Manufacturer Rebate Competition in a Supply Chain with a Common Retailer

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Abstract

We consider manufacturer rebate competition in a supply chain with two competing manufacturers selling to a common retailer. We fully characterize the manufacturers' equilibrium rebate decisions and show how they depend on parameters such as the fixed cost of a rebate program, market size, cost effectiveness of rebate, the proportion of rebate-seeking consumers in the market and competition intensity. Contrary to conventional wisdom, more intense competition induces a manufacturer to lower rebate value or stop offering rebate entirely. Without rebate, it is known that more intense competition hurts the manufacturers and benefits the retailer. With rebate, however, more intense competition could benefit the manufacturers and hurt the retailer. We find similar counter-intuitive results when there is a change in some other parameters. We also consider the case when the retailer subsidizes the manufacturers sequentially to offer rebate programs. We fully characterize the retailer's optimal subsidy strategy, and show that subsidy always benefits the retailer but may benefit or hurt the manufacturers. When the retailer wants to induce both manufacturers to offer rebate, he always prefers to subsidize the manufacturer with a higher fixed cost first. Sometimes the other manufacturer will then voluntarily offer rebate even without subsidy.

Keywords: supply chain management, rebate, manufacturer competition, incentive

1 Introduction

Rebates are very popular among consumers. The typical American household that uses rebates saves $150 a year. More than $8 billion was issued back to American households through rebates in 2010 (*PRNewswire*, 2011b). According to some surveys of UK shopper behavior (*Parago*, 2013, 2014), about 3 out of 4 shoppers want cashback rebates on appliances and electronics and 1 out of 3 shoppers are interested in rebate on consumer packaged goods. Moreover, 90% of consumers search for deals and cashback rebates before shopping and 72% shoppers are interested in seeking out rebates.

Rebates are very popular among retailers and manufacturers. According to an industrial study, 50% of retailers and 48% of manufacturers use rebates as part of their customer loyalty and promotions mix (*PRNewswire*, 2011a). Firms use rebates for various purposes, such as demand expansion, price discrimination across different consumer segments, or moving inventory. Competing manufacturers often offer rebates to the same consumer market. For example, Unilever and P&G, two major competitors in the fast moving consumer goods industry, frequently offer mail-in rebate to consumers. Canon and Epson, two major manufacturers in the electronics industry, have rebate programs and online rebate centers for rebate distribution and preprocessing. They distribute competing products such as printers through retailers like Amazon.com and offer mail-in rebate for a valid purchase. However, not all the manufacturers in the same industry offer rebates. For example, in the computer industry, Dell and HP are phasing out their rebate programs whereas other manufacturers such as Samsung and Sony continue to offer rebates (Darlin, 2006). A common feature of these rebate programs is that consumers have to redeem rebate via mail or the Internet, and as a result the redemption rate is usually less than 100%.

Because retailers could benefit from a higher demand due to manufacturers' rebate programs, they have an incentive to subsidize these programs. Some retailers do this in the form of sharing the processing cost. For instance, all the rebates issued by Epson for its products sold through Staples are collected by the retailer. Many retailers invest resources

to promote manufacturers' rebate programs in their stores or on their websites. For example, Walmart.com displays "Rebate Available" sign and the related information about manufacturer rebate on its product webpage, whereas Walmart advertises these programs at their physical outlets. Sometimes retailers selectively support the rebate programs of some, but not all, of the manufacturers in the same product category. For instance, Amazon.com provides mail-in rebate information for Epson printers on its product webpage but it does not offer any information about a similar promotion for Canon printer.

The impact of manufacturer rebate on supply chain management has been studied in the literature. Most of the papers examine a one-manufacturer-one-retailer relationship. The only exception is Demirag et al. (2011), which considers two competing supply chains with instant rebate (i.e., redemption rate is 100%). As described earlier, it is not uncommon for competing manufacturers to have different strategies in offering mail-in rebate programs (with less than 100% redemption rate) when they sell through a common retailer. It is also not uncommon for a retailer to selectively subsidize only some, but not all, of the competing manufacturers selling through its channel. Because the existing theory cannot explain these phenomena, we hope to fill this gap by specifically addressing the following research questions. What is the incentive for manufacturers to offer rebate when they sell substitutable products through a common retailer? How should a retailer subsidize these manufacturers to offer rebate? How do the answers to these questions depend on factors such as competition intensity, fixed cost of a rebate program, cost effectiveness of rebate and the proportion of rebate-seeking consumers in the market?

We consider a model with two competing manufacturers selling substitutable products through a common retailer. There are two consumer segments in the market. Consumers in the non-rebate-seeking segment are not affected by rebate programs. Consumers in the rebate-seeking segment may fail to redeem the rebate (e.g., misplace the rebate or forget to redeem) and they make purchase decisions based on expected price, which is retail price net of the rebate value perceived by these consumers. It is convenient to interpret the perceived

rebate value as a discounted rebate value due to a consumer's assessment of future redemption probability. Following other related papers in the literature (Chen et al., 2007; Cho et al., 2009), we assume that the actual redemption rate is lower than what the consumers perceive and call this the slippage effect. Jolson et al. (1987) consider a similar notion of the slippage effect where some consumers who are enticed to purchase because of the rebate fail to redeem it. Rebate allows a manufacturer to increase demand from the rebate-seeking market segment and, with the slippage effect, it is more cost effective than price reduction. Its cost effectiveness is higher when either the redemption rate is lower or the consumer perceived rebate value is higher. In our model, the manufacturers first decide whether or not to offer a rebate program with the associated fixed cost. Then they compete by determining the wholesale prices and rebate values (if a rebate program is in place). Finally the common retailer determines the retail prices for both products. We consider two cases depending on whether or not the retailer can subsidize the manufacturers' rebate programs.

First consider the case when the retailer cannot subsidize rebate programs. Without competition, a manufacturer offers rebate if the fixed cost of a rebate program is low, market size is large, rebate is cost effective, or the proportion of rebate-seeking consumers is large. With competition, both manufacturers offer rebate when their fixed costs are low, and none of them offers rebate when their fixed costs are high. Otherwise the manufacturer with a low fixed cost offers rebate whereas the one with a high fixed cost does not. When competition is less intense, it is more likely for the manufacturers to offer rebate. The effect of other parameters is similar to the case of no competition.

As mentioned in the Report on the Analysis of Loyalty Discounts and Rebates Under Unilateral Conduct Laws (*International Competition Network*, 2009), loyalty discounts and rebates are considered a legitimate form of price competition and generally pro-competitive. One would expect that more intense competition induces a manufacturer to raise rebate value to increase market share. However, our analysis shows that such a conventional wisdom is not necessarily correct. When competition is more intense, a manufacturer lowers her rebate

value or stops offering rebate entirely. This is because she has to lower wholesale price, which leads to a lower profit margin and limits her ability in offering a higher rebate value.

Without rebate, more intense competition hurts the manufacturers and benefits the retailer, whereas a smaller market size generally hurts all the firms. With rebate, however, when more intense competition or a smaller market size induces a manufacturer to cease offering rebate, it hurts the retailer, benefits a non-rebate-offering rival manufacturer, and benefits a rebate-offering rival manufacturer if competition is intense and hurts her otherwise. Without competition, a manufacturer who offers rebate program is usually hurt when rebate becomes less cost effective or the rebate-seeking segment becomes smaller. With competition, however, she could benefit if competition is intense and either of these two changes triggers the rival manufacturer to stop offering rebate. We also show that when a manufacturer stops offering rebate due to a higher fixed cost, it hurts a rebate-offering rival manufacturer if competition is not intense and benefits her otherwise. These results can be explained as follows. The retailer is worse off with less rebate programs because rebate is a cost effective way to stimulate demand. Suppose the rival manufacturer stops offering rebate and consequently lowers her wholesale price. A non-rebate-offering manufacturer is better off because the rival manufacturer now loses some pricing flexibility. For a rebate-offering manufacturer, she responds by lowering the wholesale price and adjusts her rebate to raise the expected price in the rebate-seeking segment. This intensifies competition in the non-rebate-seeking segment and softens it in the rebate-seeking segment. If competition is more intense, the manufacturers compete more fiercely in the rebate-seeking segment when both offer rebate, and therefore the positive effect of softening competition in the rebate-seeking segment dominates the negative effect of intensifying competition in the non-rebate-seeking segment.

Now consider the case when the retailer can sequentially subsidize the manufacturers to offer rebate programs. We fully characterize the retailer's optimal subsidy strategy. Subsidy always benefits the retailer but may benefit or hurt the manufacturer(s). A manufacturer

is generally worse off when the retailer subsidizes the rival manufacturer to offer rebate. The only exception is when competition is not intense and the manufacturer currently offers rebate. This can be explained by how the rival manufacturer's rebate program softens and intensifies competition in respectively the non-rebate-seeking and rebate-seeking segments, as described earlier. Interestingly, when the retailer wants to induce both manufacturers to offer rebate, he always prefers to subsidize the manufacturer with a higher fixed cost first. Sometimes the other manufacturer will then voluntarily offer rebate even without subsidy.

2 Literature Review

This paper is closely related to the literature on rebate programs in supply chain management. Most papers in this stream consider a one-manufacturer-one-retailer relationship. Gerstner and Hess (1991) investigate the roles of trade deal, manufacturer rebate and retailer rebate in motivating the retailer to sell to a market with two consumer segments. Gerstner et al. (1994) examine how the retail markup impacts supply chain decisions regarding wholesale price, rebate value and retail price. In these pioneering works, the authors use rebates and coupons interchangeably. Chen et al. (2005) interpret rebate as a state-dependent discount because it is redeemed after purchase whereas coupons are redeemed at the purchase. Therefore rebates have the ability to price discriminate within a consumer among her post-purchase states and are superior to coupons. Lu and Moorthy (2007) point out that rebates are different from coupons in when the uncertainty of the redemption cost is resolved: With coupon the uncertainty is resolved before purchase; with rebates the uncertainty is resolved after purchase. They show that rebates are more efficient in price discrimination than coupons among those who buy the product. Khouja and Jing (2010) examine a manufacturer's incentive in issuing mail-in rebates in a one-to-one supply chain and find that rebates are profitable for manufacturer if consumers are inconsistent in their valuation of rebate when and after they make the purchase.

