review, discussion and applications

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ABSTRACT

This paper investigates supply chain coordination with side-payment contracts. We first summarize specific side-payment contracts and present our review on the literature that developed general side-payment schemes to coordinate supply chains. Following our review, we discuss two criteria that a proper side-payment contract must satisfy, and accordingly introduce a decision-dependent transfer payment function and a constant transfer term. We present the condition that the transfer function must satisfy, and use Nash arbitration scheme and Shapley value to compute the constant transfer term and derive its closed-form solution. Next, we provide a five-step procedure for the development of side-payment contract, and apply it to four supply chain games: Cournot and Bertrand games, a two-retailer supply chain game with substitutable products and a one-supplier, one-retailer supply chain. More specifically, for the Cournot game, we construct a linear transfer function and a constant side-payment to coordinate two producers. For the Bertrand game, we build a nonlinear transfer function which is equivalent to a revenue-sharing contract, and show that the constant term is zero and two firms in the game equally share the system-wide profit. For a supply chain with substitutable products, we present a side-payment contract to coordinate two retailers. For a two-echelon supply chain, we develop a proper side-payment scheme that can coordinate the supply chain and also help reduce the impact of forward buying on supply chain performance.

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1. Introduction

The theory of games extensively applies to the analyses of multi-player decision problems, where the players behave, in a conflicting or cooperative situation, to seek their optimal solutions; see, for example, Benz et al. [4] and Gibbons [29]. In recent years, the management of supply chains has been an important and interesting research field. A supply chain is the sequence of organizations—their facilities, functions, and activities—that are involved in producing and delivering a product or service (see [55, Chapter 11]). As this definition implies, each supply chain member's decision may impact the benefits of other members; thus, game theory has become a primary methodological tool in supply chain analysis. For the applications of game theory in supply chain management, see two recent literature reviews: Cachon and Netessine [13] and Leng and Parlar [37]. Furthermore, many academics and practitioners now have increasing interests in the applications of game theory to the coordination of the members in supply chain games, because the management of a supply chain is mainly concerned with the *integration* of business processes across the supply chain. That is, the channel members of a supply chain should cooperate to reduce total amount of resources required to provide the necessary level of customer service to a specific segment, as Cooper et al. [19] discussed. In contrast, independent operations of the facilities in a supply chain would be of no help in improving chainwide performance. As a senior director of strategic sourcing and supply at Gap Inc., Wilkerson[1] recognized that managing a supply chain through tight relationships with the suppliers affects the success of each member in the supply chain. This reflects the significant role of coordination of the members in supply chain improvement. From the academic perspective, Thomas and Griffin [59] demonstrated the importance of supply chain coordination.

Under supply chain coordination, a decentralized channel (where each member is an independent decision maker) should perform as if it is operating in a "centralized" pattern (where the decisions are made by a single agent). To reach the goal, we should consider the

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following two natural questions: (i) What contractual mechanism should we develop so that these members' decisions are identical to the globally-optimal solutions that maximize the chainwide payoff? This question is important because of the following fact: All firms in a decentralized supply chain primarily aim at optimizing their own individual objectives rather than the chainwide objective; as a consequence, their self-serving focus may result in a deterioration of the chainwide performance. For the purpose of supply chain improvement, one could develop a proper contractual mechanism to coordinate all channel members, so that these members' individually-optimal decisions that optimize their own objectives also result in the optimal chainwide performance. (ii) How should the maximum chainwide payoff be fairly divided so that no supply chain member would have an incentive to leave the coalition? The importance of this question is justified as follows: Since the total profit (cost) of all channel members can reach its maximum (minimum) under supply chain coordination, a profit surplus (cost savings) can be generated if these channel members implement a proper contractual mechanism. A proper allocation of the surplus (savings) among these members is needed to make them better off compared to the situation without the supply chain coordination. Otherwise, the supply chain members who are worse off could lose their incentives for supply chain coordination.

As Cachon [11] discussed, we could develop an appropriate side-payment contract to coordinate the members in a supply chain. There are two publications that presented the definition of side-payment in supply chain management. In Rubin and Carter [51], a side-payment is defined as "an additional monetary transfer between supplier (buyer) and buyer (supplier) that is used as an incentive for deviating from the individual optimal policy". For a supply chain involving a seller, a buyer and a carrier, Carter and Ferrin [17] defined the transfer (side) payment as "an additional monetary transfer between any two of these three members (e.g., price reduction or surcharge, rebate, retainer fee, etc.), which is used as an incentive for a particular contract concession". According to the above two definitions, we find that the side-payment in supply chains should be a *monetary* transfer that two channel members make so as to improve the chainwide performance; so, it is also known as transfer payment, compensation, reimbursement, etc. In our paper, we assume that, for a supply chain, only pure monetary value is used to measure the objective of each member and the side-payment amount transferred between any two members. We don't consider any other measurements (e.g., the utility of each decision maker) because all supply chain members are business organizations rather than individual consumers, as indicated by Stevenson's definition [55, Chapter 11] of "supply chain".

We find some practical examples in which business organizations in supply chains transfer side-payments for supply chain coordination. In [53], Shapiro reported a real story in which the Hollywood studios and Blockbuster (which is a video store in the United States) signed a side-payment contract to coordinate the two-echelon video supply chain. Specifically, in order to entice the Hollywood studios to reduce their wholesale prices, the video store Blockbuster agreed to transfer a part of her sale revenue to those Hollywood studios who decrease their prices. This side-payment contract is well known as "revenue-sharing" contract. As another real example, Rombatch[2], a vice president of the damage research team at Genco Supply Chain Solutions, noticed a major increase in cooperation between trading partners to prevent damages within a supply chain in the last two years. In some supply chains (e.g., food supply chains), manufacturers perform their analyses to quantify the levels of damages, and determine the reimbursement rate for their retailers. This has forced the manufacturers and the retailers to work together to achieve supply chain coordination. The above two examples exhibit the real applications of side-payment contracts to supply chain improvement; but, one may note that some side-payment contracts could be illegal and be thus prohibited in practice. We assume that all side-payment contracts in our discussion are legally possible. Under this assumption, we shall perform our analysis only from the perspective of each supply chain's monetary benefit.

The last decade has witnessed a rapidly increasing number of publications related to supply chain coordination with side-payment contracts. In [11], Cachon reviewed the literature that applied *specific* side-payment contracts to coordinate two-echelon supply chains. Those specific side-payment contracts include constant wholesale pricing scheme, revenue-sharing, buyback, price-discount, sales-rebate contract, etc. In addition, many publications developed *general* side-payment contracts for supply chain coordination; later, in Section 2, we shall provide a survey on the literature with general side-payment contracts. According to our review, we find that the majority of publications concerned the development of contractual mechanism, but explicitly or implicitly assuming an *arbitrary* allocation of profit surplus or cost savings; some other publications only focused on the allocation problem, but assuming that all supply chain members *voluntarily* cooperate for supply chain coordination. To the best of our knowledge, no publication has provided a particular discussion on how a proper side-payment contract is obtained to ensure that the chainwide performance is improved and all supply chain members are also better off than in the non-cooperative case. We are thus motivated to, in this paper, present our discussion on supply chain coordination with side-payment contracts. To illustrate our discussion, we shall also consider four existing supply chain games, and develop proper side-payment contracts to coordinate these supply chains.

The remaining sections are organized as follows: Section 2 summarizes major specific side-payment contracts for supply chain coordination, and then provides a literature review on the applications of general side-payment contracts. Section 3, discusses the side-payment contract that satisfies two criteria as follows: the globally optimal solutions are identical to the equilibrium solutions of the players in two-person nonzero-sum games; and each player is better off than in a non-cooperative situation. A transfer function $L(x_1, x_2)$ and a constant transfer term γ are introduced to assure the above two criteria. We compute the constant transfer term γ, by utilizing Nash arbitration scheme and Shapley value. In Section 4, we use our analytical results (obtained in Section 3) to construct side-payment contracts for four existing supply chain games: Cournot and Bertrand games that are two classical ones in economics; and two recent supply chain games. More specifically, a linear transfer function and a constant side-payment are derived to coordinate two producers in the Cournot game. For the Bertrand game, we develop a side-payment contract consisting of a nonlinear transfer function and zero constant term, and show that the overall profit is equally divided between two players. Using a recent supply chain game by Parlar [44], we present a linear side-payment contract to coordinate a horizontal supply chain in which two retailers' products are substitutable. Using Cachon's game model [11] for a two-echelon supply chain, we develop a so-called "price-margin compensation" side-payment scheme to induce supply chain coordination and also solve the forward buying problem. In Section 5, we summarize our analysis and applications, and discuss the potential applications of side-payment contracts.

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2. Literature review

In this section, we review the literature on the applications of side-payment contracts for supply chain coordination. There is a recent publication in this area by Cachon [11] who presented a survey on some specific side-payment contacts (e.g., buyback, sales-rebate, etc.) that could induce supply chain coordination. Our review differs from [11] since we emphasize the applications of general forms of side-payment schemes in addition to a brief description of those specific contracts discussed in [11].

In [11], Cachon surveyed some publications concerned with the side-payment contracts in some specific forms; and the author used a two-echelon supply chain game in the newsvendor setting to illustrate the applications of these contracts for supply chain coordination. In this section, we briefly describe those specific contracts and list representative publications in Table 1. (Without loss of generality, we describe these specific contracts for a two-echelon supply chain involving a supplier and a retailer.) According to the review, we find that the literature with these specific contracts only focused on the development of coordination mechanisms under which the equilibrium behavior of supply chain members increases the chainwide profit to the maximum or decreases the chainwide cost to the minimum. However, very few publications considered the "fair" allocation of extra profit or cost savings generated by the coordination, so that all supply chain members are better off than in the non-cooperative context. Those publications including Cachon's models in [11] assumed that the supply chain members *arbitrarily* share the surplus; this leads to a natural question: Which allocation scheme should be adopted by supply chain members?

We next survey the literature that investigated supply chain coordination with general side-payment contracts rather than those specific side-payment contracts given in Table 1. Riordan [50] published a seminal work on the discussion of side-payment contracts in supply chain coordination. In this paper, the author considered a decentralized two-echelon supply chain with a seller and a buyer. The seller observes the production cost, determines his production quantity and announces a transfer payment schedule to the buyer. Then the buyer observes private information on market demand, chooses an order quantity and places an order with the seller. At last, the seller ships the products to the buyer who then pays according to the pre-determined transfer payment schedule. The author discussed the payment schedule and order quantity that, in equilibrium, achieves supply chain coordination. As another early publication related to side-payment scheme, Banerjee [3] developed a joint economic lot size (JELS) model to minimize total inventory holding and ordering costs of a vendor and a purchaser. Under the assumption of deterministic demand and lead times, the author showed that, to implement the JELS solution, the vendor should give some side-payment to the purchaser. Using the model in [3], Sucky [56] derived side-payment schemes that coordinate a single supplier and a single buyer. Assuming that the buyer has the stronger negotiation power than the supplier, the author considered two bargaining models: one with complete information and one with asymmetric information. For each model, the supplier transfers a side-payment to the buyer, so that the latter's order quantity decision would be suitable to the former. The author computed the side-payments for both models. However, this paper implicitly assumed that the buyer voluntarily accepts the side-payment; this may conflict with the assumption of the buyer's stronger bargaining power.

Lee and Whang [35] investigated the coordination of decentralized multi-echelon supply chains. The authors particularly analyzed an infinite horizon, two-echelon inventory model with positive replenishment delays, zero setup costs, and random external demands; and they developed a nonlinear side-payment scheme involving transfer pricing, consignment, shortage reimbursement, and an additional backlog penalty. It was shown that the serial supply chain can be coordinated under the side-payment scheme. To conduct the coordination scheme of the decentralized supply chain in [35], Porteus [48] proposed the use of responsibility tokens (RTs) as a mechanism for administering the transfer payments required to implement upstream responsibility. The authors showed that, the side-payment should be computed according to actual consequences of processing/delivering/shipping less than what was requested rather than the prediction of the consequences.

In [14], Cachon and Zipkin examined competition and coordination in a two-stage supply chain with a supplier and a retailer who independently choose base stock policies to minimize their costs. Assuming stationary stochastic demand and fixed transportation times, they developed two stochastic games and designed simple linear side-payment schemes to coordinate the supply chain. The transfer payment from the supplier to the retailer is involved since the retailer's backorders impacts the supplier's cost. Caldentey and Wein [15] developed a supply chain game where a supplier chooses the production capacity and a risk-neutral retailer adopts a base-stock replenishment policy.

Table 1
Brief description of specific side-payment contracts and representative publications

Specific side-payment contracts	Brief description	Representative publications
Constant wholesale-price	The side-payment from the retailer to the supplier is the supplier's unit wholesale price times the retailer's purchase amount. (Note that, under the contract, the wholesale price is *not* the supplier's decision variable but exogenously determined.)	Bresnahan and Reiss [8]; Gerchak and Wang [27]; Lariviere and Porteus [34]
Revenue-sharing	The side-payment from the retailer to the supplier is a percentage of the retailer's sale revenue	Cachon and Lariviere [12]; Gerchak and Wang [27]; Giannoccaro and Pontrandolfo [28]; Li [40]; Wang et al. [64]
Sales-rebate (a.k.a. Markdown Allowance)	The side-payment from the supplier to the retailer is the supplier's per unit rebate times the number of the retailer's sold items	Ferguson et al. [25]; Krishnan et al. [33]; Taylor [58]
Quantity flexibility	The side-payment from the supplier to the retailer is the supplier's compensation for the retailer's loss on the unsold items	Cachon and Lariviere [9]; Eppen and Iyer [23]; Tsay [60]; Tsay and Lovejoy [62]
Buyback (a.k.a. Return policy)	The side-payment from the supplier to the retailer is the supplier's expense of buying the retailer's unsold items back	Anupindi and Bassok [2]; Donohue [22]; Granot and Yin [31]; Pasternack [45]; Tsay [61]
Quantity discount	The side-payment from the supplier to the retailer is the supplier's reduced price times the retailer's purchase amount	Moses and Seshadri [41]; Weng [65]; Zhao and Wang [69]
Price-discount sharing (a.k.a. Bill back, or Count-recount)	The side-payment from the supplier to the retailer is the supplier's compensation (in the form of his wholesale-price reduction) to the retailer who reduces her retail price	Bernstein et al. [5]; Bernstein and Federgruen [6]; Cachon and Lariviere [12]

The authors designed a linear transfer payment contract in the cost-sharing form to coordinate the supply chain. The paper also examined a Stackelberg game with the supplier as a leader and the retailer as a follower, and compared system-wide costs, the agents' decision variables and the customer service level of the Nash, centralized and Stackelberg solutions.

