Supply Chain Analysis under a Price-Discount Incentive Scheme for Electric Vehicles[1]

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Abstract: We investigate an automobile supply chain where a manufacturer and a retailer serve heterogeneous consumers with electric vehicles (EVs) under a government's price-discount incentive scheme that involves a price discount rate and a subsidy ceiling. We show that the subsidy ceiling is more effective in influencing the optimal wholesale pricing decision of the manufacturer with a higher unit production cost. However, the discount rate is more effective for the manufacturer with a lower unit production cost. Moreover, the expected sales are increasing in the discount rate but may be decreasing in the subsidy ceiling. Analytic results indicate that an effective incentive scheme should include both a discount rate and a subsidy ceiling. We also derive the necessary condition for the most effective discount rate and subsidy ceiling that maximize the expected sales of EVs, and obtain a unique discount rate and subsidy ceiling that most effectively improve the manufacturer's incentive for EV production.

Key words: electric vehicle; supply chain; price discount; incentive scheme.

1 Introduction

The rapid development of the automobile industry in the past five decades has not only resulted in significant convenience to consumers but has also generated an increasingly serious air pollution problem. The environmental impact has pressured a number of governments to implement price-discount incentive schemes to promote the electric vehicle (EV). Under such schemes, each EV consumer can enjoy a price discount—calculated as a price discount rate times the retail price charged to the consumer—but limited to a subsidy ceiling. For example, the Spanish government provides a price discount equal to 25% of the purchase price, with a cap of €6,000, to each consumer who purchases a new electric car. Similar price-discount schemes have been implemented in other countries, such as the United Kingdom and Romania. For more details, see online Appendix A.

Motivated by the above practices, we consider a price-discount incentive scheme that includes a price discount rate and a subsidy ceiling. We investigate an EV supply chain that involves a manufacturer and a retailer under such a scheme. In practice, the retail price of an automobile may differ

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among consumers because of the fact that the price for each consumer is not set by the retailer but is determined as a result of the negotiation between the consumer and the retailer. We use cooperative game theory to compute the negotiated retail price for each consumer and derive the expected sales function. To our knowledge, no government has implemented a price-discount scheme or a similar concept for any other market that involves price negotiation between the consumer and retailer. Thus, the bargaining analysis—which is our key modeling approach—and its results are only applicable to the sale of vehicles. We then develop the manufacturer's expected profit function and maximize it to obtain a unique optimal wholesale price for the manufacturer. Using these results, we investigate the effect of both the price discount rate and subsidy ceiling on the manufacturer's optimal wholesale price, the expected sales, and the manufacturer's maximum expected profit.

In this paper, we use the stimulated EV sales to measure the reduction in air pollution, because of the following fact: Stimulating the EV sales would induce more consumers to use EVs instead of fuel vehicles. As roughly calculated by Cuenca, Gaines, and Vyas [5] and MacKay [12], the use of an EV can help reduce carbon emissions by approximately 30 tons compared with the use of a fuel vehicle. Therefore, stimulating EV sales can be used as a surrogate measure for the reduction in air pollution. Accordingly, we examine the impact of the incentive scheme on the expected sales of EVs.

Very few EV-related publications exist in the operations management area. Chocteau et al. [4] developed a cooperative game model to investigate the effect of the collaboration among commercial fleets on the adoption of EVs. Avci et al. [1] examined the adoption and environmental impact of EVs with battery swapping, and Mak et al. [13] constructed robust optimization models for the planning of battery swapping infrastructures. Sioshansi [17] investigated the incentives of consumers making charging decisions with different electricity tariffs.

Our paper is closely related to the recent publication by Huang et al. [7] who examined a government subsidy scheme for EVs in a duopoly setting in which a fuel automobile supply chain and an electric-and-fuel automobile supply chain compete for consumers. The incentive scheme in [7] only includes a fixed subsidy, whereas the price-discount scheme in our paper involves the retail price-dependent subsidy that cannot exceed a ceiling value. Moreover, Huang et al. [7] drew most of their managerial insights from numerical experiments; in our paper we analytically investigate an EV supply chain under a price-discount scheme and examine the effectiveness of the scheme in stimulating EV sales. Since both fixed-subsidy and price-discount schemes exist in practice, our paper and Huang et al. [7] complement each other in terms of research methodologies, contributions to literature, and findings for practitioners.

Unlike in [7], we do not consider any fuel vehicle (FV) but only focus on an EV supply chain. This is reasonable because of the following two facts. First, in the automobile industry, there are a number of EV supply chains where the electric vehicles mainly include REVAi, Buddy, Citroën C1 ev'ie, Fisker Automotive, Tesla Roadster, Smart ED, Wheego Whip LiFe, and so on. Therefore, the EV supply chain in our model exists in practice. Second, the sales of all EVs account for only a tiny share of the current automobile market. For example, in the United States, the share of EV sales was only 0.53% during the first six months of 2013 (see `hybridcars.com`). Thus, any issue in the EV market is unlikely to significantly affect the FV market.

Price discount is a commonly-used promotion strategy and has been widely investigated in marketing and operations management fields. For publications regarding price-discount schemes applicable to supply chain members, see Bernstein et al. [3], Klastorin et al. [9], and Wang [18]; for publications concerning price-discount schemes applicable to end consumers, see Gerstner and Hess [6], Huchzermeier et al. [8], Kurata and Liu [10], and Sheu [16]. In the abovementioned publications, price-discount schemes are implemented by a manufacturer or retailer to increase sales or coordinate a supply chain. In addition, a few publications (e.g., Jørgensen and Zaccour [15] and Ma et al. [11]) analyzed a government's subsidy. We examine the government's price-discount scheme for EVs that aims to reduce air pollution. Moreover, we consider the negotiation of retail price for each consumer, unlike in the abovementioned publications where retail price is determined by the firm.

2 Negotiated Retail Price

Under the government's price-discount incentive scheme, the government provides subsidy to the consumer who purchases an EV from a retailer. This subsidy is the minimum of the price discount (the discount rate α times the original retail price charged to the consumer) and subsidy ceiling A. To simplify our statement, we hereafter denote such a price-discount scheme by (α, A). In most practices in today's automobile market, the retail price for each consumer results from the negotiation between the consumer and the retailer. Accordingly, we apply the theory of cooperative game to determine the negotiated retail price for a consumer who is assumed to have valuation (consumption gain) θ on the EV.

A common solution to two-player games in cooperative game theory is "Nash bargaining scheme" (Nash [14]). The consumer pays retail price p, enjoys the consumption gain θ, and obtains the subsidy amounting to $\min(\alpha p, A)$ from the government when he or she trades with the retailer. Thus, the consumer's net gain is calculated as $u_c \equiv \theta - p + \min(\alpha p, A)$. The retailer orders the EV from the manufacturer at wholesale price w; thus, the retailer's profit resulting from the trade with the consumer is computed as $u_r \equiv p - w$. Noting that neither the consumer nor the retailer will obtain any gain or profit if they cannot reach an agreement in their transaction, we construct a Nash bargaining model as follows:

$$\max_p \Lambda = [\theta - p + \min(\alpha p, A)]\,(p - w) \quad \text{s.t.} \quad \theta - p + \min(\alpha p, A) \geq 0 \quad \text{and} \quad p - w \geq 0. \tag{1}$$

Therefore, if the manufacturer's wholesale price w is sufficiently high such that $w \geq A/\alpha$, then $p \geq A/\alpha$ because $p \geq w$; thus, $\min(\alpha p, A) = A$. This condition means that when $w \geq A/\alpha$, the consumer should obtain subsidy ceiling A from the government, and the discount rate α does not take effect in the incentive scheme. If $w < A/\alpha$, then the consumer obtains either price discount αp or subsidy ceiling A, depending on negotiated retail price p.

