# Managing a Supply Chain under the Impact of Customer Reviews: A Two-Period Game Analysis

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**Abstract**

We investigate a two-period problem for a supply chain involving a manufacturer and a retailer, who serve early and late customers in the first and second periods, respectively. In each period, customers make their purchase decisions and also provide their review ratings. The manufacturer determines his quality and wholesale price, and the retailer then decides on her retail price. We perform a game-theoretic analysis and find Stackelberg equilibrium for the supply chain in each period. When early customers overrate the actual quality in their ratings, the sales in the second period are higher than those when the average review rating reflects the actual quality, and the quality levels as well as the wholesale and retail prices in the two periods are no smaller than those when the actual quality is truly rated. Moreover, in both periods, the quality overstatement increases the retailer's expected profits in the two periods but may reduce the manufacturer's expected profits. We also show that the supply chain can achieve a higher profit if customers gain a greater utility per quality level. A larger value of customers' unit quality-deviation-based rating can raise the quality level as well as the expected sales and the two firms' expected profits in the second period. The supply chain can obtain a higher profit in the first period if the variance of early customers' quality expectations is larger, and it can achieve a higher profit in the second period if the variance of later customers' quality perceptions is smaller.

**Key words**: supply chain management, customer reviews, pricing, quality, game theory.

# 1 Introduction

Since the early 1990s, the information technology has advanced rapidly and allowed customers to shop online with increasing convenience and efficiency. In order to achieve high sales, many online retailers aim at meeting the expectations of customers who are likely to learn past customers' reviews on products or services prior to making purchase decisions. As the Nielsen (2013)—a global performance management company in the United States—reported, 68% of customers trust online reviews. Moreover, according to Moog (who is CEO of the Power Reviews, a firm providing the solution for collecting and sharing customer contents), even if there is only one online review for a product, then the product is 65% more likely to be sold than a product that has no review (Vega 2017). The above evidences indicate that online reviews can greatly influence potential customers' purchase decisions. As a response, online retailers may utilize these reviews to promote their products or services and achieve higher profits. For example, Amazon is the first online firm that publicly releases customer reviews with a motivation to realize more sales and obtain a higher profit, as reported by Vega (2017).

Motived by the impact of past customers' reviews on potential customers' purchases and retailers' performance, we analyze a supply chain in which a manufacturer makes a product and sells it to an online retailer, and the retailer serves an online market with the product under the impact of customer reviews. In this paper, we consider a two-period problem. In the first period, early customers do not have any information regarding past customers' reviews but only make their purchase decisions based on their quality expectations. In the second period, late customers observe the average value of early customers' ratings and make their purchase decisions. According to our analysis of customers' behaviors, we derive the expected sales and the two firms' expected profits for the two periods.

The quality improvement can naturally help raise the positiveness of customer reviews and then increase sales. However, if the manufacturer decides to improve the quality of his product, then he incurs a higher cost for quality control and may thereby increase his wholesale price. As a consequence, the retailer may also raise her retail price, which may result in lower sales and reduce the two firms' profits. It thus behooves us to examine the wholesale pricing and quality decisions for the manufacturer as well as the retail pricing decision for the retailer under the impact of customer reviews. Accordingly, we study a leader-follower game for each period, in which the manufacturer determines his quality and wholesale pricing decisions before the retailer makes her retail pricing decision. We derive a unique Stackelberg equilibrium, and find that the wholesale and retail prices in the second period are lower than those in the first period, which implies that customer reviews can help reduce the prices and benefit potential customers. This implication mainly results from the following fact: the release of early customers' average rating can help late customers more accurately perceive the actual quality and thus, late customers' purchase decisions are more dependent on the retail price in the second period. To attract late customers to buy, the manufacturer and the retailer would make their prices in the second period lower than those in the first period. As a result of the sales increase, the retailer's profit in the second period is higher than that in the first period; therefore, the retailer should have an incentive to allow customers to post their reviews, which is consistent with many retail operations (e.g., WalMart, Expedia, and iHerb). However, the manufacturer's expected profit in the second period may or may not be greater than that in the first period, which is dependent on customers' unit quality-deviation-based rating.

As in practice, early customers may overstate or may understate the actual quality. We compare our analytic results when the actual quality is overrated, truly rated, and underrated. We find that when early customers overrate the actual quality, the quality levels as well as the wholesale and retail prices in the first and second periods are higher than those when the average review rating reflects the actual quality. This occurs because the overstatement "forces" the manufacturer to improve his quality level, which increases his cost and thus induces the manufacturer and the retailer to raise their prices. In addition, when early customers overstate the actual quality, the two firms increase their pricing decisions, and the retailer's expected profits in the two periods are higher than those when early customers truly state the actual quality. However, the manufacturer experiences a lower profit in the first period and may also profit less in the second period. This exposes that the retailer prefers the quality overstatement to the quality understatement. This conforms with the practices of many retailers who may post fake reviews that are positive to the retailer herself but negative to her competitors.

We then perform sensitivity analysis to explore how the customers' utility per quality level, their unit quality-deviation-based rating, as well as the variation of early customers' quality expectations and that of late customers' quality perceptions affect Stackelberg equilibrium, the expected sales, and the two firms' expected profits in each period. We show that the supply chain can achieve a higher profit if customers gain more from each unit of the product level. If customers incur a larger cost when their quality expectations or perceptions deviate from the actual quality, then the expected sales in the two periods are increased. The retailer has a higher incentive to allow customer reviews in the market with a larger deviation rating for the second period. We also find that in the first period, the supply chain is better off if the variance of early customers' quality expectations is larger. In the second period, the supply chain achieves a higher profit if the variance of later customers' quality perceptions is smaller. This interesting, and somewhat surprising, result is mainly attributed to the following fact: before early customers buy from the retailer, they cannot observe any average rating to perceive the actual quality, and a higher variance of early customers' expectations reflects these customers' larger uncertainty on the actual quality. Thus, a higher quality level may be more effective in improving early customers' incentives to buy, which makes the supply chain better off when the variance of early customers' quality expectations is larger. Different from early customers, late customers can observe early customers' average rating, and they may not mainly base their purchase decisions on the quality. In the second period, a larger variance of late customers' quality perceptions gives rise to a higher quality level and also an increase in the retail price, which then results in less sales. It thus follows that the supply chain can benefit from a smaller variance of late customers' quality perceptions.

The remainder of this paper is organized as follows. In Section 2, we provide a review of extant representative publications related to the pricing and quality decisions under the impact of customer reviews. The review helps indicate the originality of our research problem. In Section 3, we develop a two-period game model to analyze early and late customers' purchase decisions, and also investigate the quality and pricing decisions for the manufacturer and the retailer in the supply chain. The quality overstatement and understatement are also studied. In Section 4, we perform sensitivity analysis to examine the impact of some important parameters on the supply chain. This paper ends with a summary of managerial implications and future research directions in Section 5. We relegate the proofs of all propositions and theorems to online Appendix A, where the proofs are given in the order that they appear in the main body of our paper.

## 2 Literature Review

Many researchers have published their findings related to the pricing and/or quality decisions in the situation that past customers' reviews influence potential customers' purchase decisions. Before we review major relevant publications, we start with our review of some publications concerned with customer reviews. In an early paper regarding customer reviews, Chevalier and Mayzlin (2006) analyzed the user review data collected from public web sites, and discovered that negative reviews are more effective in decreasing sales compared to positive reviews in increasing sales. If there are a number of long and positive online reviews for a product, then the sales of the product are very likely to be higher than others even when the product is a niche one. After Chevalier and Mayzlin (2006) released their results, in the past decade, a number of empirical studies (e.g., Sun 2012, Zhang, Ma, and Cartwright 2013, Lu, Ye, and Law 2014, and Li, Wu, and Mai 2018) have also revealed that customer reviews are of great significance to influence potential customers' purchase decisions and the sales at online retail stores (e.g., Amazon) that sell products or services such as tablet computers, digital cameras, books, music albums, hotels, and movies; see our summary of the major empirical findings in Table 1.

Although a number of researchers have investigated the pricing and quality decisions under the impact of customer reviews, most relevant publications only focused on one of the two decision variables, and a few jointly considered the two decisions. Next, we review representative publications regarding the pricing decision under the impact of customer reviews. Sun (2012) performed both analytical and empirical studies to investigate the impact of customer reviews on the sales and also examine the pricing decision for a retailer who sells products online. For the analytical study, Sun (2012) developed a two-period model in which early and late customers buy in the first and second periods, respectively; and the retailer determines a retail price for each period. Sun (2012) then conducted an empirical study using the data from `Amazon.com` and `BN.com`, and found that the results drawn from the analytical and empirical studies are consistent. The major finding is that the sales in the second period are increasing in the average value of early customers' ratings in the first period.

Kwark, Chen, and Raghunathan (2014) also developed analytical models to study the effects

| **Publications** (in reverse chronology) | **Product/Service** | **Empirical Data** | **Major Relevant Findings** |
|---|---|---|---|
| Li, Wu, and Mai (2018) | Tablet computers | The dataset contains 88,901 consumer reviews on 794 tablet computer products or SKUs. Each review contains the numerical rating ranging from 1 to 5. | The average review rating positively impacts sales, and it has a significant impact on the sales performance of products. |
| Lu, Ye, and Law (2014) | Hotels | Data about 1,689 hotels were collected from two major online travel agencies in China. | The average rating of online word-of-mouth (WOM) have a significant impact on sales. |
| Zhang, Ma, and Cartwright (2013) | Digital cameras | Data were collected from `Amazon.com`, which provides the sales rankings of all new point and shoot digital cameras periodically. | Both a change in price and a change in average online review rating are significantly associated with future sales. |
| Sun (2012) | Books | Data about review ratings, price, sales rank, and others were collected from `Amazon.com` and `BN.com`. | A higher average customer rating can increase the book sales. |
| Gu, Park, and Konana (2012) | Digital Cameras | Data of daily product sales rank from `Amazon.com` and of average ratings from three external websites (i.e., Cnet, DpReview, and Epinions) were collected. | The average customer ratings posted at external review sources such as `CNET.com` are strongly associated with the Amazon sales of the product. |
| Öğüt and Taş (2012) | Hotels | Data about star rating and customer rating, hotel room sales, and prices in Paris and London were collected. | Customer ratings significantly influence online sales. A 1% increase in online rating increases sales up to 2.68% (2.62%) in Paris (London). |
| Ye et al. (2011) | Hotels | Data extracted from a major online travel agency in China (i.e., `www.ctrip.com`). | Both Review ratings and room prices are important elements to predict online sales. A 10% increase in ratings can boost online bookings by more than 5%. |
| Zhu and Zhang (2010) | Consoles and games | Data on console and game sales are from NPD (a leading market research firm in video games industry) which collected data from 17 leading retail chains. | Online reviews greatly influence the sales of games, and the average rating affects the sales of less popular and online games. |
| Pathak et al. (2010) | Books | Data about price, average customer rating, and sales rank for the 5,000 top-selling books recommended by `Amazon.com` and `BN.com`. | The average review rating positively affects the sales of each top-selling books. |
| Dhar and Chang (2009) | Music albums | Data about review ratings for four weeks before and after the release dates of albums from Pause and Play (`www.pauseandplay.com`). | The average consumer ratings are statistically significant in predicting future sales. A higher average rating results in higher sales. |
| Dellarocas, Zhang, and Awad (2007) | Movies | Data about user reviews posted on Yahoo! Movies. The data set consists of 80 movies with the 689 reviews per movie on average. | The average customer rating is statistically significant as a predictor of the sales for a movie (the number of persons who watch the movie). |
| Chevalier and Mayzlin (2006) | Books | Data consisting of individual book characteristics and user review data collected from `Amazon.com` and `BN.com`. | An increase in the average rating for a book results in higher sales of the book. One-star reviews have a greater impact than five-star reviews on the same book site. |

Table 1: A summary of some empirical findings regarding the significant impact of average customer rating on customers' purchase decisions or a retailer's sales.

of online product reviews in a two-echelon supply chain involving a retailer and two competing manufacturers who sell substitutable products. In this paper, the authors analyzed consumers' purchase behaviors and obtained the pricing decisions for the retailer and the manufacturers. They found that quality information can homogenize consumers' perceived utility differences between the two products but fit information can heterogenize consumers' estimated fits to the products. In another recent study, Li and Wang (2014) treated customer reviews as a quality signal, examined the impact of pricing strategies on the reviews, and found that the sales reflect the information accuracy and a fit to customer needs. The author also disclosed that an early-period price cut can attract more customers and generate more online reviews, thus improving the accuracy of quality information. He et al. (2016) investigated the pricing and location decisions for service merchants, analyzing an agent-based competitive online-to-offline (O2O) model. In the analysis, the service merchants are profit-maximizing agents, and customers are utility-maximizing agents who can share their reviews in social networks. Yang and Dong (2018) developed a two-period model to study an online retailer's optimal rebate and pricing decisions with an aim to stimulate online customer reviews.

Then, we review representative publications regarding the quality decision under the impact of customer reviews. As we have discussed in Section 1, quality is a major factor influencing customers' use experiences and their reviews. Firms may intentionally improve product quality, because customers usually possess loyalties toward high quality products if they are confident with the quality. As a response, many firms have invested enormously to improve the quality of their products even with a target of a very high level. We learn from recent studies (Manchanda and Chintagunta 2006 and Manduchi 2010) that customer reviews have an indirect impact on the quality decision and thus, a number of firms have overstated the value of their products that do not match customer needs. In fact, when a product matches customer needs, customer reviews are useful to delivering values and effectively improving sales.