Some publications considered the impacts of information on supply chain operations, and examined the development of side-payment contracts that coordinates these supply chains. Gal-Or [26] studied the supply chain with two suppliers under complete information. The author showed that it may not be optimal for a supplier to impose pricing constraints on his retailers, and the equilibrium is sometimes achieved with a side-payment in the form of franchise fee. Ha [32] considered a two-echelon supply chain in which a supplier offers several general side-payment contracts in order to share the information about the retailer's marginal cost. Moreover, the author assumed that, under an optimal contract, the supplier employs a cutoff level policy on the buyer's marginal cost to determine whether the buyer should be induced to sign the contract. The optimal cutoff level was also characterized by Ha [32]. Weng and McClurg [67] analyzed a one-supplier one-buyer supply chain, in which the buyer orders a product from the supplier whose delivery time is uncertain, and then satisfies the random demand over a selling period of fixed length. The supplier agrees to deliver the product at the start of the selling period; but, for the case of late delivery, the supplier shall offer a discounted wholesale price to compensate the buyer. The authors showed that the price-discount transfer payment, along with the demand information sharing between these two channel members, can achieve supply chain coordination. In [20], Corbett et al. discussed different types of side-payment contracts (i.e., wholesale-pricing, general linear and non-linear side-payment schemes) for a two-echelon supply chain, in which a supplier offers a transfer payment to a buyer for sharing the information regarding the retailer's cost structure. Note that, for this problem, the side-payment is used to measure the value of information released by the buyer. The authors assumed that the supplier determines the contract parameters (i.e., the wholesale price and lump-sum side-payment) and the buyer chooses her order quantity and the retail price that influences the demand of end consumers. The optimal contract was obtained for supply chain coordination. Yue and Liu [68] analyzed a manufacturer–retailer supply chain consisting of a mix of the traditional retail channel and a direct channel. The price-dependent demand of end consumers can purchase products from either channel. The authors considered both make-to-order and make-to-stock cases, and evaluated the value of sharing demand forecasts between these two channel members. To entice the retailer to voluntarily release her information, the manufacturer may offer an appropriate price discount or side-payment to the retailer. It was shown that a proper side-payment scheme achieving channel coordination can be found in the make-to-order case whereas no side-payment can induce the retailer to share information in the make-to-stock case.

Three recent publications concerned with information sharing in supply chains considered the *fair* allocation of profit surplus or cost savings generated under supply chain coordination. These publications did not provide a proper coordination mechanism but assumed that all supply chain members' decisions make system-wide performance optimal. Li [39] examined the incentives for firms in a one-manufacturer, *N*-retailer supply chain to share information vertically for improving supply chain performance. As this paper indicated, the retailers have no incentive to share demand information with the manufacturer in the absence of a side-payment between the retailers and the manufacturer. However, when a side-payment is possible, a contract-signing game forms with demand information vertically shared by the retailers and the manufacturer. Li [39] revealed the importance of benefit allocation on encouraging a downstream member in a vertical supply chain to disclose demand information to the upstream(s). Similar to Li [39], Raghunathan [49] applied the concept of Shapley value to allocate the surplus generated by demand information sharing in a two-echelon supply chain between the manufacturer and *N* retailers, where demands at the retailers during a time period may be correlated. This paper also examined the impact of demand correlations on the manufacturer and the retailers. Leng and Parlar [38] used the nucleolus solution—which is an important concept in cooperative game theory; see Schmeidler [52]—to allocate the cost savings generated by demand information sharing in a three-echelon supply chain involving a manufacturer, a distributor and a retailer.

Ernst and Powell [24] investigated a two-echelon supply chain in which a manufacturer offers a side-payment to a retailer in order to entice the latter to increase her service level. In this paper, the demand of end consumers is sensitive to the service level, and it thus affects both the manufacturer's and the retailer's profits. Assuming that the side-payment contract is determined by the manufacturer rather than by the negotiation between two players, the authors analyzed a leader-follower (Stackelberg) game scenario as follows: The manufacturer first announces his decision on the unit transfer payment (i.e., his payment for each unit increment of the retailer's service level), and the retailer then determines her optimal service level. The side-payment from the manufacturer to the retailer is computed as $W(SL - SL_0)$, where W denotes the manufacturer's unit transfer payment, SL and SL_0 are the retailer's optimal service levels in the games with and without side-payment, respectively. Çanakoğlu and Bilgiç [16] analyzed a two-stage telecommunication supply chain with an operator and a vendor in a multi-period context. The authors introduced to their game model a transfer payment per period that the operator pays to the vendor; and they designed two coordinating side-payment contracts (i.e., revenue and investment cost sharing; quantity discount with buy-back) that make the operator choose the same investment and capacity change decisions as the centralized solution.

Several publications considered a fixed side-payment scheme (independent of decision variables) for supply chain coordination. Weng [66] studied a two-echelon supply chain with price-sensitive demand, in which a manufacturer determines his price and a distributor chooses her order quantity and price. In order for the distributor to entice the manufacturer to reduce his price, the author computed a fixed side payment transferred from the distributor to the manufacturer, which may be in the form of a franchise fee. In [30], Gjerdrum *et al.* investigated the fair allocation of system-wide profit in a two-echelon supply chain, where a plant and a distribution and sales firm determine their production plan, inventory policies, transportation plan and transfer price levels. To coordination the supply chain, the authors applied the game concept of Nash bargaining solution to allocate the chainwide profit and find a fair side-payment. Under the fair side-payment scheme, these two supply chain members are willing to cooperate for the maximization of supply chain profit. Although this paper examined the fair allocation of maximum chainwide profit, it didn't develop a proper mechanism to assure that the decisions of these channel members can induce supply chain coordination. Tan [57] presented an analytical model to analyze a supply chain where two producers cooperate on production capacity. The author used an auction mechanism to award a single buyer's order to the producer with a lower bid, and also introduced a fixed side-payment term to induce the cooperation between these two producers. Zhao and Wang [69] investigated a two-echelon decentralized supply chain in which a manufacturer produces and sells her products to the distributor who, after some further processing, sells the products to an external market. In a leader–follower setting, these two channel members make pricing and production/ordering decisions over a discrete, finite-time horizon (a selling season) to maximize their profits. In order to achieve

channel coordination, the authors developed an incentive scheme under which a manufacturer transfers a fixed lump-sum side-payment to the retailer, assuming that the retailer voluntarily accepts the side payment.

According to our review, we find that the supply chain coordination with side-payment contracts appears to be an interesting and important problem worth investigating. However, according to our review, we find that the majority of existing publications focused on the development of contractual mechanism for channel coordination but ignoring the fair allocation of profit surplus or cost savings, whereas the others investigated the allocation problems but assuming supply chain members voluntarily cooperate to improve the chainwide performance. As we discussed in Section 1, the two issues should be considered all together when we develop an appropriate side-payment contract that induces supply chain coordination. Next, in Section 3, we provide our discussion on a general idea for supply chain coordination, which is then applied, in Section 4, to the coordination of two decision makers in four supply chain games.

3. Discussion of side-payment contracts

We now consider a general two-person nonzero-sum game in which players 1 and 2 determine decisions x_1 and x_2 to maximize their objective (payoff) functions $J_1(x_1, x_2)$ and $J_2(x_1, x_2)$, respectively. In the section we characterize the equilibrium behavior of the two players by using the solution concept Nash equilibrium [43]. Denoted by (x_1^N, x_2^N), the equilibrium solution satisfies the following conditions: $J_1(x_1^N, x_2^N) \geqslant J_1(x_1, x_2^N)$ and $J_2(x_1^N, x_2^N) \geqslant J_2(x_1^N, x_2)$. As discussed in Section 1, players in a game could form a coalition to increase the system-wide payoff. In order to assure the stability of the coalition, we could develop an appropriate side-payment contract for the game, as many publications did (see our review in Section 2). Under a proper side-payment contract, the players should arrive to their equilibrium solutions that are identical to the globally-optimal solutions maximizing the overall (system-wide) payoff, and they should be also better off than in the non-cooperative game without side-payments. Accordingly, we begin by presenting two criteria that a properly-developed side-payment scheme must satisfy.

To facilitate our discussion, we let $(\tilde{x}_1^N, \tilde{x}_2^N)$ denote Nash equilibrium of the two-person game with side-payments, and let (x_1^*, x_2^*) represent the globally optimal solutions maximizing system-wide payoff function $J(x_1, x_2) = \sum_{i=1}^{2} J_i(x_1, x_2)$. Note that $(\tilde{x}_1^N, \tilde{x}_2^N)$ could be different from (x_1^N, x_2^N) that is Nash equilibrium for the original game without transfer payments.

Criterion 1. If a side-payment scheme is properly developed for a game, the resulting equilibrium solution $(\tilde{x}_1^N, \tilde{x}_2^N)$ must be identical to the globally optimal solution (x_1^*, x_2^*).

If Criterion 1 is assured by a side-payment scheme, each player would not deviate from the globally optimal solution, since the players choose their equilibrium solutions in the globally optimal case. However, only the criterion is not sufficient to guarantee stability of the two-person coalition. That is, when only Criterion 1 is satisfied, a player could become worse when the player joins the coalition. For example, for the globally optimal case, we have the maximum system-wide payoff $J(x_1^*, x_2^*) = \sum_{i=1}^{2} J_i(x_1^*, x_2^*)$. Although the inequality that $J(x_1^*, x_2^*) \geqslant J(x_1^N, x_2^N) = \sum_{i=1}^{2} J_i(x_1^N, x_2^N)$ is no doubt satisfied, we cannot assure that $J_i(x_1^*, x_2^*) \geqslant J_i(x_1^N, x_2^N)$, $i = 1, 2$ (i.e., both players are better off). As a result, we need the next criterion.

Criterion 2. If a properly-developed side-payment scheme makes the two-person coalition stable, each player must be better off than in the non-cooperative situation. That is, the properly-developed scheme must make the following inequalities satisfied: $J_i(x_1^*, x_2^*) \geqslant J_i(x_1^N, x_2^N)$, $i = 1, 2$.

Using the above criteria, we add two side-payment terms into $J_i(x_1, x_2)$ $(i = 1, 2)$, and write the resulting payoff functions (denoted by $\widetilde{J}_i(x_1, x_2)$) as follows:

$$\begin{cases} \widetilde{J}_1(x_1, x_2) = J_1(x_1, x_2) - L(x_1, x_2) - \gamma, \\ \widetilde{J}_2(x_1, x_2) = J_2(x_1, x_2) + L(x_1, x_2) + \gamma, \end{cases} \quad (1)$$

where $L(x_1, x_2)$ represents a transfer payment given by player 1 to player 2, which depends on the decisions of two players; and γ is a constant side-payment from player 1 to player 2. In the functions $\widetilde{J}_i(x_1, x_2)$, $i = 1, 2$, the terms $L(x_1, x_2)$ and γ are used to assure Criterions 1 and 2, respectively. Next, we conduct the discussion of the two side-payment terms, using a two-step approach as follows:

Step 1: Ignoring the constant term γ, we analyze the game with payoff functions $J_1(x_1, x_2) - L(x_1, x_2)$ and $J_2(x_1, x_2) + L(x_1, x_2)$, and find the appropriate form of $L(x_1, x_2)$ to assure Criterion 1;

Step 2: We use two solution concepts (i.e., Nash arbitration scheme and Shapley value) to find a unique fair allocation of overall surplus between the two players, and compute a proper value of γ to assure Criterion 2.

3.1. Transfer function $L(x_1, x_2)$

The transfer $L(x_1, x_2)$ in (1) is a function of (x_1, x_2), and it is defined as the net transfer that player 1 gives to player 2 when their decisions are x_1 and x_2. For example, consider a supply chain where retailers 1 and 2 sell a single product and compete for a common market by choosing their sale prices x_1 and x_2, respectively. Assume that retailer 2's price x_2 is fixed. When retailer 1 decreases his price x_1, he would obtain a greater market share. It may follow that an increasing sale of retailer 1 raises his payoff $J_1(x_1, x_2)$ but reduces the payoff $J_2(x_1, x_2)$ of retailer 2. For this case, in order to entice retailer 2 to stay with the coalition, retailer 1 would agree to transfer a part of his surplus to compensate retailer 2. Likewise, retailer 2 may behave similarly while retailer 1 makes his price x_1 unchanged; and to encourage the retailer 1 to cooperate, retailer 2 would also agree to share her surplus with retailer 1. The sum of the two payments transferred in reverse directions is the side-payment term $L(x_1, x_2)$. If $L(x_1, x_2) \geqslant 0$, retailer 1 has a net payment $L(x_1, x_2)$ to retailer 2; if $L(x_1, x_2) < 0$, retailer 2 transfers the amount $|L(x_1, x_2)|$ to retailer 1; otherwise, if $L(x_1, x_2) = 0$, there is no side-payment between the two retailers.

Adding $L(x_1, x_2)$ but ignoring γ, we write the payoff functions of two players as follows:

$$\begin{cases} \bar{J}_1(x_1,x_2) = \tilde{J}_1(x_1,x_2) + \gamma = J_1(x_1,x_2) - L(x_1,x_2), \\ \bar{J}_2(x_1,x_2) = \tilde{J}_2(x_1,x_2) - \gamma = J_2(x_1,x_2) + L(x_1,x_2). \end{cases}$$

By solving the above game, we find that the equilibrium solution is the same as that for the game with the payoff functions (1), since the term γ is independent of decision variables. In order to satisfy Criterion 1, we should find proper values of parameters in $L(x_1, x_2)$ to make equilibrium solution and globally optimal solution identical.