Theorem 1 Given the manufacturer's wholesale price w, we find that under the incentive scheme (α, A), the retail price for the consumer with the valuation θ, resulting from the negotiation between the consumer and the retailer, can be obtained as follows:

1. When $w \leq A/\alpha$, the consumer does not buy from the retailer if $\theta < \underline{\theta} \equiv (1-\alpha)w$; however, if $\theta \geq \underline{\theta}$, then the consumer trades with the retailer at negotiated price p^* as given below.

$$p^* = \begin{cases} p_1^* \equiv [w + \theta/(1-\alpha)]/2, & \text{if } \underline{\theta} \leq \theta \leq \hat{\theta} \equiv 2(1-\alpha)A/\alpha - (1-\alpha)w; \\ p_2^* \equiv A/\alpha, & \text{if } \hat{\theta} \leq \theta \leq 2A/\alpha - A - w; \\ p_3^* \equiv (\theta + A + w)/2, & \text{if } \theta \geq 2A/\alpha - A - w. \end{cases} \tag{2}$$

If the consumer's valuation θ is in the range $[\underline{\theta}, \hat{\theta}]$, then the amount of the subsidy awarded to the consumer is smaller than A; otherwise, if $\theta \geq \hat{\theta}$, then the consumer obtains a subsidy in amount of A.

2. When $w \geq A/\alpha$, the consumer may or may not buy from the retailer, depending on the consumer's valuation θ. Specifically, if $\theta < w - A$, then the consumer and the retailer would not reach an agreement on the retail price; however, if $\theta \geq w - A$, then the consumer and the retailer trade at the negotiated price $p^* = p_3^*$; the consumer can obtain subsidy A from the government.

Proof. For the proof of this theorem and those for all subsequent theorems, see online Appendix B. ■

In the above theorem, $\underline{\theta}$ denotes the after-discount payment made by consumers who buy from the retailer at wholesale price w. Since the wholesale price is the lowest retail price, $\underline{\theta}$ can be viewed as the smallest purchase cost for consumers. That is, if the discount for any purchase at the wholesale price does not exceed the subsidy ceiling, then a consumer's buying decision depends on whether his or her valuation is greater than the smallest purchase cost. If the consumer's valuation is less than the smallest purchase cost, then he or she cannot enjoy any positive net gain from buying an EV. Otherwise, the consumer should be willing to make a purchase. In Theorem 1, the threshold $\hat{\theta}$ represents the minimum valuation of consumers who obtain the maximum subsidy A. When a consumer's valuation is less than the minimum value, the consumer will receive subsidy αp, which is lower than subsidy ceiling A; otherwise, the consumer will receive the maximum subsidy.

We reasonably assume that θ is a random variable with p.d.f. $f(\theta)$ and c.d.f. $F(\theta)$ given that consumers' valuations differ in practice. Using Theorem 1, we find that the probability for a successful transaction can be calculated as $1 - F(w - \min(\alpha w, A))$. Assuming that B potential consumers exist in the market, we can express the expected sales as

$$D(w) = B[1 - F(w - \min(\alpha w, A))] = \begin{cases} D_1(w) \equiv B[1 - F((1-\alpha)w)], & \text{if } w \leq A/\alpha, \\ D_2(w) \equiv B[1 - F(w - A)], & \text{if } w \geq A/\alpha. \end{cases} \tag{3}$$

3 Optimal Wholesale Price and Scheme Effectiveness

3.1 Optimal Wholesale Price

The manufacturer incurs production cost c and obtains sales revenue w from the retailer for each EV unit. Therefore, the manufacturer's expected profit can be constructed as

$$\Pi_M(w) = \begin{cases} \Pi_{M1}(w) \equiv B(w - c)[1 - F((1-\alpha)w)], & \text{if } w \leq A/\alpha, \\ \Pi_{M2}(w) \equiv B(w - c)[1 - F(w - A)], & \text{if } w \geq A/\alpha. \end{cases} \tag{4}$$

The manufacturer maximizes its profit $\Pi_M(w)$ in (4) to determine the optimal wholesale price w^*. To solve the maximization problem, we need to (i) solve the maximization problems $\max_{w \leq A/\alpha} \Pi_{M1}(w)$ and $\max_{w \geq A/\alpha} \Pi_{M2}(w)$ to obtain the corresponding optimal wholesale prices w_1^* and w_2^*, respectively, and (ii) compare $\Pi_{M1}(w_1^*)$ and $\Pi_{M2}(w_2^*)$ to attain the global optimal solution w^*.

Lemma 1 If p.d.f. $f(\theta)$ is continuously differentiable and log-concave, then when we maximize $\Pi_{M1}(w)$ in (4) subject to $w \leq A/\alpha$ and when we maximize $\Pi_{M2}(w)$ in (4) subject to $w \geq A/\alpha$, the corresponding optimal wholesale prices can be determined as $w_1^* = \min(\hat{w}_1(\alpha), A/\alpha)$ and $w_2^* =$

$\max(\hat{w}_2(A), A/\alpha)$, respectively, where $\hat{w}_1(\alpha)$ and $\hat{w}_2(A)$ can be *uniquely* obtained by solving

$$\hat{w}_1(\alpha) = c + \frac{1 - F((1-\alpha)\hat{w}_1(\alpha))}{(1-\alpha) \times f((1-\alpha)\hat{w}_1(\alpha))} \quad \text{and} \quad \hat{w}_2(A) = c + \frac{1 - F(\hat{w}_2(A) - A)}{f(\hat{w}_2(A) - A)}, \tag{5}$$

respectively. Both $\hat{w}_1(\alpha)$ and $\hat{w}_2(A)$ are increasing in c. Moreover, if $\hat{w}_2(A) \geq A/\alpha$, then $\hat{w}_1(\alpha) \geq \hat{w}_2(A)$. That is, if $w_2^* = \hat{w}_2(A)$, then $w_1^* = A/\alpha$; otherwise, if $w_2^* = A/\alpha$, then w_1^* may be equal to $\hat{w}_1(\alpha)$ or A/α.

Proof. For the proof of this lemma and those for all subsequent lemmas, see online Appendix C. ■

The above lemma indicates that the uniqueness of solutions $\hat{w}_1(\alpha)$ and $\hat{w}_2(A)$ requires the continuous differentiability and log-concavity of p.d.f. $f(\theta)$. Bagnoli and Bergstrom [2] noted that a probability density function is log-concave if its logarithm is concave. The log-concavity possesses an important property: a log-concave p.d.f. can result in the quasi-concavity of objective functions in the operations management area. In this paper, the log-concavity of p.d.f. $f(\theta)$ implies that consumers do not have multiple separate most-likely valuations on the EV and that very few consumers have extremely low or high valuations. Many commonly-used probability distributions—such as uniform, normal, exponential, logistic, Laplace, Weibull, and Gamma—are continuously differentiable and log-concave. Thus, we reasonably assume that $f(\theta)$ is continuously differentiable and log-concave.

Lemma 1 indicates that $\hat{w}_1(\alpha)$ is only dependent on the price discount rate whereas $\hat{w}_2(A)$ is only dependent on the subsidy ceiling. We investigate the effects of α and A on $\hat{w}_1(\alpha)$ and $\hat{w}_2(A)$, respectively, as shown in the following lemma.