There are a number of papers that address the rebate decisions with uncertain demand. Chen et al. (2007) study the impact of manufacturer rebates on the firms' profits in a supply chain with demand uncertainty. They show that manufacturer rebates always benefit manufacturer unless the redemption rate is 100%. Aydin and Porteus (2009) extend the setting of Chen et al. (2007) to consider both channel rebates (from the manufacturer to the retailer) and manufacturer rebates with exogenous wholesale price when the demand function has multiplicative form. They show that compared with channel rebates, manufacturer rebates yield higher profit for the retailer but not necessarily for the entire supply chain. Demirag et al. (2010) consider a manufacturer's optimal rebate and retailer incentive policy when there is demand uncertainty and the retailer can price discriminate. Geng and Mallik (2011) examine a setting where both a manufacturer and a retailer decide whether to offer a mail-in rebate to consumers in a newsvendor framework.

This paper is most related to Cho et al. (2009). They consider a one-to-one supply chain and investigate how the manufacturer and the retailer independently make rebate decisions when there is a fixed cost associated with offering a rebate program. Similar to our paper, they also consider the slippage effect as the driver of a rebate program. However, they consider vertical competition between the manufacturer and the retailer in offering rebate while we consider horizontal competition between manufacturers in offering rebate. Huang et al. (2013) comment that all the rebate-dependent demand models in the supply chain literature have considered only a setting with one manufacturer and one retailer. To the best of our knowledge, this paper is the first to consider horizontal competition between manufacturers with rebate programs.

We are among the first to study manufacturer rebate competition. As far as we know, Demirag et al. (2011) is the only other paper that considers this issue. However, we focus on the cost effectiveness of rebate due to slippage effect, which is not considered by them. As a result, without competition, rebate may increase firms' profits in our model but it does not affect firms' profits in their model. Moreover, in their model, there are two competing supply

chains, the retailers make quantity instead of price decisions, the manufacturers offer both consumer rebates and retailer incentives, and rebate is instantaneous with 100% redemption rate. Thus their model setup is also quite different from ours.

Our paper is also related to the literature on manufacturer competition in a supply chain with a common retailer. Choi (1991) and Lee and Staelin (1997) study the case of linear wholesale price contracts. Cachon and Kök (2010) examine other contract forms and provide a comprehensive review of the literature. More recently, Cai et al. (2013) examine the role of probablistic selling. Bandyopadhyay and Paul (2010) and Lan et al. (2013) analyze the equilibrium return policies when demand is uncertain. Our paper contributes to this body of work by considering manufacturer competition in both wholesale prices and rebate values.

3 The Model

3.1 Model Setup

We consider a model with two manufacturers (indexed by 1 or 2) selling substitutable products through a common retailer (he). If manufacturer i (she) decides to offer a rebate program, she incurs a fixed cost F_i, which captures the costs associated with launching a rebate promotion, advertising, and distribution and processing fees.

We consider a multi-stage game with the following sequence of events:

1. Each manufacturer i decides whether to offer rebate with an associated fixed cost F_i. Let $Z_i = R$ if she offers a rebate program and $Z_i = N$ otherwise. Let n be the number of rebate programs offered, where $n = 0, 1, 2$.

2. After observing the rebate program decisions, each manufacturer i determines her wholesale price w_i, and rebate value r_i if a rebate program is in place.

3. The retailer determines the retail prices p_1 and p_2 for both products, given the wholesale prices and rebate values.

4. The manufacturers produce to meet their demands and the firms receive their payoffs.

Without loss of generality, we assume that the unit manufacturing cost, the unit selling cost and the unit rebate processing cost are constant and normalized to zero. We also assume that the manufacturers have to commit on offering a rebate program before other decisions because it takes time to launch the rebate program, e.g., setting up the website and the rebate center. Moreover, manufacturers determine wholesale prices and rebate values simultaneously because a manufacturer cannot commit on these decisions to influence the rival manufacturer's decisions.

3.2 Demand Functions

We derive the demand functions from consumers' primitive utility functions by following the approach from Vives (2001). See also Cai et al. (2012) for a similar approach. There are two consumer segments in the market. Consumers in the first segment are rebate-seeking and those in the second segment are non-rebate-seeking. Let β $(0 < \beta < 1)$ be the proportion of rebate-seeking consumers in the market. Consumers in each segment are homogeneous with identical utility functions. Without loss of generality, we normalize the consumer base to 1.

The utility function of a representative non-rebate-seeking consumer is given by:

$$U(x_1', x_2', p_1, p_2, \gamma) = (x_1' + x_2')a - \frac{1}{2}((x_1')^2 + (x_2')^2 + 2\gamma x_1' x_2') - p_1 x_1' - p_2 x_2',$$

where p_i is the retail price of product i and x_i' is the quantity of product i purchased by the consumer. Here $\gamma \in (0, 1)$ captures the substitutability between the two products, generally interpreted as the competition intensity. Given p_1 and p_2, the optimal consumption quantities x_1 and x_2 are given by:

$$\begin{aligned} x_1 &= \frac{(1-\gamma)a - p_1 + \gamma p_2}{1-\gamma^2}, \\ x_2 &= \frac{(1-\gamma)a - p_2 + \gamma p_1}{1-\gamma^2}. \end{aligned}$$

These are the demand functions for the non-rebate-seeking segment.

For the consumers in the rebate-seeking segment, they also have the same utility functions and make purchase decisions based on expected price, which is retail price net of rebate value perceived by these consumers, i.e., $p_i - br_i$. It is convenient to interpret $b \in (0, 1)$ as a consumer's assessment of future redemption probability, which is generally less than 1 because a consumer may fail to redeem the rebate (e.g., forget to redeem the rebate or deposit the check, or misplace the rebate). An alternative interpretation is that at the time when a purchase decision is made, a consumer expects to exert some effort in redeeming the rebate in the future and hence the rebate value is discounted (Aydin and Porteus, 2009; Chen et al., 2007; Cho et al., 2009).

Let $m > 0$ be the actual redemption rate of the rebate. Following other related papers in the literature (Chen et al., 2007; Cho et al., 2009), we assume that the actual redemption rate is lower than what the consumers perceive at the time they make purchase decisions, i.e., $m < b$. This is called the slippage effect. As explained by Chen et al. (2007), there is empirical evidence that consumers systematically underestimate the future effort in the context of delayed reward and the purchase decision is independent of the decision to redeem the rebate later. Let $\delta = b/m > 1$. The parameter δ can be interpreted as the cost effectiveness of the rebate program. Its cost effectiveness is higher when either the actual redemption rate is lower or the consumer perceived rebate value is higher.

The utility function of a representative rebate-seeking consumer is given by:

$$U(y_1', y_2', p_1, p_2, \gamma) = (y_1' + y_2')a - \frac{1}{2}((y_1')^2 + (y_2')^2 + 2\gamma y_1' y_2') - (p_1 - br_1)y_1' - (p_2 - br_2)y_2',$$

where p_i is the retail price of product i and y_i' is the quantity of product i purchased by the consumer. Given p_1, p_2, r_1 and r_2, the optimal consumption quantities y_1 and y_2 are given

by:

$$
\begin{aligned}
y_1 &= \frac{(1-\gamma)a - (p_1 - br_1) + \gamma(p_2 - br_2)}{1-\gamma^2}, \\
y_2 &= \frac{(1-\gamma)a - (p_2 - br_2) + \gamma(p_1 - br_1)}{1-\gamma^2}.
\end{aligned}
$$

These are the demand functions for the rebate-seeking segment.

We restrict to a decision space such that the demands in both segments, i.e., x_i and y_i, can be assumed to be positive. Cho et al. (2009) make a similar assumption that demand is always positive with either sales price or regular price. Without such an assumption, a manufacturer can always increase profit by increasing both the wholesale price and the rebate value, while maintaining a constant demand in the rebate-seeking segment and zero demand in the non-rebate-seeking segment. This is not realistic for the following reasons. First, it is against the law to charge a price that is deceptively high if the firm uses it to trick the consumers into buying the product (*Federal Trade Commission*, 2015). Second, consumers may not buy the product when they find the retail price to be unreasonably high. As pointed out by Urbany et al. (1988), according to the adaptation level and assimilation-contrast theories, a reference price affects consumer perception when it is judged acceptable or plausible relative to consumers' internal price standards. When the reference price is not plausible (i.e., it is judged to be outside the range of expected prices), consumers will reject it. Third, it rarely happens in practice that a manufacturer sells to only the rebate-seeking segment, and such a case would not be interesting anyway. In short, the assumption is needed to make our model realistic and practically interesting.

4 Single Manufacturer

In this section, we investigate the benchmark case of a single manufacturer. We remove the subscript i in our notations here and assume $\beta < 2/(\delta + 1)$ such that the manufacturer's profit function is concave and the demands in both segments are positive.

Given the manufacturer's rebate decision Z, we derive the equilibrium retail price p^Z, wholesale price w^Z and rebate value r^Z by backward induction, and based on these, we compute the retailer's equilibrium profit π_R^Z and manufacturer's equilibrium profit π_M^Z. These profits are then used to solve for the equilibrium rebate decision in the first stage.

Given w and r, the retailer maximizes his profit

$$(p - w)\left[\beta y + (1 - \beta)x\right],$$

by choosing the best-response function:

$$\hat{p}(w, r) = \frac{1}{2}(a + w + b\beta r). \tag{1}$$

The above pricing rule indicates that, when the manufacturer raises her wholesale price, she increases the (expected) retail prices for both consumer segments; when the manufacturer raises her rebate value, she increases the retail price p for the non-rebate-seeking segment but lowers the expected retail price $p - br$ for the rebate-seeking segment.

When there is no rebate program, i.e., $Z = N$, the manufacturer maximizes her profit

$$w\left[\beta y + (1 - \beta)x\right],$$

by choosing

$$w^N = \frac{a}{2}.$$

Substituting $w = w^N$ and $r = 0$ into the retailer's response function $\hat{p}(w, r)$ given by (1), we obtain the equilibrium retail price without rebate program:

$$p^N = \frac{3a}{4}.$$

When there is rebate program, i.e., $Z = R$, the manufacturer maximizes her profit

$$\beta(w - mr)y + (1 - \beta)wx,$$

by choosing

$$\begin{aligned} w^R &= \frac{[4\delta - \beta(3\delta + 1)]\, a}{h(\delta, \beta)}, \\ r^R &= \frac{(\delta - 1)a}{mh(\delta, \beta)}, \end{aligned}$$

where $h(\delta, \beta) \equiv 8\delta - \beta(\delta^2 + 6\delta + 1) > 0$. Substituting $w = w^R$ and $r = r^R$ into the retailer's response function $\hat{p}(w, r)$ given by (1), we obtain the equilibrium retail price with rebate program:

$$p^R = \frac{[6\delta - \beta(5\delta + 1)]\, a}{h(\delta, \beta)}.$$

Without rebate, the manufacturer determines the wholesale price to optimally balance between extracting a larger share of the profit margin and increasing the demand volume. With rebate, the manufacturer determines the wholesale price and the rebate value to balance her share of the profit margin, the demand volume, and the cost effectiveness of the rebate. For the non-rebate-seeking segment, she has to rely on lowering the wholesale price to induce a lower retail price to stimulate demand. For the rebate-seeking segment, it is more effective for the manufacturer to use rebate to stimulate demand instead of rely on only lowering the wholesale price. So rebate leads to a higher wholesale price, a higher price for the non-rebate-seeking segment, but a lower expected price for the rebate-seeking segment.