Theorem 1. *To assure Criterion 1, the transfer function $L(x_1, x_2)$ must satisfy the following conditions:*

$$\begin{cases} J_1(x_1^*, x_2^*) - L(x_1^*, x_2^*) \geqslant J_1(x_1, x_2^*) - L(x_1, x_2^*), \\ J_2(x_1^*, x_2^*) + L(x_1^*, x_2^*) \geqslant J_2(x_1^*, x_2) + L(x_1^*, x_2), \end{cases}$$

where $(x_1^*, x_2^*) \equiv \arg\max_{x_1, x_2} J(x_1, x_2)$.

Proof. For a proof of this theorem and all subsequent proofs, see Appendix A. □

When both $J_1(x_1, x_2)$ and $J_2(x_1, x_2)$ are concave in (x_1, x_2), we can develop the transfer term $L(x_1, x_2)$, as shown in the following theorem.

Theorem 2. *When the payoff functions of two players are concave in any point (x_1, x_2), we should introduce a concave transfer function $L(x_1, x_2)$ to assure Criterion 1. The function is modeled as follows:*

$$\frac{\partial L(x_1^*, x_2^*)}{\partial x_1} = -\frac{\partial J_2(x_1^*, x_2^*)}{\partial x_1} \quad \text{and} \quad \frac{\partial L(x_1^*, x_2^*)}{\partial x_2} = \frac{\partial J_1(x_1^*, x_2^*)}{\partial x_2}, \tag{2}$$

where (x_1^*, x_2^*) is the globally optimal solution.

A simple way to model $L(x_1, x_2)$, according to Theorem 2, is to write a linear transfer function. For instance, we may write the function as $L(x_1, x_2) = \alpha x_1 + \beta x_2$, where $\alpha \equiv \partial L(x_1, x_2)/\partial x_1$ and $\beta \equiv \partial L(x_1, x_2)/\partial x_2$. Theorem 2 indicates that $\alpha = -\partial J_2(x_1^*, x_2^*)/\partial x_1$ and $\beta = \partial J_1(x_1^*, x_2^*)/\partial x_2$ for a properly-developed $L(x_1, x_2)$ in the point (x_1^*, x_2^*). We describe the side-payment contract without constant term γ as follows: if player 1 decreases his decision x_1 by one unit, he gives a side-payment $\partial J_2(x_1^*, x_2^*)/\partial x_1$ to player 2; if player 2 decreases her decision x_2 by one unit, she transfers the amount $\partial J_1(x_1^*, x_2^*)/\partial x_2$ to player 1.

3.2. Constant transfer term, γ

Besides modeling the transfer function $L(x_1, x_2)$ to assure Criterion 1, we need a constant term γ to make Criterion 2 satisfied. The value of γ should result from the negotiation between these two players. We apply a solution concept in the theory of cooperative games to properly determine the value of γ. We find from Section 3.1 that a properly-developed $L(x_1, x_2)$ has made the equilibrium $(\tilde{x}_1^N, \tilde{x}_2^N)$ identical to the globally optimal solution (x_1^*, x_2^*). Prior to computing γ, two players have their payoffs as $\bar{J}_1(x_1^*, x_2^*) = J_1(x_1^*, x_2^*) - L(x_1^*, x_2^*)$ and $\bar{J}_2(x_1^*, x_2^*) = J_2(x_1^*, x_2^*) + L(x_1^*, x_2^*)$. Comparing them with the payoffs in terms of (x_1^N, x_2^N), we can compute the surplus/deficit for each player, generated by transferring $L(x_1, x_2)$, as $K_i = \bar{J}_i(x_1^*, x_2^*) - J_i(x_1^N, x_2^N)$, $i = 1, 2$. If $K_i \geqslant 0$, player i has the surplus K_i; otherwise, the player experiences the deficit $|K_i|$, and possibly leaves the coalition. Since $J(x_1^*, x_2^*) \geqslant J(x_1^N, x_2^N)$, the overall surplus—denoted by K—incurred by two players is computed as $K = J(x_1^*, x_2^*) - J(x_1^N, x_2^N) \geqslant 0$. In order to make the coalition stable, we should properly choose the constant term γ for the game so that both players would be "happy" with the allocation of K. Next, we focus on allocating K between two players, and compute γ accordingly.

One may notice that it is interesting to apply the theory of cooperative games to find a unique fair allocation scheme. There are two commonly-used solution concepts for the unique allocation of surplus in two-person nonzero-sum games, i.e., Nash arbitration scheme (a.k.a. Nash bargaining scheme) and Shapley value. From our review in Section 2, we find that Gjerdrum et al. [30] and Raghunathan [49] respectively used Nash arbitration scheme and Shapley value to "fairly" allocate the profit surplus in two-echelon supply chains.

Nash arbitration scheme, introduced by Nash [42], is concerned with allocating surplus on the negotiation set. Any point (allocation scheme) on the set satisfies the following two conditions (see von Neumann and Morgenstern [63]): (i) it is Pareto optimal; (ii) it is at or above the security level of both players. Here, the security level of a player—which is called *status quo* point for the player—is defined as the guaranteed payoff incurred by the player in a non-cooperative play. In [42], Nash developed four axioms: rationality; linear invariance; symmetry; and independence of irrelevant alternatives. Using the axioms, Nash showed that there is a unique solution, say, Nash arbitration scheme. The scheme can be obtained by solving

$$\max_{f_1 \geqslant f_1^0, f_2 \geqslant f_2^0} (f_1 - f_1^0)(f_2 - f_2^0), \quad \text{s.t. } (f_1, f_2) \in \mathscr{P}, \tag{3}$$

where f_i and f_i^0, respectively denote player i's allocated surplus and the security level, $i = 1, 2$; and $\mathscr{P}$ is the set of Pareto optimal solutions.

In our paper, two players bargain for the allocation of total surplus K. Since we allocate K between the two players, the Pareto optimal set $\mathscr{P}$ is written as $\mathscr{P} = \{(f_1, f_2) | f_1 + f_2 = K\}$, where $K = J(x_1^*, x_2^*) - J(x_1^N, x_2^N)$. Next, we discuss the security levels of two players (i.e., f_1^0 and f_2^0). When we involve the constant term γ in the side-payment scheme, the two players have the payoffs $\tilde{J}_1(x_1^*, x_2^*)$ and $\tilde{J}_2(x_1^*, x_2^*)$. Note that, for the game without the transfers $L(x_1, x_2)$ and γ, the players have their payoffs $J_1(x_1^N, x_2^N)$ and $J_2(x_1^N, x_2^N)$. Therefore, the allocations of K to the two players can be computed as follows:

$$\begin{aligned} f_1 &= \tilde{J}_1(x_1^*, x_2^*) - J_1(x_1^N, x_2^N) = \bar{J}_1(x_1^*, x_2^*) - J_1(x_1^N, x_2^N) - \gamma = K_1 - \gamma, \\ f_2 &= \tilde{J}_2(x_1^*, x_2^*) - J_2(x_1^N, x_2^N) = \bar{J}_2(x_1^*, x_2^*) - J_2(x_1^N, x_2^N) + \gamma = K_2 + \gamma. \end{aligned}$$

The security level in our game is the minimum allocations to the players that assure that $f_1 \geqslant 0$ and $f_2 \geqslant 0$. To make both players better off, we find that the status quo point is $(f_1^0, f_2^0) = (0, 0)$.

Theorem 3. *Nash arbitration scheme suggests that we equally allocate the system-wide surplus K between two players, i.e., $f_1 = f_2 = K/2 = [J(x_1^*, x_2^*) - J(x_1^N, x_2^N)]/2$.*

Theorem 3 shows that under Nash arbitration scheme, two players equally allocate the surplus K. Since the status quo point is $(0,0)$, the payoffs received by the two players are increased by a same amount after transfer payments. By using this scheme, we compute the constant term γ.

Theorem 4. *If we equally allocate the overall surplus K as suggested by Nash arbitration scheme, then the constant side-payment term γ is determined as follows:*

$$\gamma = \frac{\sum_{i=1}^{2}(-1)^{i+1}[J_i(x_1^*, x_2^*) + (-1)^i L(x_1^*, x_2^*) - J_i(x_1^N, x_2^N)]}{2}. \tag{4}$$

Fig. 1 shows negotiation set (which is same as Pareto optimal set $\mathscr{P}$) and status quo point for the two-person game, and indicates that the middle point on the negotiation set is the point of equally allocating K, which is used to compute γ.

Shapley value [54] has been widely used to allocate the surplus for n-person games. Even though it is usually used for a game with three or more players, some publications (e.g., Lemaire [36]) applied it to two-person games. In addition, Lemaire [36] showed that, as suggested by Shapley value, two players equally allocate the system-wide surplus. This means that the allocation scheme in terms of Shapley value is the same as that suggested by Nash arbitration scheme.

We have discussed side-payment contracts $(L(x_1, x_2), \gamma)$ for two-person games. To find a proper contract, we present the following five steps procedure:

Step 1: Compute Nash equilibrium (x_1^N, x_2^N) for the original game without side-payments.
Step 2: Compute the globally optimal solution (x_1^*, x_2^*) that maximizes the system-wide payoff function $J(x_1, x_2) = \sum_{i=1}^{2} J_i(x_1, x_2)$, and find the overall surplus $K = J(x_1^*, x_2^*) - J(x_1^N, x_2^N)$.
Step 3: Compute the Nash equilibrium $(\tilde{x}_1^N, \tilde{x}_2^N)$ for the game with the transfer function $L(x_1, x_2)$.
Step 4: Equate $(\tilde{x}_1^N, \tilde{x}_2^N)$ to (x_1^*, x_2^*) and compute the values of contract parameters in $L(x_1, x_2)$. To avoid the analytical difficulty in finding the parameters, we should choose a feasible and tractable transfer function $L(x_1, x_2)$ in Step 3. As Section 2 indicates, the majority of existing publications designed the linear transfer function $L(x_1, x_2)$.
Step 5: Use Theorem 4 to compute the constant transfer term γ.

4. Applications of side-payment contracts in supply chain coordination

We consider the development of side-payment contracts for four two-person nonzero-sum supply-chain games, in order to demonstrate the importance of side-payments in supply chain coordination. More specifically, in Section 4.1, we shall apply side-payment contracts to coordinate two players in Cournot and Bertrand games which are two classical game models in economics. Note that Cournot and Bertrand games are commonly considered as two seminal supply chain games; for a particular discussion, see, for example, Leng and Parlar [37]. In Section 4.2, we shall first use Parlar's game model in [44] to develop a proper side-payment contract to coordinate two retailers in a supply chain with substitutable products; and then use Cachon's game model in [11] to coordinate a supplier and a retailer in a two-echelon supply chain.

4.1. Side-payment contracts for Cournot and Bertrand games

We now apply our analytical results in Section 3 to two classical game models, i.e., Cournot and Bertrand games. Cournot [21] developed a two-person game where two producers determine their production quantities and sell similar products to a market with an identical price dependent of total production quantity. Bertrand [7] focused on a pricing decision problem in a two-person game involving two firms.
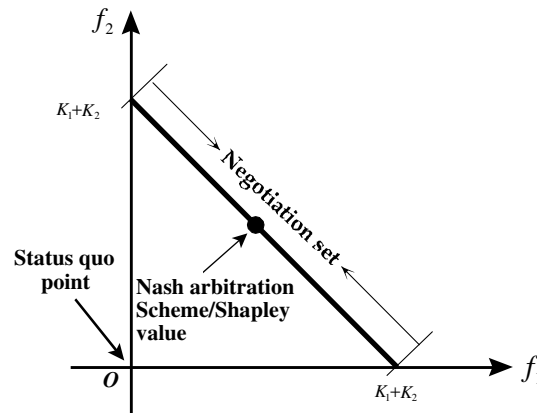
Fig. 1. The negotiation set, status quo point $(0,0)$, Nash arbitration scheme $(f_1 = f_2 = K/2)$ and Shapley value $(f_1 = f_2 = K/2)$ for the two-person game, where the system-wide surplus $K = \sum_{i=1}^{2} K_i = \sum_{i=1}^{2}[J_i(x_1^*, x_2^*) + (-1)^i L(x_1^*, x_2^*) - J_i(x_1^N, x_2^N)]$.

It is, in Bertrand game, assumed that demand is a decreasing linear functions of prices determined by the two firms, and is satisfied by the firm with a lower price. However, when the prices of two firms are identical, these two firms equally share the demand.

4.1.1. Cournot game with a side-payment contract

In [21], Cournot considered a two-player game where two producers (P_1 and P_2) serve a market with similar products. In the game P_1 and P_2, respectively choose optimal production quantities q_1 and q_2 to maximize their profit functions as follows:

$$J_1(q_1,q_2) = q_1[a - (q_1 + q_2) - c] \quad \text{and} \quad J_2(q_1,q_2) = q_2[a - (q_1 + q_2) - c], \tag{5}$$

where both producers have an identical unit production cost c; the sale prices of their similar products are the same and they are computed as $a - (q_1 + q_2)$. It is assumed that $a > q_1 + q_2$ and $a > c$. For the game, Cournot found that Nash equilibrium solution is $(q_1^N, q_2^N) = ((a-c)/3, (a-c)/3)$, and the profits for both producers are $J_1(q_1^N, q_2^N) = J_2(q_1^N, q_2^N) = (a-c)^2/9$. The equilibrium solution for Cournot game is also known as Cournot–Nash equilibrium.