Lemma 2 We find that $0 < \partial\hat{w}_1(\alpha)/\partial\alpha < \hat{w}_1(\alpha)/(1-\alpha)$—that is, $\hat{w}_1(\alpha)$ is increasing in the price discount rate α—and that both $D_1(\hat{w}_1(\alpha))$ and $\Pi_{M1}(\hat{w}_1(\alpha))$ are increasing in α. In addition, $0 < \partial\hat{w}_2(A)/\partial A < 1$, that is, $\hat{w}_2(A)$ is increasing in subsidy ceiling A; $D_2(\hat{w}_2(A))$ and $\Pi_{M2}(\hat{w}_2(A))$ are both increasing in A. ■

The above lemma indicates that $\hat{w}_1(\alpha)$ is increasing in α. When the value of A is fixed, A/α is decreasing in α. Hence, a unique value α_0 exists such that $\hat{w}_1(\alpha_0) = A/\alpha_0$. We then find that $\hat{w}_1(\alpha) > A/\alpha$ if $\alpha > \alpha_0$ and $\hat{w}_1(\alpha) < A/\alpha$ if $\alpha < \alpha_0$. Given that $\hat{w}_1(\alpha)$ uniquely satisfies an equation in (5), we replace $\hat{w}_1(\alpha_0)$ with A/α_0 and find that α_0 can be *uniquely* obtained by solving the equation below.

$$1 - F((1-\alpha_0)A/\alpha_0) - (A/\alpha_0 - c)(1-\alpha_0)f((1-\alpha_0)A/\alpha_0) = 0, \tag{6}$$

which implies that α_0 is a function of A. Therefore, given subsidy ceiling A, the optimal wholesale price for the maximization problem $\max_{w \leq A/\alpha} \Pi_{M1}(w)$ is A/α_0 if the price discount rate is α_0.

Given a value of α, as A increases, $\hat{w}_2(A)$ also increases at slope $\partial\hat{w}_2(A)/\partial A$, which is smaller than one, and A/α increases at slope $1/\alpha$, which is greater than one. Since $\hat{w}_2(A) > 0$ and $A/\alpha = 0$ when $A = 0$, a unique subsidy ceiling A_0 exists such that $\hat{w}_2(A_0) = A_0/\alpha$. If $A < A_0$, then $\hat{w}_2(A) > A/\alpha$; if $A > A_0$, then $\hat{w}_2(A) < A/\alpha$. Similar to our above discussion on α_0, we find that A_0 is a function of α and *uniquely* satisfies the following equation:

$$1 - F(A_0/\alpha - A_0) - (A_0/\alpha - c)f(A_0/\alpha - A_0) = 0. \tag{7}$$

Hence, given price discount rate α, the optimal wholesale price for the maximization problem $\max_{w \geq A/\alpha} \Pi_{M2}(w)$ is A_0/α if the subsidy ceiling is A_0.

Lemma 3 We find that for a given subsidy ceiling A, $A/\alpha_0 \geq \hat{w}_2(A)$ and $\Pi_{M1}(\hat{w}_1(\alpha_0)) < \Pi_{M2}(\hat{w}_2(A))$; for a given discount rate α, $A_0/\alpha \leq \hat{w}_1(\alpha)$ and $\Pi_{M1}(\hat{w}_1(\alpha)) > \Pi_{M2}(\hat{w}_2(A_0))$. ■

Using the above lemma, we can determine the manufacturer's globally-optimal wholesale price w^* that maximizes $\Pi_M(w)$ in (4), as provided in the following theorem.

Theorem 2 Given the price-discount incentive scheme (α, A), we find that the manufacturer's optimal wholesale price w^* can be *uniquely* determined as,

$$w^* = \begin{cases} \hat{w}_1(\alpha), & \text{if } 0 < \alpha \leq \alpha_0, \text{ or, } A \geq A_1 \equiv \alpha \hat{w}_1(\alpha); \\ A/\alpha, & \text{if } \alpha_0 \leq \alpha \leq \alpha_1 \equiv A/\hat{w}_2(A), \text{ or, } A_0 \leq A \leq A_1; \\ \hat{w}_2(A), & \text{if } \alpha_1 \leq \alpha < 1, \text{ or, } 0 < A \leq A_0. \quad ■ \end{cases} \tag{8}$$

Two *alternative* inequalities exist for each condition (stated after "if") in the theorem above. As indicated by Theorem 2, if the government adopts a discount rate α that is smaller than α_0, then subsidy ceiling A will not affect the manufacturer's optimal wholesale pricing decision, expected sales $D_1(\hat{w}_1(\alpha))$, and the manufacturer's expected profit $\Pi_{M1}(\hat{w}_1(\alpha))$. The value α_0 can thus be explained as the minimum price discount rate for the effectiveness of a given subsidy ceiling. Theorem 2 also indicates that if the discount rate α is greater than α_1 (i.e., $\alpha > \alpha_1$) or, subsidy ceiling A is less than A_0 (i.e., $A < A_0$), then the manufacturer's optimal wholesale price will be dependent on A but independent of α. Therefore, A_0 is the minimum subsidy ceiling for the effectiveness of a given price discount rate. Similar to our discussion for the case where $\alpha < \alpha_0$, we find that when $\alpha > \alpha_1$, only the subsidy ceiling in the government's incentive scheme affects the expected sales and the manufacturer's expected profit.

Remark 1 In (8), α_1 and A_1 represent the maximum effective price-discount rate and subsidy ceiling, respectively. That is, the effective price discount rate α should be smaller than or equal to α_1. A discount rate α larger than α_1 cannot influence EV sales, and should not be involved into the government incentive scheme, which should only include a subsidy ceiling. Similarly, the effectiveness of subsidy ceiling A requires that A be smaller than or equal to A_1. ■

Moreover, if the manufacturer incurs a high unit production cost c, then the values of α_0 and α_1 will be reduced, because of the following: α_0 is a unique value that satisfies $\hat{w}_1(\alpha_0) = A/\alpha_0$. As the value of c increases, $\hat{w}_1(\alpha_0) > A/\alpha_0$ because wholesale price $\hat{w}_1(\alpha)$ is increasing in c, as indicated by Lemma 1; there must exist a unique value α_0' such that $\alpha_0' < \alpha_0$ and $\hat{w}_1(\alpha_0') = A/\alpha_0'$. Similarly, we can show that if the value of c increases, then α_1 will decrease. We then find from (8) that because the value of c is high, the range $(0, \alpha_0)$—in which only discount rate α takes effect—is small; however, the range $(\alpha_1, 1)$—in which only subsidy ceiling A takes effect—is large.

Remark 2 The result above implies that if the manufacturer incurs a *higher* unit production cost, then subsidy ceiling A is *more effective* in influencing the optimal wholesale pricing decision and the expected sales. In today's automobile market, the production cost for EVs is high, mainly because the EV battery is costly. Similarly, we conclude that a *lower* unit production cost will result in a *higher effectiveness* of discount rate α in the wholesale pricing decision and the expected sales. ■

3.2 Effectiveness of the Incentive Scheme

We investigate the effect of price discount rate α and subsidy ceiling A on expected sales $D(w^*)$ and the manufacturer's expected profit $\Pi_M(w^*)$.

3.2.1 Effect of the Incentive Scheme on Expected Sales

The analysis of expected sales is very important in our paper. Using Theorem 2 and Lemma 2, we obtain the following results.