Next, we review the publications that jointly consider the pricing and quality decisions. Teng and Thompson (1996) investigated the optimal price and quality policies for the introduction of a new product. Mathios (2000) performed an empirical study for the salad industry, using the data from upscale supermarkets in the state of New York—i.e., duopoly markets where salad dressings are either high or low in fat. The results revealed that firms are willing to disclose product information when the quality of their products is able to meet customers' expectations on, e.g., the low-fat salad dressings. Koessler and Renault (2012) found that quality changes with the price swing. Full disclosure of the product information helps achieve the price equilibrium in a market and maximize the firms' profits in the market. Under the price equilibrium, top quality firms intentionally utilize customer reviews and sell their products at high prices. In addition, Board (2009) showed that a low-quality firm may set a high price and then enjoy free ride from high-quality firms, which would decide to share the product information even though a higher competition could lead the price to fall.

Without considering the impact of customer reviews, Mukhopadhyay and Setaputra (2007) developed a profit-maximization model to jointly obtain optimal quality level, price, and return policy. Xu (2009) investigated a supply chain in which a manufacturer jointly determines the wholesale price and quality of a product and the retailer determines the retail price. The author found that if the marginal revenue function is strictly concave, then the quality level is lower than that when the manufacturer sells the product directly to customers. In a recent publication, Chen et al. (2017) considered the price and quality decisions in dual-channel supply chains in which a manufacturer may sell a product through a retailer, or may directly sell it to customers, or may use a dual (retail and direct) channel. The authors examined a customer utility function to derive demand functions for the supply chain, and demonstrated that quality improvement can be realized when a new channel is introduced. They also studied the effects of the quality sensitivity parameters in different channels on price and product quality as well as profits and consumer surplus. Our review for the publications concerning the joint decisions reveals that extant publications usually analyze the decision making problems in the supply chain setting but they do not consider the impact of customer reviews.

According to our review, we can conclude that our paper significantly differs from extant publications. As briefly described in Section 1, in this paper we jointly analyze the pricing and quality decisions in a supply chain under the impact of customer reviews. We construct a two-period game problem in which, for each period, a manufacturer first determines his wholesale price and the quality level and a retailer then determines her retail price. In the first period, early customers make their purchase decisions, and also provide their review ratings. Observing the average rating, late customers decide on whether to buy or not and write their review ratings. Our problem is somewhat similar to Sun's two-period problem (2012), but they significantly differ mainly in that (i) our customer choice models are different from Sun's models (which were built based on the Hotelling model), (ii) in our paper a two-echelon supply chain (consisting of a manufacturer and a retailer) serves customers whereas Sun (2012) considered only a seller, (iii) in our paper the manufacturer makes his quality decision whereas Sun (2012) treated the quality as an exogenous variable, and (iv) we consider heterogenous customers in two periods whereas Sun (2012) considered heterogenous customers in the first period but assumed homogenous customers in the second period. Some other publications also presented similar two-period analyses, see, e.g., Kuksov and Xie (2010) and Wang et al. (2018). Nevertheless, from our analysis, we draw a number of managerial implications that were not delivered by extant publications; for a summary of the implications, see Section 5.

# 3 Customer Purchases and Supply Chain Analysis

In this section, we develop a two-period decision problem in which a unit mass of "early customers" and a unit mass of "late customer" make their purchase decisions and provide their review ratings in the first and second periods, respectively. We describe our two-period problem as follows.

1. In the first period, the manufacturer makes his quality level and wholesale pricing decisions (denoted by $q_1$ and $w_1$, respectively) and the retailer determines her retail price (denoted

by $p_1$). In reality, most firms usually adopt a quality level scale to grade their product or service quality. For example, the Rapid Survey Group (2012) released a kitchen quality guide based on a 14-points quality level scale in which points 1 and 14 represent the highest and the lowest quality levels, respectively. The Meteo France (2018) measures sea surface temperature products using a 5-points quality level scale in which the values of product levels increase from 1 ("worst") to 5 ("best"). Such points- or values-based quality level scale systems have been widely used to investigate relevant academic research problems; see, for example, Pace et al. (2014), Smith (1985), and Taşkın and Ünal (2009). Moreover, for each product, the manufacturer incurs a quality (control) cost $\tau q_1$, where $\tau > 0$ represents a unit control cost for each unit of quality level. Although, in relevant publications, the linear and quadratic function forms are commonly used, we adopt the linear function form for the quality cost in this paper because of the following facts: Sedatole (2003) performed an empirical study and showed that the linear function form is better than the quadratic function form in the measurement of quality-related control costs. In addition, a number of researchers—who include, e.g., Hong (1998), Jin and Ryan (2014, 2016), Niculescu, Wu, and Xu (2018)—have used linear quality control cost functions to investigate some quality problems.

After the two firms announce their decisions, early customers arrive to decide on whether to buy or not. Since there is no information regarding customer reviews, each early customer only has an expected value of the quality, which is simply called "quality expectation." Early customers can learn the actual quality of the product according to their use experiences. These customers write review ratings, which are based on the difference between the actual quality and their quality expectations. The retailer then posts the average customer rating online.

2. In the second period, the manufacturer determines his quality level $q_2$, and the manufacturer and the retailer make their pricing decisions (i.e., $w_2$ and $p_2$), similar to the first period. After the pricing decisions are announced, late customers decide on whether to buy or not. Different from early customers, each late customer can observe the average customer rating (provided by early customers) to perceive a quality level (which is simply called "quality perception") and make his or her purchase decision. Note that late customers' perceived quality level is the manufacturer's quality level decision in the first period rather than that in the second period. The late customers then write customer ratings, which are dependent on the difference between the actual quality and these customers' quality perceptions.

For the two-period problem, we begin by constructing and analyzing customer choice models for purchases in the two periods. That is, we first investigate early customers' decisions on purchases and review ratings. An average customer rating then appears online and helps late customers perceive the actual quality level (which is made by the manufacturer in the first period). Using the fact, we examine late customers' choice models to obtain their purchase decisions. According to our analysis of customer choice models, we can derive the sales in the first and second periods, which can be used to develop the manufacturer's and the retailer's expected profit functions in the two periods.

To find the manufacturer's and the retailer's quality and pricing decisions, we first analyze the two firms' decision-making problems in the second period, and then investigate their quality and pricing decisions in the first period. In each period, as the manufacturer makes his decision(s) before the retailer determines her price, we solve a leader-follower game to obtain the two firms' decisions in Stackelberg equilibrium.

## 3.1 Analyses of Customers' Purchases and Review Ratings

We develop and study customer choice models to determine customers' purchases in the two periods, starting with our analysis for the first period. We also compute customers' review ratings in the two periods.

### 3.1.1 Early Customers' Purchases and Review Ratings

Early customers represent a first mass of customers served by the retailer. Hence, when an early customer arrives, he or she cannot have any information regarding past customers' review ratings. Instead, the customer can only have an expected value of the product quality, to wit, quality expectation, which is denoted by $x$. The customer's expected utility resulting from the consumption of the product is $\alpha x$, where $\alpha$ means each customer's expected utility from one unit of quality level, similar to Li and Hitt (2010). The early customer's before-purchase net gain $u_e^b$ is the customer's expected utility minus the retail price in the first period (i.e., $p_1$); that is,

$$u_e^b = \alpha x - p_1. \tag{1}$$

The customer is willing to buy from the retailer if and only if $u_e^b \geq 0$, or, $x \geq x_1 \equiv p_1/\alpha$. In addition, because retail price $p_1$ is no smaller than wholesale price $w_1$ which should be greater than the manufacturer's unit quality control cost $\tau q_1$, the two firms should make their quality and pricing decisions such that $p_1 > \tau q_1$. If $\alpha < \tau$, then, when the quality level is sufficiently high, $u_e^b$ may be negative for each customer and thus, no early customer will buy. This is obviously contrary to our intuition. Therefore, it is reasonable to assume that $\alpha > \tau$.

Since early customers have different tastes and are thus heterogeneous in their expectations, we reasonably assume that $X$ is a random variable on the interval $[0, b_1]$, where $b_1 > 0$ denotes the maximum expectation. For variable $X$, the probability density function (p.d.f.) is $f(x)$ and the cumulative distribution function (c.d.f.) is $F(x)$. Here, we set the minimum expectation of early customers to be zero, which reflects the fact that some customers may view the product as one with no value. Naturally, the manufacturer should choose an actual quality $q_1$ that does not exceed maximum expectation $b_1$, i.e., $q_1 \leq b_1$. As early customers do not have any information to make their expectations, their expectations are by and large scattered very widely. This means that $b_1$ should be a sufficiently large number. For example, The Mill City Press (2018a) reported that, for short ebooks (i.e., those under 50,000 words) with average price \$3.99, customers have their quality expectations ranging from \$0.99 to \$2.99 with average value \$1.99. This means that for an ebook, the value of $b_1$ is around 70% of its retail price (approximated by 2.99/3.99), and the value of $\alpha$ is 2 (approximated by 3.99/1.99).

To ensure tractability in our subsequent analysis and find meaningful solutions, we assume that $X$ satisfies a uniform distribution; that is, $f(x) = 1/b_1$ and $F(x) = x/b_1$, for $x \in [0, b_1]$. Such an assumption is common in the analysis of consumer behaviors in relevant publications; see, for example, Sun (2012), Kwark, Chen, and Raghunathan (2014), Xu et al. (2015), Ma, Lin, and Zhao (2016), and Ma et al. (2018). If $x_1 > b_1$, or, $p_1 > \alpha b_1$, then no customer buys from the retailer. Otherwise, if $p_1 \leq \alpha b_1$, then an early customer may purchase the product. When the market size is $A > 0$, the expected sales in the first period as

$$D_1 \equiv A \int_{x_1}^{b_1} f(x)dx = A\frac{b_1 - x_1}{b_1} = A\left(1 - \frac{p_1}{\alpha b_1}\right). \tag{2}$$

The above implies that the retailer should make her retail price in the first period (i.e., $p_1$) such that $p_1 \leq \alpha b_1$.

Next, we compute early customers' review ratings. We learn from Sun (2012) that the rating of a customer with expectation $x$ (denoted by $r(x)$) is based on actual quality $q_1$ and the quality deviation (i.e., the difference between $q_1$ and the customer's before-purchase quality expectation $x$), i.e.,

$$r(x) = q_1 + \beta(q_1 - x) = (1 + \beta)q_1 - \beta x, \tag{3}$$

where $\beta > 0$ is a parameter indicating the impact of quality deviation on the customer's rating. Hereafter, we call $\beta$ the "unit quality-deviation-based rating" or simply "unit deviation rating," as in some publications by, e.g., Lee and Litkouhi (2015). We consider the deviation-based rating component in (3), because of the following fact: as revealed by some empirical studies (e.g., Li and Hitt 2010, Floyd et al. 2014, and Li, Wu, and Mai 2018), in addition to actual quality $q_1$, any difference between $q_1$ and an early customer's before-purchase expectation $x$ (i.e., $q_1 - x$) significantly affects the customer's shopping satisfaction and thus influences his or her review rating. That is, when the early customer rates the product, he or she usually considers both actual quality $q_1$ and quality deviation $q_1 - x$. As in practice, when $q_1 < x$, the customer is unhappy with the product quality and is very likely to depreciate the product, which makes the customer rate the product at a level lower than $q_1$. However, when $q_1 \geq x$, the customer is satisfied with the product quality and is thus likely to rate the product at a level equal to or higher than $q_1$. Therefore, we compute an early customer's rating as in (3), similar to Sun (2012) who developed a formula to approximate the unit deviation rating as $\beta = 2\sqrt{3V}/L$, where $V$ is the variance of customer ratings and $L$ is the difference between the highest and lowest customer ratings.

Since random variable $x$ is distributed with p.d.f. $f(x)$, we compute early customers' average rating (also known as the valence of customer reviews) as

$$\bar{r}_1 = \int_0^{b_1} r(x)f(x)dx = (1+\beta)q_1 - \beta\bar{x}, \tag{4}$$

where $\bar{x} \equiv \int_0^{b_1} xf(x)dx = b_1/2$ denotes the average value of early customers' quality expectations. The average customer rating in (4) indicates that the average review score is increasing in the product's quality level, which is consistent with a finding from Kroll, Wright, and Heiens (1999). To protect the manufacturer's and the retailer's reputation in the market, the two firms usually expect the average customer rating to be no smaller than a minimum desirable rating $\hat{r}$. To ensure that $\bar{r}_1 > \hat{r}$, the manufacturer should determine his quality level in the first period such that $q_1 \geq (2\hat{r} + \beta b_1)/[2(1+\beta)]$.

Although the average customer rating basically reflects the product quality, customers may provide their ratings to overstate or understate the actual product quality. That is, whether customer ratings overstate or understate the actual quality is dependent on the comparison between average rating $\bar{r}$ in (4) and actual quality $q_1$.

**Remark 1** If customer ratings overstate the actual quality in general, then the quality level implied by average rating $\bar{r}$, denoted by $\tilde{q}_1$, is greater than actual quality $q_1$, i.e.,

$$\tilde{q}_1 \equiv \frac{\bar{r}_1 + \beta\bar{x}}{1+\beta} = \frac{2\bar{r}_1 + \beta b_1}{2(1+\beta)} > q_1. \tag{5}$$

The above indicates that a larger average value of their ratings (i.e., $\bar{r}$) implies a higher possibility for early customers to overstate the actual quality. If $\tilde{q}_1 = q_1$, then customer ratings accurately indicate the actual quality. Otherwise, if $\tilde{q}_1 < q_1$, then customers are unsatisfied with product performance in general and their ratings thereby understate the actual quality.