Next, we develop a proper side-payment contract for the Cournot game to coordinate these two producers. Particularly, we introduce a transfer function $L(q_1,q_2)$ and a constant transfer term γ to the game. Under such a side-payment arrangement, an equilibrium solution—chosen by these two producers—maximizes the system-wide profit and both producers are better off. For the contract development, we follow the five-step procedure given in Section 3. In step 1, we use Nash equilibrium solution attained by Cournot. For step 2, we compute the globally optimal solution (q_1^*, q_2^*), by solving the system-wide profit function

$$J(q_1,q_2) = \sum_{i=1}^{2} J_i(q_1,q_2) = (q_1 + q_2)[a - (q_1 + q_2) - c].$$

We find that there are infinitely many optimal solutions, which can be written as

$$(q_1^*, q_2^*) = \{(q_1,q_2) | q_1 = \frac{a-c}{2} - z, \ q_2 = z.\},$$

where z is an arbitrary value in the range $[0, (a-c)/2]$. Hence, maximum overall profit $J(q_1^*, q_2^*)$ is $(a-c)^2/4$, and the two producers have their individual (local) profits as

$$J_1(q_1^*, q_2^*) = \frac{(a-c-2z)(a-c)}{4} \quad \text{and} \quad J_2(q_1^*, q_2^*) = \frac{(a-c)z}{2}.$$

Since $J(q_1^N, q_2^N) = J_1(q_1^N, q_2^N) + J_2(q_1^N, q_2^N) = 2(a-c)^2/9$, we obtain the system-wide surplus $K = J(q_1^*, q_2^*) - J(q_1^N, q_2^N) = (a-c)^2/36$.

To coordinate these two producers, we build a proper side-payment contract $(L(x_1,x_2), \gamma)$ for the Cournot game. It is found from (5) that the profit functions $J_1(q_1,q_2)$ and $J_2(q_1,q_2)$ are decreasing in q_2 and q_1, respectively. This means that a producer benefits if the other producer reduces his/her quantity. Specifically, P_1 would guarantee a side-payment to P_2, in order to entice the latter to decrease her quantity q_2. Similarly, the producer P_2 would give a transfer payment to P_1 if P_1 reduces q_1. Recalling that $L(q_1,q_2)$ is transferred from P_1 to P_2, we write the side-payment function as $L(q_1,q_2) = \alpha q_1 + \beta q_2$, where $\alpha \geqslant 0$ and $\beta \leqslant 0$. Since $J_1(q_1,q_2)$ and $J_2(q_1,q_2)$ are concave in any point (x_1,x_2), we can use Theorem 2 to find the values of α and β.

Theorem 5. *In order to coordinate two producers in the Cournot game, we develop the side-payment contract as follows:*

$$L(q_1,q_2) = zq_1 + (z - \frac{a-c}{2})q_2 \quad \text{and} \quad \gamma = \frac{(a-c)^2 - 4z(a-c)}{8},$$

where $z \in [0, (a-c)/2]$.

Under the side-payment contract specified in Theorem 5, two producers choose their equilibrium solutions that are identical to the globally optimal solutions, and they are both better off than in the non-cooperative situation. One may notice that the contract development depends on z. For each range in which the value of z falls, we next discuss the implementation of corresponding side-payment scheme below.

Remark 1. In order to coordinate two producers in the Cournot game, we choose a value of z and compute the corresponding $L(q_1,q_2)$ and γ for the side-payment contract. We discuss the constant term γ for different values of z. The surpluses of two producers are found as

$$K_1 = J_1(x_1^*, x_2^*) - L(x_1^*, x_2^*) - J_1(x_1^N, x_2^N) = \frac{(a-c)[5(a-c) - 18z]}{36},$$
$$K_2 = J_2(x_1^*, x_2^*) + L(x_1^*, x_2^*) - J_2(x_1^N, x_2^N) = \frac{(a-c)[9z - 2(a-c)]}{18},$$

where $z \in [0, (a-c)/2]$. We discuss the following three cases:

Case 1: $5(a-c)/18 \leqslant z \leqslant (a-c)/2$. It follows that $5(a-c) - 18z \leqslant 0$ and $9z - 2(a-c) \geqslant 0$. Thereby, $K_1 \leqslant 0$ and $K_2 \geqslant 0$, so producer P_1 has a deficit whereas P_2 has a surplus. We find from Theorem 5 that $\gamma = (a-c)[(a-c) - 4z]/8$. Since $a > c$ and

$$(a-c) - 4z = \frac{5(a-c) - 20z}{5} \leqslant \frac{5(a-c) - 18z}{5} \leqslant 0,$$

the constant transfer γ from P_1 to P_2 is non-positive, i.e., $\gamma \leqslant 0$. This means that P_1 gets the side-payment $|\gamma|$ from P_2.

Case 2: $2(a-c)/9 \leqslant z \leqslant 5(a-c)/18$. Both P_1 and P_2 have positive surpluses, i.e., $K_1 \geqslant 0$ and $K_2 \geqslant 0$. For this case, γ could be positive or negative, which depends on the value of z.
    1. When $2(a-c)/9 \leqslant z \leqslant (a-c)/4$, γ is positive. Thus, P_1 has a greater surplus than P_2, and transfers γ to P_2.
    2. When $(a-c)/4 \leqslant z \leqslant 5(a-c)/18$, γ is negative and P_1 receives $|\gamma|$ from P_2.

Case 3: $0 \leqslant z \leqslant 2(a-c)/9$. For this case, P_1 has a surplus whereas P_2 has a deficit. The value of γ is positive, so P_1 gives the side-payment γ to P_2.

4.1.2. Bertrand game with a side-payment contract

Bertrand [7] investigated the game problem of two firms (R_1 and R_2) making their pricing decisions (p_1 and p_2) for demands in a market. In the market, demand is a linear decreasing function dependent of both p_1 and p_2. Consequently, the demand function is written as $q = a - p_1 - p_2$, where $a \geqslant p_1 + p_2$. Bertrand game assumed that these two firms have an identical unit production/purchase cost c, which is no greater than p_1 and p_2, i.e., $c \leqslant p_i$, $i = 1, 2$. Since $a \geqslant p_1 + p_2$ and $p_i \geqslant c$, we find that $a \geqslant 2c$. Moreover, in this game, the market demand q is allocated to the firm whose price is lower. When $p_1 = p_2$, each firm satisfies a half of total demand (i.e., $q/2$). Hence, these two firms' profit functions are

$$
J_1(p_1, p_2) = \begin{cases} (p_1 - c)q, & p_1 < p_2, \\ \frac{1}{2}(p_1 - c)q, & p_1 = p_2, \\ 0, & p_1 > p_2, \end{cases} \quad \text{and} \quad J_2(p_1, p_2) = \begin{cases} (p_2 - c)q, & p_2 < p_1, \\ \frac{1}{2}(p_2 - c)q, & p_2 = p_1, \\ 0, & p_2 > p_1, \end{cases} \tag{6}
$$

where $q = a - p_1 - p_2$. Bertrand [7] obtained the Nash equilibrium as $(p_1^N, p_2^N) = (c, c)$. As a result, both firms have zero profits. The system-wide profit function is the sum of $J_1(p_1, p_2)$ and $J_2(p_1, p_2)$, i.e.,

$$
J(p_1, p_2) = \begin{cases} (p_1 - c)(a - p_1 - p_2), & p_1 < p_2, \\ (p_1 - c)(a - p_1 - p_2), \text{or}, (p_2 - c)(a - p_1 - p_2), & p_1 = p_2, \\ (p_2 - c)(a - p_1 - p_2), & p_1 > p_2, \end{cases}
$$

which can be simplified to

$$
J(p_1, p_2) = \begin{cases} (p_1 - c)(a - p_1 - p_2), & p_1 \leqslant p_2, \\ (p_2 - c)(a - p_1 - p_2), & p_1 \geqslant p_2. \end{cases} \tag{7}
$$

Lemma 1. *The optimal solution maximizing $J(p_1, p_2)$ in (7) is obtained as $p_1^* = p_2^* = (a + 2c)/4$.*

Using Lemma 1, we compute the maximum system-wide profit as $J(p_1^*, p_2^*) = (2c - a)^2/8$. Recall that two firms have zero profits when they choose Nash equilibrium in the non-cooperative situation. The overall surplus is $K = (2c - a)^2/8$. Next, we develop an appropriate side-payment contract, under which the coordination of the two firms is induced.

Theorem 6. *The side-payment contract for coordinating two firms in Bertrand game is designed as follows:*

$$
L(p_1, p_2) = \begin{cases} \frac{1}{2}(p_1 - c)(a - p_1 - p_2), & \text{if } p_1 < p_2 \leqslant a - p_1, \\ 0, & \text{if } p_1 = p_2, \\ -\frac{1}{2}(p_2 - c)(a - p_1 - p_2), & \text{if } p_2 < p_1 \leqslant a - p_2, \end{cases} \quad \text{and} \quad \gamma = 0.
$$

Based on the above contract for Bertrand game, two firms agree that, if a firm chooses a lower price, then the firm should equally share his/her profit with the other firm. As a result, in Bertrand game with the side-payment contract, two firms always have a half of maximum system-wide profit. For this case, the side-payments is equivalent to a "revenue-sharing" scheme. As Table 1 indicates, there are some publications concerned with the applications of revenue-sharing contracts in supply chain management.

4.2. Side-payment contracts for two recent supply-chain games

In this section, we develop proper side-payment contracts to coordinate two recent supply chain games: a two-retailer (horizontal) supply chain game with substitutable products in Parlar [44] and a two-echelon (vertical) supply chain game in Cachon [11]. In [44], Parlar developed a non-cooperative game to investigate a supply chain in which two retailers sell their substitutable products to customers; and the author showed that these two retailers can cooperate to improve the chainwide performance. In Section 4.2.1 we shall apply our approach discussed in Section 3 to the development of a proper side-payment contract that can induce the coordination of the supply chain with substitutable products. In Section 4.2.2, we shall develop a side-payment contract that coordinates a two-echelon supply chain involving a supplier and a retailer. Cachon [11] analyzed a game model, and showed that the buyback, quantity-flexibility and sales-rebate contracts cannot coordinate the supply chain, and other specific contracts such as revenue-sharing and price-discount contracts can achieve supply chain coordination *only* under some conditions. We shall demonstrate that a properly-designed side-payment contract exists for supply chain coordination. Moreover, in our side-payment contract, a transfer payment term, called *price-margin compensation* scheme, can be used to solve the forward buying problem (i.e., the retailer responds to the reduction of the supplier's wholesale price, by increasing her own profit margin rather than accordingly decreasing her retail price and passing the benefit to end consumers.)

4.2.1. Side-payment contract in a two-retailer supply chain

Parlar [44] developed a game-theoretic model to investigate competition and cooperation between two retailers (R_1 and R_2) whose products are substitutable in a market. In a single-period (newsvendor) context, retailers R_1 and R_2 determine order quantities q_1 and q_2, respectively. The author showed that a unique Nash equilibrium exists and the system-wide performance, if the two retailers cooperate, can be improved. However, the author didn't provide any contractual scheme for coordinating these two retailers but only mentioned the importance of finding a proper scheme. To coordinate the two-retailer horizontal supply chain, we now develop a proper side-payment contract to minimize the chainwide cost while both retailers are better off compared to the non-cooperative situation without the supply chain coordination.

Each retailer's expected inventory-related cost is the sum of underage and overage costs for the single period. For the standard newsvendor model, the unit underage and overage costs of the retailer R_i ($i = 1, 2$) are denoted by c_i^u and c_i^o, respectively. Since these two retailers sell substitutable products, their unit overage costs (i.e., c_1^o and c_2^o) should not be significantly different; thus, we assume that

$c_1^o + c_1^u \geqslant c_2^o$ and $c_2^o + c_2^u \geqslant c_1^o$. We let X denote random demand for retailer R_1's products with c.d.f. $F(x)$ and p.d.f $f(x)$; and we let Y denote random demand for retailer R_2's product with c.d.f. $G(y)$ and p.d.f $g(y)$. Due to the substitutability of these two retailers' products, we assume that, when a retailer's products are sold out, the customers who initially arrive to the retailer may switch to the other retailer. Accordingly, we denote by $a \in [0, 1]$ the fraction of R_1's demand which will switch to R_2 when R_1's products are sold out; and denote by $b \in [0, 1]$ the fraction of R_2's demand which will switch to R_1 when R_2's products are sold out.

Retailer R_1's cost function $J_1(q_1; q_2)$ and retailer R_2's cost function $J_2(q_2; q_1)$ are developed as

$$J_1(q_1; q_2) = c_1^o[q_1 - x - b(y - q_2)^+]^+ + c_1^u(x - q_1)^+$$
$$= c_1^o\left[\int_0^{q_1}(q_1 - x)f(x)\,dxG(q_2) + \int_0^{q_1}\int_{q_2}^{\frac{q_1 + bq_2 - x}{b}}(q_1 + bq_2 - x - by)g(y)f(x)\,dy\,dx\right] + c_1^u\int_{q_1}^{\infty}(x - q_1)f(x)\,dx, \tag{8}$$

and

$$J_2(q_2; q_1) = c_2^o[q_2 - y - a(x - q_1)^+]^+ + c_2^u(y - q_2)^+$$
$$= c_2^o\left[\int_0^{q_2}(q_2 - y)g(y)\,dyF(q_1) + \int_0^{q_2}\int_{q_1}^{\frac{aq_1 + q_2 - y}{a}}(aq_1 + q_2 - ax - y)f(x)g(y)\,dx\,dy\right] + c_2^u\int_{q_2}^{\infty}(y - q_2)g(y)\,dy. \tag{9}$$

We first analyze the non-cooperative "simultaneous-move" game, in which retailers R_1 and R_2 choose Nash-equilibrium order quantities to minimize their individual inventory-related costs.