1. If discount rate α is in the range $(0, \alpha_0)$ or equivalently, subsidy ceiling A is greater than A_1, then both the optimal wholesale price $w^* = \hat{w}_1(\alpha)$ and resulting expected sales $D(w^*)$ are increasing in α but are independent of A.
2. If $\alpha \in [\alpha_0, \alpha_1]$ or $A \in [A_0, A_1]$, then $w^* = A/\alpha$, which is decreasing in α but is increasing in A. As a result, expected sales $D(A/\alpha)$ are increasing in α but are decreasing in A.
3. If $\alpha \in (\alpha_1, 1)$ or $A \in (0, A_0)$, then wholesale price $w^* = \hat{w}_2(A)$ is independent of α and thus, expected sales $D(\hat{w}_2(A))$ do not depend on α. However, the wholesale price and expected sales are both increasing in A.

Remark 3 Following the discussion above, we find that expected sales $D(w^*)$ reaches its maximum value when $\alpha \geq \alpha_1 = A/\hat{w}_2(A)$. Noting that the incentive scheme mainly aims to stimulate EV sales for the reduction of air pollution, we find that the price discount rate should be set to $\alpha \geq \alpha_1$ to effectively increase the expected sales of the EV. However, as indicated in Remark 1, the effectiveness of the price discount rate α requires that $\alpha \leq \alpha_1$. Therefore, to maximize the EV sales, the government should set the discount rate to $\alpha = \alpha_1$. Similarly, to maximize $D(w^*)$, the government should set the subsidy ceiling to $A = A_0$. ■

The above remark indicates that for a given subsidy ceiling A, the most effective price discount rate should be $\alpha = \alpha_1 = A/\hat{w}_2(A)$; for a given discount rate α, the most effective subsidy ceiling should be $A = A_0$. The most effective price discount rate and subsidy ceiling (denoted by α^* and A^*, respectively) should satisfy the conditions $\alpha^* = A^*/\hat{w}_2(A^*)$ and $A^* = A_0(\alpha^*)$, which can be obtained by solving (7) where α is replaced with α^*. According to (5) and (7), we find that the condition $\alpha^* = A^*/\hat{w}_2(A^*)$ is equivalent to the condition $A^* = A_0(\alpha^*)$. Thus, the *most effective* incentive scheme (α^*, A^*)—that maximizes the expected EV sales—should satisfy the condition $1 - F(A^*/\alpha^* - A^*) - (A^*/\alpha^* - c)f(A^*/\alpha^* - A^*) = 0$, which is a necessary condition for the most effective scheme. Therefore, the government may need to include *both* a price discount rate *and* a subsidy ceiling in its incentive scheme to effectively stimulate EV sales.

3.2.2 Impact of the Incentive Scheme on the Manufacturer's Expected Profit

The incentive scheme should entice the manufacturer to invest in EV production. The high purchasing price of EVs, which is the main factor that discourages consumers from purchasing EVs, is partly ascribed to manufacturers' very low production volumes. Thus, we should also investigate whether the manufacturer can benefit from the incentive scheme by achieving a high expected profit and will thus have a motivation to promote EV sales for the reduction of air pollution.

Similar to our discussion on the impact of the incentive scheme on sales, we use Theorem 2 and Lemma 2 and find that (i) if $\alpha \in (0, \alpha_0)$ or $A > A_1$, then the manufacturer's expected profit $\Pi_M(w^*) = \Pi_M(\hat{w}_1(\alpha))$ is increasing in the discount rate α but is independent of subsidy ceiling A;

and (ii) if $\alpha \in (\alpha_1, 1)$ or $A \in (0, A_0)$, then the manufacturer's expected profit $\Pi_M(w^*) = \Pi_M(\hat{w}_2(A))$ is independent of α but is increasing in A. However, we cannot immediately determine the effect of $\alpha \in [\alpha_0, \alpha_1]$ and $A \in [A_0, A_1]$ on the manufacturer's expected profit. Theorem 2 indicates that, if $\alpha \in [\alpha_0, \alpha_1]$ or $A \in [A_0, A_1]$, then the manufacturer's expected profit is $\Pi_M(w^*) = \Pi_M(A/\alpha)$.

Theorem 3 If $\alpha \in [\alpha_0, \alpha_1]$ or $A \in [A_0, A_1]$, then the manufacturer's expected profit $\Pi_M(A/\alpha)$ is increasing in both discount rate α and subsidy ceiling A. ■

According to the discussion above, we can draw managerial insights as indicated in the following remark.

Remark 4 The government should select a price discount rate α that is not smaller than α_1 to maximize the manufacturer's expected profit $\Pi_M(w^*)$ and *effectively* improve the manufacturer's incentive for EV production. Remark 1 also indicates that α should not be larger than α_1, as a result of assuring the effectiveness of the discount rate. Therefore, the government should set its discount rate to $\alpha = \alpha_1 = A/\hat{w}_2(A)$ to maximize $\Pi_M(w^*)$ for the improvement of the EV manufacturer's production incentive. Similarly, to maximize $\Pi_M(w^*)$, the government should set its subsidy ceiling to $A = A_1 = \alpha\hat{w}_1(\alpha)$. ■

The above remark reveals that the most effective price discount rate and subsidy ceiling for the improvement of the EV manufacturer's production incentive (denoted by $\tilde{\alpha}^*$ and $\tilde{A}^*$, respectively) should satisfy the following conditions:

$$\tilde{\alpha}^* = \tilde{A}^*/\hat{w}_2(\tilde{A}^*) \quad \text{and} \quad \tilde{A}^* = \tilde{\alpha}^*\hat{w}_1(\tilde{\alpha}^*). \tag{9}$$

Hence, the *most effective* incentive scheme $(\tilde{\alpha}^*, \tilde{A}^*)$—that *maximizes* the manufacturer's expected profit $\Pi_M(w^*)$—can be uniquely obtained by solving the two equations in (9).

If the government adopts the unique most effective scheme $(\tilde{\alpha}^*, \tilde{A}^*)$ for improving the EV manufacturer's production incentive, then $\alpha_0 = \alpha_1 = \tilde{\alpha}^*$ and $A_0 = A_1 = \tilde{A}^*$, where α_0 and A_0 uniquely satisfy (6) and (7), respectively. Therefore, we conclude that, to most effectively improve the incentive of the EV manufacturer, the government may need to include *both* a price discount rate *and* a subsidy ceiling in its incentive scheme.

4 Conclusions

We investigated an automobile supply chain under the government's price-discount scheme that is implemented to stimulate the sales of electric vehicles. A number of analytic managerial insights were drawn. We showed that the negotiated retail price for each consumer is increasing in the wholesale price and the consumer's valuation. The retail price for the consumer with a low valuation is mainly dependent on the price discount rate, whereas the retail price for the consumer with a high valuation is mainly dependent on the subsidy ceiling. Moreover, the subsidy ceiling is more effective in influencing the optimal wholesale pricing decision of the manufacturer with a higher unit production cost, whereas the discount rate is more effective for the manufacturer with a lower unit production cost.

We also showed that for a given subsidy ceiling, only a discount rate that is smaller than the maximum effective price-discount rate can affect the EV sales; for a given discount rate, only a subsidy ceiling that is smaller than the maximum effective subsidy ceiling can affect sales. The incentive scheme may need to include *both* a price discount rate *and* a subsidy ceiling to effectively increase EV sales. Moreover, we derived a necessary condition for the most effective discount rate and subsidy ceiling that maximize the expected EV sales. We also obtained a unique discount rate and subsidy ceiling that maximize the manufacturer's expected profit and most effectively improve the manufacturer's incentive for EV production.