### 3.1.2 Late Customers' Purchases and Review Ratings

Late customers are those who can observe average customer rating $\bar{r}_1$ in (4) prior to making a purchase decision. Because average rating $\bar{r}_1$ is calculated based on early customers' reviews in the first period, each later customer can use $\bar{r}_1$ to perceive the actual quality level in the first period (i.e., $q_1$) but cannot perceive the actual quality level in the second period (i.e., $q_2$). Without any information about the second-period quality level, the late customer has to make a purchase decision in the second period based on his or her perception of the first-period quality level.

Because different customers may differ in their views on customer ratings, they may make different quality perceptions according to average customer rating $\bar{r}_1$. Nevertheless, as Li and Hitt (2010), Floyd et al. (2014), and Li, Wu, and Mai (2018) have exposed, average customer rating $\bar{r}_1$ is an indicator of actual product quality $q_1$. This implies that the overall quality perception (i.e., the average value of late customers' perceptions) should be the same as or very close to $q_1$. Late customers' perceptions are scattered around $q_1$. Accordingly, we denote a late customer's $\bar{r}_1$-based quality perception by $y \equiv q_1 + \theta$, where $\theta$ is a random variable describing the late customers' heterogeneity in their quality perceptions. The p.d.f. and c.d.f. of random variable $\theta$ are $h(\theta)$ and $H(\theta)$, respectively. As in some relevant publications by, for example, Kwark, Chen, and Raghunathan (2014), Xu et al. (2015), Ma, Lin, and Zhao (2016), and Ma et al. (2018), we assume that $\theta$ satisfies a uniform distribution. Moreover, since the average customer rating improves the accuracy of late customers' perceptions, the perceptions are usually scattered closely around actual quality $q_1$. Accordingly, $\theta$ is uniformly distributed on the interval $[-\varepsilon, \varepsilon]$, respectively, where $\varepsilon$ is a sufficiently small, positive number; that is, $h(\theta) = 1/(2\varepsilon)$ and $H(\theta) = (\theta + \varepsilon)/(2\varepsilon)$. Our model differs from Sun (2012) who assumed that all late customers can certainly perceive the actual quality, i.e., $\varepsilon = 0$.

Similar to our analysis of an early customer's purchase decision in Section 3.1.1, before making a purchase decision, the later customer forecasts that if buying, then he or she enjoys the perceived utility $\alpha y$ and pays for retail price $p_2$; thus, the customer's before-purchase net gain is computed as

$$u_l^b = \alpha y - p_2 = \alpha(q_1 + \theta) - p_2. \tag{6}$$

The late customer decides to buy from the retailer if $u_l^b \geq 0$, or, $\theta \geq \theta_1 \equiv p_2/\alpha - q_1$, and does not buy if $\theta < \theta_1$. We can compute the expected sales to late customers as

$$D_2 \equiv A \int_{\theta_1}^{\varepsilon} h(\theta) d\theta = A \frac{\varepsilon - \theta_1}{2\varepsilon} = A \frac{\varepsilon + q_1 - p_2/\alpha}{2\varepsilon}, \tag{7}$$

which is nonnegative if and only if $p_2 \leq \alpha(\varepsilon + q_1)$.

We recall from Remark 1 that early customers may overstate the actual quality when the inequality in (5) (i.e., $\tilde{q}_1 > q_1$) is satisfied or may understate the actual quality when $\tilde{q}_1 < q_1$. When early customers' ratings deviate from the actual quality, a late customer's quality perception becomes $y = \tilde{q}_1 + \theta$. If the late customer possesses the value of $\theta$ such that

$$\theta \geq \tilde{\theta}_1 \equiv p_2/\alpha - \tilde{q}_1 = p_2/\alpha - q_1 + q_1 - \tilde{q}_1 = \theta_1 + q_1 - \tilde{q}_1,$$

then the customer is willing to buy; otherwise, if $\theta < \tilde{\theta}_1$, then the customer does not buy. We note that $\tilde{\theta}_1 < \theta_1$ when early customers overstate the actual quality; but, $\tilde{\theta}_1 > \theta_1$ when early customers understate the actual quality. For the quality overstatement or understatement case, the expected sales to late customers are $\tilde{D}_2 \equiv A \int_{\tilde{\theta}_1}^{\varepsilon} h(\theta) d\theta = A(\varepsilon - \tilde{\theta}_1)/(2\varepsilon) = A(\varepsilon + \tilde{q}_1 - p_2/\alpha)/(2\varepsilon)$, which is nonnegative if and only if $p_2 \leq \alpha(\varepsilon + \tilde{q}_1)$; thus, the manufacturer and the retailer should make their quality and pricing decisions subject to $p_2 \leq \alpha(\varepsilon + \tilde{q}_1)$. Comparing $D_2$ and $\tilde{D}_2$, we have the following theorem.

**Theorem 1** *If early customers overstate the actual quality in their customer reviews, i.e., $\tilde{q}_1 > q_1$, then the expected total sales in the second period (denoted by $\tilde{D}_2$) are greater than those (i.e., $D_2$) when early customers accurately rate the actual quality (i.e., $\tilde{q}_1 = q_1$). Otherwise, if early customers understate the actual quality (i.e., $\tilde{q}_1 < q_1$), then $\tilde{D}_2 < D_2$.*

Theorem 1 indicates that a smack of exaggeration in early customers' ratings can help increase the expected sales; and on the other hand, when early customers lower review ratings to understate the actual quality (i.e., $\tilde{q}_1 < q_1$), the expected sales are reduced. Next, we compute late customers' review ratings. Similar to our calculation for early customers' ratings, the rating of a late customer with perception $y$ (denoted by $r_2(y)$) is based on actual quality $q_2$ and the quality deviation (i.e., the difference between $q_2$ and quality perception $y$), i.e.,

$$r_2(y) = q_2 + \beta(q_2 - y) = (1+\beta)q_2 - \beta y = (1+\beta)q_2 - \beta(q_1 + \theta), \tag{8}$$

where $\beta$ is the customer's unit deviation rating, as defined in Section 3.1.1. In this paper, early and late customers have an identical unit deviation rating, because these customers are in a single market that is served by the two-echelon supply chain, and are thus likely to identically or similarly respond to the difference between their quality expectations/perceptions and the actual quality. Since random variable $\theta$ is distributed with p.d.f. $h(\theta)$, we compute late customers' average rating (a. k. a. the valence of customer reviews) as

$$\bar{r}_2 = \int_{-\varepsilon}^{\varepsilon} r_2(y)h(\theta)d\theta = (1+\beta)q_2 - \beta(q_1 + \bar{\theta}), \tag{9}$$

where $\bar{\theta} \equiv \int_{-\varepsilon}^{\varepsilon} \theta h(\theta)d\theta = \varepsilon$ denotes the average value of late customers' quality perceptions.

Then, we compare early customers' average rating $\bar{r}_1$ and late customers' average rating $\bar{r}_2$. In reality, all supply chain members expect that the second-period rating $\bar{r}_2$ should be no smaller than the first-period rating $\bar{r}_1$, i.e., $\bar{r}_2 \geq \bar{r}_1$; that is, the overall customer review rating in the market does not decrease. In order to ensure that $\bar{r}_2 \geq \bar{r}_1$, the manufacturer should make his quality decision in the second period such that

$$q_2 \geq \hat{q}_2 \equiv \frac{(1+2\beta)q_1 + \beta\bar{\theta} - \beta\bar{x}}{1+\beta} = q_1 + \frac{\beta[2q_1 + (2\varepsilon - b_1)]}{2(1+\beta)}. \tag{10}$$

We note from (10) that, when the average rating is not reduced in the second period, the manufacturer's second-period quality level $q_2$ may not need to be higher than his first-period quality level $q_1$. Specifically, if $\beta[2q_1 + (2\varepsilon - b_1)]/[2(1+\beta)] < 0$, or, early customers' maximum before-purchase expectation (i.e., $b_1$) satisfies the inequality $b_1 > 2(q_1 + \varepsilon)$, then the manufacturer may determine a lower quality level in the second period. This happens mainly because a greater variance of early customers' quality expectations results in a lower average rating in the first period; as a result, the second-period rating is more likely to be higher than the first-period rating.

## 3.2 Game-Theoretic Analysis of the Supply Chain in the Two Periods

We obtain the manufacturer's and the retailer's quality and pricing decisions in the first and second periods, using a backward induction approach.

### 3.2.1 Game-Theoretic Analysis in the Second Period

In the second period, the manufacturer first determines his wholesale price $w_2$ and quality level $q_2$, and the retailer then decides on her retail price $p_2$. The decision problem is a leader-follower game. In the second period, the supply chain sells $D_2$ units of the product to late customers. The retailer's expected profit in the second period is computed as $\pi_{R2}(p_2) = (p_2 - w_2)D_2$, and the manufacturer's expected profit in the second period is $\pi_{M2}(w_2, q_2) = (w_2 - c - \tau q_2)D_2$, where $c$ denotes the manufacturer's unit acquisition cost.

**Proposition 1** *In the second period, the quality level as well as the wholesale and retail prices in Stackelberg equilibrium are obtained as*

$$\begin{cases} q_2^S(q_1) = \dfrac{2[(1+2\beta)q_1 + \beta\varepsilon] - \beta b_1}{2(1+\beta)}, \\ w_2^S(q_1) = \dfrac{2[(\alpha+2\tau)\beta + \alpha + \tau]q_1 - (b_1 - 2\varepsilon)\tau\beta + 2(1+\beta)(\alpha\varepsilon + c)}{4(1+\beta)}, \\ p_2^S(q_1) = \dfrac{2[(3\alpha+2\tau)\beta + 3\alpha + \tau]q_1 - (b_1 - 2\varepsilon)\tau\beta + 2(1+\beta)(3\alpha\varepsilon + c)}{8(1+\beta)}. \end{cases} \tag{11}$$

We learn from Proposition 1 that, when each customer draws a higher expected utility from one unit of quality level (i.e., the value of $\alpha$ increases), the manufacturer should increase his wholesale price, which then induces the retailer to charge customers a higher retail price. Moreover, if the manufacturer chooses a higher quality level in the first period, then, in order to ensure profitability, he also needs to increase his wholesale price and quality level in the second period, and the retailer also increases her retail price in the second period.

When the manufacturer and the retailer adopt their quality level and pricing decisions in Stackelberg equilibrium, the resulting sales in the second period are

$$D_2^S \equiv A\frac{2[(\alpha-2\tau)\beta + \alpha - \tau]q_1 + (b_1 - 2\varepsilon)\tau\beta + 2(1+\beta)(\alpha\varepsilon - c)}{8\alpha\varepsilon}; \tag{12}$$

and the two firms' expected profits in the second period are

$$\begin{cases} \pi_{R2}^S(q_1) \equiv \pi_{R2}(p_2^S(q_1)) = (p_2^S(q_1) - w_2^S(q_1))D_2^S, \\ \pi_{M2}^S(q_1) \equiv \pi_{M2}(w_2^S(q_1), q_2^S(q_1)) = (w_2^S(q_1) - c - \tau q_2^S(q_1))D_2^S. \end{cases} \tag{13}$$

### 3.2.2 Game-Theoretic Analysis in the First Period

In the first period, the manufacturer jointly makes his quality level and wholesale pricing decisions and the retailer then determines her retail price. Similar to our analysis for the second period, in the first period the number of early customers who buy the product is $D_1$, the retailer's expected profit in the first period is computed as $\pi_{R1}(p_1) = (p_1 - w_1)D_1$, and the manufacturer's expected profit in the first period is $\pi_{M1}(w_1, q_1) = (w_1 - c - \tau q_1)D_1$. In our two-period decision problem, both the manufacturer and the retailer aim at maximizing their *total* profits in the two periods rather than only their profits in the first period. Assuming that the discount factor of each firm's profit in the second period is equal to one, we find the manufacturer's and the retailer's objective (total profit) functions as

$$\Pi_M(w_1, q_1) \equiv \pi_{M1}(w_1, q_1) + \pi_{M2}^S(q_1) \text{ and } \Pi_R(p_1) \equiv \pi_{R1}(p_1) + \pi_{R2}^S(q_1), \tag{14}$$

where $\pi_{M1}(w_1, q_1)$ and $\pi_{M2}^S(q_1)$ (which is given in (13)) are the manufacturer's first- and second-period profits, respectively; and $\pi_{R1}(p_1)$ and $\pi_{R2}^S(q_1)$ (which is given in (13)) are the retailer's first- and second-period profits, respectively.

**Theorem 2** *In the first period, the quality level (i.e., $q_1^S$) as well as the wholesale and retail pricing decisions (i.e., $w_1^S$ and $p_1^S$) in Stackelberg equilibrium are obtained as follows:*

$$\begin{cases} q_1^S = \dfrac{2\hat{r} + \beta b_1}{2(1+\beta)}, \\ w_1^S = \dfrac{[(\tau + 2\alpha)b_1 + 2c]\beta + 2(\alpha b_1 + \hat{r}\tau + c)}{4(1+\beta)}, \\ p_1^S = \dfrac{[(\tau + 6\alpha)b_1 + 2c]\beta + 2(3\alpha b_1 + \hat{r}\tau + c)}{8(1+\beta)}. \end{cases} \tag{15}$$

We find from Theorem 2 that if early customers' quality expectations are widely spread out (i.e., the value of $b_1$ is greater), then both the quality level and the pricing decisions are increased. In practice, the value of $b_1$ is greater when early customers are more uncertain about the product quality. This exposes that a higher uncertainty of customers in their quality expectations induces the manufacturer to improve his quality level in the first period, which increases the manufacturer's quality control cost. As a response, the manufacturer raises his wholesale price to offset the reduction in his profit margin. The retailer also increases the retail price to ensure her profitability.