Theorem 7. *For the non-cooperative game without side-payments, the Nash equilibrium order quantities q_1^N and q_2^N are obtained by solving the following equations:*

$$\begin{cases} c_1^o \int_0^{q_1} G(\frac{q_1 + bq_2 - x}{b})f(x)\,dx + c_1^u F(q_1) = c_1^u, \\ c_2^o \int_0^{q_2} F(\frac{aq_1 + q_2 - y}{a})g(y)\,dy + c_2^u G(q_2) = c_2^u. \end{cases}$$

Next, we minimize the chainwide cost $J^C(q_1, q_2)$, which is the sum of $J_1(q_1; q_2)$ and $J_2(q_2; q_1)$, to obtain the globally-optimal solutions (q_1^*, q_2^*). This shall be used later for our contract development. The chainwide inventory-related cost function is given as

$$J^C(q_1, q_2) = J_1(q_1; q_2) + J_2(q_2; q_1)$$
$$= c_1^o\left[\int_0^{q_1}(q_1 - x)f(x)\,dxG(q_2) + \int_0^{q_1}\int_{q_2}^{\frac{q_1 + bq_2 - x}{b}}(q_1 + bq_2 - x - by)f(x)g(y)\,dx\,dy\right]$$
$$+ c_2^o\left[\int_0^{q_2}(q_2 - y)g(y)\,dyF(q_1) + \int_0^{q_2}\int_{q_1}^{\frac{aq_1 + q_2 - y}{a}}(aq_1 + q_2 - ax - y)f(x)g(y)\,dx\,dy\right]$$
$$+ c_1^u\int_{q_1}^{\infty}(x - q_1)f(x)\,dx + c_1^u\int_{q_2}^{\infty}(y - q_2)g(y)\,dy. \tag{10}$$

Theorem 8. *The globally optimal solutions (q_1^*, q_2^*) that minimize the chainwide cost $J^C(q_1, q_2)$ are obtained by solving the following two equations:*

$$\begin{cases} c_1^o \int_0^{q_1} G\left(\frac{q_1 + bq_2 - x}{b}\right)f(x)\,dx + c_2^o a \int_0^{q_2}\left[F(\frac{aq_1 + q_2 - y}{a}) - F(q_1)\right]g(y)\,dy = c_1^u[1 - F(q_1)], \\ c_2^o \int_0^{q_2} F(\frac{aq_1 + q_2 - y}{a})g(y)\,dy + c_1^o b \int_0^{q_1}\left[G\left(\frac{q_1 + bq_2 - x}{b}\right) - G(q_2)\right]f(x)\,dx = c_2^u[1 - G(q_2)]. \end{cases} \tag{11}$$

We now develop a proper linear side-payment contract to ensure Criterions 1 and 2. Since

$$\frac{\partial J_1(q_1; q_2)}{\partial q_2} = c_1^o b \int_0^{q_1}\left[G\left(\frac{q_1 + bq_2 - x}{b}\right) - G(q_2)\right]f(x)\,dx > 0, \tag{12}$$

retailer R_1's inventory-related cost $J_1(q_1; q_2)$ increases when retailer R_2 increases her order quantity q_2. This means that retailer R_1 can benefit from the reduction of retailer R_2's order quantity q_2. In order to entice retailer R_2 to reduce her order quantity. Thus, retailer R_1 would give a transfer payment to R_2, which depends on reduced amount of retailer R_2. Note that, if R_2's order quantity is above q_2^N, retailer R_1 would disagree on this contract and leave the coalition because R_2's order quantity is q_2^N in the non-cooperative case without side-payments. We accordingly develop this transfer function (from R_1 to R_2) as

$$T_1(q_2) \equiv \alpha(q_2^N - q_2),$$

where $\alpha > 0$, and q_2^N is obtained from Theorem 7. If retailer R_2's order size q_2 is greater than q_2^N, then $T_1(q_2) < 0$. This means that R_2 should pay a compensation to retailer R_1; otherwise, the retailer R_1 would leave the coalition. Similarly, since

$$\frac{\partial J_2(q_2; q_1)}{\partial q_1} = c_2^o a \int_0^{q_2}\left[F\left(\frac{aq_1 + q_2 - y}{a}\right) - F(q_1)\right]g(y)\,dy > 0, \tag{13}$$

retailer R_2 would give a transfer payment to entice R_1, in order to decrease his order quantity q_1. Thus, the side-payment term from R_2 to R_1 should be written as follows:

$$T_2(q_1) \equiv \beta(q_1^N - q_1),$$

where $\beta > 0$, and q_1^N is obtained from Theorem 7. If retailer R_1's order quantity q_1 is greater than q_1^N, then $T_2(q_1) < 0$; this means that R_1 should give a transfer payment to R_2.

In order to satisfy Criterion 2, we need to compute a constant side-payment term $\gamma \in (-\infty, +\infty)$. We shall use Theorem 4 to compute the proper value of γ that coordinates the supply chain. Considering three side-payment terms $T_1(q_2)$, $T_2(q_1)$ and γ, we write the retailers' objective functions under cooperation as follows:

$$J_1^C(q_1, q_2) = J_1(q_1; q_2) - T_1(q_2) + T_2(q_1) - \gamma,$$
$$J_2^C(q_1, q_2) = J_2(q_2; q_1) + T_1(q_2) - T_2(q_1) + \gamma.$$

Theorem 9. *For the two-retailer supply chain with substitutable products, the side-payment contract that induces supply chain coordination should be designed as follows:*

$$\alpha = bc_1^o \int_0^{q_1^*} \left[G\left(\frac{q_1^* + bq_2^* - x}{b} \right) - G(q_2^*) \right] f(x)\,\mathrm{d}x,$$
$$\beta = ac_2^o \int_0^{q_2^*} \left[F\left(\frac{aq_1^* + q_2^* - y}{a} \right) - F(q_1^*) \right] g(y)\,\mathrm{d}y,$$
$$\gamma = \frac{[J_1^C(q_1^*, q_2^*) - J_1(q_1^N; q_2^N)] - [J_2^C(q_1^*, q_2^*) - J_2(q_2^N; q_1^N)]}{2}.$$

The contract development in Theorem 9 are justified as follows: we find from (12) that, when retailer R_2 decreases her order quantity q_2 by one unit, retailer R_1's cost can be reduced by $\partial J_1(q_1; q_2)/\partial q_2$. This means that the cost savings of retailer R_1 is generated by retailer R_2's decision rather than by retailer R_1's own effort. Thus, in order to reflect retailer R_2's "contribution", R_1 should transfer $\partial J_1(q_1^*; q_2^*)/\partial q_2 (= \alpha)$ to R_2. Similarly, we find that retailer R_2 should give the transfer payment $\partial J_2(q_2^*; q_1^*)/\partial q_1 (= \beta)$ to R_1, if R_1 reduces his order quantity by one unit.

4.2.2. Side-payment contract in a two-echelon supply chain

We next build a proper side-payment contract to coordinate a two-echelon supply chain where a supplier determines his wholesale price and a retailer chooses her retail price and order quantity. These two supply chain members aim at maximizing their individual expected profits in the newsvendor setting. In [11], Cachon investigated whether or not a contract can be developed to coordinate the quantity and pricing decisions in the supply chain. It was assumed that the wholesale price is not the supplier's decision variable and that the supplier has ample capacity to fulfill the retailer's order under *forced compliance*. The author showed that buybacks, quantity-flexibility and sales-rebate contracts cannot coordinate the supply chain; the revenue-sharing contract can induce the coordination only if the lost-sale goodwill penalties are not considered; price discount contract can coordinate the supply chain only when the supplier doesn't incur lost-sale goodwill penalties. Cachon assumed that, if supply chain coordination is achieved, the supplier and the retailer *arbitrarily* allocate maximum chainwide profit. Next, we shall develop another side-payment contract for coordinating the two-echelon supply chain. Different from Cachon [11], we treat the wholesale price as the supplier's decision variable; this is a more realistic case since the wholesale price should be, in practice, endogenously determined rather than exogenously given. In addition, for our contract development, we consider the impact of trade promotion and forward buying on supply chain performance.

Our study on trade promotion and forward buying is motivated by the following evidence: In recent years, when a manufacturer reduces his wholesale price, some retailers don't accordingly reduce their retail prices but utilize the manufacturer's offer to improve their own profit margins. Such a response is called *forward buying* (see, for example, [10] and [18]), and it is troubling the manufacturers since the manufacturers cannot pass the benefits of price reduction to end customers through the retailers. In the consumer packaged goods (CPG) industry, trade promotion is extensively applied, as surveyed by the Gelco Trade Management Group[3]. Ailawadi et al. [1] stated that trade-promotion expenditure in the industry increased from 1983 to 1994, and its budget was more than twice the media advertising budget. Beckwith, the Gelco's President & CEO, described CPG manufacturers' two major concerns on trade promotion in a talk with the Grocery Manufacturers Association (GMA) Forum. The manufacturers are concerned with maintaining the product differentiation and passing the benefits of promotion to consumers through their retailers. More specifically, if a manufacturer provides a trade promotion (e.g., wholesale-price reduction), then the retailers may not share the promotion with consumers but use the offer to increase their own margins. On the other hand, without the trade promotion, the product differentiation cannot be maintained so that the power of brands is dimmed and the cheaper products win the consumers.

To encourage the use of trade promotion but avoid the forward buying, Ailawadi et al. [1] suggested that the manufacturers could coordinate their supply chains by using *price-up deal-down* strategies that link the wholesale price to the retail price. Using the strategies, Bernstein and Federgruen [6] introduced a linear price-discount sharing (PDS) scheme to a game model, and investigated the equilibrium behavior of decentralized supply chains involving N competing retailers in the newsvendor setting. The authors showed that the properly-designed linear PDS and buyback contracts can induce supply chain coordination. However, in [6] the wholesale price is not treated as the supplier's decision variable but it is exogenously determined by a linear price-discount function; that is, the wholesale price is linearly dependent of the retail price. In this section, we consider the forward buying problem and develop a side-payment contract to coordinate the supply chain in which the supplier makes his wholesale pricing decision.

For our analysis, we use the game model developed by Cachon [11], where w, p and q denote the supplier's wholesale price, the retailer's price and order quantity, respectively. The supplier's profit function $\pi_s(w; p, q)$ and the retailer's profit function $\pi_r(p, q; w)$ are as follows:

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[3] http://gelcotrade.com/news/2007/20070424-BestInClass.shtml. (Last accessed March 2008.)

$$\pi_s(w; p, q) = g_s\left[q - \int_0^q F(y|p)\, \mathrm{d}y\right] + (w - c_s)q - g_s\mu, \tag{14}$$

$$\pi_r(p, q; w) = (p - v + g_r)\left[q - \int_0^q F(y|p)\, \mathrm{d}y\right] - (w + c_r - v)q - g_r\mu, \tag{15}$$

where v is the retailer's unit salvage value at the end of this period; g_s and g_r are the supplier's and the retailer's lost-sales goodwill penalty costs, respectively; c_s and c_r respectively denote the supplier's unit production cost and the retailer's marginal (purchasing management-related) cost per unit such that $c_s \leqslant w$ (which ensures the supplier's non-negative profit) and $w + c_r \leqslant p$ (which ensures the retailer's non-negative profit); $F(y|p)$ is the continuous probability function (c.d.f.) of the demand that decreases stochastically in price p with mean value μ; that is, $\partial F(y|p)/\partial p > 0$. For more discussions on random demand functions in the newsvendor setting, see, for example, Petruzzi and Dada [46].

The system-wide profit function $\Pi(p, q)$ is the sum of $\pi_s(w; p, q)$ in (14) and $\pi_r(p, q; w)$ in (15), i.e.,

$$\Pi(p, q) = (p - v + g)\left[q - \int_0^q F(y|p)\, \mathrm{d}y\right] - (c - v)q - g\mu, \tag{16}$$

where $g \equiv g_s + r_r$ and $c \equiv c_s + c_r$. As Cachon [11] assumed, $v \leqslant c < p$. We notice that, since the wholesale price w is transferred from the supplier to the retailer, it is not involved in the system-wide profit (16). As Petruzzi and Dada [46] showed, the chainwide profit $\Pi(p, q)$ don't need to be concave or unimodal, and the optimal solutions p^* and q^* that maximize $\Pi(p, q)$ in (16) must exist.

We now compute the supplier's and the retailer's Nash equilibrium solutions (w^N, p^N, q^N) for the "simultaneous-move" game without side-payments, which is denoted by $\mathscr{G}$. Since $\partial \pi_s(w; p, q)/\partial w = q > 0$ and $w \leqslant p - c_r$, the supplier's best-response solution w^{BR} for any given retail price p is $w^{BR} = p - c_r$, which results in the retailer's zero profit margin under *forced compliance*. One may note that, for this case, the retailer could not make any purchase from the supplier. To solve this problem, we assume that the supplier agrees to choose a wholesale price no more than a *base* value $w^0 < p - c_r$; for an introduction to the base wholesale price w^0, see Bernstein and Federgruen [6]. When the supplier uses this approach to entice the retailer to order items from the supplier, the retailer's profit margin for each sold item should be no less than $p - w^0 - c_r$. We now momentarily assume that the value of w^0 is given *a priori*; but, we later discuss the value of w^0 when building a proper side-payment contract for supply chain coordination. Therefore, we have that $w^N = w^0$. As Cachon [11] discussed, the retailer's equilibrium solutions satisfy the following conditions:

$$\frac{\partial \pi_r(p^N, q^N; w^N)}{\partial q} = (p^N - v + g_r)[1 - F(q^N|p^N)] - (w^0 + c_r - v) = 0,$$

$$\frac{\partial \pi_r(p^N, q^N; w^N)}{\partial p} = q^N - \int_0^{q^N} F(y|p^N)\, \mathrm{d}y - (p^N - v + g_r)\int_0^{q^N} \frac{\partial F(y|p^N)}{\partial p}\, \mathrm{d}y = 0.$$

Next, we consider the development of a side-payment contract that can coordinate the supplier and the retailer in the supply chain. In order to attain proper transfer payments between these two members, we need to discuss the impacts of each member's decisions on the profits of both members. Similar to Cachon [11], we find that the supplier's expected profit $\pi_s(w; p, q)$ increases in the retailer's order quantity q since $\partial \pi_s(w; p, q)/\partial q = g_s[1 - F(q|p)\, \mathrm{d}y] + (w - c_s) > 0$. Therefore, the supplier should encourage the retailer to place a larger order; but, ordering more items from the supplier may result in a higher holding cost at the retailer. To entice the retailer to increase her order size, the supplier would make a transfer payment to the retailer. Note that, if these two supply chain members cannot reach an agreement on this transfer payment, then they would not join a coalition for supply chain coordination and instead choose their equilibrium solutions (w^N, p^N, q^N) for the non-cooperative game $\mathscr{G}$. Consequently, the supplier should be willing to make the transfer payment only when the retailer's order side is greater than q^N. We thus write this transfer payment term (given by the supplier to the retailer) as

$$T_1(q) = \alpha(q - q^N), \tag{17}$$

where the parameter $\alpha > 0$ and it represents the supplier's transfer amount (in \$) when the retailer increases her order quantity from q^N units to $q^N + 1$ units. If the retailer's order side q is less than q^N, then the term $T_1(q)$ is negative; this means that a smaller order makes the supplier worse and the retailer should transfer a payment to compensate the supplier's loss.