Overage, i.e., $p^R < r^R$, could occur in our model. That means after claiming and collecting the rebate from the manufacturer, a consumer not only gets the product for free but also earns some extra money. It is not uncommon to have overage in real life. For example, Cascade Platinum Actionpacs dishwasher detergent is sold in Walmart at \$3.97 and one can claim \$5 mail-in rebate from the manufacturer (*thefrugalfind.com*, 2015). However, it is never

optimal to charge consumers a negative expected retail price (i.e., $p - br > 0$ always holds). Overage happens when $\beta > \frac{1+\delta(6m-1)}{(5\delta+1)m}$, i.e., the proportion of rebate-seeking consumers is large, because the manufacturer relies more on selling to the rebate-seeking segment.

Based on the equilibrium decisions, we compute firms' equilibrium profits:

$$\begin{aligned} \pi_R^N &= \frac{a^2}{16}, \quad \pi_R^R = \frac{4(1-\beta)^2\delta^2 a^2}{h^2(\delta,\beta)}, \\ \pi_M^N &= \frac{a^2}{8}, \quad \pi_M^R = \frac{(1-\beta)\delta a^2}{h(\delta,\beta)}. \end{aligned}$$

We can show that r^R, w^R, p^R, π_R^R and π_M^R are increasing in β and δ. Unsurprisingly, when either the rebate is more cost effective, or there are more rebate-seeking consumers, manufacturer increases the rebate value.

Proposition 1 *(a) The manufacturer offers rebate if and only if* $F \leq \bar{F} = \pi_M^R - \pi_M^N = \frac{\beta(\delta-1)^2 a^2}{8h(\delta,\beta)}$. *(b)* $\bar{F}$ *is increasing in* a, δ *and* β. *(c)* $\pi_R^R > \pi_R^N$.

From parts (a) and (b), the manufacturer offers rebate if the fixed cost of the rebate program is low, the total market size is large, the proportion of rebate-seeking segment is high or rebate is cost effective. From part (c), the retailer always benefits when the manufacturer offers a rebate program.

Next consider the rebate decision that maximizes the total supply chain profit. It is straightforward to show that manufacturer rebate increases the supply chain profit iff

$$F \leq \bar{F}_s = \frac{\beta(\delta-1)^2(32\delta - \beta(3\delta^2 + 26\delta + 3))a^2}{16h^2(\delta,\beta)},$$

where $\bar{F}_s$ is increasing in a, δ and β. We can show that $\bar{F}_s > \bar{F}$ and it follows that the retailer can use subsidy to induce manufacturer to make the rebate decision that maximizes the total supply chain profit. When $\bar{F} < F \leq \bar{F}_s$, he will offer a subsidy of $F - \bar{F}$, to induce the manufacturer to offer rebate.

5 Two Competing Manufacturers: Price and Rebate Value Decisions

Now consider two competing manufacturers selling through a common retailer. Given manufacturers' rebate decisions (Z_1, Z_2), we solve for the equilibrium retail prices, wholesale prices and rebate values and then derive the firms' profits. The fixed cost of a rebate program is not relevant because it is a sunk cost that does not have any impact on the price and rebate value decisions. Let $\pi_{Mi}(n)$ $(n = 0, 2)$ or $\pi_{Mi}^{Z_i}(n)$ $(n = 1)$ be manufacturer i's profit, and $\pi_R(n)$ $(n = 0, 1, 2)$ be the retailer's profit when the number of rebate programs is n.

Given w_i and r_i, the retailer maximizes his profit

$$(p_1 - w_1)\left[\beta y_1 + (1-\beta)x_1\right] + (p_2 - w_2)\left[\beta y_2 + (1-\beta)x_2\right],$$

by choosing the following best-response function:

$$\hat{p}_i(w_i, r_i) = \frac{1}{2}(a + w_i + b\beta r_i). \tag{2}$$

5.1 Neither Manufacturer Offers Rebate

If neither manufacturer offers rebate program, manufacturer i maximizes her profit

$$w_i\left[\beta y_i + (1-\beta)x_i\right],$$

with the following best-response function:

$$\hat{w}_i(w_j) = \frac{(1-\gamma)a + \gamma w_j}{2}.$$

By solving $w_i = \hat{w}_i(w_j)$ and $w_j = \hat{w}_j(w_i)$ simultaneously, we obtain the equilibrium wholesale prices

$$w_i(0) = \frac{a}{2+\gamma'},$$

where $\gamma' = \frac{\gamma}{1-\gamma} \in (0, \infty)$ and γ' is increasing in γ. Substituting $w_i = w_i(0)$ and $r_i = 0$ into the retailer's response function $\hat{p}_i(w_i, r_i)$ given by (2), we obtain the equilibrium retail prices and profits

$$\begin{aligned} p_i(0) &= \frac{(3+\gamma')a}{2(2+\gamma')}, \\ \pi_R(0) &= \frac{(1+\gamma')^3 a^2}{2(1+2\gamma')(2+\gamma')^2}, \\ \pi_{Mi}(0) &= \frac{(1+\gamma')^2 a^2}{2(1+2\gamma')(2+\gamma')^2}. \end{aligned}$$

Lemma 1 *(a) $p_i(0)$ and $w_i(0)$ $(i = 1, 2)$ are decreasing in γ; (b) $\pi_{Mi}(0)$ is decreasing in γ, and $\pi_R(0)$ is increasing in γ.*

Lemma 1 is consistent with the findings in Choi (1991) that more intense competition induces lower wholesale prices, which hurts the manufacturers and benefits the retailer .

5.2 Both Manufacturers Offer Rebate

To ensure interior equilibrium solutions, we assume that $\beta < \min\left[\frac{2-\gamma}{\delta(1-\gamma)+1}, \frac{8\delta}{\delta^2+6\delta+1}\right]$ so that the manufacturers' profit functions are concave and the demands in both segments are positive.

If both manufacturers have rebate programs, manufacturer i maximizes her profit

$$\beta(w_i - mr_i)y_i + (1-\beta)w_i x_i,$$

by choosing the following best-response functions:

$$\begin{aligned}
\hat{r}_i(r_j, w_j) &= \frac{(1-\gamma)(\delta-1)a + \gamma\left[(4-3\beta-\beta\delta)br_j + (\delta-1)w_j\right]}{mh(\delta,\beta)}, \\
\hat{w}_i(r_j, w_j) &= \frac{(1-\gamma)((4-3\beta)\,\delta-\beta)a + \gamma\left[((4-3\beta)\,\delta-\beta)w_j - (\delta-1)(2-\beta)\beta br_j\right]}{h(\delta,\beta)}.
\end{aligned}$$

By solving the four equations $r_i = \hat{r}_i(r_j, w_j)$ and $w_i = \hat{w}_i(r_j, w_j)$ simultaneously, we obtain the manufacturers' equilibrium decisions:

$$\begin{aligned}
w_i(2) &= \frac{\left[2(2+\gamma')\delta - \beta((3+\gamma')\,\delta + 1 + \gamma')\right]a}{2(1-\beta)\gamma'^2\delta + (1+\gamma')h(\delta,\beta)}, \\
r_i(2) &= \frac{(1+\gamma')(\delta-1)a}{m\left[2(1-\beta)\gamma'^2\delta + (1+\gamma')h(\delta,\beta)\right]}.
\end{aligned}$$

Substituting $w_i = w_i(2)$ and $r_i = r_i(2)$ into the retailer's response function $\hat{p}_i(w_i, r_i)$ given by (2), we obtain the equilibrium prices and profits:

$$\begin{aligned}
p_i(2) &= \frac{\left[(2+\gamma')(3+\gamma')\delta - \beta(5+4\gamma'+\gamma'^2)\delta - \beta(1+\gamma')\right]a}{2(1-\beta)\gamma'^2\delta + (1+\gamma')h(\delta,\beta)}, \\
\pi_R(2) &= \frac{2(1-\beta)^2(1+\gamma')^3(2+\gamma')^2\delta^2a^2}{(1+2\gamma')\left[2(1-\beta)\gamma'^2\delta + (1+\gamma')h(\delta,\beta)\right]^2}, \\
\pi_{Mi}(2) &= \frac{(1-\beta)(1+\gamma')^2\delta a^2}{(1+2\gamma')\left[2(1-\beta)\gamma'^2\delta + (1+\gamma')h(\delta,\beta)\right]}.
\end{aligned}$$

Lemma 2 *(a) $p_i(2)$, $w_i(2)$ and $r_i(2)$ are decreasing in γ, and increasing in β and δ. (b) $\pi_{Mi}(2)$ is decreasing in γ, and increasing in β and δ; $\pi_R(2)$ is first increasing and then decreasing in γ, and increasing in β and δ.*

The effects of β and δ are consistent with those in the single manufacturer model. Because rebate is usually regarded as a form of price competition and pro-competitive, one might expect that more intense competition would induce a manufacturer to raise her rebate value to obtain a larger market share. However, our analysis shows that such a conventional wisdom is not necessarily correct. It would be true if the wholesale price is fixed. However if the manufacturer can adjust both the wholesale price and rebate value, she would lower

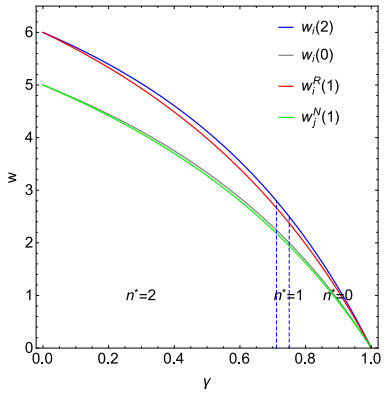
both when competition is more intense (Figures 1a, 1b). This can be explained as follows. More intense competition has opposing effects on the rebate value. On the one hand, as the wholesale price decreases, the manufacturer wants to lower the rebate value to maintain a healthy profit margin ($w - mr$) for the rebate-seeking segment. In addition, a lower rebate value induces a lower retail price which increases the demand from the non-rebate-seeking segment. On the other hand, when competition is more intense, the manufacturer wants to increase the rebate value to induce a lower perceived price to stimulate more demand from the rebate-seeking segment. However, this is less effective because a larger rebate value induces the retailer to charge a higher retail price, which dilutes the effect of rebate on the rebate-seeking segment. It turns out that the former two effects together dominate the latter one as competition intensifies.