When the manufacturer and the retailer adopt the Stackelberg equilibrium, the resulting sales in the first period are

$$D_1^S \equiv A\frac{[(2\alpha - \tau)b_1 - 2c]\beta + 2(\alpha b_1 - \hat{r}\tau - c)}{8\alpha b_1(1+\beta)}; \tag{16}$$

and the two firms' expected profits in the first period are

$$\begin{cases} \pi_{R1}^S \equiv \pi_{R1}(p_1^S) = (p_1^S - w_1^S)D_1^S, \\ \pi_{M1}^S \equiv \pi_{M1}(w_1^S, q_1^S) = (w_1^S - c - \tau q_1^S)D_1^S. \end{cases}$$

### 3.2.3 Managerial Implications of Game-Theoretic Results

Using (11) and (15), we can obtain the two firms' second-period quality and pricing decisions in Stackelberg equilibrium as

$$q_2^S = q_2^S(q_1^S),\ w_2^S = w_2^S(q_1^S), \text{ and } p_2^S = p_2^S(q_1^S). \tag{17}$$

Substituting $q_1^S$ into $D_2^S$ in (12) and the two firms' second-period profits in (13) gives the sales and profits in the second period when the firms make the quality level and pricing decisions in

Stackelberg equilibrium as in (17).

**Corollary 1** *In Stackelberg equilibrium, both the wholesale and retail prices in the second period are smaller than those in the first period. Moreover, the decrease in the wholesale price is greater than that in the retail price. We also find that, if the supply chain expects a sufficiently large value of the minimum desirable rating $\hat{r}$ such that $2[\hat{r} + \varepsilon(1+\beta)] \geq b_1$, then the manufacturer chooses a higher or the same quality level in the second period (i.e., $q_2^S \geq q_1^S$). Otherwise, the quality level in the second period may be lower than that in the first period.*

Corollary 1 indicates that both the manufacturer and the retailer make their prices in the second period lower than those in the first period, which implies that the availability of customer reviews gives rise to price decreases in the supply chain. This occurs mainly because of the following fact: when late customers can access the information about early customers' reviews, they can more accurately perceive the actual quality for their purchase decisions. Some late customers who are somewhat unsatisfied with the quality expectation may not buy in the second period. To attract those late customers to buy, the manufacturer would reduce his wholesale price to lower the retailer's cost of acquiring the product, thus encouraging the retailer to reduce the retail price without decreasing her profit margin. We also learn from Corollary 1 that $w_1^S - w_2^S > p_1^S - p_2^S$, which means that when product reviews of early customers are released to late customers, the decrease in the wholesale price is greater than the decrease in the retail price. That is, the retailer benefits from customer reviews by enjoying a higher profit margin in the second period than in the first period. Moreover, the quality level in the second period (i.e., $q_2^S$) may or may not be greater than that in the first period (i.e., $q_1^S$), which depends on $2[\hat{r}_1 + \varepsilon(1+\beta)]$ and $b_1$. Specifically, the manufacturer may increase his quality level in the second period, which depends on the minimum desirable rating. If the supply chain desires a higher customer rating, then the manufacturer needs to raise his quality level in the second period; otherwise, the quality level in the second period may be lower than that in the first period.

Next, we examine if early customers' review ratings can generate greater sales and larger profits to the manufacturer and the retailer in the second period than in the first period.

**Corollary 2** *The expected sales and the retailer's expected profit in the second period are greater than those in the first period. If unit deviation rating $\beta$ is sufficiently small, then the manufacturer enjoys a higher expected profit in the second period.*

Corollary 2 exposes that the information of average customer rating can increase the sales and help the retailer achieve a higher profit in the second period vis-à-vis those in the first period. The positive impact of customer reviews on sales has been shown by a number of empirical studies as described in Table 1, and can also illuminate some retailers' (e.g., Amazon) motivations to allow customer reviews (Vega 2017). We also learn that the retailer's expected profit when she allows early customers to review the product and releases the average customer rating to late customers is higher than that when she does not allow the product review. We can thus conclude that the retailer should have an incentive to permit customers to post their reviews, which is consistent with the fact that many retailers (e.g., WalMart, Expedia, and iHerb) have encouraged online customer reviews.

However, the manufacturer's expected profit in the second period may or may not be greater than that in the first period. That is, the manufacturer may not benefit from customer reviews, which is in agreement with the practical evidence that some manufacturers accept online customer reviews but others do not prepare any online space for customer reviews. Corollary 2 specifies a factor that influences the manufacturer's willingness to allow customer reviews. When customers' ratings are insignificantly dependent on the difference between the actual quality and their before-purchase expectations, the manufacturer may reduce his quality control cost and then achieve a higher profit. It is thus concluded that, if the unit deviation rating is sufficiently small, then the manufacturer can benefit from customer reviews.

Next, we provide a real data-based numerical example to illustrate our results.

**Example 1** Consider an ebook that is on sale at an online retail store. We learn from The Mill City Press (2018b) that most average-sized trade novels fall into the price range from \$13.95 to \$17.95, which is similar to Sun's statistic summary (2012). Therefore, the average price of books is \$15. In addition, Greenfield (2014) reported that the average profit margin for books published by the largest worldwide book publishers is roughly 10%. This means that the total cost for a book approximately accounts for 90% of its price, to wit, $c + q_i = 90\% \times \$15 = \$13.5$, for $i = 1, 2$. According to Hansen and Mowen's summary (2006), the unit quality cost for a product is 20 to 70 percent of the retail price for this product. That is, the unit cost that is not associated with the book quality (i.e., $c$) is around 30%-80% of its price; thus, it is reasonable to set $c = \$6$, which is 40% of the price \$15. To the best of our knowledge, most online retailers design a five-start review system for their customers. As Campbell (2016) reported, customers usually believe that an average rating closer to 5.0 is too good to be true; and, the ideal average rating for online retailers is between 4.2 to 4.5 stars out of 5.0. Accordingly, we can reasonably assume that the lowest desirable value of average customer rating is $\hat{r} = 4.2$.

Based on our discussion in Section 3.1.1, in the book industry, $\alpha = 2$; and the value of $b_1$ is around 70% of its retail price, i.e., $b_1 = 70\% \times \$15 = \$10.5$. Since in the second stage, all late customers can roughly ascertain the actual quality, in our example we reasonably assume that $\varepsilon = 2$, which means that the maximum deviation from the actual quality is around 7% of the retail price. This differs from Sun's assumption (2012) that all late customers identically perceive the actual quality according to the average rating, i.e., $\varepsilon = 0$. In addition, Sun (2012) developed a formula to approximate unit deviation rating $\beta$, i.e., $\beta = 2\sqrt{3V}/L$, where $V$ is the variance of customer ratings and $L$ is the difference between the highest and lowest customer ratings (i.e., $L = 4$). According to Sun's empirical data from `Amazon.com` and `BN.com`, the value of $V$ approximates 1. We can roughly set $\beta = 0.85$. Moreover, we learn from Sedatole's empirical study (2003) that the manufacturer's control cost for each unit of quality level is around 0.6, i.e., $\tau = 0.6$. Without loss of generality, we assume that the number of potential customers is $A = 10,000$.

According to Theorem 2, we find that in Stackelberg equilibrium, the quality levels in the first and second periods are $q_1^S = 4.68$ and $q_2^S = 5.35$. We compute the wholesale and retail prices in the first period as $w_1^S = \$14.91$ and $p_1^S = \$17.95$, respectively. Moreover, using (17), we compute the wholesale and retail prices in the second period as $w_2^S = \$11.28$ and $p_2^S = \$15.32$, respectively. One can observe that $w_1^S > w_2^S$, $p_1^S > p_2^S$, and $w_1^S - w_2^S > p_1^S - p_2^S$, which is consistent with our analytic results in Corollary 1. We also find that $D_1^S = 1,451 < D_2^S = 5,200$, $\pi_{M1}^S = \$8,845 < \pi_{M2}^S = \$20,007$, and $\pi_{R1}^S = \$4,422 < \pi_{R2}^S = \$19,413$, which are consistent with Corollary 2.

We recall from Remark 1 that early customers may overstate the actual quality when the inequality in (5) (i.e., $\tilde{q}_1 > q_1$) is satisfied or may understate the actual quality when $\tilde{q}_1 < q_1$. Moreover, early customers' overstatements or understatements influence the sales in the second period, as shown by Theorem 1. It is interesting and also important to investigate how early customers' reviews deviating from the actual quality affect the quality decision as well as wholesale and retail pricing decisions in Stackelberg equilibrium.

**Theorem 3** *We draw the following results.*

1. *When early customers overstate [understate] the actual quality (i.e., $\tilde{q}_1 > q_1$ [$\tilde{q}_1 < q_1$]), the equilibrium quality levels as well as the wholesale and retail prices in the first and second periods are higher [lower] than or the same as those when the actual quality is truly rated (i.e., $\tilde{q}_1 = q_1$).*

2. *The retailer's expected profits in the two periods when $\tilde{q}_1 > q_1$ [$\tilde{q}_1 < q_1$] are higher [lower] than those when $\tilde{q}_1 = q_1$. The manufacturer's expected profit in the first period is lower [higher] when $\tilde{q}_1 > q_1$ [$\tilde{q}_1 < q_1$]. If $\tilde{q}_1$ is sufficiently close to $q_1$, then the manufacturer's expected profit in the second period may be higher [lower] when $\tilde{q}_1 > q_1$ [$\tilde{q}_1 < q_1$]; otherwise, the manufacturer's expected profit in the second period may be lower [higher] when $\tilde{q}_1 > q_1$ [$\tilde{q}_1 < q_1$].*

We learn from the above theorem that the manufacturer responds to the quality overstatement by increasing his quality level in each period, which occurs mainly because a higher quality expectation "forces" the manufacturer to spend more efforts on quality control. To offset the larger quality cost, the manufacturer raises his wholesale price. The retailer also increases her retail price to ensure profitability.

The retailer enjoys a higher profit and achieves greater sales for each period in the quality exaggeration case compared with those when early customers truly rate the quality. However, the manufacturer is worse off in the first period, mainly because of the following facts: in the first period, early customers are uncertain about product quality, as there is no previous customer review for them. As a response, the manufacturer spends a significant effort on quality improvement. But, he does not charge a higher wholesale price to offset the quality cost. In the second period, the manufacturer may benefit from the quality overstatement by achieving a higher profit when there is only a modicum of such an overstatement. A greater overstatement leads the manufacturer to absorb a higher cost for quality control and thus obtain a lower profit vis-à-vis the case of true quality rating.

The above discussion exposes that the retailer desires the quality exaggeration made by early customers, whereas the manufacturer still expects the true quality evaluation in general. This is consistent with the fact that a number of retailers have "recruited" some persons to overstate the quality of the product available for sale at their websites and/or understate the product quality at their competitors' websites. Such reviews are called "fake reviews," which have widely existed in practice. For example, as the Negative Online Marketing (2017) reported, 11% to 14% of companies in the United States have paid for online reviews with the average cost $5 for each fake review.

**Example 2** We use parameter values in Example 1 to illustrate our results in Theorem 3. We first investigate the case that early customers overstate the actual quality, assuming that $\tilde{q}_1 = 5 > q_1^S = 4.68$. Using the formulas given in the proof of Theorem 3, we find that $\tilde{q}_1^S = 5.01 > q_1^S = 4.68$, $\tilde{q}_2^S = 5.74 > q_2^S = 5.35$; $\tilde{w}_1^S = \$15.33 > w_1^S = \$14.91$, $\tilde{p}_1^S = \$18.21 > p_1^S = \$17.95$; $\tilde{w}_2^S = \$12.51 > w_2^S = \$11.28$, $\tilde{p}_2^S = \$16.47 > p_2^S = \$15.32$. The results are consistent with those in Item 1 of Theorem 3. In addition, $\tilde{\pi}_{R1}^S = \$5,728 > \pi_{R1}^S = \$4,422$, $\tilde{\pi}_{R2}^S = \$21,003 > \pi_{R2}^S = \$19,413$; $\tilde{\pi}_{M1}^S = \$8,127 < \pi_{M1}^S = \$8,845$, and $\tilde{\pi}_{M2}^S = \$9,611 < \pi_{M2}^S = \$20,007$, as shown by Item 2 of Theorem 3.

When $\tilde{q}_1 = 4 < q_1^S = 4.68$, we find that $\tilde{q}_1^S = 4.01 < q_1^S = 4.68$, $\tilde{q}_2^S = 4.96 < q_2^S = 5,35$; $\tilde{w}_1^S = \$14.12 < w_1^S = \$14.91$, $\tilde{p}_1^S = \$17.04 < p_1^S = \$17.95$; $\tilde{w}_2^S = \$10.65 < w_2^S = \$11.28$, $\tilde{p}_2^S = \$14.79 < p_2^S = \$15.32$. These are the same as what are indicated by Item 1 of Theorem 3. In addition, $\tilde{\pi}_{R1}^S = \$3,877 < \pi_{R1}^S = \$4,422$, $\tilde{\pi}_{R2}^S = \$18,244 < \pi_{R2}^S = \$19,413$; $\tilde{\pi}_{M1}^S = \$9,230 > \pi_{M1}^S = \$8,845$, and $\tilde{\pi}_{M2}^S = \$21,333 > \pi_{M2}^S = \$20,007$, which are also consistent with our findings in Item 2 of Theorem 3.