It is easy to find from (14) that the supplier's expected profit $\pi_s(w; p, q)$ decreases in the retailer's pricing decision p since

$$\frac{\partial \pi_s(w; p, q)}{\partial p} = -g_s \int_0^q \frac{\partial F(y|p)}{\partial p}\, \mathrm{d}y < 0; \tag{18}$$

and find from (15) that the retailer's expected profit $\pi_r(p, q; w)$ decreases in the supplier's wholesale price w since $\partial \pi_r(p, q; w)/\partial w = -q < 0$. Thus, the supplier (the retailer) hopes that the retailer (the supplier) can reduces her retail (his wholesale) price. To encourage the retailer to decrease her retail price p, the supplier would make the side-payment to the retailer according to her reduction in the price p. Since, in the non-cooperative case, the retailer's equilibrium price is p^N, we write the side-payment transferred from the supplier to the retailer

$$T_{21}(p) = \beta_1(p^N - p), \tag{19}$$

where the parameter $\beta_1 > 0$ and it denotes the supplier's transfer amount (in \$) when the retailer decreases her retail price by \$1 from p^N to $p^N - 1$. If the retailer increases her price, then the transfer term $T_{21}(p)$ is negative and the retailer pays the amount $|T_{21}(p)|$ to supplier. This is justified as follows: When $p > p^N$, the supplier's profit is worse than that in the non-cooperative case without side-payments and the supplier would, as a result, leave the coalition. For the coalition stability, the retailer should make a transfer payment so as to entice the supplier to stay in the coalition.

We consider the transfer payment from the retailer to the supplier, which encourages the supplier to decrease his wholesale price w. Similar to the term $T_{21}(p)$ in (19), we write this transfer term as

$$T_{22}(w) = \beta_2(w^N - w) = \beta_2(w^0 - w), \tag{20}$$

where the parameter $\beta_2 > 0$ and it denotes the retailer's transfer amount (in \$) when the supplier decreases his wholesale price by \$1 from w^0 to $w^0 - 1$. If the supplier increases his wholesale price w, then the transfer term $T_{22}(w)$ is negative and the supplier pays the amount $|T_{22}(p)|$ to compensate the retailer's loss of profit.

We introduce the side-payments $T_1(q)$, $T_{21}(p)$ and $T_{22}(w)$ and a constant transfer term γ to the supply chain, analyze the game with side-payments, and then find proper values of contract parameters that induce supply chain coordination. In the "simultaneous-move" game with side-payments, denoted by $\mathscr{G}^C$, the supplier's and the retailer's expected profit functions are developed as:

$$\pi_s^C(w; p, q) = \pi_s(w; p, q) - \alpha(q - q^N) - \beta_1(p^N - p) + \beta_2(w^0 - w) - \gamma,$$
$$\pi_r^C(p, q; w) = \pi_r(p, q; w) + \alpha(q - q^N) + \beta_1(p^N - p) - \beta_2(w^0 - w) + \gamma.$$

Here, we use the superscript "C" to represent the cooperative case with side-payments. A significant advantage of our side-payment contract is that the involvement of the transfer payments $T_{21}(p)$ and $T_{22}(w)$ can prevent the occurrence of *forward buying* in the two-echelon supply chain. Our justification is given as follows: We compute

$$T_{21}(p) - T_{22}(w) = \beta_1(p^N - p) - \beta_2(w^0 - w) = (\beta_1 p^N - \beta_2 w^0) - (\beta_1 p - \beta_2 w).$$

For simplicity, we assume that $\beta \equiv \beta_1 = \beta_2$; this actually reflects the "symmetric" situation in which the supplier and the retailer bargain with each other for their compensations due to the reductions of the prices w and p. Letting $T_2(w, p) \equiv T_{21}(p) - T_{22}(w)$, we have

$$T_2(w, p) = \beta[(p^N - w^0) - (p - w)], \tag{21}$$

where $(p^N - w^0)$ and $(p - w)$ mean the profit margins of the retailer in the games $\mathscr{G}$ and $\mathscr{G}^C$, respectively. The profit margin $(p^N - w^0)$ for the game $\mathscr{G}$ (without side-payments) is considered as the retailer's *base* profit margin. If the retailer, in the game $\mathscr{G}^C$, responds to the reduction of the supplier's wholesale price w by accordingly decreasing her retail price p, then the supplier should make a transfer payment $T_2(w, p)$ to the retailer so as to compensate the latter's loss of profit margin. Otherwise, if the wholesale price w is decreased but the retailer doesn't reduce her retail price p, then the retailer's own profit margin $(p - w)$ increases and the forward buying phenomenon (i.e., the supplier's price-reduction promotion cannot be passed to consumers through the retailer) thus happens. In the forward-buying case, $T_2(w, p) < 0$, which means that the retailer should give a penalty payment $|T_2(w, p)|$ to the supplier. The term $T_2(w, p)$ in our contract is called a *profit-margin compensation* (PMC) scheme, which is explained as follows: The supplier gives the compensation \$$\beta$ to the retailer if the latter chooses a retail price to reduce her profit margin by \$1.

Using the PMC scheme $T_2(w, p)$ in (21), we re-write the supplier's and the retailer's profit functions $\pi_s^C(w; p, q)$ and $\pi_r^C(p, q; w)$ (in the game $\mathscr{G}^C$) as

$$\pi_s^C(w; p, q) = \pi_s(w; p, q) - \alpha(q - q^N) - \beta[(p^N - w^0) - (p - w)] - \gamma, \tag{22}$$
$$\pi_r^C(p, q; w) = \pi_r(p, q; w) + \alpha(q - q^N) + \beta[(p^N - w^0) - (p - w)] + \gamma. \tag{23}$$

Note that, for this specific supply chain game, the transfer function $L(w, p, q)$ defined in (1) is the sum of the transfer terms $T_1(q)$ and $T_2(w, p)$, i.e., $L(w, p, q) = T_1(q) + T_2(w, p)$. For the game $\mathscr{G}^C$ with side-payment terms $T_1(q)$, $T_2(w, p)$ and γ, the retailer's Nash equilibrium (p^{CN}, q^{CN}) must satisfy the following first-order conditions:

$$\frac{\partial \pi_r^C(p^{CN}, q^{CN}; w^{CN})}{\partial q} = (p^{CN} - v + g_r)[1 - F(q^{CN}|p^{CN})] - (w^0 + c_r - v) + \alpha = 0, \tag{24}$$

$$\frac{\partial \pi_r^C(p^{CN}, q^{CN}; w^{CN})}{\partial p} = q^{CN} - \int_0^{q^{CN}} F(y|p^{CN})\,dy - (p^{CN} - v + g_r)\int_0^{q^{CN}} \frac{\partial F(y|p^{CN})}{\partial p}\,dy - \beta = 0. \tag{25}$$

Theorem 10. *In order to coordinate the two-echelon supply chain, we should choose the contract parameters (α, β and γ) as follows:*

$$\alpha = g_s[1 - F(q^*|p^*)] + w^0 - c_s, \tag{26}$$

$$\beta = g_s \int_0^{q^*} \frac{\partial F(y|p^*)}{\partial p}\,dy, \tag{27}$$

$$\gamma = \frac{[\pi_s^C(w^{CN}; p^{CN}, q^{CN}) - \pi_s(w^N; p^N, q^N)] - [\pi_r^C(p^{CN}, q^{CN}; w^{CN}) - \pi_r(p^N, q^N; w^N)]}{2}, \tag{28}$$

where the retailer's equilibrium decisions (p^{CN}, q^{CN}) are identical to the globally-optimal solutions (p^*, q^*); and

$$w^{CN} = \begin{cases} w^0, & \text{if } q^{CN} > \beta, \\ c_s, & \text{if } q^{CN} < \beta, \\ \text{an arbitrary value in the set } [c_s, w^0], & \text{if } q^{CN} = \beta. \end{cases}$$

In Theorem 10, we find that the value of w^* depends on the comparison of q^* and β. From (27) the value of β increases in the supplier's unit lost-sale goodwill cost g_s. If $g_s = 0$ (which means that the retailer absorbs all of the lost-sale costs), then the supplier's optimal wholesale price is $w^* = w^0$. We specify managerial insights on α in (26) and β in (27) below.

The parameter $\alpha = g_s[1 - F(q^*|p^*)] + w^0 - c_s$. We notice from (26) that α is equal to the sum of the supplier's lost-sales goodwill penalty cost ($g_s[1 - F(q^*|p^*)]$) and his profit margin ($w^0 - c_s$). The managerial insight for α is given as follows: When the retailer reduces his order quantity by one unit, the supplier loses the sale of one item, and he also incurs lost-sale goodwill cost if one unit of demand cannot be satisfied. Because the supplier's loss of profit and his goodwill cost occur because the retailer reduces her order quantity, the supplier should be compensated by the retailer.

The parameter $\beta = g_s \int_0^{q^*} [\partial F(y|p^*)/\partial p]\,dy$. According to (27) we find that the parameter β is related to the supplier's lost-sale goodwill cost. If the retailer decreases her price p by Section 1, then the supplier's profit $\pi_s(w; p, q)$, according to (18), is increased by $g_s \int_0^{q^*} [\partial F(y|p^*)/\partial p]\,dy$. This means that the supplier's profit surplus is yielded by the reduction of the retailer's price p. To reflect the retailer's contribution, the supplier should make the transfer payment to the retailer.

Since our contract development, as Theorem 10 indicates, depends on the value of the *base* wholesale price w^0, we need to discuss the selection of w^0. From Theorem 10, we find that the supplier's equilibrium wholesale pricing decision w^{CN}—for the game $\mathscr{G}^C$ with side-payments terms $T_1(q)$ and $T_2(w, p)$—equals the supplier's unit production cost c_s when $q^{CN} < \beta$ or, alternatively, $q^* < \beta$, thus leading a non-positive profit to the supplier who would then lose an incentive to cooperate with the retailer for supply chain coordination. To discourage the supplier from leaving the coalition, the retailer should agree to consider the following contract term: The selection of the base wholesale price w^0 must satisfy the condition $q^{CN} \geqslant \beta$. Note that the value of q^{CN} is dependent of w^0, as shown in (24). Some other contracts such as buyback (see Cachon [11]) and constant wholesale-pricing contracts (see Bernstein and Federgruen [6]) require that $w^0 = c_s$ and the supplier obtains a non-positive profit for supply chain coordination. According to our above discussion, we find that our side-payment contract (involving a *profit-margin compensation* scheme) can be developed to coordinate the two-echelon supply chain, make two members better off than in the non-cooperative case, and also prevent the occurrence of forward buying problem.

5. Summary and concluding remarks

In many supply chains, it is worthwhile to investigate the coordination of bargaining firms for improving system-wide performance. In this paper, we provide our review, discussion and applications of side-payment contracts in two-person nonzero-sum supply chain games. Under supply chain coordination, the chainwide payoff should be maximized and both players are better off than in a non-cooperative situation.

Based on our review and discussion, we conclude that a side-payment inducing supply chain coordination must satisfy two criteria. Accordingly, we introduce two payment components to a nonzero-sum game, i.e., a transfer function $L(x_1, x_2)$ and a constant transfer term γ. The conditions that the properly-designed $L(x_1, x_2)$ must satisfy are presented. We utilize two solution concepts, Nash arbitration scheme and Shapley value, to derive the closed-form solution of γ. We show that both concepts suggest equal allocation, and derive a closed-form formula for γ. To illustrate our analytical results, we apply our side-payment arrangement to four supply chain games: two classical games (i.e., Cournot and Bertrand games) and two recent supply chain games. To coordinate two producers in Cournot game, we develop a linear transfer function and compute the value of the constant term γ. For Bertrand game, we derive a nonlinear side-payment function to coordinate two firms, and show that the scheme is equivalent to a revenue-sharing contract. After introducing the nonlinear function, we find that the surpluses of two firms are equal so that the constant transfer term γ is zero. In addition, we extend Parlar's game model [44] and develop a linear side-payment contract to coordinate two retailers who sell substitutable products to a market. Using Cachon's model [11], we derive a proper side-payment scheme that can induce supply chain coordination and also help solve the forward buying problem in a supply chain involving a supplier and a retailer. These four applications demonstrates the significant importance of side-payment contracts to the coordination for two-person nonzero-sum games.

In this paper we perform the analysis of side-payment contracts that are assumed to be legally possible in practice. Under this assumption we can only focus on how a proper contract should be developed to achieve supply chain coordination. However, this assumption is a limitation of our research because, as we mentioned in Section 1, some side-payment contracts may not be allowed by law. For instance, Plattner [47] reported that in a public market, some vendors could make illegal side-payments to stall owners in order to "purchase" additional stalls and thus "gain" greater market share; this behavior is prohibited since it results in the illegal collusion and the deterioration of "fair" competition. Because the legal possibility of side-payment contracts is important and worth investigating, a potential research topic should be concerned with the legal boundaries in side-payment contracts.

As another potential research, we could consider the application of side-payment approach to N-player games where $N \geqslant 3$. For this case, we may need to construct a cooperative game in a characteristic-value form, use some solution concepts (e.g., the core, Shapley value, nucleolus solution, etc.) in the theory of cooperative games to allocate the payoffs among the N players, and compute side-payments to implement the allocation.