Manufacturer competition benefits the consumers in the non-rebate-seeking segment because the retail price is decreasing in γ (Figure 1c). However, the impact of manufacturer competition on the consumers in the rebate-seeking segment is ambiguous because both the retail price and the rebate value are decreasing in γ. We can show that the expected retail price for the rebate-seeking consumer is first decreasing and then increasing when competition becomes more intense (Figure 1d). It can be higher with competition than without when γ is sufficiently high. In other words, manufacturer competition does not necessarily benefit consumers in the rebate-seeking segment.

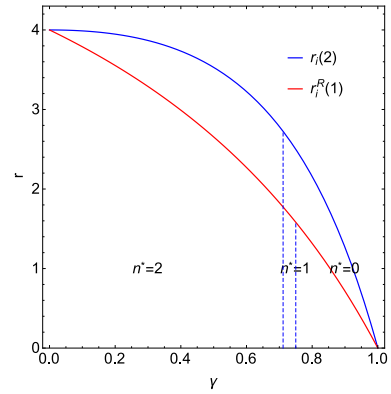
Another counter-intuitive result is that, as illustrated in Figure 2a, the retailer's profit does not always increase as competition intensifies, in contrast to the case without rebate. This is because the retailer benefits from a lower wholesale price but is hurt by a lower rebate value when competition becomes more intense. The above lemma shows that the former effect dominates the latter effect when competition is not intense, and the dominance is reversed otherwise. As shown in Figure 1b, rebate value decreases more rapidly when competition is intense.

Figure 1: Equilibrium Wholesale Price, Rebate Value and (Expected) Retail Price versus Competition Intensity ($a = 10$, $\delta = 2$, $\beta = 1/2$, $m = 1/3$, $F_1 = 1/5$, $F_2 = 1/2$)

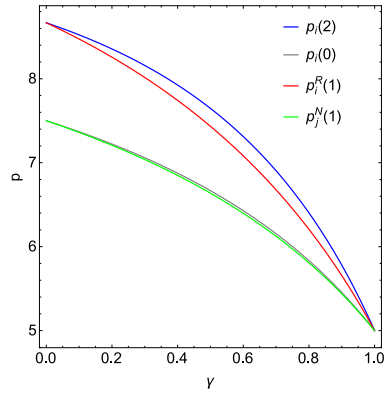
(a) Wholesale Prices



(b) Rebate Values



(c) Retail Price for Non-rebate-seeking Consumer



(d) Expected Retail Price for Rebate-seeking Consumer

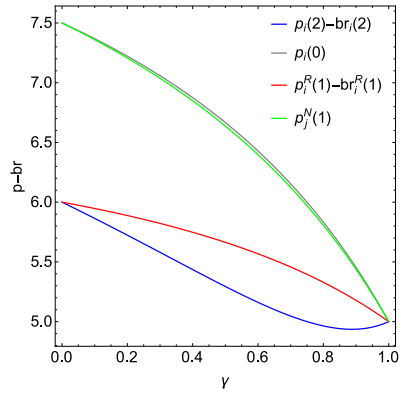
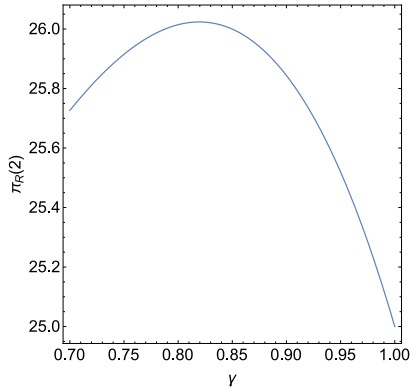
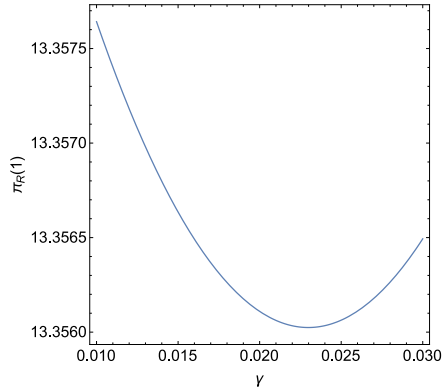


Figure 2: Retailer Profit versus Competition Intensity ($a = 10, \delta = 2$)

(a) $\pi_R(2)$ ($\beta = 4/5$)



(b) $\pi_R(1)$ ($\beta = 1/2$)



5.3 One Manufacturer Offers Rebate

Suppose manufacturer i issues rebate and manufacturer j does not. We assume that $\delta < \frac{2+\gamma}{\gamma}$, $\beta < \min\left[\frac{2}{\delta+1}, \frac{2(2+\gamma)(2-\gamma(\delta-1))\delta}{-2(\delta-1)\delta\gamma^2+(-3\delta^2+6\delta+1)\gamma+\delta^2+6\delta+1}\right]$ to ensure that the manufacturers' profit functions are concave and the demands in both segments are positive.

Manufacturer i maximizes her profit by choosing the following best-response functions:

$$\begin{aligned}
\hat{r}_i(w_j) &= \frac{(\delta-1)\left[(1-\gamma)a+\gamma w_j\right]}{mh(\delta,\beta)}, \\
\hat{w}_i(w_j) &= \frac{\left[(4-3\beta)\,\delta-\beta\right]\left[(1-\gamma)a+\gamma w_j\right]}{h(\delta,\beta)}.
\end{aligned}$$

Manufacturer j maximizes her profit by choosing following the best-response function:

$$\hat{w}_j(r_i, w_i) = \frac{(1-\gamma)a+\gamma(w_i-\beta b r_i)}{2}.$$

By solving $r_i = \hat{r}_i(w_j)$, $w_i = \hat{w}_i(w_j)$ and $w_j = \hat{w}_j(r_i, w_i)$ simultaneously, we obtain the manufacturers' equilibrium decisions:

$$\begin{aligned}
w_i^R(1) &= \frac{(2+3\gamma')\left[(4-3\beta)\,\delta-\beta\right]a}{(2+4\gamma'+\gamma'^2)h(\delta,\beta)+4(1-\beta)\gamma'^2\delta}, \\
r_i^R(1) &= \frac{(2+3\gamma')(\delta-1)a}{m\left[(2+4\gamma'+\gamma'^2)h(\delta,\beta)+4(1-\beta)\gamma'^2\delta\right]}, \\
w_j^N(1) &= \frac{\left[(1+2\gamma')h(\delta,\beta)-4(1-\beta)\gamma'\delta\right]a}{(2+4\gamma'+\gamma'^2)h(\delta,\beta)+4(1-\beta)\gamma'^2\delta},
\end{aligned}$$

where $w_i^R(1) > w_j^N(1)$. Substituting $w_i = w_i^R(1)$, $r_i = r_i^R(1)$, $w_j = w_j^N(1)$ and $r_j = 0$ into the retailer's best-response functions $\hat{p}_i(w_i, r_i)$ and $\hat{p}_j(w_j, r_j)$ given by (2), we obtain the equilibrium retail prices and profits:

$$\begin{aligned}
p_i^R(1) &= \frac{a}{2}\left[1+\frac{(2+3\gamma')(12\delta-10\beta\delta-2\beta-h(\delta,\beta))}{(2+4\gamma'+\gamma'^2)h(\delta,\beta)+4(1-\beta)\gamma'^2\delta}\right], \\
p_j^N(1) &= \frac{a}{2}\left[1+\frac{(1+2\gamma')h(\delta,\beta)-4(1-\beta)\gamma'\delta}{(2+4\gamma'+\gamma'^2)h(\delta,\beta)+4(1-\beta)\gamma'^2\delta}\right], \\
\pi_R(1) &= \frac{(1+\gamma')^3 h_1 a^2}{4(1+2\gamma')\left[(2+4\gamma'+\gamma'^2)h(\delta,\beta)+4(1-\beta)\gamma'^2\delta\right]^2},
\end{aligned}$$

$$\pi_{Mi}^R(1) = \frac{(1-\beta)(1+\gamma')^2(2+3\gamma')^2\delta h(\delta,\beta)a^2}{(1+2\gamma')\left[h(\delta,\beta)(\gamma'^2+4\gamma'+2)+4(1-\beta)\gamma'^2\delta\right]^2},$$
$$\pi_{Mj}^N(1) = \frac{(1+\gamma')^2\left[(2\gamma'+1)h(\delta,\beta)-4(1-\beta)\gamma'\delta\right]^2 a^2}{2(1+2\gamma')\left[h(\delta,\beta)(\gamma'^2+4\gamma'+2)+4(1-\beta)\gamma'^2\delta\right]^2},$$

where $h_1 = (1+3\gamma'+2\gamma'^2)h^2(\delta,\beta)+8\gamma'(2+\gamma')(1-\beta)\delta h(\delta,\beta)+32(1-\beta)^2(2+4\gamma'+\gamma'^2)\delta^2$. Rebate-seeking consumers perceive a higher expected price for product j than product i ($p_j^N(1) > p_i^R(1) - br_i^R(1)$), whereas non-rebate-seeking consumers face a higher price for product i than product j ($p_i^R(1) > p_j^N(1)$). For a rebate-seeking consumer, we can show that manufacturer i makes more profit from selling a product to her than manufacturer j, and the retailer earns more profit from selling product i to her than product j. The reverse is true for a non-rebate-seeking consumer.

Lemma 3 *(a) $p_i^R(1)$, $w_i^R(1)$ and $r_i^R(1)$ are decreasing in γ, and increasing in β and δ; $p_j^N(1)$ and $w_j^N(1)$ are decreasing in γ, β and δ. (b) $\pi_{Mi}^R(1)$ is decreasing in γ, and increasing in β and δ; $\pi_{Mj}^N(1)$ is decreasing in γ, β, and δ; $\pi_R(1)$ is first decreasing and then increasing in γ, and increasing in β and δ.*

When rebate is more cost effective, or the rebate-seeking segment is larger, manufacturer j who does not issue rebate is more disadvantaged for not having the flexibility of using both wholesale price and rebate value to influence the expected prices in both consumer segments. When only one manufacturer offers rebate program, as illustrated in Figure 2b, the retailer's profit is first decreasing and then increasing as competition becomes more intense, in contrast to the case where both manufacturers offer rebate programs. When competition is more intense, the retailer benefits from lower w_i and w_j, but is hurt by a lower r_i. Our results show that the effect of competition on retailer's profit from product i can be negative and it can dominate the positive effect of competition on his profit from product j.