# 4 Sensitivity Analysis and Managerial Discussions

We investigate the impact of some major parameters (i.e., $\alpha$, $\beta$, $b_1$, and $\varepsilon$) on the pricing and quality decisions in Stackelberg equilibrium as well as on the expected sales and the two firms' expected profits.

## 4.1 The Impact of Parameter $\alpha$

We examine the effects of parameter $\alpha$ (i.e., expected utility drawn from one unit of quality level) on the decisions in Stackelberg equilibrium, the expected sales, and the two firms' expected profits. We first perform sensitivity analysis to explore how this parameter influences the quality and pricing decisions in this supply chain.

**Corollary 3** *When customers can draw a higher utility from one unit of quality level, the manufacturer keeps his quality level unchanged but increases his wholesale prices in the two periods, and the retailer also raises her retail prices in the two periods.*

As the above corollary indicates, if customers can draw a greater utility from one unit of quality level, then the manufacturer should not reduce his effort on quality control in the two periods, and the two firms should increase their prices. This is attributed to the following fact: in the monopoly setting, the manufacturer's quality decisions are mainly dependent on the average customer ratings in the two periods. As a result, if, in each period, the impact of actual quality level and their quality expectations or perceptions on customer ratings does not change, then the manufacturer decides to maintain his quality level although customers gain more from each unit of quality level. We observe from (1) and (6) that in each period, a customer with a greater utility per quality level may still buy the product even when the retail price is higher. In order to obtain a larger profit margin, the retailer decides to increase her retail prices in the two periods. The manufacturer responds by increasing his wholesale price without reducing the retailer's profit margin.

Next, we examine how customers' utility per quality level influences the expected sales as well as the manufacturer's and the retailer's expected profits.

**Corollary 4** *When customers' utility per quality level rises, the expected sales in the two periods increase. Moreover, both the manufacturer and the retailer can achieve higher expected profits in the two periods.*

We learn from Corollary 4 that when customers can gain more from each unit of quality level, both early and late customers may decide to purchase more from the retailer. This occurs possibly because in each period, customers are uncertain about the product quality and their purchase decisions would be highly associated with their utility per quality level. If they expect more gain from the quality, then they may be more likely to buy. As Corollary 3 indicates, the retail price is increased, which could reduce the positive impact of a higher utility per quality level. Nevertheless, when customers have a higher utility from the quality, the supply chain benefits by achieving higher sales in total.

Since both the manufacturer and the retailer respond to a higher customer utility per quality level by increasing their prices, their expected profits are greater when customers gain more from each unit of quality level. This result implies that the two firms can obtain greater profits when customers benefit more from the product quality. To illustrate our results in Corollaries 3 and 4, we use the parameter values in Example 1 but increase the value of $\alpha$ from 1.5 to 2.5 in increments of 0.1, we plot Figure 1 to show the effect of customers' utility per quality level $\alpha$ on the supply chain. The implications drawn from Figure 1 are consistent with the results in Corollaries 3 and 4. Moreover, we find from this figure that regardless of what the value of $\alpha$ is, the manufacturer's and the retailer's expected profits in the second period are always larger than those in the first period. That is, the manufacturer and the retailer benefit from the increase in consumers' utility per quality level.

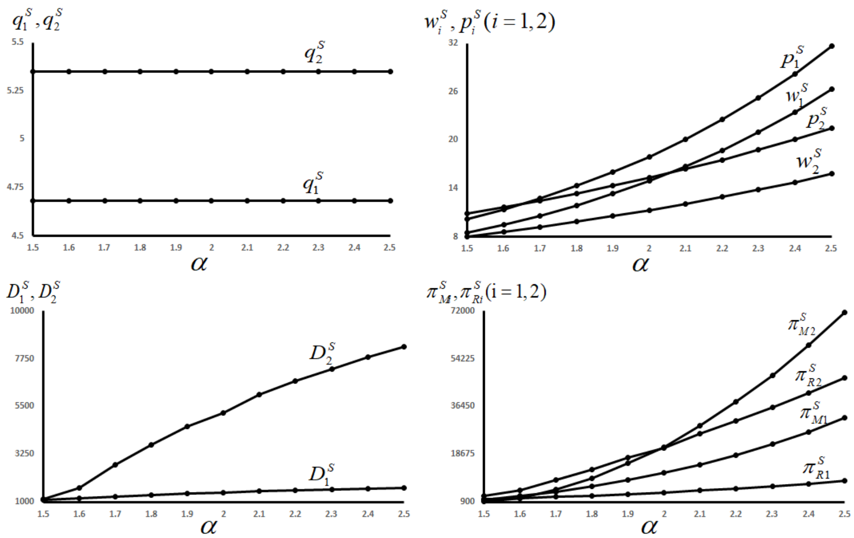


Figure 1: The impact of $\alpha$ on the supply chain.

### 4.2 The Impact of Parameter $\beta$

We investigate how the unit deviation rating (i.e., $\beta$) influences the two firms' decisions in Stackelberg equilibrium, as well as the expected sales and the expected profits in the two periods.

**Corollary 5** *When customers incur a higher unit deviation rating, the manufacturer raise his quality level in the two periods. Moreover, the wholesale and retail prices in the two periods also increase.*

A higher deviation rating implies that each customer provides a lower rating if the actual product quality cannot meet the customer's before-purchase expectation. Naturally, if the deviation rating is higher *ceteris paribus*, then customers are more likely to abandon their online shopping carts. In order to avoid this, the manufacturer should increase his quality level, which can help reduce the deviation chance. But, an increase in the quality level leads the manufacturer to incur a higher cost for quality control. To assure his profit margin, the manufacturer may increase the wholesale price in each period. In the two periods, the retailer increases her retail prices, because, in her view, a higher quality level can prevent the sales from dropping, even though the retail prices are somewhat increased.

**Corollary 6** *When customers' unit deviation rating is higher, the expected sales in the first period are reduced, and the manufacturer's and the retailer' expected profits in the first period decrease. However, in the second period, the expected sales and the two firms' expected profit increase.*

The above corollary reveals that when customers' ratings are more dependent on the difference between the actual quality and customers' quality expectations/perceptions, the supply chain experiences a sales reduction in the first period but can sell more in the second period. This is mainly ascribed to the fact that early customers in the first period cannot ascertain the actual product quality but late customers in the second period can make their quality perceptions around the actual value. This means that increasing the actual product quality cannot significantly influence early customers' purchase decisions but can greatly affect late customers' decisions. Therefore, when the manufacturer increases his wholesale price in the first period, the retailer also raises her retail price and as a result, less early customers purchase the product from the retailer. In the second period, the actual quality is almost certain to late customers. Although the manufacturer increases the wholesale price in order to offset a higher cost for quality improvement, a higher quality level still attracts more late customers to buy.

As Corollary 5 indicates, the manufacturer responds to a higher deviation rating by improving the actual quality in each period. A greater cost for quality control and a sales reduction in the first period make the manufacturer experience a profit loss in this period. The retailer's profit is also reduced as a consequence of the sales reduction. In the second period, as the unit deviation rating rises, late customers can accurately perceive the increasing quality level and buy more from the retailer, who then enjoys a higher profit. Similarly, the manufacturer can sell more, although he charges a larger wholesale price. Therefore, the manufacturer can also achieve a higher profit in the second period.

To illustrate our analytic results in Corollaries 5 and 6, we perform numerical sensitivity analysis by using the parameter values in Example 1 but increasing the value of $\beta$ from 0.8 to 0.9 in steps of 0.05. We present our numerical results in Figure 2, which are consistent with the analytic results in Corollaries 5 and 6.

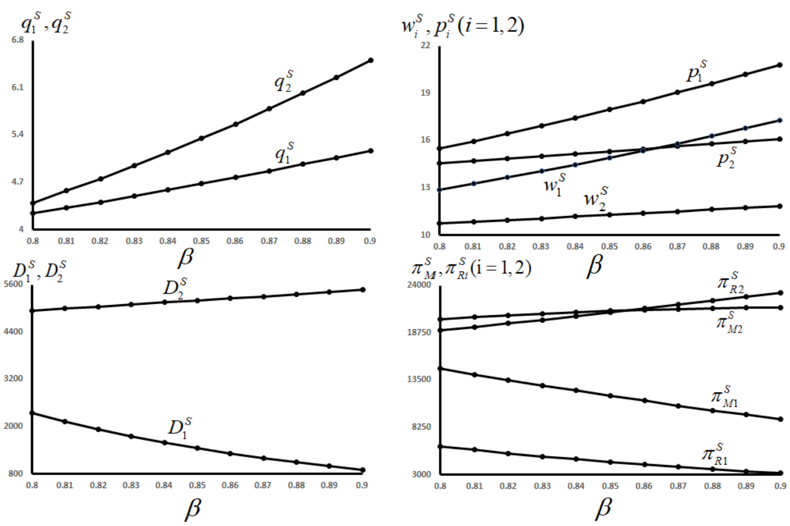


Figure 2: The impact of $\beta$ on the supply chain.

### 4.3 The Impact of Parameters $b_1$ and $\varepsilon$

We examine the impact of parameters (i.e., $b_1$ and $\varepsilon$) in the distributions of early and late customers' quality expectations/perceptions on the supply chain. One may note that the variance of early customers' quality expectations is $b_1^2/12$, in which $b_1 > 0$ is sufficiently large so as to characterize the widely-scattered expectations. Moreover, the variance of late customers' quality perceptions is $\varepsilon^2/3$, in which $\varepsilon > 0$ assumes a sufficiently small number conforming to the fact that late customers can use early customers' average rating to roughly perceive the actual quality.

**Corollary 7** *When the variance of early customers' quality expectations increases, the quality levels as well as the wholesale and retail prices in the two periods rise. Moreover, the expected sales as well as the two firms' expected profits in the two periods are increasing in the variance.*

When early customers are highly uncertain about the product quality, their quality expectations are largely spread out. The manufacturer should respond to a larger uncertainty by improving his quality level, which would help improve early customers' purchase satisfaction, thus enticing them to purchase more and also provide a higher average rating. This encourages the manufacturer to also raise the quality level in the second period. As a consequence of quality improvement, the manufacturer incurs a higher cost for quality control. To offset this, the manufacturer increases the wholesale prices in the first and second periods, which also induces the retailer to raise her retail prices in the two periods.

Although the wholesale price in the first period increases, the quality improvement can greatly attract early customers to buy; therefore, the sales in the first period are higher than those when the variance is unchanged. In the second period, although late customers are almost sure of the actual quality before they make purchase decisions, the quality improvement in the second period can help stimulate more late customers to buy from the retailer. Moreover, the manufacturer benefits from a higher variance of early customers' expectations by enjoying a higher profit in each period. This occurs because the sales increase and the manufacturer's wholesale prices in the two periods are higher. Similarly, an increase in the variance in the first period can raise the retailer's profit in each period.

We conclude from the above that the two firms can achieve higher sales and also profit more from an increase in the variance of early customers' quality expectations. This means that the variance in the first period benefits the supply chain. We plot Figure 3 to verify our results in Corollary 7. In Figure 3, the charts indicate the results for the impact of $b_1$ which is increased from 5.5 to 15.5 in steps of 1. The implications from Figure 3 are the same as what we learn from Corollary 7.

**Corollary 8** *In the second period, as the variance of late customers' quality perceptions increases, the quality level, the wholesale price, and the retail price increase. However, as a result of an increase in the variance, the expected sales as well as the manufacturer's and the retailer's expected profits decrease.*

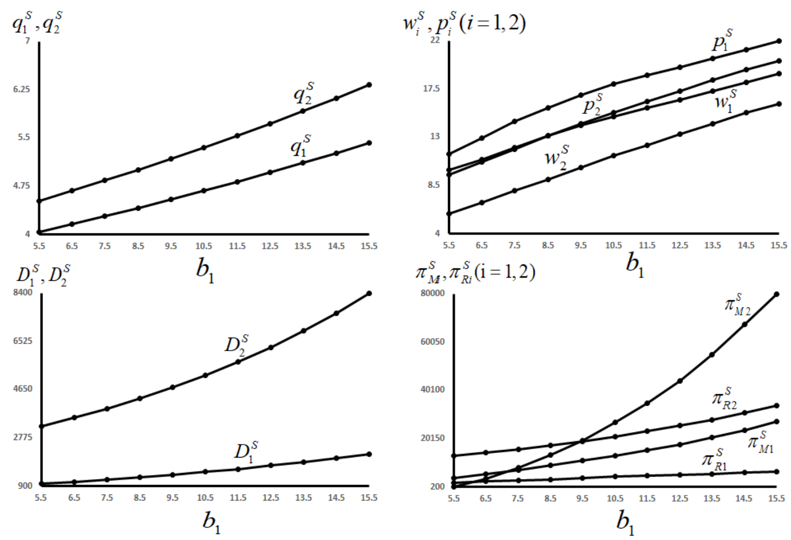

Figure 3: The impact of $b_1$ on the supply chain.