Acknowledgements

Appendix A. Proofs

Proof of Theorem 1. According to Criterion 1, the globally optimal solution (x_1^*, x_2^*), which maximizes $J(x_1, x_2) = \sum_{i=1}^2 J_i(x_1, x_2) = \sum_{i=1}^2 \bar{J}_i(x_1, x_2)$, equals equilibrium solution for the game with $L(x_1, x_2)$. That is, (x_1^*, x_2^*) must satisfy the inequalities that $J(x_1^*, x_2^*) \geqslant J(x_1, x_2)$; $\bar{J}_1(x_1^*, x_2^*) \geqslant \bar{J}_1(x_1, x_2^*)$ and $\bar{J}_2(x_1^*, x_2^*) \geqslant \bar{J}_2(x_1^*, x_2)$, for all (x_1, x_2). In conclusion, we arrive to the result. $\square$

Proof of Theorem 2. When both $J_1(x_1, x_2)$ and $J_2(x_1, x_2)$ are concave in (x_1, x_2), the Hessian of $J_i(x_1, x_2)$, denoted by $H[J_i(x_1, x_2)]$, is negative semi-definite for all (x_1, x_2), $i = 1, 2$. Hence, the globally optimal solution (x_1^*, x_2^*) is found as

$$(x_1^*, x_2^*) = \left\{ (x_1, x_2) \Big| \frac{\partial J_1(x_1, x_2)}{\partial x_1} + \frac{\partial J_2(x_1, x_2)}{\partial x_1} = 0, \frac{\partial J_1(x_1, x_2)}{\partial x_2} + \frac{\partial J_2(x_1, x_2)}{\partial x_2} = 0. \right\} \tag{29}$$

For the game with payoff functions $\bar{J}_i(x_1, x_2)$ $(i = 1, 2)$, we introduce a concave transfer function $L(x_1, x_2)$. Nash equilibrium $(\tilde{x}_1^N, \tilde{x}_2^N)$ for the game with the transfer function $L(x_1, x_2)$ is

$$(\tilde{x}_1^N, \tilde{x}_2^N) = \left\{ (x_1, x_2) \Big| \frac{\partial \bar{J}_1(x_1, x_2)}{\partial x_1} = 0, \frac{\partial \bar{J}_2(x_1, x_2)}{\partial x_2} = 0. \right\} = \left\{ (x_1, x_2) \Big| \frac{\partial J_1(x_1, x_2)}{\partial x_1} - \frac{\partial L(x_1, x_2)}{\partial x_1} = 0, \frac{\partial J_2(x_1, x_2)}{\partial x_2} + \frac{\partial L(x_1, x_2)}{\partial x_2} = 0. \right\}. \tag{30}$$

Equating $(\tilde{x}_1^N, \tilde{x}_2^N)$ in (30) to (x_1^*, x_2^*) in (29), we have

$$\frac{\partial L(x_1, x_2)}{\partial x_1} = -\frac{\partial J_2(x_1, x_2)}{\partial x_1} \quad \text{and} \quad \frac{\partial L(x_1, x_2)}{\partial x_2} = \frac{\partial J_1(x_1, x_2)}{\partial x_2}$$

in the point (x_1^*, x_2^*). Thus, we arrive to (2). $\square$

Proof of Theorem 3. Since the status quo point is $(0, 0)$, we reduce (3) to $\max_{f_1 \geqslant 0, f_2 \geqslant 0} f_1 f_2$, s.t., $(f_1, f_2) \in \mathscr{P} = \{(f_1, f_2) | f_1 + f_2 = K\}$. Solving the problem gives the result. $\square$

Proof of Theorem 4. For our game only with the transfer function $L(x_1, x_2)$, the equilibrium solutions chosen by two players are identical to the globally optimal solutions, and the surpluses of the two players are computed as $K_i = \bar{J}_i(x_1^*, x_2^*) - J_i(x_1^N, x_2^N)$, $i = 1, 2$. If a player experiences the deficit (as his/her surplus is negative), we first fill the deficit for the player. Then, according to Theorem 3, the surplus K is equally allocated to these two players. Note that γ is defined as the constant side-payment given by player 1 to player 2. We compute it by analyzing three cases below.

1. If player 1 has deficit $(K_1 < 0)$, he gets the amount $|K_1| (= -K_1)$ from player 2 to fill his deficit, and then receives the allocation $K/2$. Now, K can be thought of as player 2's leftover. (Note that $K = K_1 + K_2 \geqslant 0$ since the overall payoff in terms of (x_1^*, x_2^*) is no less than that in (x_1^N, x_2^N).) As a result, we obtain γ as $\gamma = -(|K_1| + K/2) = (K_1 - K_2)/2$, which is negative and implies that player 1 gets the side-payment $|\gamma|$ from player 2.
2. If player 2 has deficit $(K_2 < 0)$, player 1 first transfers $|K_2| (= -K_2)$ to player 2; thus, the surplus K for which two players bargain is considered as player 1's leftover. According to Theorem 3, player 1 further transfers $K/2 (= (K_1 + K_2)/2)$ to player 2. The constant term γ is computed as $\gamma = |K_2| + (K_1 + K_2)/2 = (K_1 - K_2)/2$, which is positive for the case, and represents the transfer payment from player 1 to player 2.
3. If no player has deficit $(K_i \geqslant 0, i = 1, 2)$, two players directly bargain for $K = K_1 + K_2$ and each gets $(K_1 + K_2)/2$. If $K_1 \geqslant K_2$, then the side-payment that player 1 gives to player 2 is $\gamma = K_1 - (K_1 + K_2)/2 = (K_1 - K_2)/2$; otherwise, $\gamma = -[K_2 - (K_1 + K_2)/2] = -(K_2 - K_1)/2$, which means that player 1 receives the amount of $(K_2 - K_1)/2$ from player 2. For this case, the constant side-payment is $\gamma = (K_1 - K_2)/2$.

Summarizing the above results, we can write $\gamma = (K_1 - K_2)/2$. Using the equality that $K_i = \bar{J}_i(x_1^*, x_2^*) - J_i(x_1^N, x_2^N)$, $i = 1, 2$, we can reach (4). $\square$

Proof of Theorem 5. From Theorem 2, α and β in $L(q_1, q_2)$ are obtained as

$$\alpha = -\frac{\partial J_2(q_1^*, q_2^*)}{\partial q_1} = z \quad \text{and} \quad \beta = \frac{\partial J_1(q_1^*, q_2^*)}{\partial q_2} = z - \frac{a - c}{2}.$$

According to Theorem 4 we compute the value of constant term γ as follows:

$$\gamma = \frac{\sum_{i=1}^{2}(-1)^{i+1}[J_i(x_1^*, x_2^*) + (-1)^i L(x_1^*, x_2^*) - J_i(x_1^N, x_2^N)]}{2} = \frac{\left[\frac{(a-c-2z)(a-c)}{4} - \frac{(a-c)^2}{9}\right] - \left[\frac{(a-c)z}{2} - \frac{(a-c)^2}{9}\right]}{2} = \frac{(a-c)^2 - 4z(a-c)}{8}. \quad \square$$

Proof of Lemma 1. When $p_1 \leqslant p_2$, the profit function $J(p_1, p_2)$ is $(p_1 - c)(a - p_1 - p_2)$, which is decreasing in p_2 as $\partial[(p_1 - c)(a - p_1 - p_2)]/\partial p_2 = c - p_1 < 0$. Therefore, we should decrease the price p_2, in order to increase the profit. For this case, the function $(p_1 - c)(a - p_1 - p_2)$ is maximized when p_2 is reduced to p_1.

When $p_1 \geqslant p_2$, $J(p_1, p_2) = (p_2 - c)(a - p_1 - p_2)$. Since $\partial[(p_2 - c)(a - p_1 - p_2)]/\partial p_1 = c - p_2 < 0$, decreasing the price p_1 can raise the profit $(p_2 - c)(a - p_1 - p_2)$, which reaches the maximum when p_1 is decreased to p_2.

Thus, the problem of maximizing $J(p_1, p_2)$ is equivalent to the constrained maximization problem

$$\max \quad (p_1 - c)(a - p_1 - p_2)$$
$$\text{s.t.} \quad p_1 = p_2, a \geqslant p_1 + p_2, p_1 \geqslant c \quad \text{and} \quad p_2 \geqslant c.$$

When we ignore $a \geqslant p_1 + p_2$ and $p_i \geqslant c$ $(i = 1, 2)$ and replace p_2 with p_1, the objective function is changed to $(p_1 - c)(a - 2p_1)$. The first-order derivative of this function w.r.t. p_1 is

$$\frac{d[(p_1 - c)(a - 2p_1)]}{dp_1} = -4p_1 + a + 2c. \tag{31}$$

Eq. (31) to zero and solving it for p_1, we have $p_1 = (a + 2c)/4$. Since $a \geqslant 2c$, the inequalities that $a \geqslant p_1 + p_2$ and $p_i \geqslant c$ are satisfied when $p_1 = p_2 = (a + 2c)/4$. Thus, we arrive to the globally optimal solution. $\square$

Proof of Theorem 6. From Bertrand game (6), we find that each firm attempts to choose a lower price to capture the whole market. Note that each firm's price must be greater than or equal to unit cost c. When $p_i = c$, firm i doesn't continue to decrease p_i; otherwise, the firm has a negative profit. Therefore, as Bertrand showed, Nash equilibrium for the game is (c, c), which results in zero profit for each firm. In order to win the whole market and obtain a positive profit, firm R_1 may agree to transfer a side-payment to firm R_2 if R_2 is willing to choose p_2 that is higher than p_1. In fact, when p_2 is decreased in the range $(p_1, a - p_1]$, the profit of the firm R_1 increases, since

$$\frac{\partial J_1(p_1, p_2)}{\partial p_2} = \frac{\partial[(p_1 - c)(a - p_1 - p_2)]}{\partial p_2} = c - p_1 \leqslant 0.$$

Thus, R_1 would give R_2 a side transfer so as to entice R_2 to decrease p_2 in the range. Likewise, firm R_2 would have a transfer payment to firm R_1, if R_1 decreases p_1 in the range $(p_2, a - p_2]$.

Based on our above discussion, we assume that R_1 shares his profit with R_2 for $p_2 \in (p_1, a - p_1]$, and R_2 transfers a portion of her profit to R_1 for $p_1 \in (p_2, a - p_2]$. So, we develop a nonlinear side-payment function for $L(p_1, p_2)$ (transferred from R_1 to R_2) as follows:

$$L(p_1, p_2) = \begin{cases} \alpha(p_1 - c)(a - p_1 - p_2), & \text{if } p_1 < p_2 \leqslant a - p_1, \\ 0 & \text{if } p_1 = p_2, \\ \beta(p_2 - c)(a - p_1 - p_2), & \text{if } p_2 < p_1 \leqslant a - p_2, \end{cases}$$

where $\alpha \geqslant 0$ and $\beta \leqslant 0$. Adding $L(p_1, p_2)$ to Bertrand game, we have

$$\bar{J}_1(p_1, p_2) = J_1(p_1, p_2) - L(p_1, p_2) = \begin{cases} \bar{J}_{11} = (1 - \alpha)(p_1 - c)(a - p_1 - p_2), & \text{if } p_1 < p_2 \leqslant a - p_1, \\ \bar{J}_{12} = \frac{1}{2}(p_1 - c)(a - p_1 - p_2), & \text{if } p_1 = p_2, \\ \bar{J}_{13} = -\beta(p_2 - c)(a - p_1 - p_2), & \text{if } p_2 < p_1 \leqslant a - p_2, \end{cases}$$

and

$$\bar{J}_2(p_1, p_2) = J_2(p_1, p_2) + L(p_1, p_2) = \begin{cases} \bar{J}_{21} = (1 + \beta)(p_2 - c)(a - p_1 - p_2), & \text{if } p_2 < p_1 \leqslant a - p_2, \\ \bar{J}_{22} = \frac{1}{2}(p_2 - c)(a - p_1 - p_2), & \text{if } p_2 = p_1, \\ \bar{J}_{23} = \alpha(p_1 - c)(a - p_1 - p_2), & \text{if } p_1 < p_2 \leqslant a - p_1. \end{cases}$$

We now solve Bertrand game with $L(p_1, p_2)$ to obtain the best response of each firm and find the Nash equilibrium $(\tilde{p}_1^N, \tilde{p}_2^N)$. Consider R_1's profit function $\bar{J}_1(p_1, p_2)$. For $p_1 \in (p_2, a - p_2]$, R_1 has the profit function $\bar{J}_{13}$, which is decreasing in p_1 because $\partial \bar{J}_{13}/\partial p_1 = \beta(p_2 - c) \leqslant 0$. The profit of firm R_1 increases when p_1 approaches to p_2. Note that when $p_1 = p_2$ and $-\beta \leqslant 1/2, \bar{J}_{13} \leqslant \bar{J}_{12}$. Thus, if $-\beta \leqslant 1/2$, R_1 reduces p_1 to p_2.

If $p_1 < p_2 \leqslant a - p_1$, firm R_1's profit function is $\bar{J}_{11}$. Partially differentiating $\bar{J}_{11}$ w.r.t. p_1 gives

$$\frac{\partial \bar{J}_{11}}{\partial p_1} = (1 - \alpha)(a + c - 2p_1 - p_2).$$

Equating the function to zero and solving it for p_1, we have $p_1 = (a + c - p_2)/2$. Considering the constraint $p_1 < p_2 \leqslant a - p_1$, we obtain R_1's optimal price as $\min[(p_2 - \varepsilon), (a + c - p_2)/2]$ where ε is a tiny positive number. When $\alpha = 1/2$, we find that $\bar{J}_{11}$ and $\bar{J}_{12}$ are identical, and R_1's optimal price is $\min[p_2, (a + c - p_2)/2]$. Furthermore, if $p_2 = p_1$, R_1's function $\bar{J}_{12}$ is reduced to $(p_1 - c)(a - 2p_1)/2$, which is maximized when $p_1 = (a + 2c)/4$. Thereby, p_2 is also equal to $(a + 2c)/4$. Since $a \geqslant 2c$, we find

$$p_2 - \frac{a + c - p_2}{2} = \frac{3p_2 - a - c}{2} = \frac{-a + 2c}{8} \leqslant 0.$$

Thus, $p_2 \leqslant (a + c - p_2)/2$, which implies that, when $\alpha = 1/2$, R_1's optimal price is $\min[p_2, (a + c - p_2)/2] = p_2$.

In conclusion, when $-\beta \leqslant 1/2$ and $\alpha = 1/2$, firm R_1 has his best response as $p_1^B = p_2$.