6 Two Competing Manufacturers: Rebate Decisions

In the first stage, the manufacturers simultaneously decide whether or not to pay a fixed cost to launch a rebate program. Let F_i be the fixed cost of manufacturer i. We can construct a normal game with the manufacturers as players and a payoff matrix given by Table 1. We assume $\delta < \frac{2+\gamma}{\gamma}$ and $\beta < \min\left[\frac{2}{\delta+1}, \frac{2(2+\gamma)\delta(2-\gamma(\delta-1))}{-2(\delta-1)\delta\gamma^2+(-3\delta^2+6\delta+1)\gamma+\delta^2+6\delta+1}\right]$, which ensure interior solutions for all the possible rebate decisions. We also assume that a manufacturer will offer rebate if she is indifferent to doing so or not.

Table 1: Payoff Matrix of the Stage-One Game

Manufacturer 1 ╲ Manufacturer 2	$Z_2 = R$	$Z_2 = N$
$Z_1 = R$	$\pi_{M1}(2) - F_1, \pi_{M2}(2) - F_2$	$\pi^R_{M1}(1) - F_1, \pi^N_{M2}(1)$
$Z_1 = N$	$\pi^N_{M1}(1), \pi^R_{M2}(1) - F_2$	$\pi_{M1}(0), \pi_{M2}(0)$

Let $T_1 \equiv \pi^R_{Mi}(1) - \pi_{Mi}(0)$, $T_2 \equiv \pi_{Mi}(2) - \pi^N_{Mi}(1)$, and $T_3 \equiv \pi_{Mi}(2) - \pi_{Mi}(0)$, where T_1, T_2, T_3 are functions of a, γ, β and δ. We can show that $0 < T_1 < T_2$ and $0 < T_3 < T_2$. Without loss of generality we assume $F_1 \leq F_2$. The following proposition presents the equilibrium rebate structure and it is illustrated in Figure 3.

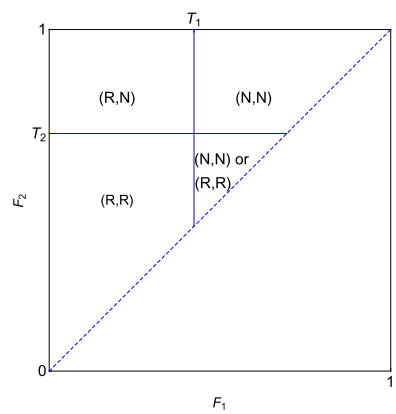
Proposition 2 *(a) Suppose $F_1 \leq F_2$. (1) If $F_1 > T_1$ and $F_2 > T_2$, (N,N) is the unique equilibrium; (2) If $F_1 < T_1$ and $F_2 \leq T_2$, (R,R) is the unique equilibrium; (3) If $T_1 \leq F_i \leq T_2$, (N,N) and (R,R) are the (only) two equilibria. (N,N) is Pareto optimal if either $T_3 < T_1$ or $T_1 < T_3 < F_i < T_2$; (R,R) is Pareto optimal if $T_1 < F_i < T_3 < T_2$; (N,N) and (R,R) do not dominate each other otherwise. (4) If $F_1 \leq T_1$ and $F_2 > T_2$, (R,N) is the unique equilibrium. (b) T_1, T_2 and T_3 are decreasing in γ, and increasing in a, β and δ.*

From part (a), both manufacturers offer rebate if their fixed costs are low, and none of them offers rebate when their fixed costs are high. Otherwise only the manufacturer with a lower fixed cost offers rebate. If $T_3 < F_i < T_1$ for $i = 1$ and 2, (R, R) is the unique equilibrium but both manufacturers would be better off with (N, N). This is the classical prisoners' dilemma. When a manufacturer offers rebate, the rival manufacturer will do so

too or else she will become disadvantaged. From part (b), when the rebate-seeking segment is larger, rebate is more cost effective or the total market size is larger, it is more likely for the manufacturers to offer rebate. However, when competition is more intense, it is less likely for the manufacturers to offer rebate as explained after Lemma 2.

Corollary 1 *Suppose $F_1 = F_2 = F$. (a) When $F < T_1$, (R,R) is the unique equilibrium; (b) When $T_1 \leq F \leq T_2$, (N,N) and (R,R) are the (only) two equilibria and (R,R) is Pareto optimal if $F < T_3$, and (N,N) is Pareto optimal otherwise; (c) When $F > T_2$, (N,N) is the unique equilibrium.*

Figure 3: Equilibrium Rebate Decisions ($a = 10, \delta = 2, \gamma = 1/2, \beta = 1/2$)



The following proposition presents some sensitivity results (i.e., how the firms' profits are affected by a parametric change when it induces a change in the equilibrium rebate decisions) and compare the retailer's profits under different numbers of rebate programs. Here an increase in u means an increase in γ or a decrease in a, δ or β. Let u_i $(i = 1, 2)$ be the inverse function of $T_i(u)$.

Proposition 3 *Let $u = \gamma, -a, -\delta,$ or $-\beta$. (a) There exist $\epsilon_2 > 0$ such that when u (F_2) increases from $u_2 - \epsilon_2$ ($T_2 - \epsilon_2$) to $u_2 + \epsilon_2$ ($T_2 + \epsilon_2$), the equilibrium changes from (R,R) to (R,N), manufacturer 1 is better off if $\gamma > \bar{\gamma}$ and worse off otherwise, and the retailer is worse off. (b) There exists $\epsilon_1 > 0$ such that when u (F_1) increases from $u_1 - \epsilon_1$ ($T_1 - \epsilon_1$) to*

$u_1 + \epsilon_1$ ($T_1 + \epsilon_1$), the equilibrium changes from (R,N) to (N,N), manufacturer 2 is better off, and the retailer is worse off. (c) $\pi_R(0) < \pi_R(1) < \pi_R(2)$.

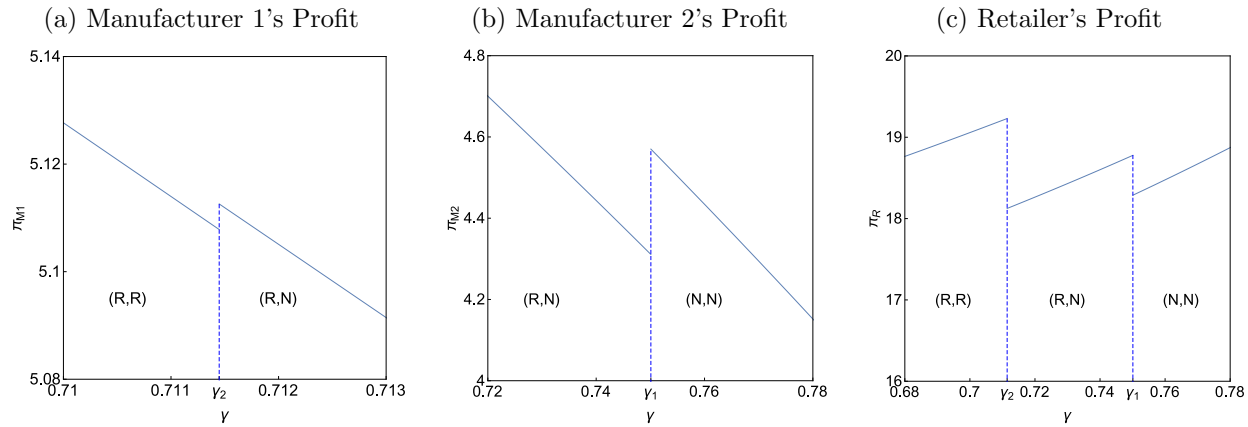
Without rebate, it is known that more intense competition or a smaller market size hurts the manufacturers. With rebate, however, from part (a), when more intense competition induces the rival manufacturer to cease offering rebate, it benefits a rebate-offering manufacturer if competition is intense, as illustrated in Figure 4a, and hurts him otherwise. Similarly, a smaller market size benefits a rebate-offering manufacturer when it induces its rival to cease offering rebate and competition is intense. These results can be explained as follows. Suppose both manufacturer offers rebate. When one manufacturer stops offering rebate, both manufacturers lower their wholesale prices ($w_j^N(1) < w_i^R(1) < w_i(2)$ in Figure 1a) and the rebate-offering manufacturer also lowers her rebate value ($r_i^R(1) < r_i(2)$ in Figure 1b). This intensifies competition in the non-rebate-seeking segment ($p_i(2) > p_i^R(1) > p_j^R(1)$ in Figure 1c) but softens it in the rebate-seeking segment ($p_j^R(1) > p_i^R(1) - br_i^R(1) > p_i(2) - br_i(2)$ in Figure 1d). When competition is more intense, the manufacturers compete more fiercely in the rebate-seeking segment when both offer rebate, and therefore the positive effect of softening competition in the rebate-seeking segment dominates the negative effect of intensifying competition in the non-rebate-seeking segment.

Without competition, a manufacturer who offers rebate is worse off when rebate becomes less cost effective or the rebate-seeking segment becomes smaller. With competition, however, a rebate-offering manufacturer could be better off if either change induces the rival manufacturer to cease offering rebate and competition is intense, as explained earlier. Part (a) also shows that when competition is not intense and the rival manufacturer stops offering rebate due to a higher fixed cost, a rebate-offering manufacturer could become worse off.

From part (b), a non-rebate-offering manufacturer benefits from more intense competition, as illustrated in Figure 4b, if it induces the rival manufacturer to cease offering rebate. This is because the rival manufacturer loses the flexibility of using rebate to influence competition. Consequently, the non-rebate-offering manufacturer raises her wholesale price

$(w_i^N(1) < w_i(0))$, whereas the rival manufacturer lowers the wholesale price $(w_i(0) < w_i^R(1))$, as illustrated in Figure 1a. Similarly, a smaller market size benefits the non-rebate-offering manufacturer when it induces the rival manufacturer to stop offering rebate.

Figure 4: Equilibrium Firms' Profits versus Competition Intensity $(a = 10, \delta = 2, \beta = 1/2, F_1 = 1/5, F_2 = 1/2)$



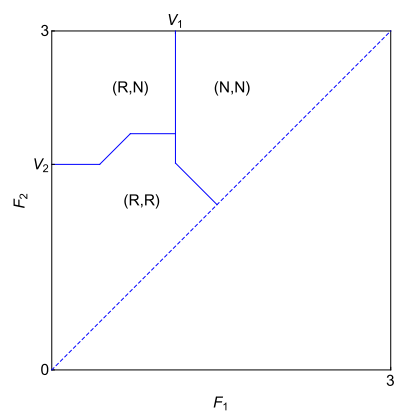
From part (c), the retailer always benefits from manufacturer rebate because it is a cost effective way to stimulate demand. This explains the result in part (a) and (b) that the retailer is worse off when a change in a parameter induces a manufacturer to stop offering rebate. This is illustrated in Figure 4c for the case of competition intensity.