The above corollary exposes that the variance of late customers' quality perceptions does not influence the supply chain in the first period but only affects the manufacturer's and the retailer's quality and pricing decisions, the expected sales, and the two firms' expected profits in the second period. This happens because in the first period, the supply chain has no information about the variance of late customers' quality perceptions in the second period and is thus mainly concerned with the variance of early customers' quality expectations. We also find from Corollary 8 that the two firms' quality and pricing decisions are increasing in the variance. We justify this result as follows: when late customers have more different views about the actual quality after they observe the average rating provided by early customers, their quality perceptions are spread out in a wider range. The manufacturer responds by improving his quality level in the second period, which increases the manufacturer's cost for quality control. As a result, the manufacturer increases his wholesale price to offset the larger quality cost. To ensure the marginal profit, the retailer also increases his retail price in the second period.

In the second period, a higher retail price discourages late customers from buying at the retail store; thus, the expected sales are decreasing in the variance. Although the wholesale and retail prices rise, the manufacturer and the retailer still suffer from a larger variance by obtaining less profits. We thereby conclude that an increase in the variance would make the supply chain worse off. That is, a smaller variance of late customers' quality perceptions—i.e., late customers have more similar perceptions according to early customers' product reviews—can benefit the supply chain.

We verify our results in Corollary 8 by plotting Figure 4, in which we present the numerical results for the impact of $\varepsilon$ which is increased from 1.5 to 2.5 in increments of 0.1. Because any change of $\varepsilon$ does not influence the decisions and the supply chain performance in the first period, we only present the results for the second period. The implications from Figure 4 are consistent with the findings given in Corollary 8.

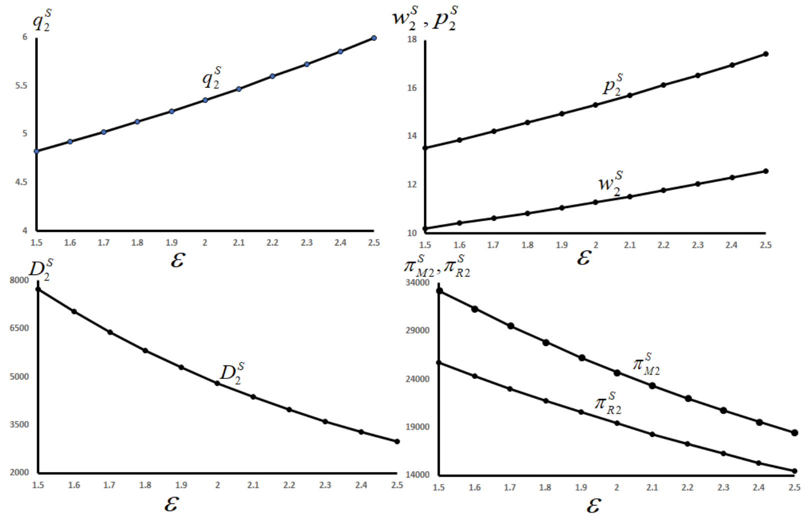


Figure 4: The impact of $\varepsilon$ on the supply chain.

## 5 Summary and Concluding Remarks

In this paper we investigate a two-period game problem for a supply chain in which, for each period, the manufacturer determines his wholesale pricing and quality decisions and the retailer then decides on her retail price when past customers' reviews influence potential customers' purchase decisions. We begin by analyzing customers' purchases and review ratings in the two-period problem. In the first period, early customers do not have any information regarding past customers' reviews but only make their purchase decisions based on their quality expectations. Moreover, early customers provide their review ratings, according to the actual quality and the difference between the actual product quality and their before-purchase expectations. The retailer releases the average review rating to late customers in the second period. Different from early customers, late customers use the average review rating to perceive the actual quality before their purchases.

According to our analysis of customers' behaviors, we derive the expected sales and the two firms' expected profits for the two periods. Then, we analyze a leader-follower game for each period, in which the manufacturer determines his quality and wholesale pricing decisions before the retailer makes her retail pricing decision. We find a unique Stackelberg equilibrium for each period, and also analyze the supply chain when early customers overstate or understate the actual quality. We then perform sensitivity analysis to explore how the customers' utility per quality level, their unit deviation rating, as well as the variation of early customers' quality expectations and that of late customers' quality perceptions influence the Stackelberg equilibrium-characterized decisions, the expected sales, and the two firms' expected profits.

According to our analysis, we draw a number of managerial insights that are verified by our realistic data-based numerical studies. Next, we discuss major insights for managerial practice.

When early customers exaggerate the actual quality in their ratings, the sales in the second period are higher than those when the average review rating reflects the actual quality. Moreover, the quality levels as well as the wholesale and retail prices in two periods are higher than or the same as those when the actual quality is truly rated. When early customers overstate the actual quality, the retailer's expected profits in the two periods are higher than those when early customers truly rate the actual quality. However, the manufacturer experiences a lower profit in the first period and may also profit less in the second period. Our result may explain the practices that the retailer prefers the quality overstatement to the quality understatement, and may thus pay for fake reviews that are positive to the retailer herself but negative to her competitors.

The wholesale and retail prices in the second period are lower than those in the first period. It thereby follows that the expected sales in the second period are higher than those in the first period. Moreover, the retailer's profit margin in the second period is higher than that in the first period, which means that the retailer profits more in the second period than in the first period. However, the manufacturer's expected profit in the second period may or may not be greater than that in the first period, which is dependent on the unit deviation rating. Specifically, the manufacturer can enjoy a higher profit if customers' ratings are insignificantly dependent on the deviation. Both the manufacturer and the retailer can obtain higher profits when customers enjoy a greater utility from each unit of quality level. Moreover, the sales are increasing in the customer utility per quality level. This result reveals that the supply chain can achieve a higher benefit if customers gain more from the quality level. If customers' ratings are lower when their quality expectations or perceptions deviate from the actual quality, then the manufacturer improves his quality levels in the two periods, and the two firms increase their prices in the two periods. The expected sales in the second period rise whereas those in the first period decrease. Moreover, the manufacturer and the retailer profit less from customers' higher deviation rating in the first period, whereas they can achieve higher expected profits in the second period.

We find that in the first period, the supply chain is better off if the variance of early customers' quality expectations is larger; but, in the second period, the supply chain achieves a higher profit if the variance of later customers' quality perceptions is smaller. Thus, the supply chain expects a larger variance of early customer's quality expectations and a smaller variance of later customers' quality perceptions. The interesting and somewhat surprising result is mainly attributed to the following reason: before early customers buy from the retailer, they cannot observe any average rating to perceive the actual quality but have to make their expectations that are usually spread in a wide range. A higher variance of early customers' expectations reflects these customers' larger uncertainty on the actual quality. This implies that a greater quality level may be more effective in improving early customers' incentives to buy. The manufacturer responds by raising his quality level and also increasing his wholesale price, which also induces the retailer to increase her retail price. Since the quality is more influential than the price in early customers' purchases, the two firms can achieve higher sales and also enjoy greater profit margins, which makes the supply chain better off when the variance of early customers' quality expectations is larger. Different from early customers, late customers can observe early customers' average rating to perceive the actual quality, which means that they may not mainly base their purchase decisions on the quality; instead, the price plays an important role in customers' purchase decisions. In the second period, although the manufacturer has to improve his quality level as a response to a higher variance of late customers' quality perceptions, the two firms increase their prices, which then results in less sales. It thus follows that in the second period, the supply chain benefits from a smaller variance of late customers' quality perceptions.

In this paper, we have several limitations and accordingly suggest some research directions. First, similar to many extant publications, for tractability, we assume a uniform distribution for early customers' quality expectations and another uniform distribution for late customers' quality perceptions. The assumptions may not hold in some practices. In future, we may relax the assumptions, and collect real data to generate a best-fit random distribution. Using the random distribution, we would perform simulation to find the optimal decisions for the supply chain with some software (e.g., Arena with OptQuest). The real data-based study may generate some new insights. For an application of such a simulation approach, see a publication by Becerril-Arreola, Leng, and Parlar (2013).

In this paper, we do not consider the product returns and the restocking fee decision for the retailer. In practice, the retailer may charge customers a restocking fee when they return their products. Since it is intractable in this paper to consider the joint pricing, quality, and restocking fee decisions, we may consider a potential research direction in which we assume either the prices or the quality as an exogenous variable rather than a decision variable. One may also consider a simulation model to investigate a supply chain in which the pricing, quality, and restocking fee decisions are jointly made. We do not consider the competition between two retailers or that between two manufacturers, which may be important to supply chain operations. In future, we may investigate the competition at an echelon in the supply chain.

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Online Supplements

**"Managing a Supply Chain under the Impact of Customer Reviews: A Two-Period Game Analysis"**

X. Wang, M. Leng, J. Song, C. Luo, S. Hui

# Appendix A Proofs

**Proof of Proposition 1.** Given wholesale price $w_2$ and quality $q_2$, we partially differentiate $\pi_{R2}(p_2)$ once and twice w.r.t. $p_2$, and find

$$\frac{\partial \pi_{R2}(p_2)}{\partial p_2} = \frac{\alpha(q_1+\varepsilon)+w_2-2p_2}{2\alpha\varepsilon} \text{ and } \frac{\partial^2 \pi_{R2}(p_2)}{\partial (p_2)^2} = -\frac{1}{\alpha\varepsilon} < 0,$$

which indicates the strict concavity of $\pi_{R2}(p_2)$. The optimal retail price is obtained as

$$p_2^*(w_2, q_1) = \frac{\alpha(\varepsilon+q_1)+w_2}{2}.$$

Next, we find the manufacturer's optimal quality and pricing decisions when the retailer chooses her best response. With $p_2^*(w_2, q_1)$, the manufacturer's expected profit becomes

$$\pi_{M2}(w_2, q_2) = (w_2 - c - \tau q_2)(\varepsilon + q_1 - p_2^*(w_2, q_1)/\alpha)/(2\varepsilon).$$

We compute the first- and second-order derivatives of $\pi_{M2}(w_2, q_2)$ w.r.t. $w_2$ as

$$\frac{\partial \pi_{M2}(w_2, q_2)}{\partial w_2} = \frac{\alpha(\varepsilon+q_1)+\tau q_2+c-2w_2}{4\alpha\varepsilon} \text{ and } \frac{\partial^2 \pi_{M2}(w_2, q_2)}{\partial (w_2)^2} = -\frac{1}{2\alpha\varepsilon} < 0,$$

which indicates the strict concavity of $\pi_{M2}(w_2, q_2)$. We then find that

$$w_2^*(q_2) = \frac{\alpha(\varepsilon+q_1)+\tau q_2+c}{2};$$

and rewrite the manufacturer's expected profit as

$$\begin{aligned}\pi_{M2}(q_2) &\equiv \pi_{M2}(w_2^*(q_2), q_2) \\ &= (w_2^*(q_2) - c - \tau q_2)(\varepsilon + q_1 - p_2^*(w_2^*(q_2), q_1)/\alpha)/(2\varepsilon) \\ &= \frac{[\alpha(\varepsilon+q_1) - \tau q_2 - c]^2}{16\alpha\varepsilon}.\end{aligned}$$

In addition, using $w_2^*(q_2)$, we find the resulting retail price in the second period as

$$p_2^*(w_2^*(q_2), q_1) = \frac{3\alpha(\varepsilon+q_1)+\tau q_2+c}{4},$$

which is smaller than or equal to $\alpha(\varepsilon+q_1)$, if the second-period quality decision $q_2$ is made such that $\tau q_2 + c \le \alpha(\varepsilon + q_1)$. Partially differentiating $\pi_{M2}(q_2)$ once w.r.t. $q_2$ yields

$$\frac{\partial \pi_{M2}(q_2)}{\partial q_2} = -\frac{\tau[\alpha(\varepsilon + q_1) - \tau q_2 - c]}{8\alpha\varepsilon} < 0,$$

which means that $\pi_{M2}(q_2)$ is decreasing in $q_2$. Noting from (10) that $q_2 \ge \tilde{q}_2$, we obtain the manufacturer's optimal quality decision as $q_2 = \tilde{q}_2$. Therefore, we find the wholesale and retail prices as well as the quality decision in Stackelberg equilibrium as shown in this proposition. ■

**Proof of Theorem 2.** Similar to our supply chain analysis for the second period in Section 3.2.1, we first find the optimal retail price as a response to the manufacturer's decisions, and then compute Stackelberg equilibrium. Given wholesale price $w_1$ and quality level $q_1$, we partially differentiate $\Pi_R(p_1)$ once and twice w.r.t. $p_1$, and find

$$\frac{\partial \Pi_R(p_1)}{\partial p_1} = \frac{\alpha b_1 - 2p_1 + w_1}{\alpha b_1} \text{ and } \frac{\partial^2 \Pi_R(p_1)}{\partial (p_1)^2} = -\frac{2}{\alpha b_1} < 0,$$

which indicates the strict concavity of $\Pi_R(p_1)$. We thus have the $w_1$-dependent optimal retail price as

$$p_1^*(w_1) = \frac{\alpha b_1 + w_1}{2}. \tag{18}$$

We next analyze the manufacturer's wholesale pricing and quality decisions in the first period. We start with the optimal wholesale pricing decision given a quality level. Then, we substitute the optimal quality-dependent wholesale price into the manufacturer's expected profit function, and maximize it for the optimal quality level.