We investigated an EV supply chain in which only EV is produced and sold. This may not be consistent with most scenarios in the automobile industry. As Huang et al. [7] mentioned, an analysis for a supply chain that involves EVs and fuel vehicles should be more realistic. In the future, we plan to analyze the effect of price-discount schemes on a supply chain that produces both automobile types.

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Online Supplements
“Supply Chain Analysis under a Price-Discount Incentive Scheme for Electric Vehicles”
C. Luo, M. Leng, J. Huang, L. Liang

Appendix A Price-Discount Incentive Schemes for Electric Vehicles

The rapid development of the automobile industry in the past five decades has not only resulted in significant convenience to consumers but has also generated an increasingly serious air pollution problem. The environmental impact has pressured a number of governments to promote the electric vehicle (EV), which has not yet dominated the automobile market mainly because the lack of electric charging stations and high purchase price discourage most consumers from purchasing such vehicles.

As a response to the high price of electric vehicles—which is partly attributed to very low production volume, some governments have implemented incentive schemes to provide each consumer a price discount and thus improve EV sales. In Romania, the government provides subsidy amounting to 25% of the retail price, capped at €5,000, to each consumer who purchases a new electric car. Since January 2011, the British government has implemented a price discount scheme similar to that of the Romanian government. In Spain's incentive scheme, each consumer who buys a new electric car can enjoy a price discount equal to 25% of the purchase price (before tax), with a cap of €6,000; each consumer who purchases an electric bus or van can obtain a price discount in amount of 25% of the purchase price (up to €15,000 and €30,000 for the bus and the van, respectively). For more information on price-discount incentive schemes for EVs, see `Wikipedia.org` [19].

The situations above indicate that price-discount incentive schemes (e.g., in Romania, Spain, and the United Kingdom) consist of a price discount rate and a subsidy ceiling, which is the maximum subsidy for each EV consumer.

Appendix B Proofs of Theorems

Proof of Theorem 1. When $w \leq A/\alpha$, we consider these two cases: $\alpha p \leq A$, and $\alpha p \geq A$. We begin by analyzing the case wherein $\alpha p \leq A$.

1. When $\alpha p \leq A$, we temporarily ignore the constraints, write the Nash bargaining model as $\Lambda = (\theta - p + \alpha p)(p - w)$, and maximize it to obtain the corresponding optimal retail price as $p_1^* = [w + \theta/(1-\alpha)]/2$. From (1), we find that, to satisfy the constraints, retail price p must be determined such that $w \leq p \leq \theta/(1-\alpha)$, which is satisfied if $\theta \geq \underline{\theta} = (1-\alpha)w$. Otherwise, if $\theta < \underline{\theta}$, then the constraints in (1) cannot be satisfied and the consumer does not buy from the retailer.
   Assuming that $\theta \geq \underline{\theta}$, we find that if $\theta \leq \hat{\theta}$ where $\hat{\theta}$ is defined as in (2), then p_1^* must be smaller than or equal to A/α; thus, $p^* = p_1^*$. If $\theta \geq \hat{\theta}$, then $p^* = p_2^* = A/\alpha$. Summarizing the above, we obtain
$$p^* = \begin{cases} p_1^*, & \text{if}\, \underline{\theta} \leq \theta \leq \hat{\theta}, \\ p_2^*, & \text{if}\, \theta \geq \hat{\theta}. \end{cases}$$
2. When $\alpha p \geq A$, temporarily ignoring the constraints, we find that the Nash bargaining model in (1) can be re-written as $\Lambda = (\theta - p + A)(p - w)$, which is maximized at point $p_3^* = (\theta + A + w)/2$. To satisfy the constraints, we require that retail price p be determined such that $w \leq p \leq \theta + A$,

which is assured when $\theta \geq w - A$. If $\theta < w - A$, then the consumer and the retailer cannot reach an agreement on their transaction.

Under the condition that $\theta \geq w - A$, we find that, if $\theta \geq 2A/\alpha - A - w$, then $p^* = p_3^*$; if $\theta \leq 2A/\alpha - A - w$, then $p^* = p_2^* = A/\alpha$. In summary, we obtain

$$p^* = \begin{cases} p_2^*, & \text{if } \theta \leq 2A/\alpha - A - w, \\ p_3^*, & \text{if } \theta \geq 2A/\alpha - A - w. \end{cases}$$

In order to find the optimal retail price p^* for the constrained maximization problem in (1), we need to compare the case wherein $\alpha p \leq A$ and the case wherein $\alpha p \geq A$. Because $w \leq A/\alpha$, $\hat{\theta} - (2A/\alpha - A - w) = \alpha(w - A/\alpha) \leq 0$. The following five possibilities must be considered.

1. If $\theta < \underline{\theta}$, then the consumer and retailer cannot reach an agreement over the retail price.
2. If $\underline{\theta} \leq \theta \leq A/\alpha - A$, then the consumer and retailer will complete their transaction at retail price as p_1^* for the case where $\alpha p \leq A$; however, the two players cannot reach an agreement on the retail price for the case where $\alpha p > A$. Thus, when $\underline{\theta} \leq \theta \leq A/\alpha - A$, the consumer will buy an EV from the retailer at price $p^* = p_1^*$.
3. If $A/\alpha - A \leq \theta \leq \hat{\theta}$, then $\alpha p_1^* \leq A$. According to the analysis above, the consumer and retailer will set the retail price as p_1^* for the case where $\alpha p \leq A$; however, the two players should choose retail price $p_2^* = A/\alpha$ for the case where $\alpha p \geq A$. Hence, the consumer will buy an EV from the retailer at price $p^* = p_1^*$.
4. If $\hat{\theta} \leq \theta \leq 2A/\alpha - A - w$, then the consumer and retailer will set the retail price as $p_2^* = A/\alpha$ for both the case where $\alpha p \leq A$ and the case where $\alpha p \geq A$. Therefore, the consumer and the retailer will reach an agreement on price $p^* = p_2^*$.
5. If $\theta \geq 2A/\alpha - A - w$, then $\alpha p_3^* \geq A$. The consumer and retailer will determine the retail price as p_2^* for the case where $\alpha p \leq A$. However, the two players should decide to complete their transaction at retail price p_3^* for the case where $\alpha p \geq A$. Hence, the consumer will buy an EV at price $p^* = p_3^*$.

Similarly, when $w \geq A/\alpha$, we can determine the negotiated retail price as given in this theorem. We thus complete our proof. ■

Proof of Theorem 2. When $0 < \alpha \leq \alpha_0$, $\hat{w}_1(\alpha) \leq A/\alpha$, which can be alternatively written as $A \geq A_1 = \alpha\hat{w}_1(\alpha)$. Thus, the optimal wholesale price for the problem $\max_{w \leq A/\alpha} \Pi_{M1}(w)$ is $w_1^* = \min(\hat{w}_1(\alpha), A/\alpha) = \hat{w}_1(\alpha)$. As Lemma 3 indicates that for a given subsidy ceiling A, $A/\alpha_0 \geq \hat{w}_2(A)$, we find that $A/\alpha \geq \hat{w}_2(A)$; thus, the optimal wholesale price for the problem $\max_{w \geq A/\alpha} \Pi_{M2}(w)$ is $w_2^* = \max(\hat{w}_2(A), A/\alpha) = A/\alpha$. $\Pi_{M1}(\hat{w}_1(\alpha)) \geq \Pi_{M1}(A/\alpha) = \Pi_{M2}(A/\alpha)$, which means that if $0 < \alpha \leq \alpha_0$ or, $A \geq A_1 \equiv \alpha\hat{w}_1(\alpha)$, then the optimal wholesale price is $w^* = \hat{w}_1(\alpha)$.