Our results have important implications to rebate programs in practice. Proposition 2 may explain why a manufacturer (e.g., Samsung) offers rebate while its rival (e.g., HP) does not. This could be because they face different fixed costs, and the competition intensity is neither too high nor too low such that it is only profitable for some but not all of the manufacturers to offer rebate. Proposition 3 shows that a change in the business environment (e.g., rebate becomes more cost effective) that normally benefits a rebate-offering manufacturer could hurt her if it motivates a rival to start offering rebate. It also suggests that a retailer should be cautious in taking actions to intensify the competition between its vendors, because such actions might induce a vendor to cease offering rebate and hurt the retailer.

7 Retailer Subsidizes Manufacturer Rebate

In this section we study how the retailer should offer subsidy to induce the manufacturer(s) to offer more rebate programs. We assume that the retailer makes offers to the manufacturers sequentially. (We have also considered the case of simultaneous offering and, because the results are qualitatively similar, details are omitted.) Let S_i $(i = 1, 2)$ be the subsidy offered to manufacturer i. Let $V_1 = \pi_R(1) - \pi_R(0) + T_1$ and $V_2 = \pi_R(2) - \pi_R(1) + T_2$. Figure 5 illustrates the equilibrium rebate decisions. In Figure 6, we impose the equilibrium structure of Figure 3 into Figure 5 to highlight the changes due to subsidy. The corresponding regions are denoted by A to I, and their definitions are given in Appendix.

Figure 5: Equilibrium Rebate Decisions with Retailer's Subsidy ($a = 10, \delta = 2, \beta = 1/2, \gamma = 1/2$)



Proposition 4 *The equilibrium rebate and subsidy decisions are given in Table 2. When the retailer uses subsidy to induce the equilibrium to change from (N,N) to (R,R), he always prefers to subsidize the manufacturer with a higher fixed cost (i.e., manufacturer 2) first.*

Compared with the case of no subsidy, the manufacturers are obviously more likely to offer rebate. The retailer may subsidize manufacturer 1 to induce a change in the equilibrium rebate decisions from (N, N) to (R, N) (Region H), or subsidize manufacturer 2 to induce a change from (R, N) to (R, R) (Region C). If he wants to induce a change from (N, N) to (R, R), he always prefers to subsidize manufacturer 2 first because she has a higher fixed cost.

Figure 6: Effect of Retailer's Subsidy ($a = 10, \delta = 2, \beta = 1/2, \gamma = 1/2$)

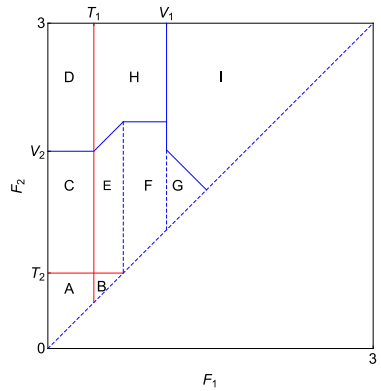


Table 2: Equilibrium Rebate and Subsidy Decisions

Rebate decisions without subsidy	Rebate decisions with subsidy	Subsidy values	Regions
(R, R)	(R, R)	$S_2 = 0,\ S_1 = 0$	A
(R, R) and (N, N)	(R, R)	$S_2 = 0,\ S_1 = 0$	B
(R, N)	(R, R)	$S_2 = F_2 - T_2,\ S_1 = 0$	C
	(R, N)	$S_2 = 0,\ S_1 = 0$	D
(N, N)	(R, R)	$S_2 = F_2 - T_2,\ S_1 = 0$	E
		$S_2 = F_2 - T_2,\ S_1 = F_1 - T_2$	F
		$S_2 = F_2 - T_3,\ S_1 = F_1 - T_2$	G
	(R, N)	$S_2 = 0,\ S_1 = F_1 - T_1$	H
	(N, N)	$S_2 = 0,\ S_1 = 0$	I

By doing so, he can either induce manufacturer 1 to voluntarily offer rebate (Region E), or lower the total subsidy cost to both manufacturers (Region F). This is because after the first manufacturer has accepted the subsidy, it is easier to make the second manufacturer offer rebate if she is manufacturer 1 instead of manufacturer 2. We can show that if the retailer subsidizes manufacturer 1 first, it is impossible to induce manufacturer 2 to voluntarily offer rebate. In Region B, without subsidy, both equilibria (R, R) and (N, N) exist. With subsidy, the retailer can ensure that (R, R) is the unique equilibrium by first paying an arbitrarily small subsidy to manufacturer 2 for offering rebate, and then manufacturer 1 will follow by voluntarily offering rebate too.

As observed in Section 1, both Epson and Cannon offer rebate but Amazon.com promotes only Epson's rebate program. This could be explained by Proposition 4, which shows that

a retailer may need to subsidize only one manufacturer to induce both to offer rebate.

Contrary to the single manufacturer case, subsidy does not always induce the rebate decisions that maximize supply chain profit. It always benefits the retailer because he offers subsidy only when it is profitable. The following proposition examines the impact of subsidy on the manufacturers' profits when it induces a change in the equilibrium rebate decisions.

Proposition 5 *(a) In Region C, when retailer's subsidy induces the equilibrium to change from (R,N) to (R,R), manufacturer 1 is better off if $\gamma < \bar{\gamma}$ and worse off otherwise. (b) In Region H, when retailer's subsidy induces the equilibrium to change from (N,N) to (R,N), manufacturer 2 is worse off. (c) In Regions E, F and G, when retailer's subsidy induces the equilibrium to change from (N,N) to (R,R), manufacturer 1 is strictly worse off and manufacturer 2 is weakly worse off.*

From parts (a) and (b), the results are similar to those in Proposition 3, because the manufacturer who is not subsidized does not change her rebate decision. Suppose the retailer subsidizes a manufacturer to offer rebate. If the rival manufacturer is not currently offering rebate (Region H), she is worse off. If the rival manufacturer is currently offering rebate (Region C), she is better off if competition is not intense and worse off otherwise, as explained in the previous section. Now consider part (c). When the retailer uses subsidy to induce both manufacturers to offer rebate programs (Regions E, F, G), manufacturer 1 is always strictly worse off. Manufacturer 2 is also strictly worse off in Regions E and F, but indifferent in Region G. In Regions E and F, a subsidized manufacturer can be worse off with subsidy than without because if she does not accept the offer while the rival manufacturer does, she would be worse off than the case when both do not offer rebate. In Region G, manufacturer 2, who is subsidized, is indifferent because the retailer would not offer any subsidy to manufacturer 1 if manufacturer 2 rejects the offer.

Besides the regions characterized by Proposition 5, it would be interesting to consider Region B too. Without subsidy, both (N, N) and (R, R) can be an equilibrium. Proposition 2 provides conditions under which (1) (N, N) is Pareto optimal, (2) (R, R) is Pareto optimal,

and (3) (N, N) and (R, R) do not dominate each other. It is natural to pick the Pareto optimal equilibrium, if it exists, as the outcome of a game because it is preferred by all the players (Cachon and Netessine, 2004). With subsidy, from Table 2, the retailer uses an arbitrarily small subsidy to induce (R, R) to be the unique equilibrium. In case (1), both manufacturers become worse off. In case (2), they are indifferent. In case (3), it is easy to show that manufacturer 1 prefers (R, R) whereas manufacturer 2 prefers (N, N). Thus manufacturer 1 becomes better off and manufacturer 2 becomes worse off.

8 Conclusion

In this paper, we study manufacturer rebate competition in a supply chain with a common retailer. We identify fixed cost of a rebate program, market size, cost effectiveness of rebate, proportion of rebate-seeking consumers and competition intensity as the key performance drivers and characterize how they affect the firms' decisions and their profits. Our analysis reveals some novel results that contradict conventional wisdom. For instance, we find that more intense competition induces a manufacturer to lower rebate value or stop offering rebate entirely. When the latter occurs, it hurts the retailer, benefits a non-rebate-offering rival manufacturer, and benefits a rebate-offering rival manufacturer if competition is intense and hurts her otherwise. Similarly, a manufacturer could benefit when market size becomes smaller. A rebate-offering manufacturer could benefit when either rebate becomes less cost effective or the proportion of rebate-seeking consumers becomes smaller, and could be hurt when the rival's fixed cost becomes higher. We also find that when the retailer subsidizes the manufacturers to offer rebate programs, it always benefits the retailer but may benefit or hurt the manufacturers. When the retailer wants to induce both manufacturers to offer rebate, he always prefers to subsidize the manufacturer with a higher fixed cost first. Sometimes the other manufacturer will then voluntarily offer rebate even without subsidy.

In our model, we assume full information and focus on the slippage effect of rebate. In

practice, the retailer may have superior demand information and rebate could offer other benefits such as price discrimination or moving unwanted inventory. Because these issues require very different modes of analysis, we leave them for future research.