Using $p_1^*(w_1)$ in (18), we have the manufacturer's expected profit function as

$$\Pi_M(w_1, q_1) = (w_1 - c - \tau q_1)[1 - p_1^*(w_1)/(\alpha b_1)] + \pi_{M2}^S(q_1).$$

We differentiate $\Pi_M(w_1, q_1)$ once and twice w.r.t. $w_1$ as

$$\begin{aligned} \frac{\partial \Pi_M(w_1, q_1)}{\partial w_1} &= 1 - (\alpha b_1 + w_1)/(2\alpha b_1) - (-\tau q_1 - c + w_1)/(2\alpha b_1), \\ \frac{\partial^2 \Pi_M(w_1, q_1)}{\partial (w_1)^2} &= -1/(\alpha b_1) < 0, \end{aligned}$$

which means the strict concavity of $\Pi_M(w_1, q_1)$ w.r.t. $w_1$, when the value of $q_1$ is not changed. Therefore, given quality level $q_1$, the manufacturer's $q_1$-dependent optimal wholesale price is

$$w_1^*(q) = \frac{\alpha b_1 + \tau q_1 + c}{2}.$$

As a result, the manufacturer's expected profit becomes $\Pi_M(q_1) \equiv \Pi_M(w_1^*(q_1), q_1)$. Since the first-order derivative of $\Pi_M(q_1)$ w.r.t. $q_1$ is too complicated, we cannot find whether their signs are positive or negative, and instead compute the second-order derivative of this objective function as

$$\begin{aligned}\frac{\partial^2 \Pi_M(q_1)}{\partial (q_1)^2} &= [2(\alpha-2\tau)^2\beta^2 + 4(\alpha^2 - 3\alpha\tau + 2\tau^2)\beta + 2(\alpha-\tau)^2]b_1 + 4\varepsilon\tau^2(1+\beta)^2 \\ &> 0,\end{aligned}$$

which means that $\Pi_M(q_1)$ is convex on $q_1$. We should determine the range in which the value of $q_1$ can change. We learn from Section 3.1.1 that $q_1 \geq (2\hat{r} + \beta b_1)/[2(1+\beta)]$, where $\hat{r}$ is the lowest average customer rating acceptable to the retailer. In addition, we find that the sales in the second period are

$$D_2 = \frac{\alpha b_1 - \tau q_1 - c}{4\alpha b_1},$$

which is non-negative when $q_1 \leq (\alpha b_1 - c)/\tau$. Therefore, the range for $q_1$ is $[(2\hat{r} + \beta b_1)/[2(1+\beta)], (\alpha b_1 - c)/\tau]$. When $q_1 = (\alpha b_1 - c)/\tau$, $\Pi_M(q_1) = 0$; but, when $q_1 = (2\hat{r} + \beta b_1)/[2(1+\beta)]$, $\Pi_M(q_1) > 0$. Thus, $q_1^S = (2\hat{r} + \beta b_1)/[2(1+\beta)]$. We then compute the first-period wholesale and retail prices in Stackelberg equilibrium as given in (15). ■

**Proof of Corollary 1.** Using (15) and (17), we compute

$$\begin{aligned} w_2^S - w_1^S &= \frac{-1}{4(1+\beta)^2}\{[\alpha b_1 - 2(\alpha+\tau)\varepsilon]\beta^2 + [\alpha(3b_1 - 4\varepsilon - 2\hat{r}) \\ &\quad +\tau(b_1 - 2\varepsilon - 2\hat{r})]\beta + 2\alpha(b_1 - \varepsilon - \hat{r})\}.\end{aligned}$$

Since $b_1$ is a sufficiently large number and $\varepsilon$ is a sufficiently small number, $w_2^S - w_1^S < 0$. We also compute

$$\begin{aligned} p_2^S - p_1^S &= \frac{-1}{8(1+\beta)^2}\{[\alpha b_1 - 2(3\alpha+\tau)\varepsilon]\beta^2 + [\alpha(9b_1 - 12\varepsilon - 6\hat{r}) \\ &\quad +\tau(b_1 - 2\varepsilon - 2\hat{r})]\beta + 6\alpha(b_1 - \varepsilon - \hat{r})\}.\end{aligned}$$

which is negative. Thus, $p_2^S < p_1^S$. Moreover, it is easy to find that $p_1^S - p_2^S < w_1^S - w_2^S$. Then, we find that

$$q_2^S - q_1^S = -\frac{\beta\{b_1 - 2[\hat{r} + \varepsilon(1+\beta)]\}}{2(1+\beta)^2},$$

which may or may not be positive, depending on $b_1$ and $2[\hat{r}+\varepsilon(1+\beta)]$. We thus prove this corollary. ■

**Proof of Corollary 2.** We first compare the expected sales in the first and second periods, which are $D_1^S$ and $D_2^S$, respectively. Using $q_1^S$ as given in (15), we compute

$$\begin{aligned} D_2^S - D_1^S &= \frac{1}{16\alpha b_1\varepsilon(1+\beta)^2}\{\beta[\alpha+(\alpha-\tau)\beta]b_1^2+[-2(\alpha\varepsilon+c)\beta^2 \\ &\quad +2(\alpha\hat{r}-2\alpha\varepsilon-2\hat{r}\tau-2c)\beta+2(\hat{r}\alpha-\hat{r}\tau-c-\alpha\varepsilon)]b_1 \\ &\quad +4\varepsilon(1+\beta)(\beta c+\hat{r}\tau+c)\} \\ &> 0. \end{aligned}$$

In period $i$ ($i = 1, 2$) the retailer's profit is $\pi_{Ri}^S = (p_i^S - w_i^S)D_i^S$. As $D_2^S > D_1^S$ and Corollary 1 indicates that $w_1^S - w_2^S > p_1^S - p_2^S$, or, $p_2^S - w_2^S > p_1^S - w_1^S$, we find that $\pi_{R2}^S > \pi_{R1}^S$.

Next, we compare the manufacturer's profits in the two periods. One may note that both $\pi_{M1}^S$ and $\pi_{M2}^S$ are too complicated to be compared. In fact, $\pi_{M1}^S$ may be greater than, may be equal to, or may be less than $\pi_{M2}^S$. Nevertheless, we find that the denominator of $\pi_{M2}^S - \pi_{M1}^S$ is $16(1+\beta)^4\alpha\varepsilon > 0$. Thus, we examine the numerator of $\pi_{M2}^S - \pi_{M1}^S$, which is denoted by $N(\pi_{M2}^S - \pi_{M1}^S)$. Since $N(\pi_{M2}^S - \pi_{M1}^S)$ is a very complicated expression involving a very large number of terms, we consider a special case in which the value of $\beta$ is very small. If the value of $\beta$ approaches zero, i.e., $\beta \to 0^+$, then

$$\begin{aligned} \lim_{\beta\to 0^+} N(\pi_{M2}^S - \pi_{M1}^S) &= [(\varepsilon+\hat{r})^2\alpha^2+(2(\varepsilon-\hat{r}))(\hat{r}\tau+c)\alpha+(\hat{r}\tau+c)^2]b_1-2\alpha^2b_1^2\varepsilon-2\varepsilon(\hat{r}\tau+c)^2 \\ &> 0. \end{aligned}$$

Therefore, $\pi_{M2}^S > \pi_{M1}^S$, when the value of $\beta$ is sufficiently small. ■

**Proof of Theorem 3.** We first investigate the supply chain when $\tilde{q}_1 > q_1$. For this case, $\tilde{q}_1^S > q_1^S = (2\hat{r} + \beta b_1)/[2(1+\beta)]$. The quality and pricing decisions in the second period are

$$\begin{cases} \tilde{q}_2^S = \dfrac{2[(1+2\beta)\tilde{q}_1^S+\beta\varepsilon]-\beta b_1}{2(1+\beta)}, \\ \tilde{w}_2^S = \dfrac{2[(\alpha+2\tau)\beta+\alpha+\tau]\tilde{q}_1^S-(b_1-2\varepsilon)\tau\beta+2(1+\beta)(\alpha\varepsilon+c)}{4(1+\beta)}, \\ \tilde{p}_2^S = \dfrac{2[(3\alpha+2\tau)\beta+3\alpha+\tau]\tilde{q}_1^S-(b_1-2\varepsilon)\tau\beta+2(1+\beta)(3\alpha\varepsilon+c)}{8(1+\beta)}. \end{cases}$$

Obviously, $\tilde{q}_2^S > q_2^S$, $\tilde{w}_2^S > w_2^S$, and $\tilde{p}_2^S > p_2^S$. In the second period, the retailer's expected profit is

$$\begin{aligned} \tilde{\pi}_{R2}^S &= (\tilde{p}_2^S - \tilde{w}_2^S)\tilde{D}_2^S \\ &= \left\{\frac{2\alpha\varepsilon(1+\beta)+2[\alpha(1+\beta)-\tau(1+2\beta)]\tilde{q}_1^S+\beta\tau(b_1-2\varepsilon)-c}{8(1+\beta)}\right\} \\ &\quad \times\frac{2[(\alpha-2\tau)\beta+\alpha-\tau]\tilde{q}_1^S++\beta\tau(b_1-2\varepsilon)+2(1+\beta)(\alpha\varepsilon-c)}{8\alpha\varepsilon} \\ &> \pi_{R12}^S, \end{aligned}$$

which means that the retailer's expected profit in the second period is higher for the quality overstatement case. We compute manufacturer's expected profits in the second period as

$$\tilde{\pi}^S_{M2} = (\tilde{w}^S_2 - c - \tau \tilde{q}^S_2)\tilde{D}^S_2 = \frac{[(\alpha \tilde{q}^S_1 - \tau \tilde{q}^S_2) + \alpha\varepsilon - c]^2}{16\varepsilon\alpha},$$

which may or may not be greater than $\pi^S_{M2}$, depending on the comparison between $q^S_i$ and $\tilde{q}^S_i$ $(i = 1, 2)$. If $\tilde{q}_1$ is sufficiently close to $q$, then $\tilde{\pi}^S_{M2} > \pi^S_{M2}$. Otherwise, $\tilde{\pi}^S_{M2}$ could be smaller than $\pi^S_{M2}$.

In addition, because $\tilde{q}^S_1 \geq q^S_1$, the wholesale and retail prices in the first period are

$$\tilde{w}^S_1 = \frac{\alpha b_1 + \tau \tilde{q}^S_1 + c}{2} > w^S_1 \text{ and } \tilde{p}^S_1 = \frac{\alpha b_1 + \tilde{w}^S_1}{2} > p^S_1.$$

We also compute the retailer's expected profit in the first period as

$$\tilde{\pi}^S_{R1} = (\tilde{p}^S_1 - \tilde{w}^S_1)\tilde{D}^S_1 = \frac{(\tau \tilde{q}^S_1 - \alpha b_1 + c)^2}{16\alpha b_1} > \pi^S_{R1},$$

and calculate the manufacturer's expected profits in the first period as

$$\tilde{\pi}^S_{M1} = (\tilde{w}^S_1 - c - \tau \tilde{q}^S_1)\tilde{D}^S_1 = \frac{(\tau \tilde{q}^S_1 - \alpha b_1 + c)^2}{8\alpha b_1} < \pi^S_{M1}.$$

Similarly, when $\tilde{q} < q$, we can obtain the results as shown in this theorem. ■

**Proof of Corollary 3.** We learn from (15) that the manufacturer's quality level $q^S_1$ in the first period does not change with the value of parameter $\alpha$, and also find from (17) that the quality level $q^S_2$ in the second period also does not change as the value of parameter $\alpha$ varies. We then compute the first-order derivative of the first-period wholesale and retail prices w.r.t. $\alpha$ as $\partial w^S_1/\partial\alpha = b_1/2 > 0$ and $\partial p^S_1/\partial\alpha = 3b_1/4 > 0$. In addition, we compute the first-order derivative of $w^S_2$ w.r.t. $\alpha$ as $\partial w^S_2/\partial\alpha = (q^S_1 + \varepsilon)/2 > 0$, and find the first-order derivative of $p^S_2$ w.r.t. $\alpha$ as $\partial p^S_2/\partial\alpha = 3(q^S_1 + \varepsilon)/4 > 0$. ■

**Proof of Corollary 4.** Using (16), we can find the first-period expected sales (i.e., $D^S_1$). Differentiating $D^S_1$ once w.r.t. $\alpha$ yields

$$\frac{\partial D^S_1}{\partial\alpha} = \frac{\beta(\tau b_1 + 2c) + 2(\hat{r}\tau + c)}{8\alpha^2 b_1(1+\beta)} > 0.$$

Then, we partially differentiate the second-period expected sales (i.e., $D^S_2$) in (12) and find

$$\frac{\partial D^S_2}{\partial\alpha} = \frac{\beta^2[\tau(b_1 + 2\varepsilon) + 2c] + \beta[\tau(4\hat{r} + 2\varepsilon) + 4c] + 2(\hat{r}\tau + c)}{8\alpha^2\varepsilon(1+\beta)} > 0.$$

We then compute the first-order derivatives of $\pi_{M1}^S$ and $\pi_{M2}^S$ w.r.t. $\alpha$ as

$$\begin{aligned}\frac{\partial \pi_{M1}^S}{\partial \alpha} &= \left(\frac{\partial w_1^S}{\partial \alpha} - \tau \frac{\partial q_1^S}{\partial \alpha}\right) D_1^S + (w_1^S - c - \tau q_1^S)\frac{\partial D_1^S}{\partial \alpha} \\ &= \frac{\partial w_1^S}{\partial \alpha} D_1^S + (w_1^S - c - \tau q_1^S)\frac{\partial D_1^S}{\partial \alpha} \\ &> 0,\end{aligned}$$

and

$$\frac{\partial \pi_{M2}^S}{\partial \alpha} = \frac{\partial w_2^S}{\partial \alpha} D_2^S + (w_2^S - c - \tau q_2^S)\frac{\partial D_2^S}{\partial \alpha} > 0.$$