When $\alpha_0 \leq \alpha \leq \alpha_1 = A/\hat{w}_2(A)$, we find that $A \geq A_0$ and $\hat{w}_2(A) \leq A/\alpha$; and $\hat{w}_1(\alpha) \geq A/\alpha$, or alternatively, $A \leq A_1$. Thus, the optimal wholesale prices for the problems $\max_{w \leq A/\alpha} \Pi_{M1}(w)$ and $\max_{w \geq A/\alpha} \Pi_{M2}(w)$ are obtained as $w_1^* = \min(\hat{w}_1(\alpha), A/\alpha) = A/\alpha$ and $w_2^* = \max(\hat{w}_2(A), A/\alpha) = A/\alpha$, respectively; the optimal wholesale price is $w^* = A/\alpha$.

When $\alpha_1 \leq \alpha < 1$, we find that $\hat{w}_2(A) \geq A/\alpha$ or $A \leq A_0$. Thus, the optimal wholesale prices for the problems $\max_{w \leq A/\alpha} \Pi_{M1}(w)$ and $\max_{w \geq A/\alpha} \Pi_{M2}(w)$ are obtained as $w_1^* = \min(\hat{w}_1(\alpha), A/\alpha) = A/\alpha$ and $w_2^* = \max(\hat{w}_2(A), A/\alpha) = \hat{w}_2(A)$, respectively. Since $\Pi_{M1}(A/\alpha) = \Pi_{M2}(A/\alpha) \leq \Pi_{M2}(\hat{w}_2(A))$, the optimal wholesale price is $w^* = \hat{w}_2(A)$.

We thus prove this theorem. ■

Proof of Theorem 3. We begin by investigating the effect of subsidy ceiling A on $\Pi_M(A/\alpha)$. We learn from the proof of Lemma 1 that both $\Pi_{M1}(w)$ and $\Pi_{M2}(w)$ are log-concave functions with maximum points at $\hat{w}_1(\alpha)$ and $\hat{w}_2(A)$, respectively. It also follows from the comparison between $\Pi_{M1}(w)$ and $\Pi_{M2}(w)$ in (4) that $\Pi_{M1}(w) < \Pi_{M2}(w)$ when $w < A/\alpha$; $\Pi_{M1}(w) = \Pi_{M2}(w)$ when $w = A/\alpha$; and $\Pi_{M1}(w) > \Pi_{M2}(w)$ when $w > A/\alpha$. This means that $\Pi_{M1}(w)$ and $\Pi_{M2}(w)$ intersect at point $w = A/\alpha$.

Lemma 3 indicates that for a given value of discount rate α, $A_0/\alpha \leq \hat{w}_1(\alpha)$ and $\Pi_{M1}(\hat{w}_1(\alpha)) > \Pi_{M2}(\hat{w}_2(A_0))$. When $A = A_0$, the optimal value $\hat{w}_2(A_0)$ that maximizes $\Pi_{M2}(w)$ is equal to A_0/α, and $\Pi_{M1}(w)$ and $\Pi_{M2}(w)$ intersect at point $w = \hat{w}_2(A_0)$, which corresponds to Case (1) in Figure 1(a). In this figure, the solid and dotted lines represent $\Pi_{M1}(w)$ and $\Pi_{M2}(w)$, respectively. In addition, when $A = A_1 = \alpha\hat{w}_1(\alpha)$, $\Pi_{M1}(w)$ and $\Pi_{M2}(w)$ intersect at point $w = \hat{w}_1(\alpha)$, where $\Pi_{M1}(w)$ reaches its maximum value. This corresponds to Case (3) in Figure 1(a).

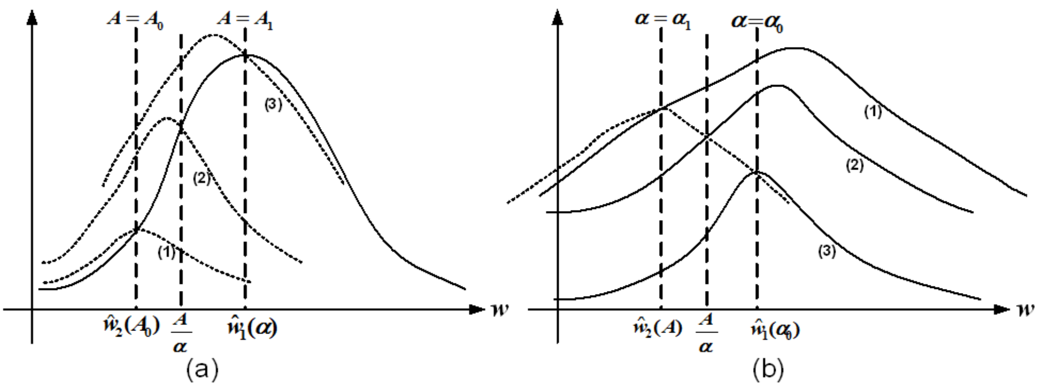

Figure 1: The effects of A and α on the manufacturer's maximum expected profit. The solid and dotted lines represent $\Pi_{M1}(w)$ and $\Pi_{M2}(w)$, respectively.

When $A \in [A_0, A_1]$, $\Pi_{M1}(w)$ and $\Pi_{M2}(w)$ intersect at point $w = A/\alpha$; see the corresponding Case (2) in Figure 1(a). $\Pi_{M1}(w)$ is independent of A; however, both $\hat{w}_2(A)$ and $\Pi_{M2}(\hat{w}_2(A))$ are increasing in A, as shown in Lemma 2. In Figure 1(a), increasing the value of A in the range $[A_0, A_1]$ will not alter curve $\Pi_{M1}(w)$ but will increase the values of $\hat{w}_2(A)$ and $\Pi_{M2}(\hat{w}_2(A))$. Therefore, point $w = A/\alpha$ at which $\Pi_{M1}(w)$ and $\Pi_{M2}(w)$ intersect will also increase when the value of A increases. Noting from Theorem 2 that $w^* = A/\alpha$ if $A \in [A_0, A_1]$, we find that the manufacturer's maximum expected profit $\Pi_M(w^*)$ is increasing in A.

Similarly, we can plot Figure 1(b) to show that as price discount rate α increases in the range $[\alpha_0, \alpha_1]$, the value of A/α—at which $\Pi_{M1}(w)$ and $\Pi_{M2}(w)$ intersect—decreases and the manufacturer's maximum expected profit $\Pi_M(w^*) = \Pi_M(A/\alpha)$ increases. This theorem is thus proven. ■

Appendix C Proofs of Lemmas

Proof of Lemma 1. We consider the maximization problem $\max_{w \leq A/\alpha} \Pi_{M1}(w)$. Temporarily ignoring the constraint that $w \leq A/\alpha$, we take the first- and second-order derivatives of $\Pi_{M1}(w)$ with respect to w and obtain

$$\partial\Pi_{M1}(w)/\partial w = B[1 - F((1-\alpha)w)] - B(w-c)(1-\alpha)f((1-\alpha)w), \tag{10}$$

and

$$\partial^2\Pi_{M1}(w)/\partial w^2 = -2B(1-\alpha)f((1-\alpha)w) - B(w-c)(1-\alpha)^2 f'((1-\alpha)w).$$