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Online Appendix. Proofs

Proof of Proposition 1. Part (a) is from equilibrium manufacturer profits with and without rebate program. For part (b), it is clear that $\bar{F}$ is increasing in a and β based on the expression in part (a). $\bar{F}$ is increasing in δ because $\partial\bar{F}/\partial\delta = \beta(1-\beta)(\delta^2 - 1)a^2/h^2(\beta,\delta) > 0$. Part (c) is true because we can verify $\pi_R^R - \pi_R^N = \beta(\delta-1)^2(2h(\beta,\delta) + \beta(\delta-1)^2)a^2/[16h^2(\beta,\delta)] > 0$. □

Proof of Lemma 1. For part (a), notice that $p_i = \frac{a}{2}(1+\frac{1}{2+\gamma'})$ and $w_i = a/(2+\gamma')$ are decreasing in γ'. Then p_i and w_i are decreasing in γ because γ' is increasing in γ. For part (b), it is clear that $\pi_R(0)$ is increasing in γ' and therefore is increasing in γ. The conclusion with respect to manufacturer is true because $\partial\pi_{Mi}(0)/\partial\gamma = -[1-(1-\gamma)\gamma]a^2/[(2-\gamma)^3(1+\gamma)^2] < 0$. □

Proof of Lemma 2. For part (a), notice that

$$\begin{aligned}
\partial w_i/\partial\beta &= \frac{2(2-\gamma)(1-\gamma)(\delta-1)\delta(1+(1-\gamma)\delta)a}{\left[(2\gamma^2\delta+(\delta^2+6\delta+1)(1-\gamma))\beta-2(2-\gamma)^2\delta\right]^2} > 0,\\
\partial r_i/\partial\beta &= \frac{(1-\gamma)(\delta-1)(2\gamma^2\delta+(1-\gamma)(1+6\delta+\delta^2))a}{m\left[(2\gamma^2\delta+(\delta^2+6\delta+1)(1-\gamma))\beta-2(2-\gamma)^2\delta\right]^2} > 0.
\end{aligned}$$

Then $\partial p_i/\partial\beta > 0$ because $p_i = (a+w_i+b\beta r_i)/2$. We can verify that $\partial w_i/\partial\delta$ equals $[-(3-2\gamma)(1-\gamma)\delta^2 - 2(1-\gamma)\delta - 3 + \gamma]\beta + 2(2-\gamma)[(1-\gamma)\delta^2+1]$ multiplied by a positive factor. It suffices to show that the above term is positive. That term is a linear function of β with a positive intercept, and is always positive because it is positive at $\beta = 1$. Similarly, $\partial r_i/\partial\delta$ equals $[-2\gamma^2 + (1-\gamma)(\delta^2-2\delta-7)]\beta + 2(2-\gamma)^2$ multiplied by a positive factor. Again, this term is a linear function of β with a positive intercept. Then it is always positive because it is positive at $\beta = 1$, so is $\partial r_i/\partial\delta$. We have $\partial p_i/\partial\delta > 0$ because $p_i = (a+w_i+b\beta r_i)/2$. The sensitivity results with respect to γ can be obtained similarly.

Most conclusions in part (b) can be obtained by taking derivatives of the equilibrium

firm profits and checking the signs, and we only show $\pi_R(2)$ is first increasing and then decreasing in γ. We can check that $\partial\pi_R(2)/\partial\gamma$ equals $6\delta(1-\beta)\gamma^2+(\beta(\delta^2+22\delta+1)-24\delta)\gamma-(5\delta^2+14\delta+5)\beta+24\delta$ multiplied by a positive factor. The above term is a quadratic function of γ with the following properties: (i) The coefficient of γ^2 is positive; (ii) The quadratic function is positive when $\gamma=0$; (iii) The quadratic function is decreasing in γ when $\gamma=1$. Therefore, $\partial\pi_R(2)/\partial\gamma$ crosses zero at most once and from above. Furthermore, the threshold is the smaller root of the quadratic function $\hat{\gamma}=\frac{-\beta\left(\delta^2+22\delta+1\right)+24\delta-(\delta-1)\sqrt{\beta(\beta(\delta^2-74\delta+1)+72\delta)}}{12\delta(1-\beta)}$. It follows that $\pi_R(2)$ increases in γ when $\gamma<\hat{\gamma}$ and decreases in γ otherwise. □

Proof of Lemma 3. Part (a) is straightforward by checking the signs of corresponding derivatives. Part (b) is similar except the statement that $\pi_R(1)$ is first decreasing and then increasing in γ needs further discussion. We can show that $d\pi_R(1)/d\gamma$ equals a positive term multiplied by a quartic function of γ. The quartic function has the following three properties: (i) It is negative when $\gamma=0$; (ii) It is positive when $\gamma=1$; (iii) Its derivative with respect to γ is always positive under the conditions to ensure interior solutions. To see (iii), notice that (1) the derivative in (iii) is a cubic function of γ with a positive coefficient of γ^3; (2) when $\gamma=0$ and $\gamma=1$, the cubic function is positive with a positive slope; (3) The second derivative of the cubic function is positive. Therefore, the cubic function is always positive. Then the result with respect to $d\pi_R(1)/d\gamma$ follows directly from (i), (ii) and (iii). □

Proof of Proposition 2. For part (a), we first show $T_1<T_2$. It is easy to check that T_2-T_1 equals $d_2\beta^2+d_1\beta+d_0$ multiplied by a positive factor, where

$$\begin{aligned}
d_2 &= -2\delta\left(\delta^2+4\delta+1\right)\gamma^5+\left(\delta^2+6\delta+1\right)^2(4-3\gamma)\\
&\quad+\left(\delta^2+6\delta+1\right)\left(2\delta\gamma^4+2\left(\delta^2+4\delta+1\right)\gamma^3-(\delta+3)(3\delta+1)\gamma^2\right),\\
d_1 &= -2\delta(2-\gamma)\left(\left(\delta^2+10\delta+1\right)\gamma^4-4\left(3\delta^2+14\delta+3\right)\gamma^2+\left(\delta^2+6\delta+1\right)\left(16-4\gamma+\gamma^3\right)\right),\\
d_0 &= 4(4-3\gamma)\left(4-\gamma^2\right)^2\delta^2.
\end{aligned}$$

The coefficient of β^2 of the above quadratic function is positive (i.e., $d_2 > 0$). We can verify that at both $\beta = 0$ and $\beta = 2/(1+\delta)$, the quadratic function is positive with a negative slope. Therefore, the quadratic function is always positive for $\beta \in (0, 2/(1+\delta))$ and we have $T_1 < T_2$. Second, we show that $T_3 < T_2$. We can verify that $T_2 - T_3$ equals $[2\gamma^2(\delta+1)^2 - \gamma(\delta-1)^2 - 4(\delta^2+6\delta+1)]\,\beta + 8\,(4-\gamma^2)\,\delta$, multiplied by a positive factor. The above expression is a linear function of β with a positive intercept. When $\beta = 2/(1+\delta)$, the expression equals $2(\delta-1)((12-\gamma-2\gamma^2)\,\delta + 4 + \gamma - 2\gamma^2)/(1+\delta) > 0$. Hence $T_3 < T_2$. Then the results follow from the definitions of equilibrium and T_i and the above ordering results.

Now we prove part (b). From Lemmas 3 and 4, we can easily check that $T_1 = \pi^R_{Mi}(1) - \pi_{Mi}(0)$, $T_2 = \pi_{Mi}(2) - \pi^N_{Mi}(1)$ and $T_3 = \pi_{Mi}(2) - \pi_{Mi}(0)$ are increasing in β and δ. We take a two-step procedure to show T_1 is decreasing in γ. First, $\partial T_1/\partial\gamma$ equals a quadratic function of β multiplied by a positive factor, and the coefficient of β^2 of the quadratic function is negative. Second, we can check that at both $\beta = 0$ and $\beta = 2/(1+\delta)$, the quadratic function is negative with a positive slope. Therefore, under the condition for interior solution, we always have $\partial T_1/\partial\gamma < 0$. Similarly, we can show that $\partial T_2/\partial\gamma$ equals a cubic function of β multiplied by a positive factor. The cubic function has the following properties: It is negative at both $\beta = 0$ and $\beta = 2/(1+\delta)$; It is increasing in β for $\beta \in (0, 2/(1+\delta))$. Then the cubic function is always negative and therefore T_2 is decreasing in γ. We can show that $\partial T_3/\partial\gamma$ equals a linear function of β multiplied by a positive factor. The linear function is negative at both $\beta = 0$ and $\beta = 2/(1+\delta)$, and hence is always negative. Therefore, T_3 is decreasing in γ. Because we can show that T_1, T_2, $T_3 > 0$, the sensitivity results of T_i with respect to a follow directly. □

Proof of Corollary 1. The results follow directly from Proposition 2. □

Proof of Proposition 3. For part (a), notice that $\pi_{M1}(2) - \pi^R_{M1}(1)$ equals $[\beta\,(\delta^2+4\delta+1) - 6\delta]\gamma^2 + [\beta\,(\delta^2+6\delta+1) - 8\delta]\gamma - \beta\,(\delta^2+6\delta+1) + 8\delta$ multiplied by a positive factor. The

above term is a quadratic function of γ with following properties: (i) The coefficient of γ^2 is negative; (ii) The quadratic function is positive when $\gamma = 0$ and negative when $\gamma = 1$. Therefore, the quadratic function passes zero exactly once from above. The threshold is the larger root of the quadratic function $\bar{\gamma} = \frac{-(8\delta-\beta(\delta^2+6\delta+1))+\sqrt{(8\delta-\beta(\delta^2+6\delta+1))(32\delta-\beta(5\delta^2+22\delta+5))}}{2(6\delta-\beta(\delta^2+4\delta+1))}$. It follows that $\pi_{M1}(2) > \pi^R_{M1}(1)$ iff $\gamma < \bar{\gamma}$. For part (b), $\pi^N_{M1}(1) < \pi_{M1}(0)$ follows from $T_3 < T_2$ $(\pi_{Mi}(2) - \pi_{Mi}(0) < \pi_{Mi}(2) - \pi^N_{Mi}(1))$. Now we consider part (c) and show $\pi_R(2) > \pi_R(1)$ first. $\pi_R(2) - \pi_R(1)$ equals a cubic function of β multiplied by a positive factor. The cubic function has the following properties: (i) The function is positive and has a negative slope when $\beta = 0$ and $\beta = 1$; (ii) The second derivative of the cubic function with respect to β is positive for $\beta \in (0, 1)$. Then it follows that the cubic function is always positive. Next, we show $\pi_R(1) > \pi_R(0)$. $\pi_R(1) - \pi_R(0)$ equals $[2(\delta+1)^2\gamma^3 - 4(\delta^2+6\delta+1)\gamma - (\delta^2+14\delta+1)(4-\gamma^2)]\beta + 8\delta(2-\gamma)(2+\gamma)^2$ multiplied by a positive factor. The above function is linear in β and positive when $\beta = 0$ and $\beta = 2/(1+\delta)$. Then the result follows. □

Proof of Proposition 4. First, we show $T_1 < T_2 < V_1 < V_2$. Actually, $V_1 - T_2$ equals a quadratic function of β multiplied by a positive factor. The quadratic function has a positive coefficient of β^2, and its value and slope are both positive at $\beta = 0$. Hence both the quadratic function and $V_1 - T_2$ are always positive. In addition, we can check that $V_2 - V_1$ equals a cubic function of β multiplied by a positive factor. The cubic function has following properties: (i) The value of the cubic function is positive but the slope is negative when $\beta = 0$ and $\beta = 2/(1+\delta)$. (ii) The second derivative of the cubic function with respect to β is always negative for $\beta \in (0, 2/(1+\delta))$. From (i) and (ii) both the cubic function and $V_2 - V_1$ are always positive.

Second, we consider the case when the retailer offers subsidy with manufacturer 2 and manufacturer 1 sequentially.