Next, we differentiate $\pi_{R1}^S$ once w.r.t. $\alpha$ and find

$$\frac{\partial \pi_{R1}^S}{\partial \alpha} = \frac{\{\beta[b_1(\alpha+\tau/2)+c]+\hat{r}\tau+\alpha b_1+c\}\{\beta[b_1(\alpha-\tau/2)-c]+\alpha b_1-\hat{r}\tau-c\}}{16 b_1 \alpha^2(1+\beta)^2} > 0.$$

We also differentiate $\pi_{R2}^S$ once w.r.t. $\alpha$ as

$$\begin{aligned}\frac{\partial \pi_{R2}^S}{\partial \alpha} &= \frac{1}{16\varepsilon\alpha^2(1+\beta)^3}\{[\alpha(b_1/2+\varepsilon)+\tau b_1/2+\tau\varepsilon+c]\beta^2 \\ &\quad +[\alpha(\hat{r}+b_1/2+2\varepsilon)+2\hat{r}\tau+\tau\varepsilon+2c]\beta+\alpha(\hat{r}+\varepsilon)+\hat{r}\tau+c\} \\ &\quad \times\{[(\alpha-\tau)b_1/2+(\alpha-\tau)\varepsilon-c]\beta^2+[\alpha(\hat{r}+b_1/2+2\varepsilon)-2\hat{r}\tau-\tau\varepsilon-2c]\beta \\ &\quad +\alpha(\hat{r}+\varepsilon)-\hat{r}\tau-c\} \\ &> 0.\end{aligned}$$

This corollary is thus proved. ■

**Proof of Corollary 5.** We partially differentiate $q_1^S$ once w.r.t. $\beta$ as

$$\frac{\partial q_1^S}{\partial \beta} = \frac{b_1 - 2\hat{r}}{2(1+\beta)^2} > 0,$$

which means that quality level $q_1^S$ in the first period is increasing in the unit deviation rating. Partially differentiating $q_2^S$ once w.r.t. $\beta$ gives

$$\frac{\partial q_2^S}{\partial \beta} = \frac{\beta(b_1+\varepsilon-2\hat{r})+\varepsilon}{(1+\beta)^3} > 0,$$

which implies that quality level $q_2^S$ in the second period is also increasing in $\beta$.

Taking the first-order partial derivatives of $w_1^S$ and $w_2^S$ w.r.t. $\beta$, we have

$$\frac{\partial w_1^S}{\partial \beta} = \frac{\tau(b_1 - 2\hat{r})}{4(1+\beta)^2} > 0,$$

and

$$\frac{\partial w_2^S}{\partial \beta} = \frac{\beta[\alpha(b_1 - 2\hat{r}) + 4\tau(b_1/2 + \varepsilon/2 - \hat{r})] + \alpha(b_1 - 2\hat{r}) + 2\tau\varepsilon}{4(1+\beta)^3} > 0.$$

The first-order partial derivatives of $p_1^S$ and $p_2^S$ w.r.t. $\beta$ are

$$\frac{\partial p_1^S}{\partial \beta} = \frac{\tau(b_1 - 2\hat{r})}{8(1+\beta)^2} > 0,$$

and

$$\frac{\partial p_2^S}{\partial \beta} = \frac{\beta[\alpha(3b_1 - 6\hat{r}) + 4\tau(b_1/2 + \varepsilon/2 - \hat{r})] + \alpha(3b_1 - 6\hat{r}) + 2\tau\varepsilon}{4(1+\beta)^3} > 0.$$

This corollary is thus proved. ■

**Proof of Corollary 6.** Partially differentiating the first- and second-period expected sales $D_1^S$ and $D_2^S$ once w.r.t. $\beta$ gives

$$\frac{\partial D_1^S}{\partial \beta} = -\frac{\tau(b_1 - 2\hat{r})}{8\alpha b_1(1+\beta)^2} < 0,$$

and

$$\begin{aligned} \frac{\partial D_2^S}{\partial \beta} &= \frac{1}{8\alpha\varepsilon(1+\beta)^2}\{\beta^2[2\varepsilon(\alpha-\tau) - b_1\tau + \alpha b_1 - 2c] + \beta[4\varepsilon(\alpha-\tau) \\ &\quad +2b_1(\alpha-\tau) - 4c] + 2\varepsilon(\alpha-\tau) - 2\hat{r}\tau + \alpha b_1 - 2c\} \\ &> 0. \end{aligned}$$

We take the first-order partial derivative of $\pi_{M1}^S$ w.r.t. $\beta$, and find

$$\frac{\partial \pi_{M1}^S}{\partial \beta} = -\frac{\tau(b_1/2 - \hat{r})\{b_1[\alpha + \beta(\alpha - \tau/2)] - \beta c - \hat{r}\tau - c\}}{4\alpha b_1(1+\beta)^3} < 0.$$

The first-order partial derivative of $\pi_{R1}^S$ w.r.t. $\beta$ is

$$\frac{\partial \pi_{R1}^S}{\partial \beta} = -\frac{\tau(b_1/2 - \hat{r})\{b_1[\alpha + \beta(\alpha - \tau/2)] - \beta c - \hat{r}\tau - c\}}{8\alpha b_1(1+\beta)^3} < 0.$$

Since $\partial\pi_{M2}^S/\partial\beta$ involves a very large number of terms, we only examine the sign of the coefficient of $b_1^2$, denoted by $\xi_1$, because the value of $b_1$ is sufficiently large and the positive sign of its coefficient implies that $\partial\pi_{M2}^S/\partial\beta > 0$. The coefficient of $b_1^2$ is

$$\xi_1 = [(\alpha-\tau)\beta^2 + (3\alpha - 4\tau)\beta + 2\alpha] \times [(\alpha-\tau)\beta^2 + \alpha\beta] > 0,$$

which means that $\pi_{M2}^S$ is increasing in $\beta$. Similarly, we find that $\partial\pi_{R2}^S/\partial\beta > 0$. ■

**Proof of Corollary 7.** We differentiate quality level $q_i^S$ $(i = 1, 2)$ once w.r.t. $b_1$, and find that $\partial q_1^S/\partial b_1 = \beta/[2(1+\beta)] > 0$ and $\partial q_2^S/\partial b_1 = \beta^2/[2(1+\beta)^2] > 0$. The first-order derivatives of $w_i^S$

and $p_i^S$ $(i=1,2)$ w.r.t. $b_1$ are computed as follows:

$$\frac{\partial w_1^S}{\partial b_1}=\frac{2\alpha+\beta(2\alpha+\tau)}{4(1+\beta)}>0 \text{ and } \frac{\partial p_1^S}{\partial b_1}=\frac{6\alpha+\beta(6\alpha+\tau)}{8(1+\beta)}>0;$$

and

$$\frac{\partial w_2^S}{\partial b_1}=\frac{\beta[\alpha+\beta(\alpha+\tau)]}{4(1+\beta)^2}>0 \text{ and } \frac{\partial p_2^S}{\partial b_1}=\frac{\beta[3\alpha+\beta(3\alpha+\tau)]}{8(1+\beta)^2}>0.$$

We calculate the first-order derivative of the sales in the first period (i.e., $D_1^S$) w.r.t. $b_1$ as $\partial D_1^S/\partial b_1=(\beta c+\hat{r}\tau+c)/[4\alpha b_1^2(1+\beta)]>0$, and obtain the first-order derivative of the sales in the second period (i.e., $D_2^S$) w.r.t. $b_1$ as $\partial D_2^S/\partial b_1=\beta[\alpha+\beta(\alpha-\tau)]/[8\alpha\varepsilon(1+\beta)]>0$. Then, we find the first-order derivatives of $\pi_{M1}^S$ w.r.t. $b_1$ as

$$\frac{\partial \pi_{M1}^S}{\partial b_1}=\frac{\{\beta[b_1(\alpha-\tau/2)-c]+\alpha b_1-\hat{r}\tau-c\}\times\{\beta[b_1(\alpha-\tau/2)+c]+\alpha b_1+\hat{r}\tau+c\}}{8\alpha b_1^2(1+\beta)^2}>0,$$

and compute the first-order derivatives of $\pi_{R1}^S$ w.r.t. $b_1$ as

$$\frac{\partial \pi_{R1}^S}{\partial b_1}=\frac{\{\beta[b_1(\alpha-\tau/2)-c]+\alpha b_1-\hat{r}\tau-c\}\times\{\beta[b_1(\alpha-\tau/2)+c]+\alpha b_1+\hat{r}\tau+c\}}{16\alpha b_1^2(1+\beta)^2}>0.$$

Next, we compute the first-order derivatives of $\pi_{M2}^S$ and $\pi_{R2}^S$ w.r.t. $b_1$ as

$$\begin{aligned}\frac{\partial \pi_{M2}^S}{\partial b_1} &= \frac{\beta[\alpha+\beta(\alpha-\tau)]}{8\alpha\varepsilon(1+\beta)^3}\{\beta^2[(\alpha-\tau)(b_1/2+\varepsilon)-c]\\ &\quad +\beta[\alpha(b_1/2+\hat{r}+2\varepsilon)-2\hat{r}\tau-\tau\varepsilon-2c]+\alpha(\hat{r}+\varepsilon)-\hat{r}\tau-c\}\\ &> 0,\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \pi_{R2}^S}{\partial b_1} &= \frac{\beta[\alpha+\beta(\alpha-\tau)]}{16\alpha\varepsilon(1+\beta)^3}\{\beta^2[(\alpha-\tau)(b_1/2+\varepsilon)-c]\\ &\quad +\beta[\alpha(b_1/2+\hat{r}+2\varepsilon)-2\hat{r}\tau-\tau\varepsilon-2c]+\alpha(\hat{r}+\varepsilon)-\hat{r}\tau-c\}\\ &> 0.\end{aligned}$$

This corollary is thus proved. ■

**Proof of Corollary 8.** We note from our analytical results in Section 3.2 that the first-period decisions (i.e., $q_1^S$; $w_1^S$ and $p_1^S$) as well as the sales (i.e., $D_1^S$) and the two firms' expected profits (i.e., $\pi_{M1}^S$ and $\pi_{R1}^S$) are all independent of the variance of late customers' quality perceptions in the second period. Next, we examine the impact of the variance on the quality and pricing decisions, the expected sales, and the expected profits in the second period.

We find that $\partial q_2^S/\partial\varepsilon = \beta/(1+\beta) > 0$, which means that $q_2^S$ is increasing in $\varepsilon$. In addition,

$$\frac{\partial w_2^S}{\partial\varepsilon} = \frac{\alpha+\beta(\alpha+\tau)}{2(1+\beta)} > 0 \text{ and } \frac{\partial p_2^S}{\partial\varepsilon} = \frac{3\alpha+\beta(3\alpha+\tau)}{4(1+\beta)} > 0.$$

Partially differentiating $D_2^S$ once w.r.t. $\varepsilon$ gives

$$\frac{\partial D_2^S}{\partial\varepsilon} = -\frac{\beta^2[b_1(\alpha-\tau)-2c]+\beta[\alpha(2\hat{r}+b_1)-4\hat{r}\tau-4c]+2(\alpha-\tau)\hat{r}-2c}{8\alpha\varepsilon^2(1+\beta)} < 0,$$

which implies that $D_2^S$ is decreasing in variance $\varepsilon$. We calculate the first-order derivative of $\pi_{M2}^S$ w.r.t. $\varepsilon$ as

$$\begin{aligned}\frac{\partial\pi_{M2}^S}{\partial\varepsilon} &= \frac{-1}{8\alpha\varepsilon^2(1+\beta)}\{\beta^2[(\alpha-\tau)(b_1/2+\varepsilon)-c]\\ &\quad +\beta[\alpha(b_1/2+\hat{r}+2\varepsilon)-2\hat{r}\tau-\tau\varepsilon-2c]+\hat{r}(\alpha-\tau)+\alpha\varepsilon-c\}\\ &\quad \times\{\beta^2[(\alpha-\tau)(b_1/2-\varepsilon)-c]\\ &\quad +\beta[\alpha(b_1/2+\hat{r}-2\varepsilon)-2\hat{r}\tau+\tau\varepsilon-2c]+\hat{r}(\alpha-\tau)-\alpha\varepsilon-c\}\\ &< 0,\end{aligned}$$

which indicates that $\pi_{M2}^S$ is decreasing in $\varepsilon$. In addition, we find

$$\begin{aligned}\frac{\partial\pi_{R2}^S}{\partial\varepsilon} &= \frac{-1}{16\alpha\varepsilon^2(1+\beta)}\{\beta^2[(\alpha-\tau)(b_1/2+\varepsilon)-c]+\beta[\alpha(b_1/2+\hat{r}+2\varepsilon)-2\hat{r}\tau\\ &\quad -\tau\varepsilon-2c]+\hat{r}(\alpha-\tau)+\alpha\varepsilon-c\}\times\{\beta^2[(\alpha-\tau)(b_1/2-\varepsilon)-c]\\ &\quad +\beta[\alpha(b_1/2+\hat{r}-2\varepsilon)-2\hat{r}\tau+\tau\varepsilon-2c]+\hat{r}(\alpha-\tau)-\alpha\varepsilon-c\}\\ &< 0,\end{aligned}$$

which means that as the value of $\varepsilon$ increases, $\pi_{R2}^S$ decreases. ■