Solving the first-order condition that $\partial\Pi_{M1}(w)/\partial w = 0$ yields $\hat{w}_1(\alpha)$ as in (5). Substituting the above solution into the second-order derivative, we obtain

$$\left.\frac{\partial^2\Pi_{M1}(w)}{\partial w^2}\right|_{w=\hat{w}_1(\alpha)} = -B(1-\alpha)\frac{2[f((1-\alpha)\hat{w}_1(\alpha))]^2 + f'((1-\alpha)\hat{w}_1(\alpha))[1-F((1-\alpha)\hat{w}_1(\alpha))]}{f((1-\alpha)\hat{w}_1(\alpha))},$$

which is negative if p.d.f. $f(\theta)$ is differentiable and log-concave. That is, assuming the differentiability and log-concavity of $f(\theta)$, we find that at the value of w that satisfies $\partial\Pi_{M1}(w)/\partial w = 0$, the second-order derivative of $\Pi_{M1}(w)$ is negative, namely, $\partial^2\Pi_{M1}(w)/\partial w^2|_{\partial\Pi_{M1}(w)/\partial w=0} < 0$. Thus, $\Pi_{M1}(w)$ is a quasi-concave (unimodal) function of w, and the optimal price w_1^* that maximizes $\Pi_{M1}(w)$ subject to $w \leq A/\alpha$ can be uniquely obtained as $w_1^* = \min(\hat{w}_1(\alpha), A/\alpha)$.

We then solve the maximization problem $\max_{w\geq A/\alpha}\Pi_{M2}(w)$. Similarly, we calculate the first- and second-order derivatives of $\Pi_{M2}(w)$ w.r.t. w, and find that

$$\begin{aligned}\partial\Pi_{M2}(w)/\partial w &= B[1-F(w-A)-(w-c)f(w-A)],\\ \partial^2\Pi_{M2}(w)/\partial w^2 &= -B[2f(w-A)+(w-c)f'(w-A)].\end{aligned}$$

Equating $\partial\Pi_{M2}(w)/\partial w$ to zero and solving it for w, we obtain $\hat{w}_2(A)$ as in (5). Substituting $\hat{w}_2(A)$ into the second-order derivative yields

$$\left.\frac{\partial^2\Pi_{M2}(w)}{\partial w^2}\right|_{w=\hat{w}_2(A)} = -B\frac{2\left[f(\hat{w}_2(A)-A)\right]^2 + [1-F(\hat{w}_2(A)-A)]f'(\hat{w}_2(A)-A)}{f(\hat{w}_2(A)-A)},$$

which is negative if p.d.f. $f(\theta)$ is continuously differentiable and log-concave. That is, $\Pi_{M2}(w)$ is a quasi-concave (unimodal) function with a unique optimal wholesale price that maximizes $\Pi_{M2}(w)$. The optimal solution for the constrained problem $\max_{w\geq A/\alpha}\Pi_{M2}(w)$ can then be obtained as $w_2^* = \max(\hat{w}_2(A), A/\alpha)$.

Next, we investigate the effect of the manufacturer's unit production cost c on $\hat{w}_1(\alpha)$ and $\hat{w}_2(A)$. We differentiate $\partial\Pi_{M1}(w)/\partial w$ in (10) once w.r.t. c, and obtain

$$\frac{\partial^2\Pi_{M1}(w)}{\partial w\partial c} = B(1-\alpha)f((1-\alpha)w) > 0,$$

which means that $\Pi_{M1}(w)$ is a *supermodular* function of w and c. It thus follows that $\hat{w}_1(\alpha)$ is increasing in c. Similarly, we can show the *supermodularity* property of the function $\Pi_{M2}(w)$, which implies that $\hat{w}_2(A)$ is increasing in c.

We subsequently discuss $w_1^* = \min(\hat{w}_1(\alpha), A/\alpha)$ and $w_2^* = \max(\hat{w}_2(A), A/\alpha)$. We consider the following four cases:

1. If $\hat{w}_1(\alpha) \leq A/\alpha$ and $\hat{w}_2(A) \leq A/\alpha$, then $w_1^* = \hat{w}_1(\alpha) \leq w_2^* = A/\alpha$.
2. If $\hat{w}_1(\alpha) \geq A/\alpha$ and $\hat{w}_2(A) \leq A/\alpha$, then $w_1^* = A/\alpha = w_2^* = A/\alpha$.
3. If $\hat{w}_1(\alpha) \leq A/\alpha$ and $\hat{w}_2(A) \geq A/\alpha$, then $w_1^* = \hat{w}_1(\alpha) \leq A/\alpha \leq \hat{w}_2(A) = w_2^*$, which is actually

contrary to the following fact: We re-write $\hat{w}_2(A)$ in (5) as,

$$\hat{w}_2(A) = c + \frac{1 - F(\hat{w}_2(A) - A)}{f(\hat{w}_2(A) - A)} = c + \frac{1 - F\left(\left(1 - \frac{A}{\hat{w}_2(A)}\right)\hat{w}_2(A)\right)}{f\left(\left(1 - \frac{A}{\hat{w}_2(A)}\right)\hat{w}_2(A)\right)}.$$

If $\hat{w}_2(A) \geq A/\alpha$, then $1 - A/\hat{w}_2(A) \geq 1 - \alpha$ and

$$\frac{1 - F(\hat{w}_2(A) - A)}{f(\hat{w}_2(A) - A)} \leq \frac{1 - F((1-\alpha)\hat{w}_2(A))}{f((1-\alpha)\hat{w}_2(A))} \leq \frac{1 - F((1-\alpha)\hat{w}_2(A))}{(1-\alpha) \times f((1-\alpha)\hat{w}_2(A))},$$

where the first inequality results from the continuous differentiability and log-concavity of p.d.f. $f(\theta)$. Therefore, we obtain

$$\hat{w}_2(A) \leq c + \frac{1 - F((1-\alpha)\hat{w}_2(A))}{(1-\alpha) \times f((1-\alpha)\hat{w}_2(A))};$$

as a result,

$$\left.\frac{\partial \Pi_{M1}(w)}{\partial w}\right|_{w=\hat{w}_2(A)} = B[1 - F((1-\alpha)\hat{w}_2(A))] - B(\hat{w}_2(A) - c)(1-\alpha)f((1-\alpha)\hat{w}_2(A)) \geq 0,$$

Noting that $\hat{w}_1(\alpha)$ can be uniquely obtained by solving the first-order condition $\partial \Pi_{M1}(w)/\partial w = 0$, namely, $\partial \Pi_{M1}(w)/\partial w|_{w=\hat{w}_1(\alpha)} = 0$, we find that when $\hat{w}_2(A) \geq A/\alpha$, $\hat{w}_2(A) \leq \hat{w}_1(\alpha)$ because $\Pi_{M1}(w)$ is a quasi-concave function with a unique optimal solution $\hat{w}_1(\alpha)$ that maximizes $\Pi_{M1}(w)$. However, such a result contradicts the case where $\hat{w}_1(\alpha) \leq A/\alpha \leq \hat{w}_2(A)$, which means that this case does *not* occur.

4. If $\hat{w}_1(\alpha) \geq A/\alpha$ and $\hat{w}_2(A) \geq A/\alpha$, then $w_1^* = A/\alpha \leq w_2^* = \hat{w}_2(A)$.