Similarly, when $-\beta = 1/2$ and $\alpha \leqslant 1/2$, we find the best response for firm R_2 as follows: $p_2^B = p_1$. Using our best-response analysis, we find that when $\alpha = -\beta = 1/2$, $(\tilde{p}_1^N, \tilde{p}_2^N) = ((a + 2c)/4, (a + 2c)/4)$. In order to make $(\tilde{p}_1^N, \tilde{p}_2^N)$ identical to (p_1^*, p_2^*) (which is given in Lemma 1), we choose the transfer payment $L(p_1, p_2)$ with $\alpha = -\beta = 1/2$.

Next, we compute the constant transfer payment γ. When two firms choose (p_1^*, p_2^*) in equilibrium, their profits are computed as: $\bar{J}_i(p_1^*, p_2^*) = (2c - a)^2/16$, $i = 1, 2$. Therefore,

$$K_1 = \bar{J}_1(x_1^*, x_2^*) - J_1(x_1^N, x_2^N) = \bar{J}_1(p_1^*, p_2^*) = \frac{(2c - a)^2}{16},$$

$$K_2 = \bar{J}_2(x_1^*, x_2^*) - J_2(x_1^N, x_2^N) = \bar{J}_2(p_1^*, p_2^*) = \frac{(2c - a)^2}{16}.$$

We find from Theorem 4 that $\gamma = K_1 - K_2 = 0$. This means that after involving the transfer function $L(p_1, p_2)$, both Criterions 1 and 2 are satisfied. $\quad\square$

Proof of Theorem 7. The first- and second-order partial derivatives of retailer R_1's cost function $J_1(q_1; q_2)$ w.r.t. q_1 are given as follows:

$$\frac{\partial J_1(q_1; q_2)}{\partial q_1} = c_1^o \int_0^{q_1} G\left(\frac{q_1 + bq_2 - x}{b}\right) f(x) \, dx - c_1^u[1 - F(q_1)],$$

$$\frac{\partial^2 J_1(q_1; q_2)}{\partial q_1^2} = c_1^o[f(q_1)G(q_2) + \frac{1}{b} \int_0^{q_1} g\left(\frac{q_1 + bq_2 - x}{b}\right) f(x) \, dx] + c_1^u f(q_1) > 0, \tag{32}$$

which implies that the function $J_1(q_1; q_2)$ is strictly convex in q_1. Therefore, by equating $\partial J_1(q_1; q_2)/\partial q_1$ in (32) to zero and solving it for q_1, we can find R_1's best-response function $q_1^{BR}(q_2)$ for any value of q_2.

Similarly, the first- and second-order partial derivatives of retailer R_2's cost function $J_2(q_2; q_1)$ w.r.t. q_2 are as follows:

$$\frac{\partial J_2(q_2; q_1)}{\partial q_2} = c_2^o \int_0^{q_2} F\left(\frac{aq_1 + q_2 - y}{a}\right) g(y) \, dy - c_2^u[1 - G(q_2)],$$

$$\frac{\partial^2 J_2(q_2; q_1)}{\partial q_2^2} = c_2^o[g(q_2)F(q_1) + \frac{1}{a} \int_0^{q_2} f\left(\frac{aq_1 + q_2 - y}{a}\right) g(y) \, dy] + c_2^u g(q_2) > 0, \tag{33}$$

which implies that the function $J_2(q_2; q_1)$ is strictly convex in q_2. Therefore, by equating $\partial J_2(q_2; q_1)/\partial q_2$ in (33) to zero and solving it for q_2, we can find R_2's best-response function $q_2^{BR}(q_1)$ for any value of q_1.

In order to obtain the Nash equilibrium solutions (q_1^N, q_2^N), we need to solve the following two equations: $q_1^{BR}(q_2) = q_1$ (or alternatively, $\partial J_1(q_1; q_2)/\partial q_1 = 0$) and $q_2^{BR}(q_1) = q_2$ (or alternatively, $\partial J_2(q_2; q_1)/\partial q_2 = 0$); for this approach, see Cachon and Netessine [13] and Leng and Parlar [37]. We thus reach the result. $\square$

Proof of Theorem 8. The first- and second-order partial derivatives of $J^C(q_1, q_2)$ w.r.t. q_1 are respectively given as

$$\frac{\partial J^C(q_1, q_2)}{\partial q_1} = c_1^o \int_0^{q_1} G(\frac{q_1 + bq_2 - x}{b})f(x)\,dx - c_1^u[1 - F(q_1)] + c_2^o a \int_0^{q_2} \left[F\left(\frac{aq_1 + q_2 - y}{a}\right) - F(q_1)\right]g(y)\,dy,$$

and

$$\frac{\partial^2 J^C(q_1, q_2)}{\partial q_1^2} = c_1^o \left[f(q_1)G(q_2) + \frac{1}{b}\int_0^{q_1} g\frac{q_1 + bq_2 - x}{b}f(x)\,dx\right] + c_1^u f(q_1) + c_2^o a \int_0^{q_2} \left[f\left(\frac{aq_1 + q_2 - y}{a}\right) - f(q_1)\right]g(y)\,dy$$

$$= (c_1^o - ac_2^o)f(q_1)G(q_2) + c_1^u f(q_1) + c_1^o \frac{1}{b}\int_0^{q_1} g\left(\frac{q_1 + bq_2 - x}{b}\right)f(x)\,dx + ac_2^o \int_0^{q_2} f\left(\frac{aq_1 + q_2 - y}{a}\right)g(y)\,dy$$

$$> (c_1^o + c_1^u - ac_2^o)f(q_1)G(q_2) + c_1^o \frac{1}{b}\int_0^{q_1} g\left(\frac{q_1 + bq_2 - x}{b}\right)f(x)\,dx + ac_2^o \int_0^{q_2} f\left(\frac{aq_1 + q_2 - y}{a}\right)g(y)\,dy > 0.$$

The first- and second-order derivatives of $J^C(q_1, q_2)$ w.r.t. q_2 are as follows:

$$\frac{\partial J^C(q_1, q_2)}{\partial q_2} = c_1^o b \int_0^{q_1} \left[G\left(\frac{q_1 + bq_2 - x}{b}\right) - G(q_2)\right]f(x)\,dx + c_2^o \int_0^{q_2} F\left(\frac{aq_1 + q_2 - y}{a}\right)g(y)\,dy - c_2^u[1 - G(q_2)],$$

and

$$\frac{\partial^2 J^C(q_1, q_2)}{\partial q_2^2} = c_1^o b \int_0^{q_1} \left[g\left(\frac{q_1 + bq_2 - x}{b}\right) - g(q_2)\right]f(x)\,dx + c_2^o\left[g(q_2)F(q_1) + \frac{1}{a}\int_0^{q_2} f\left(\frac{aq_1 + q_2 - y}{a}\right)g(y)\,dy\right] + c_2^u g(q_2)$$

$$= (c_2^o - bc_1^o)g(q_2)F(q_1) + c_2^u g(q_2) + bc_1^o \int_0^{q_1} g\left(\frac{q_1 + bq_2 - x}{b}\right)f(x)\,dx + c_2^o \frac{1}{a}\int_0^{q_2} f\left(\frac{aq_1 + q_2 - y}{a}\right)g(y)\,dy$$

$$> (c_2^o + c_2^u - bc_1^o)g(q_2)F(q_1) + bc_1^o \int_0^{q_1} g\left(\frac{q_1 + bq_2 - x}{b}\right)f(x)\,dx + c_2^o \frac{1}{a}\int_0^{q_2} f\left(\frac{aq_1 + q_2 - y}{a}\right)g(y)\,dy > 0.$$

Moreover, we obtain the mixed partial derivative of $J^C(q_1, q_2)$ as

$$\frac{\partial^2 J^C(q_1, q_2)}{\partial q_1 \partial q_2} = c_1^o \int_0^{q_1} g(\frac{q_1 + bq_2 - x}{b})f(x)\,dx + c_2^o \int_0^{q_2} f(\frac{aq_1 + q_2 - y}{a})g(y)\,dy.$$

In order to determine the convexity of the chainwide function $J^C(q_1, q_2)$, we need to find whether or not the Hessian matrix is positive definite. Note that the first principle minors $\partial J^C(q_1, q_2)/\partial q_i > 0$, for $i = 1, 2$. We compute the second principle minor as

$$\frac{\partial^2 J^C(q_1, q_2)}{\partial q_1^2}\frac{\partial^2 J^C(q_1, q_2)}{\partial q_2^2} - \left(\frac{\partial^2 J^C(q_1, q_2)}{\partial q_1 \partial q_2}\right)^2 = \left[bc_1^o \int_0^{q_1} g\left(\frac{q_1 + bq_2 - x}{b}\right)f(x)\,dx + \frac{1}{a}c_2^o \int_0^{q_2} f\left(\frac{aq_1 + q_2 - y}{a}\right)g(y)\,dy\right] \times [(c_1^o - ac_2^o)G(q_2)$$

$$+ c_1^u]f(q_1) + \left[\frac{1}{b}c_1^o \int_0^{q_1} g\left(\frac{q_1 + bq_2 - x}{b}\right)f(x)\,dx + ac_2^o \int_0^{q_2} f(\frac{aq_1 + q_2 - y}{a})g(y)\,dy\right] \times [(c_2^o$$

$$- bc_1^o)F(q_1) + c_2^u]g(q_2) + [(c_1^o - ac_2^o)G(q_2) + c_1^u][(c_2^o - bc_1^o)F(q_1) + c_2^u]f(q_1)g(q_2)$$

$$+ \left(\frac{1}{ab} + ab - 2\right)c_1^o c_2^o \left[\int_0^{q_1} g\left(\frac{q_1 + bq_2 - x}{b}\right)f(x)\,dx\right]\left[\int_0^{q_2} f\left(\frac{aq_1 + q_2 - y}{a}\right)g(y)\,dy\right].$$

Since $(c_1^o - ac_2^o)G(q_2) + c_1^u > (c_1^o + c_1^u - ac_2^o)G(q_2) > 0$, $(c_2^o - bc_1^o)F(q_1) + c_2^u > (c_2^o + c_2^u - bc_1^o)F(q_1) > 0$, and $1/ab + ab \geqslant 2$, we find that

$$\frac{\partial^2 J^C(q_1, q_2)}{\partial q_1^2}\frac{\partial^2 J^C(q_1, q_2)}{\partial q_2^2} > \left(\frac{\partial^2 J^C(q_1, q_2)}{\partial q_1 \partial q_2}\right)^2;$$

thus, $J^C(q_1, q_2)$ is jointly convex in q_1 and q_2, and the globally-optimal solutions (q_1^*, q_2^*) can be obtained by solving the following equations: $\partial J^C(q_1, q_2)/\partial q_i = 0$, $i = 1, 2$. It follows that the solutions (q_1^*, q_2^*) satisfy the equations (11). $\square$

Proof of Theorem 9. The first- and second-order derivatives of $J_1^C(q_1, q_2)$ w.r.t. q_1 are

$$\frac{\partial J_1^C(q_1, q_2)}{\partial q_1} = \frac{\partial J_1(q_1; q_2)}{\partial q_1} - \beta \quad \text{and} \quad \frac{\partial^2 J_1^C(q_1, q_2)}{\partial q_1^2} = \frac{\partial^2 J_1(q_1; q_2)}{\partial q_1^2} > 0;$$

and the first- and second-order derivatives of $J_2^C(q_1, q_2)$ w.r.t. q_2 are

$$\frac{\partial J_2^C(q_1, q_2)}{\partial q_2} = \frac{\partial J_2(q_2; q_1)}{\partial q_2} - \alpha \quad \text{and} \quad \frac{\partial^2 J_2^C(q_1, q_2)}{\partial q_2^2} = \frac{\partial^2 J_2(q_2; q_1)}{\partial q_2^2} > 0.$$

Thus, Nash equilibrium for the game with side-payments can be obtained by solving the equations

$$\frac{\partial J_1(q_1; q_2)}{\partial q_1} - \beta = 0 \quad \text{and} \quad \frac{\partial J_2(q_2; q_1)}{\partial q_2} - \alpha = 0;$$

or alternatively,

$$
\begin{cases}
c_1^o \int_0^{q_1} G\left(\frac{q_1 + bq_2 - x}{b}\right) f(x)\, dx + c_1^u F(q_1) = c_1^u + \beta, \\
c_2^o \int_0^{q_2} F\left(\frac{aq_1 + q_2 - y}{a}\right) g(y)\, dy + c_2^u G(q_2) = c_2^u + \alpha.
\end{cases}
\tag{34}
$$

In order to find the properly-designed values of α and β, the equation set (11) should be identical to (34). Thus we find α and β. Using Theorem 4 we can also compute γ. $\quad\square$

Proof of Theorem 10. As Petruzzi and Dada [46] showed, the globally-optimal solutions (p^*, q^*) must exist and satisfy the following first-order conditions:

$$
\frac{\partial \Pi(p^*, q^*)}{\partial q} = (p^* - v + g)[1 - F(q^*|p^*)] - (c - v) = 0,
\tag{35}
$$

$$
\frac{\partial \Pi(p^*, q^*)}{\partial p} = q^* - \int_0^{q^*} F(y|p^*)\, dy - (p^* - v + g) \int_0^{q^*} \frac{\partial F(y|p^*)}{\partial p}\, dy = 0.
\tag{36}
$$

If the structure of the first-order condition (35) (for the supply chain-wide optimization) is the same as that of condition (24) (for the game $\mathscr{G}^C$) and the structure of condition (36) is the same as that of the condition (25), then the Nash equilibria (p^{CN}, q^{CN}) must be identical to the globally optimal solutions (p^*, q^*) and the supply chain is coordinated. Thus, we compare (35) and (24) to compute α as shown in (26), and compare (36) with (25) to compute β as shown in (27). By using Theorem 4 we compute the constant transfer γ as shown in (28).

The first-order partial derivative of $\pi_s^C(w; p, q)$ in (22) is $q - \beta$, which may be positive (if $q > \beta$), zero (if $q = \beta$) or negative (if $q < \beta$). When $q > \beta$, the optimal wholesale price of the supplier is equal to w^0; when $q = \beta$, the optimal wholesale price is an arbitrary value in the set $[c_s, w^0]$; when $q < \beta$, the optimal wholesale price equals c_s, which implies that the supplier's profit, for this case, is non-positive. $\quad\square$

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