We discuss the contracting outcome with manufacturer 1, for any given rebate decision

of manufacturer 2. (i) Suppose $Z_2 = R$ and manufacturer 1 is offered S_1 to issue rebate. Manufacturer 1 accepts the offer iff $S_1 \geq F_1 - T_2$ (i.e., $\pi_{M1}(2) - F_1 + S_1 \geq \pi^N_{M1}(1)$). If $F_1 \leq T_2$, $S_1^* = 0$ and $Z_1^* = R$. If $F_1 > T_2$, the retailer offers either $S_1 = F_1 - T_2$ to induce rebate program and no subsidy otherwise. The retailer is willing to offer subsidy iff $F_1 \leq V_2$ (i.e., $\pi_R(2) - S_2 - F_1 + T_2 \geq \pi_R(1) - S_2$). Then if $T_2 < F_1 \leq V_2$, then $S_1^* = F_1 - T_2$, $Z_1^* = R$; if $F_1 > V_2$, then $S_1^* = 0$, $Z_1^* = N$. (ii) Suppose $Z_2 = N$ and manufacturer 1 is offered S_1 to issue rebate. Manufacturer 1 accepts the offer iff $S_1 \geq F_1 - T_1$ (i.e., $\pi^R_{M1}(1) - F_1 + S_1 \geq \pi_{M1}(0)$). If $F_1 \leq T_1$, $S_1^* = 0$, $Z_1^* = R$. If $F_1 > T_1$, the retailer is willing to offer subsidy iff $F_1 \leq V_1$ (i.e., $\pi_R(1) - F_1 + T_1 \geq \pi_R(0)$). If $T_1 < F_1 \leq V_1$, then $S_1^* = F_1 - T_1$, $Z_1^* = R$; If $F_1 > V_1$, then $S_1^* = 0$, $Z_1^* = N$.

In anticipation of the above response from manufacturer 1, we discuss the retailer's contracting outcome with manufacturer 2 and consider the following five regions:

(1) $F_1 \leq T_1$: If $Z_2 = R$, then $S_1^* = 0$, $Z_1^* = R$; If $Z_2 = N$, then $S_1^* = 0$, $Z_1^* = R$. Manufacturer 2 accepts the offer for rebate S_2 iff $S_2 \geq F_2 - T_2$ (i.e., $\pi_{M2}(2) - F_2 + S_2 \geq \pi^N_{M2}(1)$). If $F_2 \leq T_2$, then $S_2^* = 0$, $Z_2^* = R$ and $S_1^* = 0$, $Z_1^* = R$. If $F_2 > T_2$, the retailer is willing to offer subsidy iff $F_2 \leq V_2$ (i.e., $\pi_R(2) - F_2 + T_2 \geq \pi_R(1)$). If $T_2 < F_2 \leq V_2$, then $S_2^* = F_2 - T_2$, $Z_2^* = R$, $S_1^* = 0$, $Z_1^* = R$; If $F_2 > V_2$, then $S_2^* = 0$, $Z_2^* = N$, $S_1^* = 0$, $Z_1^* = R$.

(2) $T_1 < F_1 \leq T_2$: If $Z_2 = R$, then $S_1^* = 0$, $Z_1^* = R$; If $Z_2 = N$, then $S_1^* = F_1 - T_1$, $Z_1^* = R$. Manufacturer 2 accepts the offer for rebate S_2 iff $S_2 \geq F_2 - T_2$ (i.e., $\pi_{M2}(2) - F_2 + S_2 \geq \pi^N_{M2}(1)$). If $F_2 \leq T_2$, then $S_2^* = 0$, $Z_2^* = R$ and $S_1^* = 0$, $Z_1^* = R$. If $F_2 > T_2$, the retailer is willing to offer subsidy iff $F_2 \leq V_2 - T_1 + F_1$ (i.e., $\pi_R(2) - F_2 + T_2 \geq \pi_R(1) - F_1 + T_1$). If $T_2 < F_2 \leq V_2 - T_1 + F_1$, then $S_2^* = F_2 - T_2$, $Z_2^* = R$, $S_1^* = 0$, $Z_1^* = R$; If $F_2 > V_2 - T_1 + F_1$, then $S_2^* = 0$, $Z_2^* = N$, $S_1^* = F_1 - T_1$, $Z_1^* = R$.

(3) $T_2 < F_1 \leq V_1$: If $Z_2 = R$, then $S_1^* = F_1 - T_2$, $Z_1^* = R$; If $Z_2 = N$, then $S_1^* = F_1 - T_1$, $Z_1^* = R$. Manufacturer 2 accepts the offer for rebate S_2 iff $S_2 \geq F_2 - T_2$ (i.e., $\pi_{M2}(2) - F_2 + S_2 \geq \pi^N_{M2}(1)$). Since $F_2 \geq F_1 > T_2$, we have $F_2 - T_2 \geq 0$. The retailer is willing to offer subsidy iff $F_2 \leq V_2 + T_2 - T_1$ (i.e., $\pi_R(2) - F_2 + T_2 - F_1 + T_2 \geq \pi_R(1) - F_1 + T_1$). If

$T_2 < F_2 \leq V_2+T_2-T_1$, then $S_2^* = F_2-T_2$, $Z_2^* = R$, $S_1^* = F_1-T_2$, $Z_1^* = R$; If $F_2 > V_2+T_2-T_1$, then $S_2^* = 0$, $Z_2^* = N$, $S_1^* = F_1 - T_1$, $Z_1^* = R$.

(4) $V_1 < F_1 \leq V_2$: If $Z_2 = R$, then $S_1^* = F_1-T_2$, $Z_1^* = R$; If $Z_2 = N$, then $S_1^* = 0$, $Z_1^* = N$. Manufacturer 2 accepts the offer for rebate S_2 iff $S_2 \geq F_2 - T_3$ (i.e., $\pi_{M2}(2) - F_2 + S_2 \geq \pi_{M2}(0)$). From $F_2 \geq (F_1 > T_2) > T_3$, we have $F_2 - T_3 \geq 0$. The retailer is willing to offer subsidy iff $F_2 \leq \pi_R(2) - \pi_R(0) + T_3 + T_2 - F_1$ (i.e., $\pi_R(2) - F_1 + T_2 - F_2 + T_3 \geq \pi_R(0)$). If $(T_3 <)F_1 \leq F_2 \leq \pi_R(2) - \pi_R(0) + T_3 + T_2 - F_1$, then $S_2^* = F_2 - T_3$, $Z_2^* = R$, $S_1^* = F_1 - T_2$, $Z_1^* = R$; If $F_2 > \pi_R(2) - \pi_R(0) + T_3 + T_2 - F_1$, then $S_2^* = 0$, $Z_2^* = N$, $S_1^* = 0$, $Z_1^* = N$. Note that the regions are non-empty because $V_1 < V_2 + T_3 - T_1$. Actually $V_2 - V_1 + T_3 - T_1$ equals a cubic function of β multiplied by a positive factor. The cubic function of β has following properties: (i) The value of the function is positive and the slope is negative when $\beta = 0$ and $\beta = 2/(1+\delta)$; (ii) The second order derivative of the cubic function is positive for $\beta \in (0, 2/(1+\delta))$. It follows that the cubic function is always positive based on (i) and (ii).

(5) $F_1 > V_2$: If $Z_2 = R$, then $S_1^* = 0$, $Z_1^* = N$; If $Z_2 = N$, then $S_1^* = 0$, $Z_1^* = N$. Manufacturer 2 accepts the offer for rebate S_2 iff $S_2 \geq F_2 - T_1$ (i.e., $\pi_{M2}^R(1) - F_2 + S_2 \geq \pi_{M2}(0)$). Since $F_2(\geq F_1 > V_2) > V_1$ (i.e., $\pi_R(1) - F_2 + T_1 < \pi_R(0)$), we have $S_2^* = 0$, $Z_2^* = N$, $S_1^* = 0$, $Z_1^* = N$.

Third, we can similarly discuss the case when the retailer offers subsidy with manufacturer 1 and manufacturer 2 sequentially. Compare the two sequential offers, and we can verify that the retailer always prefers to offer with manufacturer 2 the first.

We define the following regions

$$
\begin{aligned}
A &\equiv \{(F_1, F_2)|F_1 \leq T_1, F_2 \leq T_2\}, \\
B &\equiv \{(F_1, F_2)|T_1 < F_1 \leq T_2, F_2 \leq T_2\}, \\
C &\equiv \{(F_1, F_2)|F_1 \leq T_1, T_2 < F_2 \leq V_2\}, \\
D &\equiv \{(F_1, F_2)|F_1 \leq T_1, F_2 > V_2\}, \\
E &\equiv \{(F_1, F_2)|T_1 < F_1 \leq T_2, F_2 \leq V_2 - T_1 + F_1\}, \\
F &\equiv \{(F_1, F_2)|T_2 < F_1 \leq V_1, T_2 < F_2 \leq V_2 + T_2 - T_1\}, \\
G &\equiv \{(F_1, F_2)|V_1 < F_1 \leq V_2, F_1 \leq F_2 \leq \pi_R(2) - \pi_R(0) + T_3 + T_2 - F_1\}, \\
H &\equiv \{(F_1, F_2)|T_1 < F_1 \leq V_1, F_2 > \min(V_2 - T_1 + F_1, V_2 + T_2 - T_1)\}, \\
I &\equiv \{(F_1, F_2)|V_1 < F_1, F_2 > \pi_R(2) - \pi_R(0) + T_3 + T_2 - F_1\},
\end{aligned}
$$

and the results follow directly based on the above analysis. □

Proof of Proposition 5. The results for parts (a) and (b) follow directly from the proofs of Propositions 3 and 4. Now we show part (c) and consider Regions E, F and G one by one. In Region E, manufacturer 1 gets $\pi_{M1}(0)$ without subsidy and $\pi_{M1}(2) - F_1$ with subsidy, and obviously $\pi_{M1}(2) - F_1 < \pi_{M1}(0)$. Manufacturer 2 gets $\pi_{M2}(0)$ without subsidy and $\pi^N_{M2}(1)$ $(\pi_{M2}(2) - F_2 + F_2 - T_2)$ with subsidy, and obviously $\pi^N_{M2}(1) < \pi_{M2}(0)$. Both manufacturers are strictly worse off in Region E. In Region F, manufacturer i gets $\pi_{Mi}(0)$ without subsidy and $\pi^N_{Mi}(1)$ $(\pi_{Mi}(2) - F_i + F_i - T_2)$ with subsidy, and each manufacturer is strictly worse off. In Region G, manufacturer 1 is strictly worse off. Manufacturer 2 is indifferent because she gets $\pi_{M2}(0)$ without and with subsidy $(\pi_{M2}(2) - F_2 + F_2 - T_3)$. □