We thus prove this lemma. ■

Proof of Lemma 2. We begin by proving the effect of α on $\hat{w}_1(\alpha)$, $D_1(\hat{w}_1(\alpha))$, and $\Pi_{M1}(\hat{w}_1(\alpha))$. Differentiating $\hat{w}_1(\alpha)$ in (5) once with respect to α, we obtain

$$\begin{aligned} & \frac{\partial \hat{w}_1(\alpha)}{\partial \alpha} \\ = \; & \frac{(2\hat{w}_1(\alpha) - c)f((1-\alpha)\hat{w}_1(\alpha)) + (1-\alpha)\{[\hat{w}_1(\alpha)]^2 - \hat{w}_1(\alpha)c\}f'((1-\alpha)\hat{w}_1(\alpha))}{2(1-\alpha)f((1-\alpha)\hat{w}_1(\alpha)) + (1-\alpha)^2(\hat{w}_1(\alpha) - c)f'((1-\alpha)\hat{w}_1(\alpha))}. \end{aligned} \tag{11}$$

Similar to the proof of Lemma 1, the denominator of the fraction above is positive. Noting the first-order condition that $(1-\alpha)(\hat{w}_1(\alpha) - c) = [1 - F((1-\alpha)\hat{w}_1(\alpha))]/f((1-\alpha)\hat{w}_1(\alpha))$, we re-write the numerator of the fraction in (11) as

$$\begin{aligned} & (2\hat{w}_1(\alpha) - c)f((1-\alpha)\hat{w}_1(\alpha)) + (1-\alpha)\{[\hat{w}_1(\alpha)]^2 - \hat{w}_1(\alpha)c\}f'((1-\alpha)\hat{w}_1(\alpha)) \\ = \; & (\hat{w}_1(\alpha) - c)f((1-\alpha)\hat{w}_1(\alpha)) + \hat{w}_1(\alpha)[f((1-\alpha)\hat{w}_1(\alpha)) + (1-\alpha)(\hat{w}_1(\alpha) - c) \\ & \times f'((1-\alpha)\hat{w}_1(\alpha))] \\ = \; & (\hat{w}_1(\alpha) - c)f((1-\alpha)\hat{w}_1(\alpha)) + \hat{w}_1(\alpha) \\ & \times \left\{\frac{[f((1-\alpha)\hat{w}_1(\alpha))]^2 + [1 - F((1-\alpha)\hat{w}_1(\alpha))]f'((1-\alpha)\hat{w}_1(\alpha))}{f[(1-\alpha)\hat{w}_1(\alpha)]}\right\}, \end{aligned}$$

which is positive because of the differentiability and log-concavity of p.d.f. $f(\theta)$. In addition,

$$\frac{\partial \hat{w}_1(\alpha)}{\partial \alpha} < \frac{2\hat{w}_1(\alpha)f((1-\alpha)\hat{w}_1(\alpha)) + (1-\alpha)\{[\hat{w}_1(\alpha)]^2 - \hat{w}_1(\alpha)c\}f'((1-\alpha)\hat{w}_1(\alpha))}{2(1-\alpha)f((1-\alpha)\hat{w}_1(\alpha)) + (1-\alpha)^2[\hat{w}_1(\alpha) - c]f'((1-\alpha)\hat{w}_1(\alpha))} = \frac{\hat{w}_1(\alpha)}{1-\alpha}.$$

Then, differentiating $D_1(\hat{w}_1(\alpha))$ once w.r.t. α, we obtain

$$\frac{\partial D_1(\hat{w}_1(\alpha))}{\partial \alpha} = -Bf((1-\alpha)\hat{w}_1(\alpha))\left[-\hat{w}_1(\alpha) + (1-\alpha)\frac{\partial \hat{w}_1(\alpha)}{\partial \alpha}\right] > 0,$$

because $\partial \hat{w}_1(\alpha)/\partial \alpha < \hat{w}_1(\alpha)/(1-\alpha)$. Thus, $D_1(\hat{w}_1(\alpha))$ is increasing in α, and $\Pi_{M1}(\hat{w}_1(\alpha)) = (\hat{w}_1(\alpha) - c)D_1(\hat{w}_1(\alpha))$ is also increasing in α.

Next, we investigate the effect of A on $\hat{w}_2(A)$, $D_2(\hat{w}_2(A))$, and $\Pi_{M2}(\hat{w}_2(A))$. We calculate the first-order derivative of $\hat{w}_2(A)$ in (5) with respect to A and use the first-order condition. We then obtain

$$\frac{\partial \hat{w}_2(A)}{\partial A} = -\frac{[f(\hat{w}_2(A) - A)]^2 + [1 - F(\hat{w}_2(A) - A)]f'(\hat{w}_2(A) - A)}{[f(\hat{w}_2(A) - A)]^2}\left(\frac{\partial \hat{w}_2(A)}{\partial A} - 1\right).$$

The log-concavity of p.d.f. $f(\theta)$ implies that $[f(\hat{w}_2(A) - A)]^2 + [1 - F(\hat{w}_2(A) - A)]f'(\hat{w}_2(A) - A) > 0$. Hence, $\partial \hat{w}_2(A)/\partial A$ and $(\partial \hat{w}_2(A)/\partial A - 1)$ must have different signs; thus, $0 < \partial \hat{w}_2(A)/\partial A < 1$.

We then differentiate $D_2(\hat{w}_2(A))$ once w.r.t. A, and find that

$$\partial[D_2(\hat{w}_2(A))]/\partial A = Bf(\hat{w}_2(A) - A)(1 - \partial \hat{w}_2(A)/\partial A) > 0,$$

which means that $D_2(\hat{w}_2(A))$ is increasing in A. Thus, $\Pi_{M2}(\hat{w}_2(A)) = [\hat{w}_2(A) - c]D_2(\hat{w}_2(A))$ is also increasing in A. The lemma is thus proven. ■

Proof of Lemma 3. Suppose that $A/\alpha_0 < \hat{w}_2(A)$ for a given subsidy ceiling A. Given that $\hat{w}_2(A)$ is independent of α, α_1 must exist such that $\alpha_1 < \alpha_0$ and $A/\alpha_1 = \hat{w}_2(A)$, which implies that

$$\begin{aligned} \Pi_{M1}(\hat{w}_1(\alpha_1)) &= B(\hat{w}_1(\alpha_1) - c)[1 - F((1-\alpha)\hat{w}_1(\alpha_1))], \\ \Pi_{M2}(\hat{w}_2(A)) &= B(\hat{w}_2(A) - c)[1 - F((1-\alpha)\hat{w}_2(A))]. \end{aligned}$$

Since $\hat{w}_1(\alpha_1)$ is the optimal solution that maximizes $B(w-c)[1-F((1-\alpha)w)]$, we obtain $\Pi_{M1}(\hat{w}_1(\alpha_1)) > \Pi_{M2}(\hat{w}_2(A))$. Similarly, $\Pi_{M1}(\hat{w}_1(\alpha_0)) < \Pi_{M2}(\hat{w}_2(A))$ because $A/\alpha_0 = w_1(\alpha_0)$. It thus follows that $\Pi_{M1}(\hat{w}_1(\alpha_1)) > \Pi_{M1}(\hat{w}_1(\alpha_0))$. However, because $\alpha_0 > \alpha_1$, $\Pi_{M1}(\hat{w}_1(\alpha_0)) > \Pi_{M1}(\hat{w}_1(\alpha_1))$, since $\Pi_{M1}(\hat{w}_1(\alpha))$ is increasing in α, as shown in Lemma 2. This result is contrary to the above. Therefore, $A/\alpha_0 \geq \hat{w}_2(A)$ and $\Pi_{M1}(\hat{w}_1(\alpha_0)) < \Pi_{M2}(\hat{w}_2(A))$.

Using a similar argument, we find that $A_0/\alpha \leq \hat{w}_1(\alpha)$ and $\Pi_{M1}(\hat{w}_1(\alpha)) > \Pi_{M2}(\hat{w}_2(A_0))$ for a given value of α. The lemma is thus proven. ■